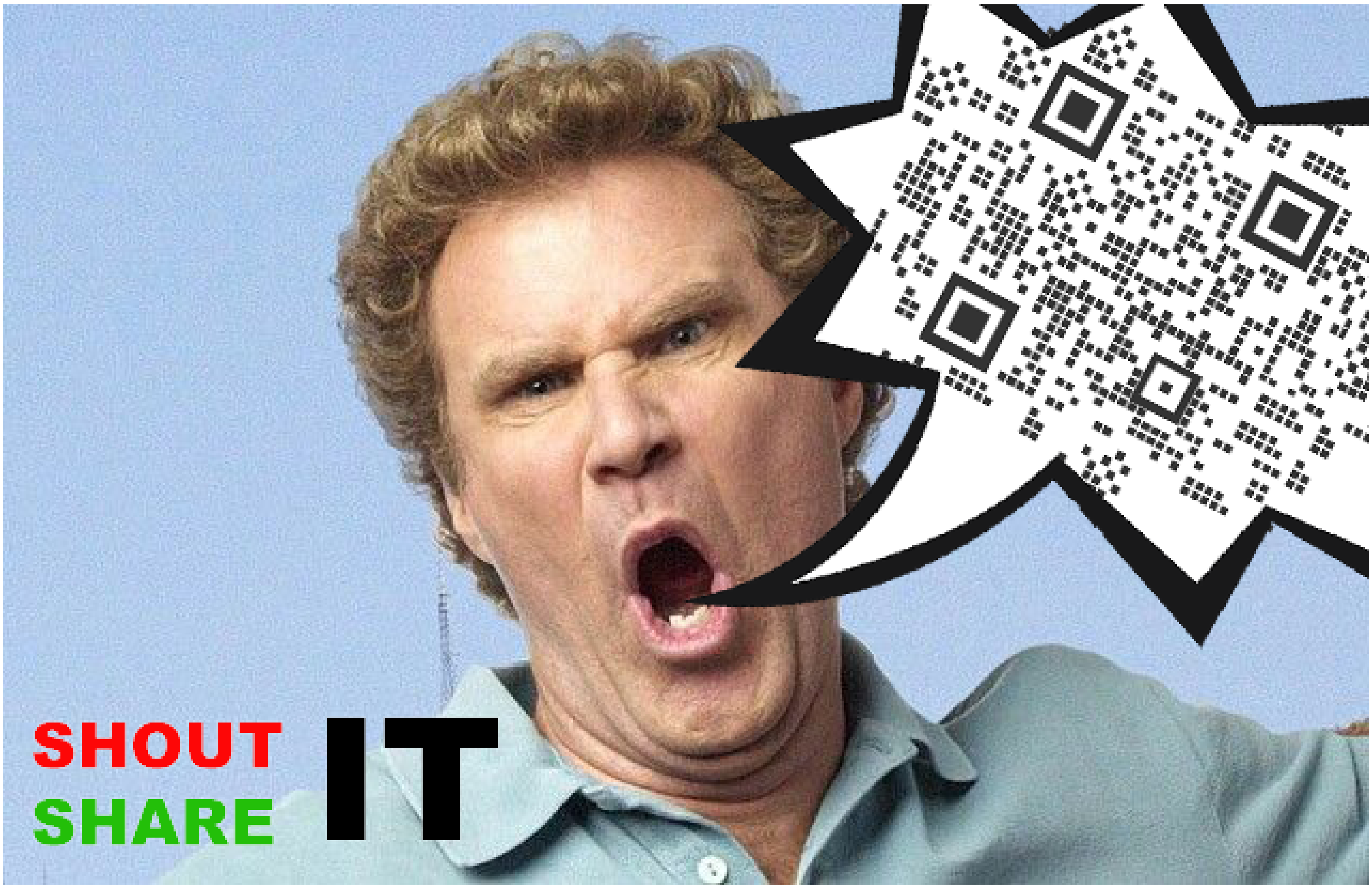
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BIOMETRIKA

A JOURNAL FOR THE STATISTICAL STUDY OF BIOLOGICAL PROBLEMS

FOUNDED BY
W. F. R. WELDON, FRANCIS GALTON AND KARL PEARSON

EDITED BY
KARL PEARSON

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BIOMETRIKA

ON THE ASSOCIATION OF TEMPERATURE, PULSE AND RESPIRATION WITH PHYSIQUE AND INTELLIGENCE IN CRIMINALS: A STUDY IN CRIMINAL ANTHROPOMETRY

BY MADELINE H. WHITING

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(1) *Introductory.* In a recent investigation as to school children by Williams, Bell and Pearson*, it was shown that in a series of seven schools temperature was negatively correlated with weight for constant age, and further in six out of the seven school series negatively correlated with stature for constant age. The correlations were on the whole small, but definitely significant. In the great public schools the temperature appeared, allowing for age, to be lower than in the elementary schools. An association was thus suggested between intelligence and temperature, the more intelligent having the lower temperature. This result might well be considered as spurious, and due to differences of nurture, particularly to differences of nutrition. On the other hand various writers have asserted that *low* temperatures are associated with *low* intelligence, and it is usually stated

---

* "A Statistical Study of Oral Temperatures in School Children," *Drapers' Company Research Memoirs. Studies in National Deterioration*, No. IV (Cambridge University Press). See pp. 57 and 59.

that the temperature of idiots is lower than that of normals. Such a conclusion might easily be reached if great care were not taken in keeping the mouths of the mentally defective closed. Norsworthy has given a descending scale of temperature from brightest girls to mediocre girls and so through mental defectives to idiots*. But the numbers dealt with were very inadequate, and the time of insertion very short. It appeared therefore well worth while considering further certain temperature problems, and the Biometric Laboratory was lucky enough to have placed at its disposal observations by Dr Charles Goring on 500 convicts. Of these convicts 400 were of normal intelligence and 100 were classed as weak-minded. In the bulk of cases the temperatures were taken twice at an interval of 14 days and upwards, the temperature of the room was also taken. Further the pulse and respiration were observed on both occasions. The examinations were all made at the same time, about a quarter of an hour after early dinner, and in the same condition,—the prisoner would be sitting in his cell, either reading, writing or doing nothing. When the thermometer had been inserted under his tongue, the medical officer would count the frequency of his pulse during one minute, and then would count respirations during one minute by observation of abdominal movement; the prisoner himself had of course no knowledge that this observation was being made. Besides the above data, the age, stature, weight, nature of the crime and previous convictions, the existence of special disease, the general health, the physical condition with regard to nutrition and muscularity, as well as the class of prison labour were all recorded. Only some of these data are discussed in the present paper.

Of the characters dealt with, stature, age, weight, pulse, respiration and temperature are quantitatively determined, intelligence is in two classes, normal and feeble-minded; the other data are in broad categories. Thus General Health is classed as "Poor," "Indifferent," or "Good"; for physical condition there are two divisions, each in alternate categories, (i) "Fat" or "Thin" and (ii) "Muscular" or "Weak"; labour done by the convict is classed into "Heavy" and "Light"; and this forms an adequate classification by which to test health conditions and other physical categories.

(2) *Temperature.* The temperatures were oral, taken with a Kew certified minute thermometer inserted under tongue with closed mouth for three minutes. None of the subjects were hospital invalids, all at the time of the observations were getting about, but of course a good many were suffering from chronic disease or disorders of one kind or another. The following classification of these diseases and disorders was made:

I. Syphilis: disease apparently inactive at time of observation.

II. Chronic Bronchitis or Asthma, or both: free from attack at time of observation.

---

* "The Psychology of Mentally Defective Children," *Archives of Psychology*, November, 1906. See also Pearson: *Mendelism and Mental Defect*, No. III (Cambridge University Press), p. 27.

III. Organic Heart Disease.

IV. Chronic Tubercular Disease: quiescent at time of observation.

V. Cardio-vascular degenerative changes.

VI. Epilepsy.

VII. Functional Heart Disorders without organic disease, and described as "weak heart."

VIII. Kidney degenerative changes: indicated by albuminuria, etc.

IX. Other pathological states: Necrosis of bone of old standing; diabetes, active at time of observation; tabes dorsalis (active); general paralysis of the insane (active); chronic rheumatism (active); urethral stricture, resulting from gonorrhoea; previously insane; goître (active); cirrhosis of liver (active); hemiplegia, resulting from a stroke.

927 temperatures were taken on 500 individuals, 427 being repeated temperatures, the intervals between observations varying from 2–3 weeks to 2–3 months. 326 temperatures were those of criminals suffering from the diseases and disorders classed as above, the remaining 601 temperatures may be taken as those of fairly normal individuals, differentiated only from the same class of the population by the fact of their criminality and by the action of prison diet, discipline and labour. Out of the 400 mentally normal convicts 34·5 % were suffering from one or another disorder, and of the 100 weak-minded convicts, 27 %. This is explained by the fact that Parkhurst, where the observations were made, is largely a prison to which the physically or mentally feeble are sent, and accordingly it is more than probable that the weak-minded in general suffer at least as much from disorder and disease as the normal-minded criminal. Taking apparently inactive syphilis alone we find 12·75 % of the normal-minded and 11·00 % of the weak-minded* suffering from non-active disease, a nearer ratio of equality. It would be of great value if a Wassermann test could be made throughout a large convict establishment. It seems probable that at least 10 % of the inmates would be found to be syphilitic. Making due allowance for the class from which the bulk of criminals are drawn, this does not suggest that much more than 5 %–6 % of the non-criminal population are likely to be affected, and compares on the whole favourably with the high estimates published of the syphilised population in Germany†. The following table sets forth our results for mean temperatures:

---

* One case of general paralysis of the insane included, and in both classes the probably syphilitic also counted.

† Fritz Lenz: "Ueber die Verbreitung der Lues...," *Archiv für Rassen- und Gesellschaftsbiologie*, Bd. VII. S. 306–327, 1910.

TABLE A. *Mean Temperatures of Various Classes.*

Class	No. of Temperatures	Mean Temperatures
"Weak Heart" (VII)	20	98·585 ± ·068
Weak-minded	191	98·547 ± ·022
Epilepsy (VI)	27	98·537 ± ·059
Cardio-vascular changes (V)	36	98·408 ± ·050
Normally Healthy Criminals	601	98·383 ± ·013
Whole Population	927	98·373 ± ·011
Chronic Bronchitis, Asthma (II) ...	51	98·353 ± ·043
Kidney changes (VIII)	14	98·350 ± ·082
Syphilis (I)	97	98·342 ± ·031
Organic Heart Disease (III)	32	98·278 ± ·054
Chronic Tubercular Disease (IV) ...	22	98·241 ± ·065
Other pathological conditions (IX) ...	27	98·170 ± ·059

Now it must be at once admitted that many of the series here are so small, that we cannot say that the temperature differences, still less the general temperature order, are significant, but certain differences do appear significant. For example, the weak-minded do appear to have a significantly *higher* average temperature than normally healthy criminals. The difference,

$$\text{weak-minded} - \text{normally healthy} = \cdot 164 \pm \cdot 026,$$

is clearly significant numerically, but this does not show that the higher temperature is inherent in the weak-mindedness. We have already seen that of the weak-minded 27 % were suffering from one form or another of disorder, while we are comparing them against a normally healthy criminal population. The only groups, however, that could tend to *raise* the temperature of the weak-minded are V, VI and VII. To these groups the weak-minded contribute

V.	Cardio-vascular degenerative changes:	4	temperatures.
VI.	Epilepsy:	16	,,
VII.	"Weak Heart":	0	,,

These 20 temperatures have therefore been removed and we find:

$$\text{Mean temperature of weak-minded} = 98\cdot 541 \pm \cdot 021.$$

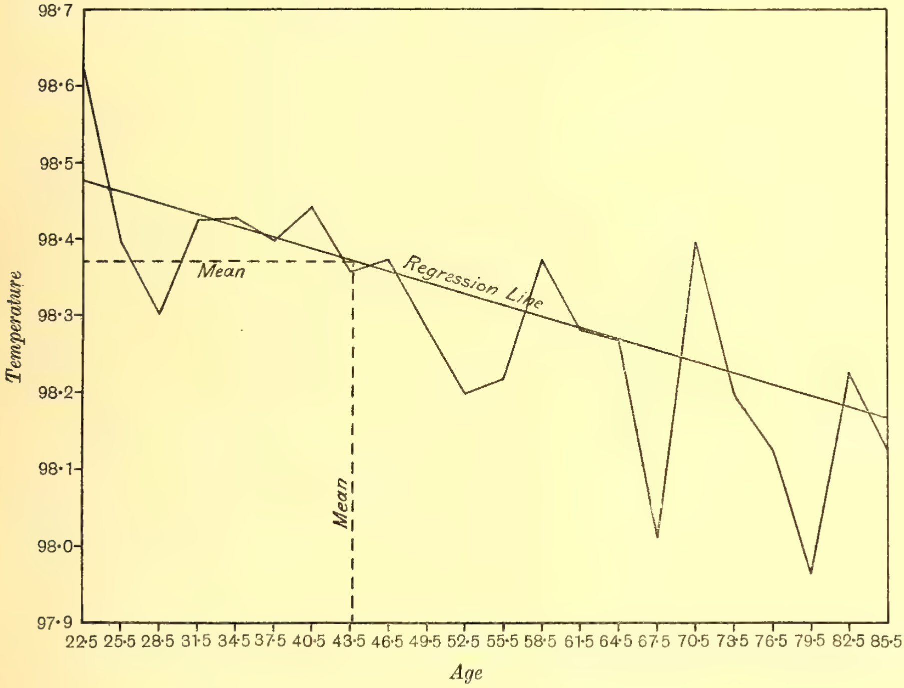
Thus the removal of the epileptic has only lowered very slightly the temperature of the weak-minded group. That has indeed 8·4 % of the temperatures of the epileptic as against only 1·2 % in the group of normal-minded criminals*, but the difference of temperature is only in the very smallest degree due to this excess of epilepsy. One point, however, of some apparent importance had it been based on more numerous data was observed, namely, that the 18 temperatures of the weak-minded who were suffering from syphilis, apparently inactive at the time of observation, gave a high mean, i.e. 98·683, as against the fact that the general syphilised population had if anything a rather subnormal temperature; but the removal of these 18 temperatures gave for the weak-minded population a mean-temperature of 98·533 ± ·021, which although slightly reduced is still significantly greater than that of the normal criminal population. I think we must conclude that none of the observed pathological states is the source of the higher temperature of the weak-minded, but whatever that source is, it is intensified in the presence of either epilepsy or syphilis*.

---

* Reckoned on individuals, not on temperatures observed, the percentages are 9·0 % and 1·5 % respectively.

In the paper by Williams, Bell and Pearson, already cited, it is shown that body temperature falls with age up to the adult stage. From the present data we see that from age 22·5 to age 85·5 the fall is continued. Table XXXIV gives the data and Diagram I shows the plotted regression line. If T be the temperature in degrees Fahrenheit, A the age in years, the equation to the regression line is:

DIAGRAM I. Regression Line of Temperature on Age for Criminals.



$$T = 98{\cdot}5915 - {\cdot}004989\,A,$$

which indicates that the temperature falls ·005 or $\frac{1}{200}$ of a degree per year. The problem then arises—can the difference of temperature between the weak-minded and normal criminal be solely an age difference? An examination of Table B shows that the mean age of the weak-minded is 38·16 years and of the normal-minded 44·86 years, or the former are younger by 6·70 years. This would correspond only to an average greater temperature of ·034 degrees as against the observed difference of ·164 degrees. The temperature difference of weak- and normal-minded is therefore not due to age difference. This is confirmed in Diagram II, where

---

* Weak-minded epileptics, 98·669; criminal epileptics in general, 98·537; weak-minded syphilitics, 98·683; criminal syphilitics in general, 98·342. These results are open to the interpretation that the weak-mindedness may be owing to cerebral injury due to the disease, and that cerebral disorders are more apt than other forms of these diseases to be accompanied by higher temperatures.

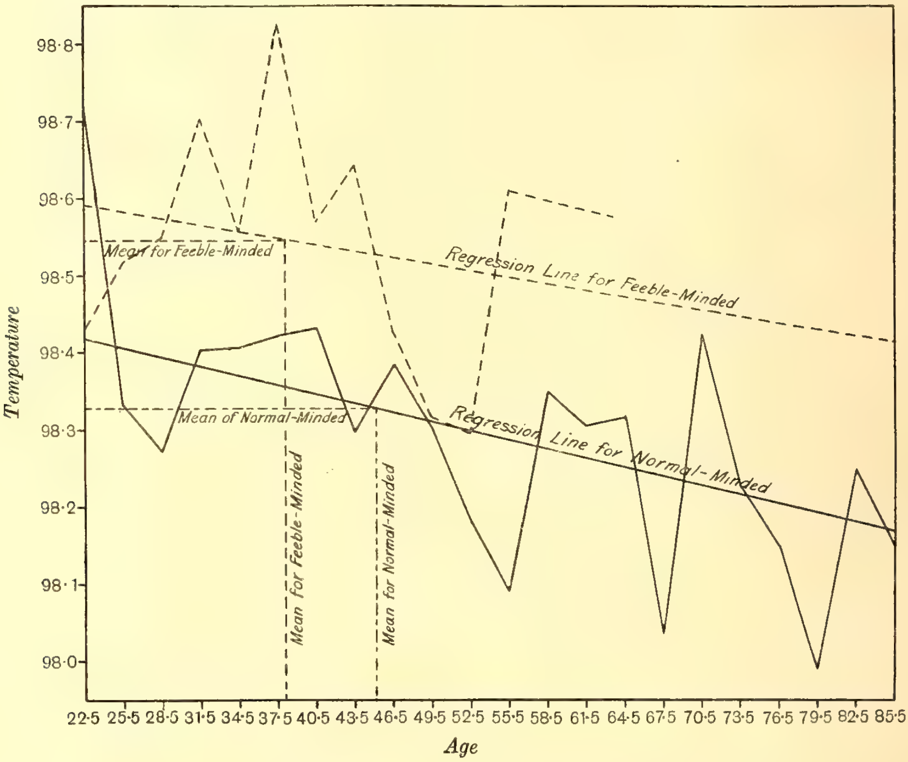


DIAGRAM II. Regression Lines of Temperature on Age for Feeble- and Normal-Minded.

the observations and the calculated regression lines for the weak-minded and the normal-minded groups* are plotted alongside each other. Notwithstanding the irregularities due to the smallness of the sample, it will be seen that at *every* age the mentally defective have a higher temperature than the normal-minded, and further that, as averaged out by the regression lines, the difference at each age is sensibly constant (the regression lines are practically parallel) and equal to 0°·2.

We conclude that weak-minded adults have a higher temperature than normal-minded adults and in this respect differ from mentally defective children who are stated by Professor Norsworthy* to have a lower temperature than normal-minded children, or else Norsworthy's material by including idiots was differentiated from weak-minded criminals, or again was unreliable owing to the brief period of insertion of the thermometer, and possibly to insufficient care in keeping the mouths of the mentally defective children closed during the observation. It is not possible in our case to account for this temperature difference by either (i) difference of age† or (ii) difference of observed pathological states other than those summed up in the word weak-minded.

---

* The equation to the regression line of the weak-minded group is $T = 98{\cdot}6535 - {\cdot}002796A$, and of the normal-minded group $T = 98{\cdot}5063 - {\cdot}003939A$.

One of the noteworthy points of Table A is that when we pass from one or other type of heart trouble (V or VII) and from the epileptic and weak-minded groups to chronic and organic disorders and diseases of a degree of activity which does not involve hospital segregation, we find, without laying too great stress on the individual order, that they tend on the whole to subnormal temperatures.

(3) *Further Differentiated Characters of the Weak-minded and Normal-minded Populations.*

A study of Table B shows that the weak-minded criminal is differentiated from the normal-minded‡ in a number of other characters beyond body temperature.

TABLE B.

Table of Means, Standard Deviations and Coefficients of Correlation of the Chief Variates dealt with.

Variate	Unit	Mean		Standard Deviation		Coefficient of Variation	
		Whole Population	Weak-minded	Whole Population	Weak-minded	Whole Population	Weak-minded
Age	Years	$43{\cdot}518 \pm {\cdot}438$	$38{\cdot}160 \pm {\cdot}717$	$14{\cdot}510 \pm {\cdot}309$	$10{\cdot}627 \pm {\cdot}507$	$33{\cdot}34 \pm {\cdot}79$	$27{\cdot}85 \pm 1{\cdot}43$
Temperature	Degs. Fahr.	$98{\cdot}373 \pm {\cdot}011$	$98{\cdot}547 \pm {\cdot}022$	${\cdot}486 \pm {\cdot}008$§	${\cdot}454 \pm {\cdot}016$	${\cdot}49 \pm {\cdot}01$	${\cdot}46 \pm {\cdot}02$
Pulse	Per minute	$74{\cdot}215 \pm {\cdot}245$	$77{\cdot}620 \pm {\cdot}578$	$11{\cdot}062 \pm {\cdot}173$	$11{\cdot}852 \pm {\cdot}409$	$14{\cdot}89 \pm {\cdot}24$	$15{\cdot}27 \pm {\cdot}54$
Respiration	Per minute	$18{\cdot}985 \pm {\cdot}075$	$19{\cdot}679 \pm {\cdot}168$	$3{\cdot}380 \pm {\cdot}053$	$3{\cdot}433 \pm {\cdot}119$	$17{\cdot}80 \pm {\cdot}29$	$17{\cdot}45 \pm {\cdot}62$
Height	Inches	$65{\cdot}281 \pm {\cdot}082$	$64{\cdot}788 \pm {\cdot}185$	$2{\cdot}682 \pm {\cdot}058$	$2{\cdot}706 \pm {\cdot}131$	$4{\cdot}11 \pm {\cdot}09$	$4{\cdot}18 \pm {\cdot}20$
Weight	Lbs.	$142{\cdot}147 \pm {\cdot}516$	$134{\cdot}397 \pm {\cdot}951$	$16{\cdot}996 \pm {\cdot}365$	$13{\cdot}881 \pm {\cdot}672$	$11{\cdot}96 \pm {\cdot}26$	$10{\cdot}33 \pm {\cdot}51$

---

* *loc. cit.* on our p. 2.

† This is confirmed by the correlation $-{\cdot}229 \pm [{\cdot}030]$ for mentality and temperature for constant age. See Table C.

‡ The term "normal-minded" is throughout this paper used of that portion of the criminal population not classed as weak-minded, and is not intended to serve as the basis of any opinion as to whether the non-weak-minded criminal is or is not mentally differentiated from the general non criminal normal-minded population.

§ It will be noticed that the value of this standard deviation is in very close agreement with those found for first and second temperatures in the case of children by Williams, Bell and Pearson. See *loc. cit.* p. 10, where $\sigma_{t_1} = {\cdot}5343$ and $\sigma_{t_2} = {\cdot}4803$.

For if he significantly exceeds or falls short of, on the average, the *whole* population, he will *a fortiori* be differentiated from the normal-minded population*. We have the following results:

Difference of Means: (Weak-minded—whole population)	Temperature	·174 ± ·025.
	Pulse	3·205 ± ·628.
	Respiration	·694 ± ·184.
	Height	– ·493 ± ·202.
	Weight	– 7·750 ± 1·305.

We conclude from these results that while the weak-minded criminal probably but not very certainly has a somewhat less stature†, he has quite markedly less weight, and this is combined with a higher temperature, a quicker pulse and a quicker respiration.

We now come to precisely the same point as we reached in the problem of differential temperatures. We ask: How far are these differences explicable on the basis of the recognised difference in age between the weak-minded and the total population? From Table C (p. 11) we extract the following correlations and we give the corresponding regressions:

Variates	Correlations	Regression of Second Variate on Age		
		Per Year	Per 5·36 Years	Comparable Difference
Age and Temperature	– ·150 ± ·022	– ·005,026	– ·0269	·174
Age and Pulse	+ ·121 ± ·022	+ ·092,400	+ ·4951	3·205
Age and Respiration	+ ·077 ± ·022	+ ·017,887	+ ·0958	·694
Age and Height	+ ·023 ± ·030	+ ·004,177	+ ·0224	– ·493
Age and Weight	+ ·136 ± ·030	+ ·159,532	+ ·8548	– 7·750

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* Let the means, total frequencies and standard deviations of the general, the normal-minded and the weak-minded population be $\bar{g}$, $\bar{n}$ and $\bar{w}$, f_g, f_n, f_w, σ_g, σ_n, and σ_w, and let β be the multiple (say, about 3) which a quantity must be of its probable error to be considered significant. Then in the usual way $\bar{n}-\bar{w}$ is significant if

$$\bar{g}-\bar{w} \text{ is} = \text{or} > \beta\,(\cdot 67449)\sqrt{\frac{\sigma_g^2}{f_g}+\frac{\sigma_w^2}{f_w}\left(1-\frac{2f_w}{f_g}\right)}\Big/\sqrt{1+\frac{(\cdot 67449\beta)^2 f_w}{f_g\,(f_g-f_w)}}.$$

Now with $\beta=3$ and our numbers, roughly $f_w=190$, $f_g=930$, we have

$$\sqrt{1+\frac{(\cdot 67449\beta)^2 f_w}{f_g\,(f_g-f_w)}}=1\cdot 0005,$$

and this expression may be therefore taken as unity. Hence if we compare

$$\bar{g}-\bar{w} \text{ with } \cdot 67449\sqrt{\frac{\sigma_g^2}{f_g}+\frac{\sigma_w^2}{f_w}},$$

and it is significant, it will certainly be significant if compared with

$$\cdot 67449\sqrt{\frac{\sigma_g^2}{f_g}+\frac{\sigma_w^2}{f_w}\left(1-\frac{2f_w}{f_g}\right)},$$

which is a smaller quantity. See *Biometrika*, Vol. v. pp. 181–183.

† Worked out from the fuller formula the result is $-\cdot 493 \pm \cdot 143$, or the difference is greater than three times its probable error and accordingly may be considered probably significant.

Now it will be seen that while all these correlations are significant except age and height*, none of them are of any considerable magnitude, or capable of producing any appreciable influence on the observed comparable differences. Further since pulse and respiration become very slightly quicker with age, they tend to emphasise and not to reduce the observed differences between these characters in weak-minded and normal-minded. Age might account for one-ninth of the observed difference in weight, or roughly for less than 1 lb. out of the nearly 8 lbs. difference observed; and, as before, age accounts for a very inconsiderable portion of the observed temperature difference. We are thus justified in concluding that the observed physical and physiological differences are quite independent of age differences†. We are able to confirm these results by another method of investigation, which well illustrates the insignificance of the age correction. In our opinion "weak-mindedness" must depend largely on the personal equation of the recorder‡, and there is no definite bounding line between the weak- and the normal-minded. In other words we may consider mental state as an approximately continuous variate, and one not widely divergent from the Gaussian in frequency distribution. Now it is known that in school-children the feeble-minded do not amount to 2 %§; and it is probable that in the community at large their number does not even reach this. On the other hand the criminal class is largely recruited from the less-intelligent section of the general population. Among inebriate and prostitute women probably 40 % to 60 % may be classed as feeble-minded||, and though among male criminals such a high percentage is not reached, they must bulk very considerably. Goring in his invaluable *Report*¶ considers the percentage to be 10 to 15. In our present data we have 100 weak-minded individuals and 400 normal-minded, or we have taken 20 % and calculated the correlations between intelligence and the physical and physiological characters on this basis. But it is of interest to observe what order of change would be made in our results by taking a lesser percentage. Consider, for example, Table XXXV for intelligence and temperature. Here the temperatures of the weak-minded

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* Of course the correlation of age and height is largely misleading, the younger criminals are still growing, the older are slowly losing height. Thus the regression is really skew. For example, the value of the correlation ratio η for height on age is ·147. If, however, we take the correlation of age and height for criminals of over 26 years, at which age other investigations (see *Biometrika*, Vol. I. p. 47) show that height begins to diminish, the negative correlation is only −·01195 corresponding to merely a loss of ·0022 inches per year of past prime life, and therefore quite negligible.

† Except in the matter of age and weight Table B shows us that judged either by absolute or relative variability there is no practical difference in variability between weak-minded and normal-minded criminals.

‡ On the continuity of Weak-mindedness, see *Questions of the Day and the Fray*, No. VIII. "The Continuity of Mental Defect," by Karl Pearson and Gustav A. Jaederholm; No. IX. "On the Graduated Character of Mental Defect," by Karl Pearson (Cambridge University Press).

§ See *A Preliminary Study of Alcoholism in Adults*, by Amy Barrington and Karl Pearson (Cambridge University Press), p. 9.

|| *A Second Study of Extreme Alcoholism in Adults*, by David Heron (Cambridge University Press), p. 15.

¶ See *The English Convict, a Statistical Study* (Wyman and Sons), p. 254.

form 20·6 % of the 927 recorded temperatures; we will raise the normal-minded temperatures to 1719 by multiplying that row by the factor 2·336 retaining, however, only unit frequencies. We find approximately:

Table XXXV, modified to give 10 % weak-minded only.

Temperature.

Intelligence	*96·5—96·6*	*96·7—96·8*	*96·9—97·0*	*97·1—97·2*	*97·3—97·4*	*97·5—97·6*	*97·7—97·8*	*97·9—98·0*	*98·1—98·2*	*98·3—98·4*	*98·5—98·6*	*98·7—98·8*	*98·9—99·0*	*99·1—99·2*	*99·3—99·4*	*99·5—99·6*	*99·7—99·8*	*99·9—100·0*	Totals
Weak-minded	—	—	—	—	1	4	10	12	24	25	34	31	24	15	8	2	1	—	191
Normal-minded	2	5	4	7	49	70	145	201	236	299	248	236	114	61	23	12	5	2	1719
Totals	2	5	4	7	50	74	155	213	260	324	282	267	138	76	31	14	6	2	1910

Here the correlation r (found by the bi-serial method) is for this 10 % of the weak-minded

$$r = -\cdot231 \pm \cdot030,$$

whereas with the 20 % value of the weak-minded

$$r = -\cdot258 \pm \cdot030.$$

We conclude from this result that even halving the percentage of the feeble-minded will only alter the correlation by less than its probable error, and that accordingly for the purpose we have in hand the bi-serial values of the correlations based on 20 % of weak-mindedness will give sufficiently accurate results.

The reader is now asked to examine the last six lines of Table C. He will observe that, while nowhere very important, all the correlations of mentality with age, temperature, pulse, respiration, height and weight are significant; that the most important is weight, and that the signs are in agreement with the conclusions already reached, i.e. those with less mentality, or the feeble-minded, have less weight, height and age*, but greater temperature and quicker pulse and respiration. Examining the second column, which gives for the same variates the partial correlations for constant age, we see that no very substantial changes are made in the crude values; they all remain still significant, and this could have been anticipated by merely noting the small age and variate total correlations at the top of the first column. We conclude again that the physical and physiological characters dealt with are significantly related to feeble-mindedness, but the relationships are of such a low degree that they would be of absolutely no service for the purpose of

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* The extent to which this lesser age of the weak-minded criminal is due (*a*) to earlier average conviction, resulting from earlier crime or easier capture, (*b*) to more frequent transfer to other institutions, or (*c*) to earlier death is of course not determinable from the present data. But it has to be borne in mind that fraudulent offenders convicted late in life belong to a different social class, and are rarely feeble-minded. This class has usually greater stature and weight.

TABLE C. *Physiological, Physical and Mental Correlations.*

Variates Correlated	Total Correlations	Partial Correlations									
		Age Constant	Temperature Constant	Pulse Constant	Respiration Constant	Height Constant	Weight Constant	Age and Respiration Constant	Age and Weight Constant	Age and Height Constant	Weight and Height Constant
Age and Temperature	−·150 ±·022	—	—	−·195 ±·021	−·142 ±·030	−·151 ±·021	−·158 ±·021	—	—	—	−·160 ±·021
Age and Pulse	+·121 ±·022	—	+·174 ±·022	—	+·117 ±·022	+·124 ±·022	+·107 ±·022	—	—	—	+·097 ±·022
Age and Respiration	+·077 ±·022	—	—	—	—	+·081 ±·022	+·090 ±·022	—	—	—	+·083 ±·022
Age and Height	+·023 ±·030	—	—	—	—	—	−·070 ±·030	—	—	—	—
Age and Weight	+·136 ±·030	—	—	—	—	+·151 ±·030	—	—	—	—	—
Temperature and Pulse	+·288 ±·020	+·312 ±·020	—	—	+·283 ±·020	+·289 ±·020	+·286 ±·020	+·305 ±·020	+·308 ±·020	+·314 ±·020	+·286 ±·020
Temperature and Respiration	+·142 ±·022	+·156 ±·022	—	+·130 ±·022	—	+·144 ±·022	+·146 ±·022	—	+·163 ±·022	+·158 ±·022	+·144 ±·022
Temperature and Height	+·003 ±·022	+·007 ±·022	—	—	—	—	−·027 ±·022	—	−·039 ±·022	—	—
Temperature and Weight	+·043 ±·022	+·065 ±·022	—	—	—	+·051 ±·022	—	—	—	+·075 ±·022	—
Pulse and Respiration	+·060 ±·022	+·070 ±·022	+·020 ±·022	—	—	+·049 ±·022	+·071 ±·022	—	+·081 ±·022	+·059 ±·022	+·052 ±·022
Pulse and Height	−·078 ±·022	−·081 ±·022	—	—	—	—	−·178 ±·022	—	−·172 ±·022	—	—
Pulse and Weight	+·114 ±·022	+·099 ±·022	—	—	—	+·197 ±·021	—	—	—	+·181 ±·022	—
Respiration and Height	−·144 ±·022	−·146 ±·022	—	—	—	—	−·113 ±·022	—	−·108 ±·022	—	—
Respiration and Weight	−·089 ±·022	−·101 ±·022	—	—	—	+·007 ±·022	—	—	—	−·020 ±·022	—
Height and Weight	+·580 ±·020	+·583 ±·020	—	—	—	—	—	—	—	—	—
Mentality and Age	+·264 ±·041	—	—	—	—	+·263 ±[·041]	+·234 ±[·041]	—	—	—	+·230 ±[·041]
Mentality and Temperature	−·258 ±·020	−·229 ±[·030]	—	—	—	−·262 ±[·030]	−·288 ±[·030]	—	−·262 ±[·030]	−·233 ±[·030]	−·289 ±[·030]
Mentality and Pulse	−·223 ±·030	−·266 ±[·030]	—	—	—	−·214 ±[·030]	−·276 ±[·030]	—	−·313 ±[·029]	−·264 ±[·030]	−·289 ±[·030]
Mentality and Respiration	−·148 ±·031	−·175 ±[·031]	—	—	—	−·129 ±[·031]	−·127 ±[·031]	—	−·153 ±[·031]	−·160 ±[·031]	−·133 ±[·031]
Mentality and Height	+·154 ±·043	+·153 ±[·043]	—	—	—	—	−·044 ±[·044]	—	−·031 ±[·044]	—	—
Mentality and Weight	+·324 ±·040	+·304 ±[·040]	+·347 ±[·040]	+·361 ±[·040]	+·315 ±[·040]	+·291 ±[·041]	—	—	—	+·274 ±[·041]	—

The partial correlations were all calculated from values of the total correlations to four figures. The probable errors of the bi-serial correlations were calculated by Soper's formula (see *Biometrika*, Vol. x. p. 384). Those of the partial coefficients involving bi-serial values of total coefficients were calculated on the assumption that the partial coefficients had been calculated directly from bi-serial tables and are placed in squared brackets to indicate that they are only approximative.

diagnosis, or for discriminating the weak-minded from the normal-minded. An examination of the Tables XXXVII, XXXVIII and XLII, XLIII shows that the range of the normal in every case is practically coextensive with, or more extensive than, that of the feeble-minded. This is in good accordance with the results already reached by Jaederholm and Pearson from other data: see *Questions of the Day and Fray*, VIII. "The Continuity of Mental Defect," and IX. "On the Graduated Character of Mental Defect." With regard to the further partial correlations, it will be seen that the relationships between the physiological variates (temperature, pulse and respiration) and mentality are not much modified by making the physical variates constant. The relationship between mentality and temperature reaches its maximum for constant weight and height, but the gain is only about its probable error; that for mentality and pulse is more considerable being three times the probable error for constant age and weight; while that for mentality and respiration is a maximum for constant age and height, but the increase is less than half the probable error. Finally it may be noted that the relationship of mentality to weight is somewhat intensified if the temperature and pulse rates be made constant.

In no case, however, do we reach correlations of the slightest importance for diagnostic purposes.

(4) *On the Interphysiological Correlations.*

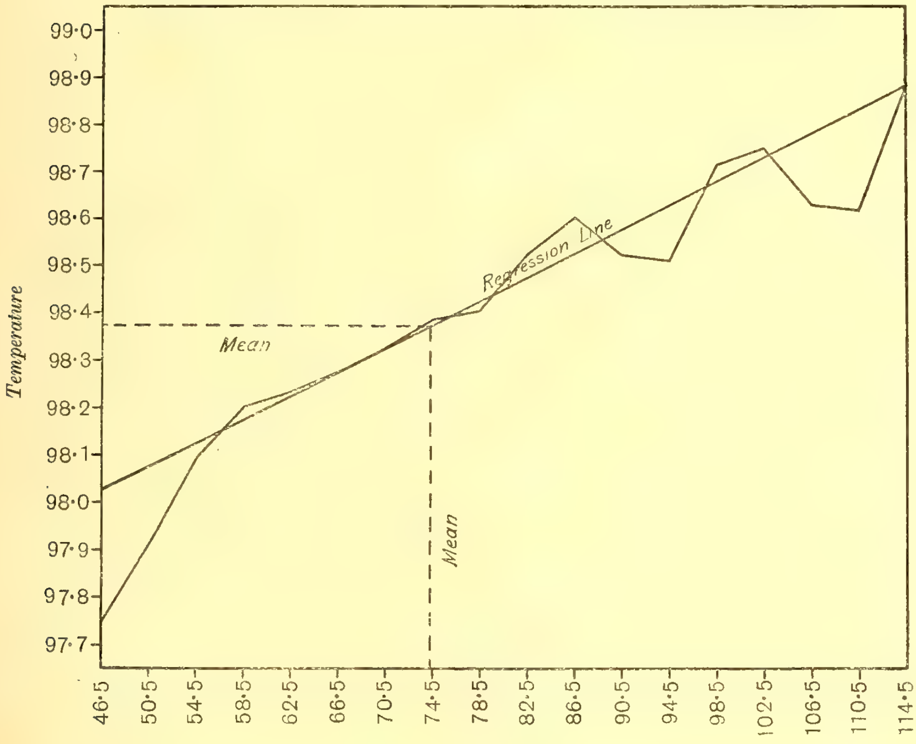
The next point to which we must turn our attention is the relationship between temperature, pulse and respiration. We have the following series of correlations from Table C.

Correlated Pair	Crude Correlations	Partial Correlations for Constant Age and Weight	Partial Correlations for Constant Age and Height
Temperature (*T*) and Pulse (*P*)	$+\cdot 288 \pm \cdot 020$	$+\cdot 308 \pm \cdot 020$	$+\cdot 314 \pm \cdot 020$
Temperature (*T*) and Respiration (*R*)	$+\cdot 142 \pm \cdot 022$	$+\cdot 163 \pm \cdot 022$	$+\cdot 158 \pm \cdot 022$
Pulse (*P*) and Respiration (*R*)	$+\cdot 060 \pm \cdot 022$	$+\cdot 081 \pm \cdot 022$	$+\cdot 059 \pm \cdot 022$

All these correlations are significant, but the sole one of any importance, and even that is of little value, is the relation between temperature and pulse. The only comparative material is that of Körösy reduced by Julia Bell (*Biometrika*, Vol. VIII. p. 232). She found for pulse and respiration $+\cdot 108 \pm \cdot 042$, in excellent agreement with our present value. Her values for pulse and height, $-\cdot 072 \pm \cdot 042$, and for respiration and height, $-\cdot 042 \pm \cdot 042$, are also in accordance with ours considering their probable errors. The smallness of the relation between pulse and respiration is noteworthy, and none of these relations are substantially modified if we make age and weight or age and height constant. In fact temperature, as an examination of Table C shows, has no relation to either height or weight, while greater stature slightly lessens, greater weight slightly increases the pulse rate. On the other hand the "bantams"—judged by both stature and weight—have a somewhat quicker respiration. But for all practical purposes height and weight may be neglected in considering these physiological variates. It may be of interest to table the actual association of an increase of any one of these variates on the other two.

Probable temperature T_p for a given pulse P: $T_p = 97{\cdot}4336 + {\cdot}01266P$.
Probable temperature T'_p for a given respiration R: $T'_p = 97{\cdot}9860 + {\cdot}02040R$.
Probable pulse P_p for a given temperature T: $P_p = 6{\cdot}5551T - 570{\cdot}6300$*.
Probable pulse P'_p for a given respiration R: $P'_p = 70{\cdot}4986 + {\cdot}19575R$.
Probable respiration R_p for a given temperature T: $R_p = {\cdot}98563T - 77{\cdot}9747$.
Probable respiration R'_p for a given pulse P: $R'_p = 17{\cdot}6291 + {\cdot}01827P$.

DIAGRAM III. Regression Line of Temperature on Pulse for Criminals.



In other words the temperature increases by ·127° for every increase of 10 beats per minute of the pulse and by ·204° for every increase of 10 respirations per minute; the pulse-rate quickens 6·56 beats per minute for each degree of temperature and 1·96 beats per minute for every increase of 10 respirations per minute. Lastly the respiration-rate quickens ·986 respirations per minute for each degree of temperature, and ·183 respirations per minute for every 10 beats per minute increase of the pulse.

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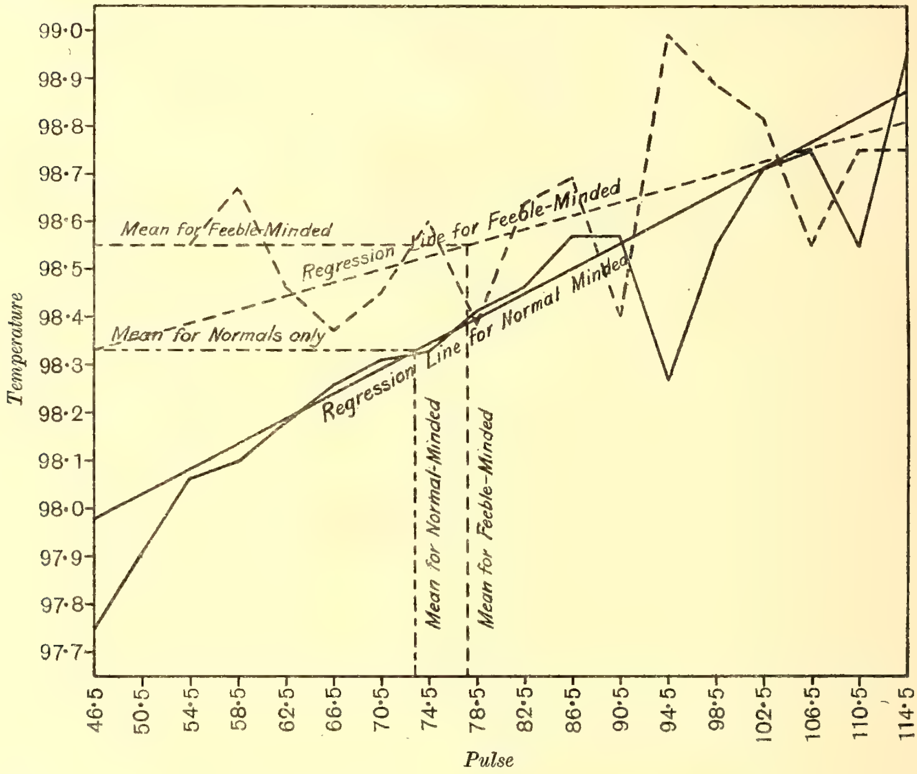
* For the normal-minded criminals only this line is $P_p = 6{\cdot}4180T - 557{\cdot}7481$.

A further interesting point is that, if we could possibly suppose these relations for criminals, whose disorders, if any, do not enforce hospital treatment, to extend beyond the range of values observed, then the pulse would cease at a body temperature $T_m = 87^\circ{\cdot}05$* and the respiration would cease at a body temperature $T'_m = 79^\circ{\cdot}11$. We have no right, however, to extend our *linear* regressions to

Diagram IV. Regression Lines of Temperature on Pulse for Feeble- and Normal-Minded.



these extreme cases; all we can say is that they suggest cessation of pulse before that of respiration, and raise the problem of whether the temperature at death, T_m, is or is not on the average as much below normal temperature as is indicated by the above vague figures.

In Diagram III the regression line of temperature on pulse is drawn to test the general accuracy of linearity in describing this relationship. It will be seen that graphically the accordance is good, the group 45–48 of pulse-rate (see Table XXXIX) has only two entries and the deviation is non-significant. Algebraically for temperature on pulse $\eta = {\cdot}2951$ (without corrections) to be compared with $r = {\cdot}2865$ (without corrections) $= {\cdot}2881$ after correction.

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* Deduced from normal-minded criminals only $T_m = 86^\circ{\cdot}90$.

In Diagram IV the weak-minded have been separated from the normal-minded criminals and the separate regression lines of temperature on pulse are given for both classes. The equations to these lines are:

For normal-minded criminals: $T_p = 97{\cdot}3651 + {\cdot}01315P$,
For weak-minded criminals: $T_p = 97{\cdot}9976 + {\cdot}00708P$,

and the correlations are respectively $\cdot2906 \pm \cdot0228$ and $\cdot1847 \pm \cdot0471$; thus we see that there is less correlation in the case of the weak-minded and further that the regression lines are no longer, as in the case of temperature on age, parallel. While the weak-minded of low pulse-rate have a higher temperature than the corresponding group of normal-minded, the weak-minded with high pulse-rate tend to have equal or perhaps slightly inferior temperature to the corresponding group of normal-minded.

(5) *On the Correlations between the Physiological and the Physical Variates.* We have already seen that various points of the physique of these criminals were noted by the medical officer. In the first place their capacity for hard labour; actually the class of work on which each criminal was employed was given, but it seemed sufficient for our present purposes to divide these into hard and light labour. In the next place we have the appreciation of muscularity given by the two categories muscular and weak. Physical condition was also marked by the classification fat and thin, corresponding to good and bad state of nutrition. Lastly the general health as appreciated medically was classed as good, indifferent and poor. We made the division Good and Not-good, and used in all cases four-fold tables, where the bi-serial r was not obtainable.

The following correlations of the physical characters were reached:

TABLE D.

Correlations of Variates measuring Physique.

	General Health (Good and Not-good)	Capacity for Labour (Set to Hard or Light Work)	Muscularity (Muscular or Weak)	Nutrition (Fat or Thin)
General Health ...	—	+·770±·027	+·639±·038	+·451±·043
Capacity for labour	+·770±·027	—	+·878±·019	+·369±·048
Muscularity	+·639±·038	+·878±·019	—	+·199±·047
Nutrition (Fat or Thin)	+·451±·043	+·369±·048	+·199±·047	—

It will be seen at once that these correlations are of a very high order, much beyond anything we have hitherto reached. Indeed it is fairly clear that the Medical Officer's general appreciation of health or muscularity tallies closely with the prison official's appreciation of the type of labour to which the convict can be set. We are justified therefore in considering how far these variates of physique, which clearly have intrinsic value far beyond the difficulties inherent in the personal equation of the medical examiner, are associated with the physiological variates.

TABLE E.

Correlation of Physiological and Physical Variates.

	Temperature	Pulse	Respiration
General Health ...	+·108±·029	–·116±·029	+·025±·029
Capacity for Labour	+·098±·028	–·010±·028	–·098±·028
Muscularity	+·126±·030	–·179±·030	–·130±·030
Nutrition	+·019±·028	+·093±·028	+·096±·028
Height	+·003±·022	–·078±·022	–·144±·022
Weight	+·043±·022	+·114±·022	–·089±·022
Age	–·150±·022	+·121±·022	+·136±·030

Now this table is really very noteworthy for all the correlations are low, even when significant. In other words the general physical condition has very little relation to the physiological variates. In a person, not ill enough for hospital treatment, temperature, pulse, and respiration would hardly be a differential measure of general health, much less of the goodness or badness of the physique in general. It is true that the person in good health has a rather higher temperature and rather lower pulse-rate than one in poor or indifferent health. But there is no significant difference in respiration and the correlations are so low that not only no rough measures of temperature and pulse would aid diagnosis, but really fine numerical determinations would not be of any discriminating value. In capacity for hard labour the pulse plays no part, but it is associated with a slightly higher temperature and a slightly slower respiration. Muscularity is associated with higher temperature, slower pulse and slower respiration. Fatness with higher respiration and pulse, but has no apparent relation to temperature. Height and weight also have no sensible relation to temperature, the larger men have a slower respiration, but the effect of tallness is to slacken pulse, of greater weight to quicken it. Pulse and respiration quicken with age, but temperature falls. In every case, however, these associations are so small that they would be incapable of appreciation except as the mean results of large numbers of accurate records. For ordinary every day experience we can only conclude that nothing can be judged from the physiological variates of physique or from physique of the physiological variates.

(6) *On the Relation of External Temperature to the Physiological Variates.* In the memoir by Williams, Bell and Pearson, it has been shown that such variations as occur in seasonal change of external temperature do not influence the body temperature *of one and the same* individual*. The present material enables us to confirm this result for fairly long series of temperatures on different individuals. We have correlated external temperature with body temperature, pulse and respiration, and find the following values:

Correlation of External Temperature and Body Temperature:	$-\cdot 002 \pm \cdot 023$.
Correlation of External Temperature and Pulse:	$+\cdot 108 \pm \cdot 023$.
Correlation of External Temperature and Respiration:	$+\cdot 067 \pm \cdot 023$.

Thus body and external temperatures are to the extent of seasonal change unrelated. Pulse quickens and respiration quickens in rather warmer weather. But again these environmental influences are of a very unimportant nature, and the associations are even less than those between the physiological and physical characters.

(7) *On the Individuality of the Physiological Variates.* We have seen that temperature, pulse and respiration are variates, which have in reality very little association with physique as measured by such anthropometric characters as age, height or weight; nor are they, within the limits of non-hospital state, at all closely associated with general health or nutrition. Further they are practically unrelated to occupation (nature of labour) or environment (external temperature) as far as we have been hitherto able to measure these factors. Does this therefore signify that they are quantities rapidly varying, changed by a variety of causes not yet appreciated, and of no essential importance to the individual, as having no close relation to any of the bigger measurable factors of personality, physique or environment? The answer to this question is, perhaps, best sought by asking which of all the characters we have dealt with is most closely related to the physiological variates? There cannot be a doubt that the answer must be the mentality of the individual, and this factor of individuality we know to be one that is remarkably persistent and personal. The majority of the weak-minded are congenitally weak-minded, and but few of them really change their intellectual grade in the course of their life-time. Those who believe that intellectual power is the product of environment, or on the other hand that it is closely related to physique, or to any anthropometric measurement of the external body, have yet to demonstrate their position, and many facts have been collected which are markedly opposed to it†. There is much evidence to show that the chief mental characters flow from congenital and hereditary potentialities. Can it be that these physiological variates have a like origin? The actual demonstration of this cannot, perhaps, be completed on the basis of our present data, but very weighty evidence for its truth can, however, be deduced. We have already indicated that in the majority of cases the observation of temperature, pulse and respiration was repeated at an interval

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* See *loc. cit.* pp. 5–7.

† See *R. S. Proc.* Vol. 69, pp. 333–342, Vol. 71, pp. 106–114; *Biometrika*, Vol. III. p. 39, et seq., Vol. V. pp. 105–146. Letters in *British Medical Journal*, January 27—March 17, 1906, etc.

varying from 2–3 weeks to 2–3 months. We are thus able to correlate first and second temperatures, first and second pulses and first and second respirations, and the results are of very marked significance. We find the following values of the correlations:

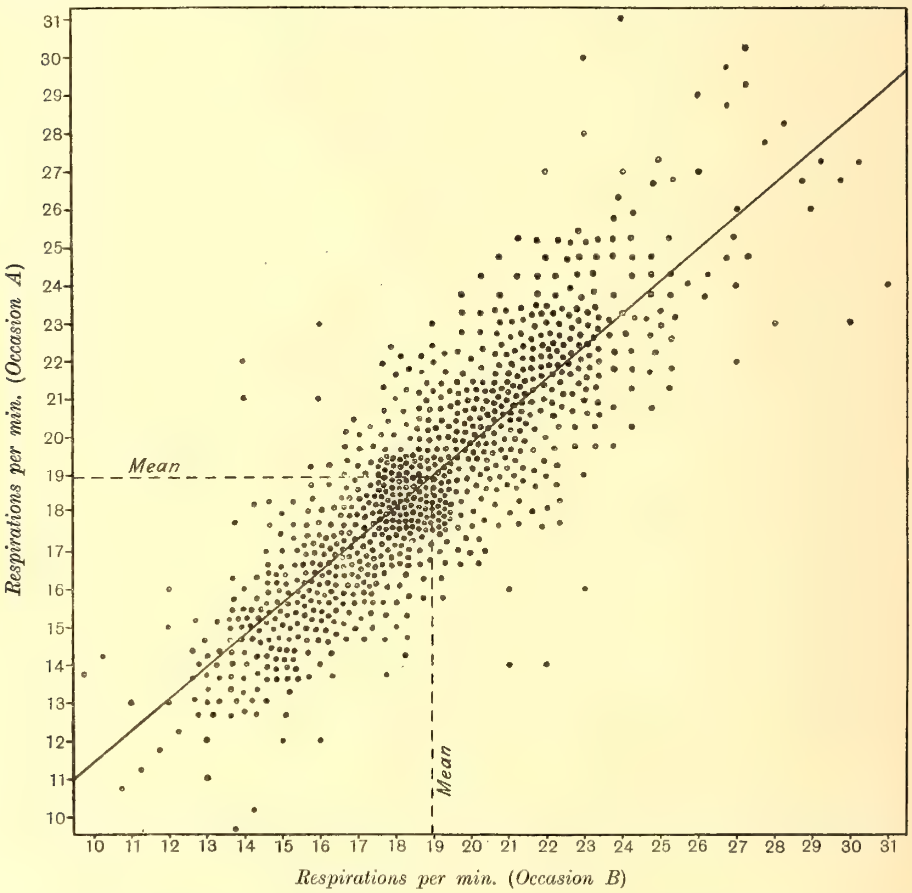
First and Second Temperature: + ·395 ± ·019*.

First and Second Pulse Rate: + ·644 ± ·012.

First and Second Respiration Rate: + ·856 ± ·006.

The accompanying Diagram V indicates the distribution of observations of First and Second Respiration and their clustering about the regression line.

Diagram V. Scatter Diagram and Regression Line for Respirations before and after an interval.



It will be evident at once from the above results that our physiological variates —above all respiration-rate, but markedly also pulse-rate—are essentially constant characters of the individual, characters which change only slightly with age, and which are only moderately linked with each other and with mentality. They are not subject to much influence from environment or occupation, and it appears to us that respiration-rate would probably form as good a mark of individual identity as most anthropometric characters and, since there is a relatively low rate of correlation between these physiological variates, their combined records might be even of value as a supplement* to finger prints in the problem of identification. The above values undoubtedly indicate that the degree of individuality which we find in the external measurable features of the body and in the mental characteristics is maintained, and just possibly exceeded, in the manner in which the various organs of the body carry on their functions; in other words the "individual" is not only characterised by an individualised framework and an individualised controlling faculty for that framework, but its metabolic processes are individualised also. The individual is such not only by his existence in space, but by his manner of existing in time. The individuality in the functioning of organs is as great as the physical differences in the organs themselves. In the future it may well be that heredity will be as effectively studied by measuring the functioning of organs as by measuring the organs themselves. The clinical thermometer and the sygmograph may gain new scientific value when they are introduced into the field of genetics.

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* The value found for first and second temperatures in the case of children by Williams, Bell and Pearson (*loc. cit.* p 10) was +·5238 ± ·0346.

I wish, in conclusion, to express my indebtedness to Professor Karl Pearson for his continuous supervision throughout the course of the work and to thank him for many ideas and suggestions set forth in this paper.

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* Of course two persons will have the same respiration but hardly an absolute sameness of finger prints. What is meant here is that if finger prints have to be placed in definite descriptive "categories"—a loop of so many ridges or a subclass of arches—these categories will hardly be more individual than the respiration-rate. Again a pulse tracing would correspond for individuality more closely to an actual finger imprint, than a pulse-rate which would represent a certain category of pulse tracings like a finger print class represents a certain type of finger prints.

APPENDIX. CORRELATION TABLES.

TABLE I.

Temperature.

Respirations per minute.	*96·5**	*96·7*	*96·9*	*97·1*	*97·3*	*97·5*	*97·7*	*97·9*	*98·1*	*98·3*	*98·5*	*98·7*	*98·9*	*99·1*	*99·3*	*99·5*	*99·7*	*99·9*	Totals
10	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	2
11	—	—	—	—	—	—	—	—	1	1	—	1	—	—	—	—	—	—	3
12	—	—	—	1	—	—	1	3	—	1	1	—	—	—	—	—	—	—	7
13	—	—	—	—	1	2	4	1	2	5	—	1	1	—	1	1	—	—	19
14	—	—	1	1	1	4	6	5	7	6	4	9	2	1	—	—	—	—	47
15	—	1	1	—	3	5	8	7	8	11	8	5	3	6	1	—	—	—	67
16	—	—	—	—	2	5	5	6	14	15	9	12	9	3	2	—	—	—	82
17	—	—	—	1	2	4	2	10	11	13	19	11	7	3	2	—	—	—	85
18	1	—	—	—	6	4	9	10	14	25	16	20	4	7	2	—	—	1	119
19	—	—	—	—	1	2	12	17	15	13	24	12	8	4	3	—	1	—	112
20	—	—	—	—	—	4	5	14	18	15	8	14	4	2	1	—	—	—	85
21	—	—	—	—	1	1	4	8	10	14	20	13	9	4	3	2	—	—	89
22	—	1	—	—	1	2	4	3	8	12	13	11	11	3	2	—	—	—	71
23	—	—	—	—	1	1	6	7	6	8	6	6	8	3	1	1	1	—	55
24	—	—	—	—	3	—	3	1	2	3	3	6	1	3	—	1	—	—	26
25	—	—	—	—	—	—	—	3	4	4	4	6	2	2	—	1	—	—	26
26	—	—	—	—	—	—	—	—	1	1	1	1	1	—	—	1	—	—	6
27	—	—	—	—	—	—	1	—	2	3	2	1	1	—	—	—	—	—	10
28	—	—	—	—	—	—	—	1	—	2	—	1	1	—	—	—	—	—	5
29	—	—	—	—	—	—	2	—	1	—	—	—	1	—	—	—	—	—	4
30	—	—	—	—	—	—	—	1	—	—	1	1	—	—	—	—	—	—	3
31	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
Totals	1	2	2	3	22	34	72	97	124	153	140	132	73	41	18	7	2	1	924

		*96·5**	*96·7*	*96·9*	*97·1*	*97·3*	*97·5*	*97·7*	*97·9*	*98·1*	*98·3*	*98·5*	*98·7*	*98·9*	*99·1*	*99·3*	*99·5*	*99·7*	*99·9*	Totals
TABLE II. General Health.	Good ...	0	1	2	3	13	24	41	59	81	101	103	95	47	27	13	6	3	1	620
	Poor ...	1	1	—	—	9	10	31	37	44	49	35	36	24	13	4	1	—	—	295
TABLE III. Nutrition.	Thin ...	1	1	—	2	11	18	47	54	68	86	78	82	37	27	9	5	1	—	527
	Fat ...	—	1	2	1	11	16	25	42	57	64	60	49	34	13	8	2	2	1	388
TABLE IV. Muscularity.	Muscular	—	1	2	1	16	23	55	67	92	108	117	106	53	34	13	5	3	1	697
	Weak	1	1	—	2	6	11	17	29	33	42	21	25	18	6	4	2	—	—	218
	Totals for each Table	1	2	2	3	22	34	72	96	125	150	138	131	71	40	17	7	3	1	915

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* 96·5 and 96·6, the temperatures were only read to tenths.

TABLE V.

Pulse-Beats per minute.

Respirations per minute.

	45—48	49—52	53—56	57—60	61—64	65—68	69—72	73—76	77—80	81—84	85—88	89—92	93—96	97—100	101—104	105—108	109—112	113—116	Totals
10	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	2
11	—	—	—	—	—	—	—	2	1	—	—	—	—	—	—	—	—	—	3
12	—	—	—	1	1	3	2	—	—	—	—	—	—	—	—	—	—	—	7
13	—	—	1	2	2	2	3	3	3	1	1	1	—	—	—	—	—	—	19
14	—	—	1	4	7	13	7	7	2	2	1	—	—	—	—	1	1	1	47
15	1	—	2	9	4	20	8	9	5	3	3	1	1	—	1	1	—	—	68
16	—	2	2	4	15	13	10	12	8	4	3	4	2	1	1	—	—	1	82
17	—	—	2	6	9	13	14	11	7	11	5	2	2	—	1	—	2	—	85
18	1	2	—	11	13	19	15	15	12	13	10	2	3	2	—	1	—	—	119
19	—	—	1	8	6	19	17	18	17	10	8	3	1	2	2	—	—	—	112
20	—	—	3	2	6	10	15	14	10	6	9	7	1	1	1	—	—	—	85
21	—	—	2	2	14	9	8	15	12	8	7	6	—	5	—	1	—	—	89
22	—	—	1	3	2	18	2	10	10	15	4	4	—	2	—	—	—	—	71
23	—	—	—	4	6	6	6	5	7	2	6	7	2	4	—	—	—	—	55
24	—	—	—	—	1	2	4	5	6	2	1	2	2	—	—	—	—	—	25
25	—	—	—	—	1	2	6	4	4	2	1	3	—	—	1	1	—	1	26
26	—	—	—	—	—	—	1	—	1	1	1	—	1	—	1	—	—	—	6
27	—	—	1	1	1	—	1	1	2	3	—	—	—	—	—	—	—	—	10
28	—	—	—	—	1	—	1	—	—	1	1	—	—	—	1	—	—	—	5
29	—	1	1	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	4
30	—	—	—	—	—	1	—	—	1	—	1	—	—	—	—	—	—	—	3
31	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	2	5	17	57	90	150	120	131	109	86	62	42	15	18	9	5	3	3	924

		45—48	49—52	53—56	57—60	61—64	65—68	69—72	73—76	77—80	81—84	85—88	89—92	93—96	97—100	101—104	105—108	109—112	113—116	Totals
TABLE VI. Good Health.	Good ...	1	4	9	41	66	109	80	93	71	57	37	24	7	9	5	4	1	2	620
	Poor ...	1	1	8	16	24	40	36	39	37	27	23	18	7	9	4	1	2	1	294
TABLE VII. Nutrition.	Thin ...	—	2	12	32	60	77	64	62	62	48	42	28	10	10	6	5	3	3	526
	Fat ...	2	3	5	25	30	72	52	70	46	36	18	14	4	8	3	—	—	—	388
TABLE VIII. Muscularity.	Muscular	1	5	13	45	75	126	98	95	83	56	36	25	9	14	8	5	1	2	697
	Weak ...	1	—	4	12	15	23	18	37	25	28	24	17	5	4	1	—	2	1	217
	Totals for each Table	2	5	17	57	90	149	116	132	108	84	60	42	14	18	9	5	3	3	914

TABLE IX.

Respirations per minute (Occasion A).

Respirations per minute (Occasion B).	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Totals
10	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
11	—	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
12	—	—	2	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
13	—	1	1	6	8	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	19
14	2	—	—	8	8	16	5	—	2	—	—	1	1	—	—	—	—	—	—	—	—	—	43
15	—	—	1	3	16	14	15	8	5	—	—	—	—	—	—	—	—	—	—	—	—	—	62
16	—	—	1	—	5	15	20	18	7	4	—	1	—	1	—	—	—	—	—	—	—	—	72
17	—	—	—	—	—	8	18	22	19	9	7	—	—	—	—	—	—	—	—	—	—	—	83
18	—	—	—	—	2	5	7	19	32	31	7	5	5	—	—	—	—	—	—	—	—	—	113
19	—	—	—	—	—	—	4	9	31	24	15	11	6	1	—	—	—	—	—	—	—	—	101
20	—	—	—	—	—	—	—	7	7	15	20	16	5	4	2	—	—	—	—	—	—	—	76
21	—	—	—	—	1	—	1	—	5	11	16	20	12	9	2	2	—	—	—	—	—	—	79
22	—	—	—	—	1	—	—	—	5	6	5	12	18	13	4	3	—	1	—	—	—	—	68
23	—	—	—	—	—	—	1	—	—	1	4	9	13	12	5	6	—	—	1	—	1	—	53
24	—	—	—	—	—	—	—	—	—	—	2	2	4	5	2	4	3	1	—	—	—	1	24
25	—	—	—	—	—	—	—	—	—	—	—	2	3	6	4	2	—	3	—	—	—	—	20
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	—	—	1	—	1	—	—	5
27	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	3	1	—	—	2	2	—	10
28	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	2	—	—	—	3
29	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	3
30	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2	—	—	—	—	3
31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals	2	3	5	19	43	62	72	83	113	101	76	79	68	53	24	20	5	10	3	3	3	1	848

		10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
TABLE X. General Health.	Good ...	1	2	6	12	30	40	53	59	89	84	51	52	47	36	15	19	2	9	4	3	3	1	618
	Poor ...	1	1	1	7	18	27	27	25	29	26	34	35	22	18	10	7	4	1	1	1	—	—	295
TABLE XI. Nutrition.	Thin ...	1	1	4	8	25	36	46	48	63	62	57	46	41	36	16	15	5	5	4	3	2	1	525
	Fat ...	1	2	3	11	23	31	34	36	55	48	28	41	28	18	9	11	1	5	1	1	1	—	388
TABLE XII. Muscularity.	Muscular	2	3	7	16	34	51	74	66	89	83	62	65	50	43	14	16	2	7	4	3	3	1	695
	Weak ...	—	—	—	3	14	16	6	18	29	27	23	22	19	11	11	10	4	3	1	1	—	—	218
	Totals for each Table	2	3	7	19	48	67	80	84	118	110	85	87	69	54	25	26	6	10	5	4	3	1	913

TABLE XIII.

Temperature (Occasion A).

Temperature (Occasion B).	96·5	96·7	96·9	97·1	97·3	97·5	97·7	97·9	98·1	98·3	98·5	98·7	98·9	99·1	99·3	99·5	99·7	99·9	Totals
96·5—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
96·7—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	2
96·9—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
97·1—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	2
97·3—	—	—	—	—	2	2	2	5	2	4	—	1	2	—	—	—	—	—	20
97·5—	—	—	—	1	2	4	4	8	3	7	2	1	1	—	—	—	—	—	33
97·7—	—	—	—	—	2	4	20	11	7	9	7	4	1	1	—	1	—	—	67
97·9—	—	—	—	—	5	8	11	24	14	10	7	6	4	—	1	—	—	—	90
98·1—	—	1	—	1	2	3	7	14	34	19	17	13	3	3	1	—	—	1	119
98·3—	—	1	—	—	4	7	9	10	19	28	23	22	14	4	2	—	—	—	143
98·5—	—	—	1	—	—	2	7	7	17	23	36	21	5	8	5	—	—	—	132
98·7—	1	—	—	—	1	1	4	6	13	22	21	26	8	11	2	2	2	—	120
98·9—	—	—	—	—	2	1	1	4	3	14	5	8	18	2	5	2	—	—	65
99·1—	—	—	—	—	—	—	1	—	3	4	8	11	2	2	1	—	—	—	32
99·3—	—	—	—	—	—	—	—	1	1	2	5	2	5	1	—	—	1	—	18
99·5—	—	—	—	—	—	—	1	—	—	—	—	2	2	—	—	—	—	—	5
99·7—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	1	—	—	—	3
99·9—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	1	2	1	2	20	33	67	90	119	143	132	120	65	32	18	5	3	1	854

TABLE XIV.

Prison Labour.

General Health.	Hard	Light	Totals
Good	260	75	335
Poor	33	124	157
Totals	293	199	492

TABLE XV.

Muscularity.

	Musc.	Weak	Totals
Good	298	37	335
Poor	81	77	158
Totals	379	114	493

TABLE XVI.

Muscularity.

Prison Labour.	Musc.	Weak	Totals
Hard	286	7	293
Light	93	106	199
Totals	379	113	492

TABLE XVII.

Height in inches.

Weight in lbs.	58—	59—	60—	61—	62—	63—	64—	65—	66—	67—	68—	69—	70—	71—	72—	73—	74—	75—	76—	Totals
90—99	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
100—109	—	1	—	—	1	1	—	—	—	—	1	—	—	—	—	—	—	—	—	4
110—119	1	—	5	9	5	6	3	1	—	—	1	—	—	—	—	—	—	—	—	31
120—129	1	2	3	7	15	12	13	5	8	2	1	—	—	—	—	—	—	—	—	69
130—139	—	—	2	14	12	25	30	19	12	8	5	1	—	—	—	—	—	—	—	128
140—149	—	1	4	5	3	13	21	18	22	18	6	6	2	1	—	—	—	—	—	120
150—159	—	—	—	—	1	6	2	10	11	14	10	7	4	1	2	—	—	—	—	68
160—169	—	—	—	1	2	—	1	4	3	12	7	6	4	—	—	—	—	—	—	40
170—179	—	—	—	1	—	—	1	3	2	5	2	3	3	1	—	—	—	—	—	21
180—189	—	—	—	—	—	—	—	—	1	1	—	1	1	—	—	—	—	—	—	4
190—199	—	—	—	—	—	—	1	—	—	—	—	1	1	—	—	—	—	—	—	3
200—209	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
210—219	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
220—229	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2

TABLE XVIII.

Mentality.

	58—	59—	60—	61—	62—	63—	64—	65—	66—	67—	68—	69—	70—	71—	72—	73—	74—	75—	76—	Totals
Normal ...	1	2	10	29	30	47	61	54	49	42	28	23	14	2	2	—	—	—	1	395
Weak-Minded	1	2	4	8	11	16	11	6	10	18	5	3	1	1	—	—	—	—	—	97
Totals for each Table	2	4	14	37	41	63	72	60	59	60	33	26	15	3	2	—	—	—	1	492

TABLE XIX.

Pulse (Occasion A).

Pulse (Occasion B).	45—48	49—52	53—56	57—60	61—64	65—68	69—72	73—76	77—80	81—84	85—88	89—92	93—96	97—100	101—104	105—108	109—112	113—116	Totals
45—48	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
49—52	1	—	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	5
53—56	—	2	2	6	1	3	1	2	—	—	—	—	—	—	—	—	—	—	17
57—60	1	1	6	12	17	7	1	2	3	2	—	—	—	—	—	—	—	—	52
61—64	—	1	1	17	16	20	14	7	2	3	1	—	—	—	—	—	—	—	82
65—68	—	—	3	7	20	42	27	14	14	6	5	1	2	—	—	—	—	—	141
69—72	—	—	1	1	14	27	28	18	12	5	2	—	—	—	—	1	—	—	109
73—76	—	—	2	2	7	14	18	36	15	8	5	7	2	2	2	1	—	—	121
77—80	—	—	—	3	2	14	12	15	26	20	5	4	—	1	—	—	—	—	102
81—84	—	—	—	2	3	6	5	8	20	16	14	3	—	2	—	—	—	—	79
85—88	—	—	—	—	1	5	2	5	5	14	12	9	1	1	1	2	1	1	60
89—92	—	—	—	—	—	1	—	7	4	3	9	6	3	3	1	—	1	—	38
93—96	—	—	—	—	—	2	—	2	—	—	1	3	2	—	2	—	—	—	12
97—100	—	—	—	—	—	—	—	2	1	2	1	3	—	4	1	1	1	—	16
101—104	—	—	—	—	—	—	—	2	—	—	1	1	2	1	—	—	—	—	7
105—108	—	—	—	—	—	—	1	1	—	—	2	—	—	1	—	—	—	—	5
109—112	—	—	—	—	—	—	—	—	—	—	1	1	—	1	—	—	—	—	3
113—116	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals	2	5	17	52	82	141	109	121	102	79	60	38	12	16	7	5	3	1	952

TABLE XX.

Weight in lbs.	Age.																						Totals
	21—	24—	27—	30—	33—	36—	39—	42—	45—	48—	51—	54—	57—	60—	63—	66—	69—	72—	75—	78—	81—	84—	
90—99	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
100—109	—	—	1	1	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	4
110—119	—	5	5	3	2	2	2	4	1	1	—	—	—	2	—	1	1	1	1	—	—	—	31
120—129	3	7	4	12	4	10	1	1	3	3	2	2	3	2	4	2	1	3	1	1	—	—	69
130—139	2	8	12	15	16	7	13	10	6	7	5	5	1	1	8	6	2	3	1	—	—	—	128
140—149	3	9	13	16	14	12	10	5	3	2	5	6	5	5	6	2	2	3	—	—	—	—	121
150—159	2	1	11	2	8	7	8	8	5	—	1	1	2	—	3	3	2	1	—	1	1	1	68
160—169	—	2	—	7	4	2	5	5	1	4	2	—	1	4	1	1	1	—	—	—	—	—	40
170—179	—	2	—	1	—	2	3	—	2	—	1	2	1	4	1	—	1	—	—	1	—	—	21
180—189	—	—	—	—	—	1	—	—	—	—	—	—	1	1	1	—	—	—	—	—	—	—	4
190—199	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	—	—	—	—	—	—	—	3
200—209	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
210—219	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
220—229	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	2
Totals	10	34	46	57	48	43	42	33	21	18	17	18	14	21	25	15	11	11	4	3	1	1	493

TABLE XXI.

Height in inches.																							
58—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
59—	—	—	—	2	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	4
60—	2	2	2	—	—	1	2	1	1	—	—	—	—	2	—	—	—	1	—	—	—	—	14
61—	—	2	2	3	4	3	6	6	2	1	—	—	1	3	1	—	1	1	—	1	—	—	37
62—	—	2	3	4	5	7	1	—	3	4	2	2	1	2	1	2	1	1	—	—	—	—	41
63—	—	6	4	13	8	4	2	5	3	—	2	3	3	2	4	3	—	—	1	—	—	—	63
64—	3	7	7	9	6	3	4	3	1	3	4	4	—	2	8	2	1	3	2	—	—	—	72
65—	2	4	9	6	6	4	5	4	1	4	1	2	—	—	3	3	2	3	—	1	—	—	60
66—	1	2	5	4	5	8	9	2	2	2	1	3	5	—	2	3	2	1	—	1	1	—	59
67—	—	3	8	4	4	6	5	5	3	—	5	—	1	7	5	1	1	1	—	—	—	1	60
68—	—	1	4	5	2	3	3	4	—	2	2	2	2	1	—	1	1	—	—	—	—	—	33
69—	1	3	2	4	4	2	3	1	1	2	—	1	—	2	—	—	—	—	—	—	—	—	26
70—	—	1	—	—	2	2	1	2	4	—	—	1	1	—	1	—	—	—	—	—	—	—	15
71—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	3
72—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
73—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
76—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	10	34	46	56	48	43	42	33	21	18	17	18	14	21	25	15	11	11	4	3	1	1	492

TABLE XXII.

Temperature.

Weight in lbs.	*96·5*	*96·7*	*96·9*	*97·1*	*97·3*	*97·5*	*97·7*	*97·9*	*98·1*	*98·3*	*98·5*	*98·7*	*98·9*	*99·1*	*99·3*	*99·5*	*99·7*	*99·9*	Totals
90—99	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	2
100—109	—	—	—	—	—	—	—	2	2	1	—	2	1	—	—	—	—	—	8
110—119	—	—	—	—	1	—	3	7	9	7	9	12	9	2	—	—	—	—	59
120—129	—	1	—	—	6	4	6	17	13	27	21	16	9	3	3	1	1	—	128
130—139	—	—	1	—	6	6	23	22	33	38	39	28	21	13	7	1	1	—	239
140—149	1	—	—	2	6	12	14	27	27	37	26	32	16	11	5	3	—	1	220
150—159	—	1	—	1	2	8	11	9	16	21	24	21	6	4	1	—	1	—	126
160—169	—	—	—	—	—	1	11	5	14	7	15	10	6	3	—	1	—	—	73
170—179	—	—	1	—	1	2	2	6	4	7	3	6	2	4	1	1	—	—	40
180—189	—	—	—	—	—	1	1	1	2	1	—	2	—	—	—	—	—	—	8
190—199	—	—	—	—	—	—	1	—	1	3	—	1	—	—	—	—	—	—	6
200—209	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	2
210—219	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
220—229	—	—	—	—	—	—	—	—	3	—	1	—	—	—	—	—	—	—	4
Totals	1	2	2	3	22	34	72	96	125	150	138	131	71	40	17	7	3	1	915

TABLE XXIII.

Height in inches.																			
58—	—	—	—	—	1	—	—	—	—	—	1	—	2	—	—	—	—	—	4
59—	—	—	—	—	—	—	—	2	—	2	1	1	2	—	—	—	—	—	8
60—	—	—	—	—	1	1	—	1	5	5	2	6	3	1	1	—	1	—	27
61—	—	—	—	—	2	3	3	8	8	7	7	11	8	6	2	1	—	—	66
62—	—	—	—	—	1	1	6	7	14	16	12	10	7	2	1	—	—	—	77
63—	—	1	—	1	6	4	8	11	22	19	20	18	7	2	2	—	—	—	121
64—	1	—	—	—	2	6	14	17	14	26	19	14	8	7	2	1	1	—	132
65—	—	—	—	1	4	6	8	18	13	20	13	12	10	3	4	—	—	—	112
66—	—	—	—	—	1	6	15	12	8	18	23	17	7	3	1	2	—	—	113
67—	—	1	—	—	2	2	10	13	11	18	19	19	5	6	1	1	1	1	110
68—	—	—	1	—	1	2	3	3	17	6	8	11	4	3	1	—	—	—	60
69—	—	—	—	—	1	1	4	2	6	9	8	5	5	3	1	2	—	—	47
70—	—	—	1	1	—	—	1	2	3	3	4	5	2	3	—	—	—	—	25
71—	—	—	—	—	—	—	—	—	2	1	—	—	1	1	1	—	—	—	6
72—	—	—	—	—	—	2	—	—	—	—	1	1	—	—	—	—	—	—	4
73—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
76—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	2
Totals	1	2	2	3	22	34	72	96	125	150	138	130	71	40	17	7	3	1	914

TABLE XXIV.

Respirations per minute.

Weight in lbs.	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Totals
90—99	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	2
100—109	—	—	—	—	—	—	—	2	2	—	3	—	—	—	1	—	—	—	—	—	—	—	8
110—119	—	—	—	1	2	2	5	3	10	8	4	4	4	8	3	2	—	1	1	—	1	—	59
120—129	—	—	—	3	2	5	11	18	16	12	15	9	14	11	4	2	2	—	—	1	1	—	126
130—139	1	—	4	2	20	19	25	17	29	30	19	22	15	11	5	10	2	5	—	2	1	—	239
140—149	—	—	1	3	10	20	19	18	32	29	21	25	13	14	3	8	—	1	3	—	—	—	220
150—159	1	1	2	3	5	13	12	13	14	14	12	13	9	3	3	2	1	2	1	1	—	1	126
160—169	—	2	—	1	7	2	3	6	6	9	7	10	7	5	4	2	1	1	—	—	—	—	73
170—179	—	—	—	5	—	3	2	5	5	6	4	4	4	—	2	—	—	—	—	—	—	—	40
180—189	—	—	—	—	—	—	2	1	3	—	—	—	1	1	—	—	—	—	—	—	—	—	8
190—199	—	—	—	—	—	2	—	1	1	2	—	—	—	—	—	—	—	—	—	—	—	—	6
200—209	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
210—219	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
220—229	—	—	—	—	1	—	1	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	4
Totals	2	3	7	19	48	67	80	84	118	110	85	87	69	54	25	26	6	10	5	4	3	1	913

TABLE XXV.

Height in inches.	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Totals
58—	—	—	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
59—	—	—	—	—	—	—	—	1	2	1	2	—	—	—	—	1	—	1	—	—	—	—	8
60—	—	—	—	1	1	—	4	3	4	3	2	3	2	2	1	—	—	—	—	—	—	—	26
61—	—	—	—	—	3	2	5	4	8	3	8	10	5	7	3	7	1	—	—	—	—	—	66
62—	—	—	—	2	4	6	5	7	5	8	7	8	6	9	3	1	—	2	—	2	1	—	76
63—	1	—	—	2	5	6	11	12	22	17	4	9	10	7	4	5	—	2	1	—	2	1	121
64—	—	—	4	2	7	10	9	13	23	17	17	6	8	8	4	3	—	1	—	—	—	—	132
65—	—	—	1	1	8	12	10	11	8	13	12	15	7	3	3	3	1	1	2	1	—	—	112
66—	—	—	—	3	5	12	13	5	18	19	10	9	11	4	1	2	—	1	—	—	—	—	113
67—	1	—	—	1	5	5	12	5	12	16	8	11	10	9	5	4	3	1	1	1	—	—	110
68—	—	3	—	4	7	3	5	13	9	2	5	7	2	—	—	—	—	—	—	—	—	—	60
69—	—	—	—	3	1	4	3	2	4	10	4	8	3	4	—	—	—	—	1	—	—	—	47
70—	—	—	2	—	—	4	2	3	3	—	5	1	4	—	1	—	—	—	—	—	—	—	25
71—	—	—	—	—	1	2	—	1	—	—	—	—	—	—	—	—	1	1	—	—	—	—	6
72—	—	—	—	—	1	1	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	4
73—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
76—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	2
Totals	2	3	7	19	48	67	80	84	118	110	84	87	69	54	25	26	6	10	5	4	3	1	912

TABLE XXVI.

Pulse.

Weight in lbs.	*45—48*	*49—52*	*53—56*	*57—60*	*61—64*	*65—68*	*69—72*	*73—76*	*77—80*	*81—84*	*85—88*	*89—92*	*93—96*	*97—100*	*101—104*	*105—108*	*109—112*	*113—116*	Totals
90—99	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	2
100—109	—	—	—	—	—	1	1	1	1	1	3	—	—	—	—	—	—	—	8
110—119	—	—	—	1	3	7	6	6	7	10	5	5	1	3	1	1	—	2	58
120—129	—	—	2	9	18	17	14	12	17	10	11	7	3	3	3	—	2	—	128
130—139	—	1	4	23	24	39	28	29	29	22	16	8	5	5	1	4	—	1	239
140—149	—	1	8	16	22	41	25	43	14	22	12	9	3	1	3	—	—	—	220
150—159	1	2	1	5	15	26	16	15	19	8	4	7	—	5	1	—	1	—	126
160—169	—	1	—	2	6	6	15	11	15	5	7	3	1	1	—	—	—	—	73
170—179	—	—	2	—	1	6	7	13	4	3	1	3	—	—	—	—	—	—	40
180—189	—	—	—	—	—	2	2	—	2	2	—	—	—	—	—	—	—	—	8
190—199	1	—	—	1	1	1	—	—	—	—	1	—	1	—	—	—	—	—	6
200—209	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	2
210—219	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
220—229	—	—	—	—	—	2	2	—	—	—	—	—	—	—	—	—	—	—	4
Totals	2	5	17	57	90	149	116	132	108	84	60	42	14	18	9	5	3	3	914

TABLE XXVII.

Height in inches.	*45—48*	*49—52*	*53—56*	*57—60*	*61—64*	*65—68*	*69—72*	*73—76*	*77—80*	*81—84*	*85—88*	*89—92*	*93—96*	*97—100*	*101—104*	*105—108*	*109—112*	*113—116*	Totals
58—	—	—	—	—	—	1	—	—	2	1	—	—	—	—	—	—	—	—	4
59—	—	—	—	1	1	1	1	—	1	1	2	—	—	—	—	—	—	—	8
60—	—	—	—	—	—	9	1	4	3	5	3	2	—	—	—	—	—	—	27
61—	—	—	3	8	4	11	3	5	13	3	2	5	2	3	1	1	—	1	65
62—	—	1	2	6	14	8	12	13	5	5	4	3	2	—	1	—	1	—	77
63—	—	—	—	6	10	16	15	18	11	18	14	6	1	4	2	—	—	—	121
64—	1	3	2	10	18	25	14	17	13	12	5	5	1	2	—	2	1	1	132
65—	—	—	1	6	11	17	17	23	11	7	4	5	6	2	1	1	—	—	112
66—	—	1	4	9	10	15	15	18	16	10	7	4	1	1	1	1	—	—	113
67—	—	—	1	4	8	17	16	18	14	8	7	9	—	5	2	—	1	—	110
68—	—	—	1	3	6	11	9	8	7	5	6	1	—	1	1	—	—	1	60
69—	1	—	2	3	5	12	8	5	5	2	4	—	—	—	—	—	—	—	47
70—	—	—	1	—	1	1	4	2	5	7	2	1	1	—	—	—	—	—	25
71—	—	—	—	1	1	1	—	—	2	—	—	1	—	—	—	—	—	—	6
72—	—	—	—	—	1	2	1	—	—	—	—	—	—	—	—	—	—	—	4
73—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
76—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	2	5	17	57	90	149	116	131	108	84	60	42	14	18	9	5	3	3	913

TABLE XXVIII.

Pulse.

Age.	45—48	49—52	53—56	57—60	61—64	65—68	69—72	73—76	77—80	81—84	85—88	89—92	93—96	97—100	101—104	105—108	109—112	113—116	Totals
21—23	—	—	—	1	2	2	3	3	1	2	2	1	—	—	—	—	—	—	17
24—26	—	—	1	6	5	9	9	12	6	5	4	1	1	3	1	—	—	—	63
27—29	1	2	2	7	15	16	10	5	6	7	5	3	1	1	1	—	1	1	84
30—32	—	1	3	7	9	25	17	17	5	7	6	3	1	3	—	3	—	—	107
33—35	—	—	1	8	4	17	12	16	15	7	3	5	—	1	—	—	—	—	89
36—38	—	—	1	3	11	13	16	12	9	6	4	3	2	—	1	—	—	—	81
39—41	—	—	—	5	11	11	10	12	14	6	4	1	1	2	—	1	1	—	79
42—44	—	—	1	3	3	10	4	6	7	9	7	—	3	4	2	—	—	2	61
45—47	—	—	1	3	5	7	4	2	7	5	3	1	—	—	—	—	—	—	38
48—50	—	—	1	2	5	8	3	3	4	1	1	3	2	—	—	—	—	—	33
51—53	—	—	—	—	3	3	3	8	6	3	2	1	—	—	1	—	—	—	30
54—56	—	—	1	4	2	1	4	8	3	4	3	2	1	1	—	—	—	—	34
57—59	—	1	1	—	3	1	1	2	4	4	3	3	1	—	2	—	—	—	26
60—62	1	1	1	2	3	6	9	7	2	1	3	4	—	1	—	—	—	—	41
63—65	—	—	—	1	2	10	7	7	5	7	4	5	1	—	—	1	1	—	51
66—68	—	—	1	5	2	4	2	3	3	4	3	1	1	—	1	—	—	—	30
69—71	—	—	—	—	—	1	4	4	6	3	3	2	—	2	—	—	—	—	25
72—74	—	—	2	—	4	3	1	2	4	3	1	1	—	—	—	—	—	—	21
75—77	—	—	—	—	—	3	1	—	—	1	1	1	—	—	—	—	—	—	7
78—80	—	—	—	—	—	—	—	1	2	—	1	1	—	—	—	—	—	—	5
81—83	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	2
84—86	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	2
Totals	2	5	17	57	90	150	120	132	109	86	63	42	15	18	9	5	3	3	926

TABLE XXIX.

Mentality.	Weak-Minded	—	—	1	10	16	19	21	29	25	20	17	12	5	8	3	3	1	1	191
	Normal ...	2	5	16	47	74	131	99	103	84	66	46	30	10	10	6	2	2	2	735
	Totals	2	5	17	57	90	150	120	132	109	86	63	42	15	18	9	5	3	3	926

TABLE XXX.

Labour.	Hard ...	1	3	7	36	54	99	77	80	61	41	28	16	8	11	6	3	—	2	533
	Light ...	1	2	10	21	36	50	38	52	47	42	32	26	6	7	3	2	3	1	379
	Totals	2	5	17	57	90	149	115	132	108	83	60	42	14	18	9	5	3	3	912

TABLE XXXI.

Respirations per minute.

Age.	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	Totals
21—23	—	—	—	—	—	—	1	3	2	3	2	3	1	—	1	1	—	—	—	—	—	—	17
24—26	—	—	2	3	—	1	7	6	7	10	4	4	8	7	1	1	—	1	—	—	—	—	62
27—29	1	—	—	3	5	5	11	11	10	10	8	9	4	2	2	3	—	—	1	—	—	—	85
30—32	—	2	—	—	6	16	6	15	17	16	8	6	8	1	1	3	—	1	—	—	1	—	107
33—35	—	—	—	—	5	7	8	5	11	11	7	9	7	8	3	4	1	2	—	—	1	—	89
36—38	—	—	2	1	1	6	12	4	15	11	9	5	2	7	5	—	1	—	—	—	—	—	81
39—41	1	1	—	4	6	7	9	6	8	7	4	9	3	4	2	3	1	1	2	1	—	—	79
42—44	—	—	—	—	4	4	6	5	8	11	6	8	5	2	1	—	—	—	—	—	—	1	61
45—47	—	—	1	—	1	1	3	3	3	3	6	2	6	2	2	1	—	1	—	1	1	—	37
48—50	—	—	—	—	1	3	2	2	4	4	2	7	3	4	—	1	—	—	—	—	—	—	33
51—53	—	—	—	3	3	3	2	2	3	4	—	4	2	2	—	2	—	—	—	—	—	—	30
54—56	—	—	1	1	2	4	4	4	7	2	2	2	1	3	—	—	—	—	1	—	—	—	34
57—59	—	—	—	1	1	1	4	5	—	3	3	6	1	—	—	—	1	—	—	—	—	—	26
60—62	—	—	—	1	2	1	—	4	6	3	6	4	5	5	1	1	—	1	—	1	—	—	41
63—65	—	—	1	2	6	5	2	4	9	3	3	5	4	4	1	1	—	1	—	—	—	—	51
66—68	—	—	—	—	3	2	4	1	4	2	3	2	2	1	2	1	1	1	—	1	—	—	30
69—71	—	—	—	—	—	—	—	2	2	2	7	3	4	1	1	1	1	1	—	—	—	—	25
72—74	—	—	—	—	2	1	1	3	3	2	3	—	2	—	2	2	—	—	—	—	—	—	21
75—77	—	—	—	—	—	—	—	—	—	2	—	—	1	2	1	—	—	—	1	—	—	—	7
78—80	—	—	—	—	—	—	—	—	—	3	1	—	—	—	—	1	—	—	—	—	—	—	5
81—83	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	2
84—86	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	2
Totals	2	3	7	19	48	67	82	85	119	112	85	89	71	55	26	26	6	10	5	4	3	1	925

TABLE XXXII.

Mentality.	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	Totals
Weak-Minded	2	—	—	2	8	8	15	18	20	25	16	22	16	14	6	5	3	6	2	1	1	—	190
Normal ...	—	3	7	17	40	59	67	67	99	87	69	67	55	41	20	21	3	4	3	3	2	1	735
Totals	2	3	7	19	48	67	82	85	119	112	85	89	71	55	26	26	6	10	5	4	3	1	925

TABLE XXXIII.

Labour.	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	Totals
Hard ...	1	2	6	9	21	34	60	52	74	69	45	51	39	30	13	13	2	4	4	—	2	1	532
Light ...	1	1	1	9	26	33	20	32	44	41	40	36	30	24	12	13	4	6	1	4	1	—	379
Totals	2	3	7	18	47	67	80	84	118	110	85	87	69	54	25	26	6	10	5	4	3	1	911

TABLE XXXIV.

Temperature.

		96·5	96·7	96·9	97·1	97·3	97·5	97·7	97·9	98·1	98·3	98·5	98·7	98·9	99·1	99·3	99·5	99·7	99·9	Totals
Age.	21—23	—	—	—	—	1	—	1	—	2	—	3	4	3	2	—	1	—	—	17
	24—26	—	—	—	1	1	3	6	5	6	9	11	8	5	4	3	—	1	—	63
	27—29	—	—	1	—	5	4	4	9	10	14	15	14	3	5	—	—	1	—	85
	30—32	—	—	—	—	2	5	4	8	15	15	23	17	7	7	4	—	—	—	107
	33—35	—	—	—	—	2	4	5	7	9	12	21	14	7	4	1	2	1	—	89
	36—38	—	—	—	—	3	—	6	9	10	13	12	14	7	5	2	—	—	—	81
	39—41	—	1	—	—	—	2	5	9	7	12	14	9	12	6	1	1	—	—	79
	42—44	—	—	—	—	2	—	6	7	8	12	8	9	5	2	1	1	—	—	61
	45—47	—	—	—	—	—	1	1	5	8	7	4	8	3	—	1	—	—	—	38
	48—50	—	—	—	—	—	1	5	4	5	7	3	4	2	2	—	—	—	—	33
	51—53	—	—	—	—	—	5	5	4	2	5	1	4	3	—	—	—	—	1	30
	54—56	—	—	1	—	4	—	1	6	6	7	1	2	2	1	3	—	—	—	34
	57—59	—	—	—	—	—	—	5	—	5	5	4	4	1	—	1	1	—	—	26
	60—62	—	—	—	1	—	3	4	5	5	6	7	3	5	2	—	—	—	—	41
	63—65	—	1	—	—	1	1	2	7	8	12	6	9	3	1	—	—	—	—	51
	66—68	1	—	—	1	—	3	5	3	6	7	1	3	—	—	—	—	—	—	30
	69—71	—	—	—	—	—	—	2	2	5	4	4	4	4	—	—	—	—	—	25
	72—74	—	—	—	—	1	2	2	3	4	5	—	1	1	—	1	1	—	—	21
	75—77	—	—	—	—	—	—	1	2	2	—	2	—	—	—	—	—	—	—	7
	78—80	—	—	—	—	—	—	1	2	2	—	—	—	—	—	—	—	—	—	5
	81—83	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	2
	84—86	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	2
	Totals	1	2	2	3	22	34	72	98	125	153	140	132	73	41	18	7	3	1	927

TABLE XXXV.

		96·5	96·7	96·9	97·1	97·3	97·5	97·7	97·9	98·1	98·3	98·5	98·7	98·9	99·1	99·3	99·5	99·7	99·9	Totals
Mentality.	Weak-Minded	—	—	—	—	1	4	10	12	24	25	34	31	24	15	8	2	1	—	191
	Normal ...	1	2	2	3	21	30	62	86	101	128	106	101	49	26	10	5	2	1	736
	Totals	1	2	2	3	22	34	72	98	125	153	140	132	73	41	18	7	3	1	927

TABLE XXXVI.

		96·5	96·7	96·9	97·1	97·3	97·5	97·7	97·9	98·1	98·3	98·5	98·7	98·9	99·1	99·3	99·5	99·7	99·9	Totals
Labour.	Hard ...	—	1	2	1	15	19	36	48	66	88	94	77	37	27	13	5	3	1	533
	Light ...	1	1	—	2	7	14	36	48	59	62	44	53	34	13	4	2	—	—	380
	Totals	1	2	2	3	22	33	72	96	125	150	138	130	71	40	17	7	3	1	913

TABLE XXXVII.

Normal-minded only.

Temperature.	Age. 21—	24—	27—	30—	33—	36—	39—	42—	45—	48—	51—	54—	57—	60—	63—	66—	69—	72—	75—	78—	81—	84—	Totals
96·5—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
96·7—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	2
96·9—	—	—	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
97·1—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	3
97·3—	—	1	5	2	2	3	—	2	—	—	—	4	—	—	1	—	—	1	—	—	—	—	21
97·5—	—	1	4	5	4	—	2	—	1	—	4	—	—	3	1	3	—	2	—	—	—	—	30
97·7—	1	3	4	4	5	6	2	6	1	4	4	—	5	4	1	5	2	2	1	1	1	—	62
97·9—	—	3	9	5	5	9	8	7	3	4	2	6	—	5	7	3	2	3	2	2	—	1	86
98·1—	1	3	8	14	5	10	5	4	7	2	1	6	5	5	6	6	5	4	2	2	—	—	101
98·3—	—	5	10	15	9	12	10	12	5	4	3	3	5	6	12	7	4	5	—	—	—	1	128
98·5—	3	6	9	20	11	11	11	5	4	1	1	1	4	7	5	1	4	—	2	—	—	—	106
98·7—	3	5	11	14	10	12	8	5	5	3	1	—	4	3	8	3	4	1	—	—	1	—	101
98·9—	1	2	2	5	4	4	7	3	2	2	2	1	1	5	3	—	4	1	—	—	—	—	49
99·1—	2	2	4	4	3	5	2	1	—	1	—	—	—	2	—	—	—	—	—	—	—	—	26
99·3—	—	—	—	2	1	1	1	—	1	—	—	2	1	—	—	—	—	1	—	—	—	—	10
99·5—	1	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	5
99·7—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
99·9—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
Totals	12	32	68	90	61	73	58	46	29	21	19	24	25	41	45	30	25	21	7	5	2	2	736

TABLE XXXVIII.

Weak-minded only.

Temperature.	Age. 21—	24—	27—	30—	33—	36—	39—	42—	45—	48—	51—	54—	57—	60—	63—	Totals
97·3—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
97·5—	—	2	—	—	—	—	—	—	—	1	1	—	—	—	—	4
97·7—	—	3	—	—	—	—	3	—	—	1	1	1	—	—	1	10
97·9—	—	2	—	3	2	—	1	—	2	—	2	—	—	—	—	12
98·1—	1	3	2	1	4	—	2	4	1	3	1	—	—	—	2	24
98·3—	—	4	4	—	3	1	2	—	2	3	2	4	—	—	—	25
98·5—	—	5	6	3	10	1	3	3	—	2	—	—	—	—	1	34
98·7—	1	3	3	3	4	2	1	4	3	1	3	2	—	—	1	31
98·9—	2	3	1	2	3	3	5	2	1	—	1	1	—	—	—	24
99·1—	—	2	1	3	1	—	4	1	—	1	—	1	—	—	1	15
99·3—	—	3	—	2	—	1	—	1	—	—	—	1	—	—	—	8
99·5—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	2
99·7—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	5	31	17	17	28	8	21	15	9	12	11	10	1	—	6	191

TABLE XXXIX.

Temperature.

Pulse.	96·5	96·7	96·9	97·1	97·3	97·5	97·7	97·9	98·1	98·3	98·5	98·7	98·9	99·1	99·3	99·5	99·7	99·9	Totals
45—48	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	2
49—52	—	—	—	—	1	—	1	—	3	—	—	—	—	—	—	—	—	—	5
53—56	—	—	—	—	—	2	3	2	4	4	1	1	—	—	—	—	—	—	17
57—60	1	—	1	—	2	3	7	6	9	13	6	3	2	1	3	—	—	—	57
61—64	—	1	—	—	1	7	9	14	9	21	14	6	6	2	—	—	—	—	90
65—68	—	—	—	2	5	10	11	19	19	28	21	23	9	1	1	1	—	—	150
69—72	—	—	—	—	4	2	7	20	17	21	20	20	5	3	1	—	—	—	120
73—76	—	—	1	1	3	6	12	11	18	14	18	23	14	9	1	1	—	—	132
77—80	—	—	—	—	4	2	9	9	20	16	16	13	8	7	2	1	2	—	109
81—84	—	1	—	—	—	2	3	9	12	12	13	10	14	6	2	2	—	—	86
85—88	—	—	—	—	—	—	2	3	7	13	12	12	5	3	3	1	1	1	63
89—92	—	—	—	—	—	—	4	2	6	3	10	8	4	5	—	—	—	—	42
93—96	—	—	—	—	1	—	2	1	1	3	1	1	1	1	2	1	—	—	15
97—100	—	—	—	—	—	—	—	1	—	2	6	3	3	1	2	—	—	—	18
101—104	—	—	—	—	—	—	—	—	—	2	2	2	1	1	1	—	—	—	9
105—108	—	—	—	—	—	—	—	1	—	—	—	3	1	—	—	—	—	—	5
109—112	—	—	—	—	—	—	—	—	—	1	—	2	—	—	—	—	—	—	3
113—116	—	—	—	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	3
Totals	1	2	2	3	21	34	72	98	125	153	140	132	73	41	18	7	3	1	926

TABLE XL.

Weight in lbs.

Mentality.	90—	100—	110—	120—	130—	140—	150—	160—	170—	180—	190—	200—	210—	220—	Totals
Normal ...	—	3	18	51	94	106	58	35	21	4	3	1	—	2	396
Weak-Minded	1	1	13	18	34	15	10	5	—	—	—	—	—	—	97
Totals	1	4	31	69	128	121	68	40	21	4	3	1	—	2	493

TABLE XLI.

Age.

Mentality.	21—	24—	27—	30—	33—	36—	39—	42—	45—	48—	51—	54—	57—	60—	63—	66—	69—	72—	75—	78—	81—	84—	Totals
Normal ...	7	18	37	48	34	41	32	26	16	12	11	13	13	21	23	15	13	11	4	3	1	1	400
Weak-Minded	3	16	9	9	14	4	11	8	5	6	6	5	1	—	3	—	—	—	—	—	—	—	100
Totals	10	34	46	57	48	45	43	34	21	18	17	18	14	21	26	15	13	11	4	3	1	1	500

TABLE XLII.

Normal-minded only.

Temperature.

Pulse.	*96·5*	*96·7*	*96·9*	*97·1*	*97·3*	*97·5*	*97·7*	*97·9*	*98·1*	*98·3*	*98·5*	*98·7*	*98·9*	*99·1*	*99·3*	*99·5*	*99·7*	*99·9*	Totals
45—48	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	2
49—52	—	—	—	—	1	—	1	—	3	—	—	—	—	—	—	—	—	—	5
53—56	—	—	—	—	—	2	3	2	4	4	—	1	—	—	—	—	—	—	16
57—60	1	—	1	—	2	2	7	6	9	11	3	3	1	—	1	—	—	—	47
61—64	—	1	—	—	1	7	9	13	6	16	10	6	4	1	—	—	—	—	74
65—68	—	—	—	2	5	9	9	18	16	26	16	20	7	1	1	1	—	—	131
69—72	—	—	—	—	4	2	5	18	14	16	16	18	3	3	—	—	—	—	99
73—76	—	—	1	1	3	4	12	10	14	12	13	16	11	5	1	—	—	—	103
77—80	—	—	—	—	3	2	6	5	16	13	15	11	5	3	2	1	2	—	84
81—84	—	1	—	—	—	2	3	8	9	11	9	5	10	4	2	2	—	—	66
85—88	—	—	—	—	—	—	1	3	4	11	10	9	3	2	1	1	—	1	46
89—92	—	—	—	—	—	—	2	1	5	2	7	5	3	5	—	—	—	—	30
93—96	—	—	—	—	1	—	2	1	1	2	—	1	1	—	1	—	—	—	10
97—100	—	—	—	—	—	—	—	1	—	1	5	2	1	—	—	—	—	—	10
101—104	—	—	—	—	—	—	—	—	—	2	2	—	—	1	1	—	—	—	6
105—108	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	2
109—112	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	2
113—116	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
Totals	1	2	2	3	20	30	62	86	101	128	106	101	49	26	10	5	2	1	735

TABLE XLIII.

Weak-minded only.

Temperature.

Pulse.	*97·3*	*97·5*	*97·7*	*97·9*	*98·1*	*98·3*	*98·5*	*98·7*	*98·9*	*99·1*	*99·3*	*99·5*	*99·7*	Totals
53—56	—	—	—	—	—	—	1	—	—	—	—	—	—	1
57—60	—	1	—	—	—	2	3	—	1	1	2	—	—	10
61—64	—	—	—	1	3	5	4	—	2	1	—	—	—	16
65—68	—	1	2	1	3	2	5	3	2	—	—	—	—	19
69—72	—	—	2	2	3	5	4	2	2	—	1	—	—	21
73—76	—	2	—	1	4	2	5	7	3	4	—	1	—	29
77—80	1	—	3	4	4	3	1	2	3	4	—	—	—	25
81—84	—	—	—	1	3	1	4	5	4	2	—	—	—	20
85—88	—	—	1	—	3	2	2	3	2	1	2	—	1	17
89—92	—	—	2	1	1	1	3	3	1	—	—	—	—	12
93—96	—	—	—	—	—	1	1	—	—	1	1	1	—	5
97—100	—	—	—	—	—	1	1	1	2	1	2	—	—	8
101—104	—	—	—	—	—	—	—	2	1	—	—	—	—	3
105—108	—	—	—	1	—	—	—	1	1	—	—	—	—	3
109—112	—	—	—	—	—	—	—	1	—	—	—	—	—	1
113—116	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	1	4	10	12	24	25	34	31	24	15	8	2	1	191

TABLE XLIV.

External Temperature.

Temperature.

	40—	42—	44—	46—	48—	50—	52—	54—	56—	58—	60—	62—	64—	66—	68—	70—	72—	74—	76—	78—	Totals
96·5—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
96·7—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
96·9—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	2
97·1—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	3
97·3—	—	—	1	—	—	1	1	1	—	3	4	1	1	—	2	—	1	2	—	1	19
97·5—	—	—	4	1	—	—	1	4	1	1	3	1	3	1	2	1	5	—	—	1	29
97·7—	—	—	3	3	—	1	1	3	3	5	10	1	8	4	5	2	8	4	—	1	62
97·9—	—	—	8	1	—	4	1	3	3	12	14	7	6	5	4	4	11	7	—	2	92
98·1—	1	—	8	4	—	4	4	1	4	17	21	6	9	9	8	3	6	9	2	1	117
98·3—	3	—	9	5	—	6	4	4	5	16	14	11	11	8	11	3	18	9	2	2	141
98·5—	2	—	11	4	—	6	5	5	5	11	16	16	16	2	6	4	11	4	—	1	125
98·7—	2	—	5	3	—	5	7	4	8	11	13	11	12	12	9	3	14	1	2	1	123
98·9—	—	—	2	—	—	—	4	1	4	7	5	6	8	3	11	1	9	2	1	—	64
99·1—	—	—	2	—	—	—	2	—	2	5	4	5	2	1	3	—	3	1	2	—	32
99·3—	—	—	—	—	—	—	—	—	2	1	5	2	3	—	1	—	—	1	—	—	15
99·5—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	1	1	1	—	—	—	5
99·7—	—	—	—	—	—	—	—	—	1	1	—	—	1	—	—	—	—	—	—	—	3
99·9—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
Totals	8	—	55	21	—	29	30	27	39	90	109	67	80	46	65	22	88	41	9	10	836

TABLE XLV.

Pulse.

45—48	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	2
49—52	—	—	—	—	—	1	1	—	—	1	—	—	—	—	1	1	—	—	—	—	5
53—56	—	—	—	1	—	1	1	—	1	—	3	1	2	—	2	—	3	—	—	1	16
57—60	1	—	3	—	—	3	1	6	1	5	14	3	3	2	5	—	3	2	—	—	52
61—64	2	—	4	2	—	5	2	5	1	11	10	4	5	6	8	1	9	3	1	—	79
65—68	—	—	10	4	—	2	5	6	8	19	17	10	11	7	5	2	20	3	1	2	132
69—72	3	—	10	3	—	4	5	1	3	10	16	12	11	7	6	4	8	4	—	3	110
73—76	1	—	9	2	—	6	5	3	7	15	17	5	12	9	9	2	12	5	1	1	121
77—80	—	—	6	4	—	1	7	2	5	9	9	11	12	3	8	1	9	5	4	2	98
81—84	—	—	5	2	—	3	2	1	4	12	9	10	6	4	4	4	6	6	—	—	78
85—88	—	—	4	1	—	2	—	2	3	3	8	4	6	4	5	2	7	7	—	—	58
89—92	1	—	1	2	—	—	1	1	1	3	3	2	4	1	5	4	5	4	1	—	39
93—96	—	—	1	—	—	—	—	—	1	—	2	—	2	—	1	—	3	—	—	1	11
97—100	—	—	1	—	—	1	—	—	—	1	—	3	3	2	3	—	2	—	1	—	17
101—104	—	—	1	—	—	—	—	—	1	—	—	1	1	—	1	1	1	1	—	—	8
105—108	—	—	—	—	—	—	—	—	1	1	1	—	—	—	2	—	—	—	—	—	5
109—112	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	2
113—116	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	2
Totals	8	—	55	21	—	29	30	27	39	90	109	66	80	46	65	22	88	41	9	10	835

TABLE XLVI.

Respirations per minute.

External Temperature.

	10	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	Totals
40—	—	—	—	—	—	—	2	—	—	1	3	1	—	—	—	—	—	—	1	—	—	—	8
42—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
44—	—	—	—	3	2	4	3	5	5	6	3	9	7	4	2	2	—	—	—	—	—	—	55
46—	—	—	—	1	—	1	3	2	5	1	1	—	3	1	1	1	—	—	—	—	1	—	21
48—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
50—	—	—	—	1	2	1	3	3	5	4	3	4	—	1	—	1	—	—	1	—	—	—	29
52—	—	—	—	—	—	4	3	3	3	3	4	4	3	—	1	1	—	—	—	—	—	1	30
54—	—	—	2	—	3	3	4	2	4	2	3	3	—	—	—	—	—	1	—	—	—	—	27
56—	—	—	—	2	2	4	4	3	7	3	3	4	2	2	2	1	—	—	—	—	—	—	39
58—	—	—	—	3	7	8	8	8	17	12	5	12	6	2	1	1	—	—	—	—	—	—	90
60—	1	—	2	1	5	8	12	11	11	20	7	10	10	7	2	1	—	—	—	1	—	—	109
62—	—	—	—	—	4	2	10	8	10	6	2	3	9	4	1	3	2	2	—	—	1	—	67
64—	—	—	—	2	5	6	3	11	7	11	7	9	5	6	5	—	—	1	—	—	—	—	78
66—	—	1	—	1	3	2	1	4	8	2	11	3	4	2	1	2	—	—	—	—	1	—	46
68—	1	—	—	—	2	6	2	6	8	8	4	10	3	2	3	2	—	3	3	2	—	—	65
70—	—	—	—	—	1	—	—	1	3	3	2	1	4	1	—	3	2	—	—	1	—	—	22
72—	—	1	1	—	4	6	11	8	5	12	9	6	8	10	3	3	1	—	—	—	—	—	88
74—	—	—	1	1	2	—	4	—	5	4	5	2	4	5	2	2	1	3	—	—	—	—	41
76—	—	—	—	—	—	—	2	1	2	—	3	—	—	1	—	—	—	—	—	—	—	—	9
78—	—	1	—	—	—	1	1	1	1	2	2	—	—	1	—	—	—	—	—	—	—	—	10
Totals	2	3	6	15	42	56	76	77	106	100	77	81	68	49	24	23	6	10	5	4	3	1	834

TABLE XLVII. Muscularity.

Nutrition.

	Musc.	Weak	Totals
Thin	206	77	283
Fat	173	37	210
Totals	379	114	493

TABLE XLVIII. Prison Labour.

	Hard	Light	Totals
Thin	140	142	282
Fat	153	57	210
Totals	293	199	492

TABLE XLIX. General Health.

Nutrition.

	Good	Poor	Totals
Thin	161	122	283
Fat	174	36	210
Totals	335	158	493

ALPHABETICAL INDEX TO CORRELATION TABLES IN APPENDIX.

Correlated Variates.

Age: with Height, XXI; with Mentality, XLI; with Pulse, XXVIII; with Respiration, XXXI; with Temperature (Total population), XXXIV; with Temperature (Normal-minded only), XXXVII; with Temperature (Weak-minded only), XXXVIII; with Weight, XX.

Health: with Labour, XIV; with Muscularity, XV; with Nutrition, XLIX; with Pulse, VI; with Respiration, X; with Temperature, II.

Height: with Age, XXI; with Mentality, XVIII; with Pulse, XXVII; with Respiration, XXV; with Temperature, XXIII; with Weight, XVII.

Labour: with Health, XIV; with Muscularity, XVI; with Nutrition, XLVIII; with Pulse, XXX; with Respiration, XXXIII; with Temperature, XXXVI.

Mentality: with Age, XLI; with Height, XVIII; with Pulse, XXIX; with Respiration, XXXII; with Temperature, XXXV; with Weight, XL.

Muscularity: with Health, XV; with Labour, XVI; with Nutrition, XLVII; with Pulse, VIII; with Respiration, XII; with Temperature, VI.

Nutrition: with Health, XLIX; with Labour, XLVIII; with Muscularity, XLVII; with Pulse, VII; with Respiration, XI; with Temperature, III.

Pulse: with Age, XXVIII; with Health, VI; with Height, XXVII; with Labour, XXX; with Mentality, XXIX; with Muscularity, VIII; with Nutrition, VII; (Occasion A) with Pulse (Occasion B), XIX; with Respiration, V; with Temperature (Total population), XXXIX; with Temperature (Normal-minded only), XLII; with Temperature (Weak-minded only), XLIII; with Temperature (external), XLV; with Weight, XXVI.

Respiration: with Age, XXXI; with Health, X; with Height, XXV; with Labour, XXXIII; with Mentality, XXXII; with Muscularity, VIII; with Nutrition, XI; with Pulse, V; (Occasion A) with Respiration (Occasion B), IX; with Temperature, I; with Temperature (external), XLVI; with Weight, XXIV.

Temperature: with Age (Total population), XXXIV; with Age (Normal-minded only), XXXVII; with Age (Weak-minded only), XXXVIII; with Health, II; with Height, XXIII; with Labour, XXXVI; with Mentality, XXXV; with Muscularity, IV; with Nutrition, III; with Pulse (Total population), XXXIX; with Pulse (Normal-minded only), XLII; with Pulse (Weak-minded only), XLIII; with Respiration, I; (Occasion A) with Temperature (Occasion B), XIII; with Temperature (external), XLIV; with Weight, XXII.

Weight: with Age, XX; with Height, XVII; with Mentality, XL; with Pulse, XXVI; with Respiration, XXIV; with Temperature, XXII.

VARIATION AND CORRELATION OF THE NUMBER OF UMBEL RAYS OF SOME UMBELLIFERAE*.

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(1) Since the investigations by Ludwig[†] into the variation of the number of umbel rays of the umbels of Umbelliferae, but little is to be found about this subject in biological literature, although the reading of the chapter "Summationskurven der Umbelliferen" of the article above-mentioned might suggest various problems.

For example, in the variation curves of Ludwig the numbers of the Fibonnacci series do not exclusively appear as maxima, but in some cases also the numbers of the first parallel series (1, 3, 4, 7, 11, etc.) appear. Vogler[‡] also finds in his investigations into *Astrantia major* that principal series as well as parallel series occur in this case.

The few investigations by Ludwig on the Umbelliferae are hardly sufficient to allow us to state that modes which arise in the numbers of the first parallel series of these variation curves are with any degree of certainty only apparent modes, or to assert that they have resulted from the mixing of plants of different local races. On the contrary it is very probable that these numbers of the first parallel series are a characteristic phenomenon of the variation curves of the number of umbel rays of many species of Umbelliferae.

Of the many countings which we made in the summer of 1912 to find out something more about this subject, only those relating to three species of plants, *Pastinaca sativa*, *Anthriscus sylvestris* and *Sium latifolium*, are used here.

Only numbers of umbel rays of terminal umbels and those of lateral umbels of the first order were counted.

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[*] Our thanks are due to Prof. Dr G. van Iterson who kindly revised this paper.

[†] *Bot. Centralblatt*, Bd. LXIV.

[‡] *Beihefte zum botanischen Centralblatt*, Bd. XXIV.

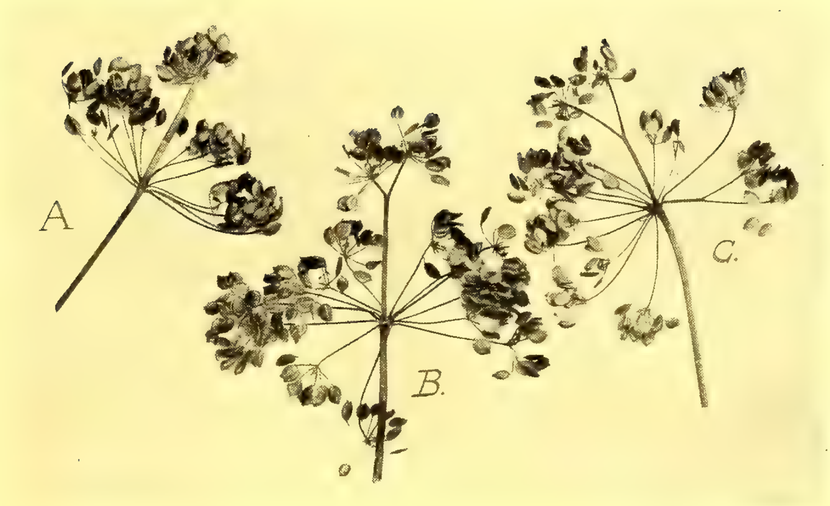

FIG. 1. *A* represents an umbel with one fasciation of two rays which are united to the top, *B* an umbel with one fasciation of two rays which are not united to the top, *C* an umbel with two fasciations each of two rays with union somewhat lower.

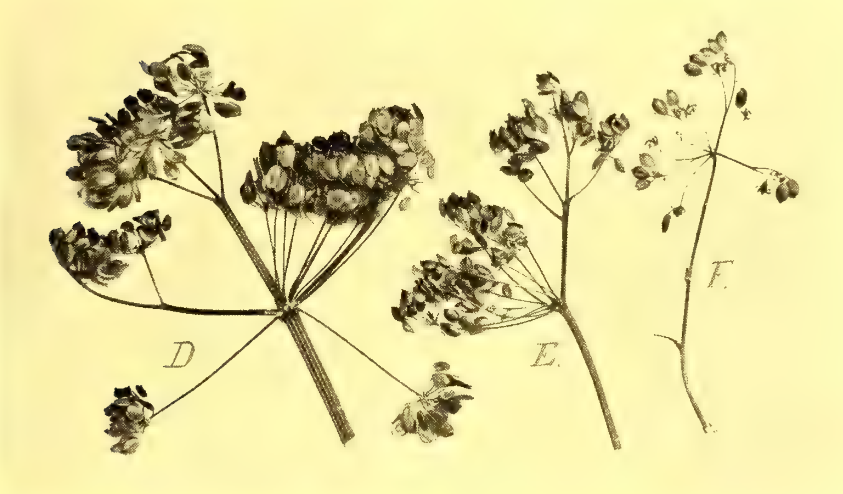

FIG. 2. *D* represents an umbel with two fasciations, one of two and the other of three rays, *E* an umbel with one fasciation of four rays, *F* an umbel with three fasciations each of two umbel rays.

Special Fasciations of *Pastinaca sativa*.

(2) Countings of *Pastinaca sativa.*

Variation of the number of umbel rays of the terminal umbel.

In Table I are arranged, according to the increasing numbers of umbel rays, terminal umbels of two groups of plants growing in different places; the first group of 341 plants came from clayey and peaty waysides in the neighbourhood of Delft (Holland), the second of 335 plants for the greater part from sheltered, for the smaller part from exposed spots in the duneland not far from the village of Lisse (Holland). The countings were made in the months of July and August of the year 1912.

In counting, a large number of fasciations were observed in the umbels. Among 515 plants, of which both the terminal and lateral umbels were counted, 167 were observed to have fasciations. As many rays were taken into account as there would have been, if no fasciation had occurred. Plate I, figures 1 and 2, shows some very interesting fasciations.

Variation of the number of umbel rays of the lateral umbels of the first order and correlation of the numbers of umbel rays of terminal umbels and that of lateral umbels.

The countings of umbel rays of lateral umbels have been combined in two different ways. In the first place lateral umbels belonging to plants with the same number of rays in the terminal umbel have been united into groups; and in the second place those which belonged to plants with the same number of lateral umbels.

In Table II, 3836 lateral umbels of 515 plants have been divided into groups, each of which has been placed in a horizontal row and contains all the lateral umbels of plants with the same number of rays in the terminal umbels.

The mean number of umbel rays of the lateral umbels is greater than that of the terminal umbels. For that reason terminal and lateral umbels are considered separately.

In Table III the lateral umbels are divided into groups in such a way that all the lateral umbels of plants with the same number of lateral umbels are arranged in one horizontal row.

The distribution of the number of umbel rays of the 3836 lateral umbels is represented graphically in Fig. 3.

The distribution, which is given in the first and the last column of Table III and which gives the number of lateral umbels per plant, has not been studied in detail because it is of no importance for this investigation. The constants $\beta_1 = 0{\cdot}144$ and $\beta_2 = 3{\cdot}485$, the "critical function" $= 0{\cdot}208$ and the curve which will fit best to this distribution is a Pearson curve of Type IV.

TABLE I.

Variation of the number of umbel rays of the terminal umbel of Pastinaca sativa.

Number of umbel rays of the terminal umbel	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	Totals
Number of terminal umbels, Delft	3	11	31	53	49	**55**	42	**53**	23	10	4	2	1	3	—	1	—	—	—	—	—	341
Number of terminal umbels, Lisse	2	14	29	34	**59**	53	40	32	22	12	**13**	9	7	2	3	2	—	—	1	—	1	335
Number of terminal umbels, Totals	5	25	60	87	**108**	**108**	82	**85**	45	22	17	11	8	5	3	3	—	—	1	—	1	676

Mean of umbel rays for 676 terminal umbels = 11·325.
S.D. of umbel rays for 676 terminal umbels = 2·746.

TABLE II.

Correlation of the number of umbel rays of terminal umbels and lateral umbels of Pastinaca sativa.

Number of umbel rays of the terminal umbel.	Number of lateral umbels with umbel rays																								Total number of lateral umbels	Number of plants
	5	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*		
6	—	1	3	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6	2
7	—	4	4	26	27	**38**	14	6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	119	21
8	2	5	13	39	46	**58**	49	20	20	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	256	43
9	—	9	10	36	57	**76**	73	55	32	7	—	1	—	—	—	—	—	—	—	—	—	—	—	—	356	56
10	1	—	5	36	50	109	**152**	110	101	34	10	4	2	—	—	—	—	—	—	—	—	—	—	—	614	89
11	—	—	3	16	43	85	**147**	136	111	55	27	10	2	**5**	—	—	—	—	—	—	—	—	—	—	640	87
12	—	—	—	3	19	65	79	94	**104**	59	34	17	9	7	1	—	—	—	—	—	—	—	—	—	491	65
13	—	1	2	**12**	10	26	49	64	**87**	79	**83**	38	19	11	—	4	—	—	—	—	—	—	—	—	485	59
14	—	—	—	—	1	4	14	27	44	**54**	49	**52**	21	6	4	—	—	—	—	—	—	—	—	—	276	30
15	—	—	—	1	2	**17**	11	14	21	**26**	**26**	20	18	12	6	1	—	—	—	—	—	—	—	—	175	20
16	—	—	—	—	—	—	1	3	5	9	**28**	18	20	**30**	19	7	1	1	—	—	—	—	—	—	142	15
17	—	—	—	—	—	—	—	2	4	9	3	12	13	17	12	4	6	4	3	1	—	—	—	—	90	9
18	—	—	—	—	—	—	2	3	3	5	7	8	5	10	8	12	7	3	5	3	—	—	—	—	81	7
19	—	—	—	—	—	—	—	—	—	1	2	8	4	10	5	4	2	3	1	1	—	—	—	—	41	4
20	—	—	—	—	—	—	—	—	1	4	4	5	6	6	1	—	—	—	—	—	—	—	—	—	27	3
21	—	—	—	—	—	—	—	—	3	2	2	2	1	—	1	5	2	4	2	—	—	—	—	—	24	3
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
24	—	—	—	—	—	—	—	—	1	—	—	2	—	1	1	—	—	—	—	—	—	—	—	—	5	1
25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	4	—	2	1	8	1
Totals	3	20	40	171	255	478	**591**	534	**537**	348	275	197	120	115	58	37	18	15	11	6	4	—	2	1	3836	515

Mean of umbel rays for 515 lateral umbels = 12·549.
S.D. of umbel rays for 515 lateral umbels = 3·007.
Coefficient of correlation = 0·7269.

TABLE III.

Correlation of the number of lateral umbels per plant and the number of umbel rays of lateral umbels of Pastinaca sativa.

Number of lateral umbels per plant.	Number of lateral umbels with umbel rays																								Total number of lateral umbels	Number of plants
	5	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*		
2	—	—	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4	2
3	—	—	3	11	8	4	3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	30	10
4	—	6	9	17	24	22	22	7	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	108	27
5	—	5	6	27	41	**80**	49	24	14	5	—	4	—	1	1	—	—	—	—	—	—	—	—	—	257	52
6	—	6	9	38	65	**102**	**102**	60	42	14	4	—	—	—	—	—	—	—	—	—	—	—	—	—	442	74
7	2	2	8	45	64	109	**127**	108	90	49	43	23	12	7	1	—	—	—	—	—	—	—	—	—	690	99
8	1	—	2	15	33	79	106	155	**160**	88	65	42	17	13	8	1	—	—	—	1	4	—	2	1	793	100
9	—	—	—	8	11	53	102	92	**120**	79	60	37	33	31	8	**12**	2	4	2	—	—	—	—	—	654	73
10	—	1	2	7	—	8	36	43	**59**	**60**	37	27	17	**30**	12	4	3	2	—	—	—	—	—	—	348	35
11	—	—	—	1	5	14	**24**	22	**25**	11	**26**	18	18	**22**	17	7	3	4	1	1	—	—	—	—	219	20
12	—	—	—	—	2	6	**17**	14	15	**31**	25	**31**	14	3	2	**7**	1	—	—	—	—	—	—	—	168	14
13	—	—	—	—	—	—	1	2	1	—	5	4	2	2	4	5	9	5	8	4	—	—	—	—	52	4
14	—	—	—	—	1	1	2	2	3	2	7	6	3	—	—	—	—	—	—	—	—	—	—	—	27	2
15	—	—	—	—	—	—	—	4	7	9	3	5	4	6	5	1	—	—	—	—	—	—	—	—	44	3
Totals	3	20	40	171	255	478	**591**	534	**537**	348	275	197	120	115	58	37	18	15	11	6	4	—	2	1	3836	515

If we consider the different distributions in the horizontal rows of Tables II and III in which the maximum numbers are printed in heavy figures, we become convinced that besides the numbers of the principal series such as 8 and 13 (and multiples of numbers of this series such as 15 and 16), also numbers of the first parallel series such as 11 and 18 (and perhaps multiples of numbers of that series, 14 for example) are not infrequent.

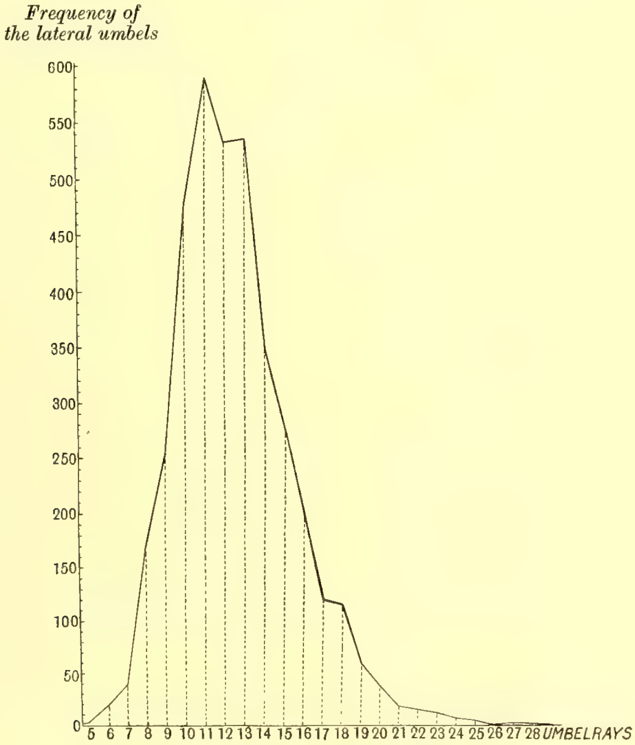

Fig. 3. *Pastinaca sativa.*

It is certainly very remarkable that with the increasing number of rays of the terminal umbel or with the increasing number of lateral umbels per plant, the lateral umbels that occur as maxima in a horizontal distribution possess a number of umbel rays that leaps up from 11 in one to 13 in another horizontal distribution; in other words:

The number of umbel rays 12 does not occur as a maximum in any of the horizontal groups of the two correlation tables.

(3) COUNTINGS OF *ANTHRISCUS SYLVESTRIS.*

Variation of the number of umbel rays of the terminal umbel.

A little preliminary investigation showed that the number of umbel rays of the terminal umbel of *Anthriscus sylvestris* is very small and that in most cases this umbel consists of 1, 2, 3 or 4 umbel rays and in some cases of 5, or is entirely missing.

Afterwards four countings were made:

(*a*) On 502 plants in the neighbourhood of Beverwijk on May 16th, 1912.

(*b*) On 305 plants in the neighbourhood of Lisse on May 26th and July 16th, 1912.

(*c*) On 77 plants partly in the neighbourhood of Lisse, partly in that of Bentveld, on July 6th, July 19th and July 27th, 1912.

(*d*) On 46 plants from the Schapenweg in the neighbourhood of Rijswijk on May 19th, 1912.

The result of these countings of the number of umbel rays of 930 terminal umbels is given in the following table.

TABLE IV.

Variation of the number of umbel rays of the terminal umbel of Anthriscus sylvestris.

Number of umbel rays of the terminal umbel	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	Totals
Number of terminal umbels. Group (*a*)	9	107	**196**	143	40	4	1	1	—	1	—	502
Number of terminal umbels. Group (*b*)	16	107	**112**	49	14	3	—	3	—	—	1	305
Number of terminal umbels. Group (*c*)	1	**26**	23	23	1	—	2	1	—	—	—	77
Number of terminal umbels. Group (*d*)	—	13	**15**	13	4	1	—	—	—	—	—	46
Number of terminal umbels. Totals	26	253	**346**	228	59	8	3	5	—	1	1	930

Mean of umbel rays for 930 terminal umbels = 2·125.
S.D. of umbel rays for 930 terminal umbels = 1·119.

The graphical representation in Fig. 4 of the distribution in the last row of Table IV is an asymmetrical polygon with mode at 2 umbel rays. The point which is obtained by taking the terminal umbels with 0 umbel rays into account accords with the one that we can expect when we know only the other points and causes the variation polygon to become rather symmetrical with regard to the vertical through the mode.

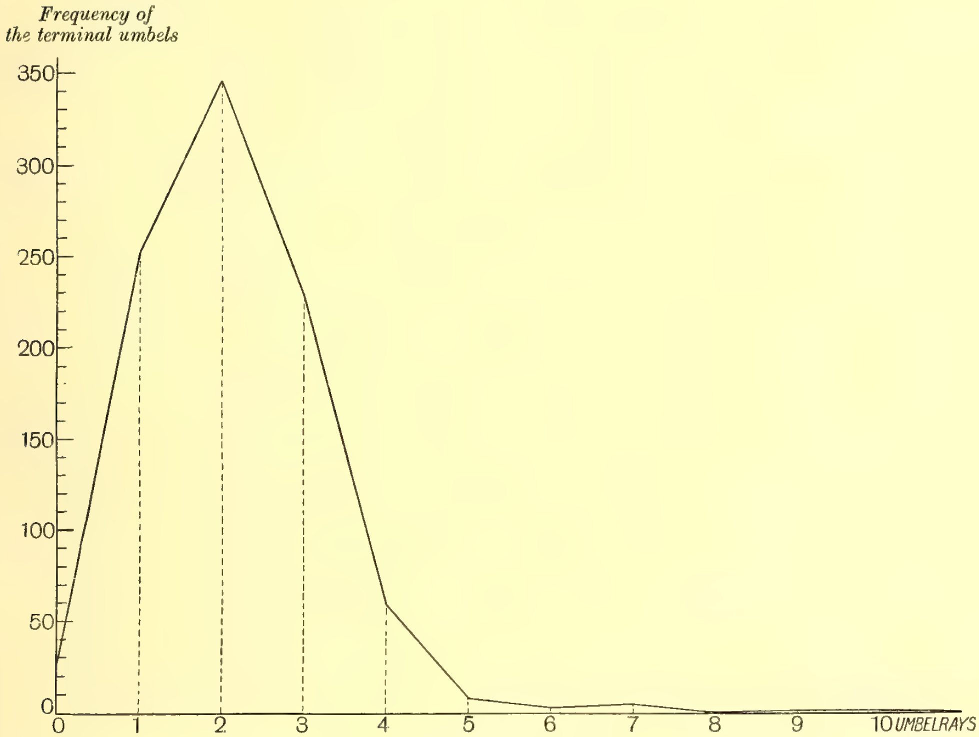

FIG. 4. *Anthriscus sylvestris.*

Variation of the number of umbel rays of the lateral umbels of the first order and correlation of the number of umbel rays of terminal umbels and that of lateral umbels.

Of 235 plants (being the same series for which the rays of the terminal umbels were counted and which were included in the preceding distributions) the number of umbel rays of the lateral umbels was determined.

Again these lateral umbels were divided into groups, in the first place into groups belonging to plants with the same number of rays in the terminal umbel, and in the second place into groups belonging to plants with the same number of lateral umbels.

In the following table have been assembled the results of the countings on 1632 lateral umbels of these 235 plants in which the first sub-division was taken. It is easy to see that the numbers 8 and 10 are predominant.

TABLE V. *Correlation of the number of umbel rays of terminal umbels and lateral umbels of* Anthriscus sylvestris.

Number of umbel rays of the terminal umbel.	Number of lateral umbels with umbel rays 3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	Total number of lateral umbels	Number of plants
0	—	2	6	5	11	19	13	13	2	—	—	—	—	—	—	—	—	71	9
1	2	23	27	37	55	**135**	119	107	44	17	1	—	—	—	—	—	—	567	77
2	3	5	15	22	43	**108**	97	**127**	73	49	15	5	—	—	—	—	—	562	82
3	1	3	6	7	19	30	45	**82**	68	30	24	7	2	3	—	—	—	327	50
4	—	1	—	—	4	2	3	8	17	7	4	3	1	—	—	—	—	50	9
5	1	1	—	—	2	5	—	2	5	1	1	2	1	—	1	—	—	22	3
6	—	1	—	3	2	3	1	1	1	—	1	—	—	—	—	—	—	13	2
7	—	—	1	1	1	2	1	4	2	1	1	1	1	1	1	1	1	20	3
Totals	7	36	55	75	137	**304**	279	**344**	212	105	47	18	5	4	2	1	1	1632	235

Mean of umbel rays for 235 lateral umbels = 9·127.
S.D. of umbel rays for 235 lateral umbels = 2·150. Coefficient of correlation = 0·3033.

The mean number of umbel rays of the lateral umbels is much greater than the mean number of rays of the terminal umbels. For that reason terminal and lateral umbels were considered separately. This great difference of the average numbers to a certain degree accounts for the fact that the coefficient of correlation is low, much lower than with *Pastinaca sativa.*

The second sub-division of the lateral umbels in groups belonging to plants with the same number of lateral umbels gives the following result:

TABLE VI. *Correlation of the number of lateral umbels per plant and the number of umbel rays of lateral umbels of* Anthriscus sylvestris.

Number of lateral umbels per plant.	Number of lateral umbels with umbel rays 3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	Total number of lateral umbels	Number of plants
2	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1
3	—	—	—	—	2	1	—	—	—	—	—	—	—	—	—	—	—	3	1
4	—	—	—	—	—	—	—	1	3	2	—	2	—	—	—	—	—	8	2
5	—	—	1	2	8	19	22	**24**	23	6	5	—	1	1	1	1	1	115	23
6	—	3	5	6	28	56	61	**74**	48	34	18	6	2	—	1	—	—	342	57
7	1	11	22	27	41	**125**	98	**119**	69	30	8	1	—	—	—	—	—	552	79
8	4	10	12	23	36	**55**	54	**75**	47	22	15	9	2	3	—	—	—	367	46
9	—	8	11	10	15	32	36	**43**	17	7	1	—	—	—	—	—	—	180	20
10	2	3	2	4	6	**15**	7	8	3	—	—	—	—	—	—	—	—	50	5
11	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
13	—	1	2	1	1	1	1	—	2	4	—	—	—	—	—	—	—	13	1
Totals	7	36	55	75	137	**304**	279	**344**	212	105	47	18	5	4	2	1	1	1632	235

In this table also the numbers 8 and 10 are conspicuous just as in the subdivisions not reproduced here of each horizontal row in the above table, which were made according to the number of rays of the terminal umbel.

It is remarkable that there is not a single case of *Anthriscus sylvestris* in which a number of 9 umbel rays in a horizontal row of one of the Tables V and VI occurs as a maximum, whereas the numbers 8 and 10 not infrequently occur as modes.

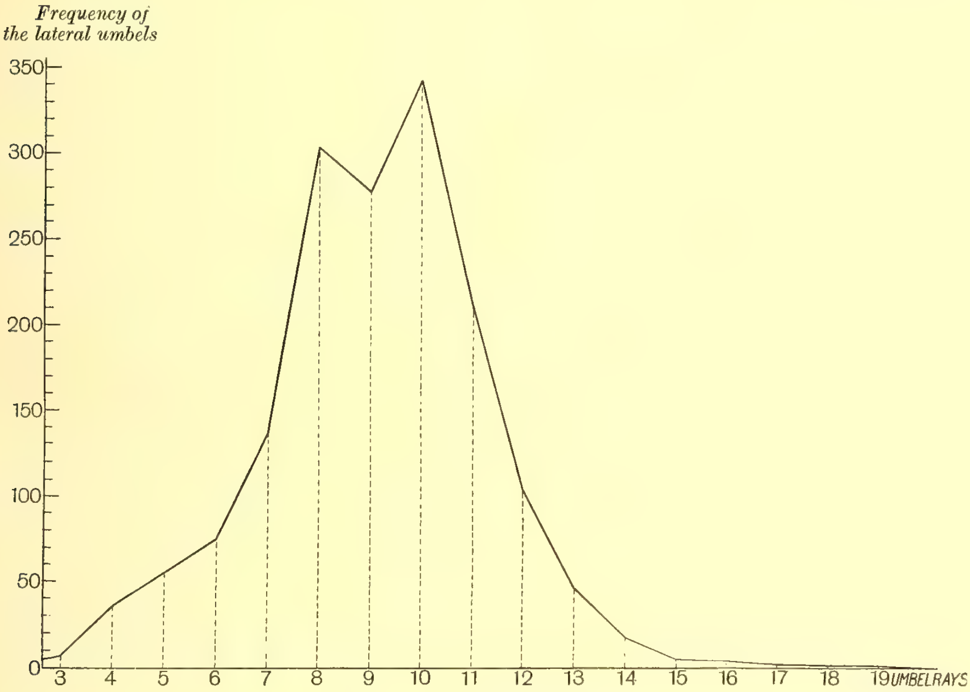

FIG. 5. *Anthriscus sylvestris.*

Figure 5 gives a graphical representation of the variability of the number of umbel rays of these 1632 lateral umbels.

(4) COUNTINGS OF *SIUM LATIFOLIUM.*

Variation of the number of umbel rays of the terminal umbel.

The number of plants examined of *Sium latifolium* was much smaller than the number of plants of the preceding species, which were submitted to examination. Only in the case of 127 plants were the umbel rays of terminal umbels and lateral umbels counted, and of one only of the umbel rays of the lateral umbels, because the terminal umbel was missing.

The distribution of the numbers of umbel rays of 127 terminal umbels was as follows:

TABLE VII.

Variation of the number of umbel rays of the terminal umbel of Sium latifolium.

Number of umbel rays of the terminal umbel	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	Total
Number of terminal umbels	3	4	3	6	6	1	6	8	3	5	7	10	4	4	6	9	10	9	8	5	4	4	1	1	127

Mean of umbel rays for 127 terminal umbels = 25·638.
S.D. of umbel rays for 127 terminal umbels = 5·948.

Conclusions could not be drawn from this distribution, the number of variates being too small with respect to the great range of variation.

Variation of the number of umbel rays of the lateral umbels.

The lateral umbels again were sub-divided into groups belonging to plants with the same number of rays in the terminal umbels and into groups belonging to plants with the same number of lateral umbels. As the former groups were very small, they can give little or no information about preference for certain numbers. The latter groups have been united in Table VIII.

Though the small number of umbels, of which the rays were counted, and the great range of variability make it necessary to be prudent in drawing conclusions, we think it probable that not only the numbers of the principal series are of importance in this case, but also other numbers as for instance 18 and 29.

Correlations.

The coefficient of correlation for the number of rays of terminal and lateral umbels was as follows:

For *Pastinaca sativa*	0·7269 ± 0·0051.
,, *Anthriscus sylvestris* ...	0·3033 ± 0·0152.
,, *Sium latifolium*	0·8693 ± 0·0070.

It is remarkable that this coefficient of correlation is different for these three species of plants, and it would be worth examining if this coefficient has a constant value within certain limits for different local races of the same species of plants. In that case a number would have been found, characteristic of each particular species.

TABLE VIII.

Correlation of the number of lateral umbels per plant and the number of umbel rays of lateral umbels of Sium latifolium.

Number of lateral umbels per plant.	Number of lateral umbels with umbel rays																														Total number of lateral umbels	Number of plants
	13	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*		
2	1	4	0	1	0	0	1	2	0	1	0	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	11	6
3	2	1	0	4	2	3	1	4	5	3	3	3	4	2	1	2	6	4	2	1	—	—	—	—	—	—	—	—	—	—	53	19
4	—	—	—	3	3	8	6	3	10	11	7	4	9	5	7	3	2	1	6	3	5	3	1	2	1	—	—	1	—	—	104	27
5	—	—	—	—	4	2	0	3	5	4	9	11	6	4	9	6	7	4	12	15	6	8	12	8	5	3	3	1	1	—	148	33
6	—	—	—	1	0	1	0	2	1	2	2	5	4	6	5	5	9	3	5	14	19	21	16	10	12	13	10	7	4	—	177	32
7	—	—	—	—	—	—	—	2	3	—	—	—	1	1	—	5	0	0	2	2	6	3	5	8	13	4	4	8	1	1	69	11
Totals	3	5	0	9	9	14	8	16	24	21	21	24	24	18	22	21	24	12	27	35	36	35	34	28	31	20	17	17	6	1	562	128

Mean of umbel rays for 562 lateral umbels = 29·416.
S.D. of umbel rays for 562 lateral umbels = 6·782.
Coefficient of correlation = 0·8693.

Conclusions.

The result of this investigation is that even in relatively small groups of sufficiently homogeneous material of *Pastinaca sativa* numbers (or multiples of numbers) of the principal series, or numbers (or multiples of numbers) of the first parallel series, or both, occur as maxima in distributions of the number of rays of terminal and of lateral umbels.

The maximum numbers of umbel rays that occur in the horizontal rows of the correlation tables do not shift regularly from top to bottom, and in the case of *Pastinaca sativa* the number of 12 rays does not occur as a maximum in the groups of which the plants have the same number of rays in the terminal umbel or in those of which the plants have the same number of lateral umbels; on the contrary the numbers 11 and 13 occur as maxima.

It is therefore not probable that the numbers of the first principal series should be caused by the mixing of different local races of plants, in each of which numbers of the principal series occur as maxima, as Ludwig thinks is the case with other Umbelliferae. It is more probable that the numbers of the first parallel series must be considered as characteristic of the variation of many species of Umbelliferae.

ON THE PARTIAL CORRELATION-RATIO

PART II. NUMERICAL

Being Part II of a Thesis for the degree of D.Sc. in the University of London

By L. ISSERLIS, B.A.

1. The object of the present paper is to provide numerical illustrations of some of the results in the first part*. Suitable material is not easily obtained, as many tables dealing with three variable characters are artificially curtailed as regards one or more of them. I am indebted to Miss Ethel M. Elderton for having kindly placed at my disposal manuscript tables of the heights to the nearest inch and weights to the nearest pound of the boys in group A (comprising schools in the poorest parts of the city) of her investigations into the heights and weights of school children in Glasgow†.

I have arranged the material in tables which differ from Tables I–IX of Miss Elderton's paper in the following respects:

(i) A *uniform* grouping of 5 lb. intervals for weight and 3 inch intervals for height is adopted at all ages;

(ii) 384 boys of central age 5 (i.e. ages 4·5 to 5·5 years), omitted in Miss Elderton's tables, are here included, and two boys (age 6·5 to 7·5 years) whose height was below 21·5 inches, included in her tables, are omitted here.

These tables are cut off artificially by the regulations of the School Board with regard to school leaving age and the admission age of infants.

Tables I–X contain the data, the frequency distributions of height and weight being given for each of the ages from 5 to 14 of the 11,382 boys.

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* "On the Partial Correlation Ratio," Part I, Theoretical, by L. Isserlis. *Biometrika*, Vol. x, 1914, pp. 391–411. (The title of the first part was badly chosen, the generalized H there introduced is a multiple one, providing a measure of the dependence of one variable on two or more other variables. An example of a partial η would be ${}_x\eta_{z\cdot y}$, being the correlation ratio of z on y for a given value of x. Pearson has recently published very simple formulae connecting the partial correlation ratio with the multiple correlation ratio: see *R. S. Proc.* Vol. 91, A, pp. 492–8. They suffice to show that a knowledge of the multiple correlation ratios leads at once to the partial correlation ratios, and to that extent justify my title.)

† "Height and Weight of School Children in Glasgow," by Ethel M. Elderton. *Biometrika*, Vol. x, 1914, pp. 288–339.

Tables XI, XII, XIII give the distributions of height and weight, height and age, and age and weight for the whole of the individuals in the preceding ten tables. They are summaries. The *direct* calculation of one of the three generalized H's, say that of age on height and weight, requires the use of Tables I–X, while the various approximations given in the paper cited can be obtained from the summaries in Tables XI–XIII.

I have calculated the correlation ratio of age on height and weight, and that of weight on age and height directly, and compared them with the values obtained as approximations from the summaries.

The direct calculation is laborious and the results show that the various approximation formulae are in general sufficiently accurate to enable us to dispense with this heavy labour.

Throughout the present paper, I denote the age by 'z' with an arbitrary origin at 10 years, the height by 'x' with the origin at 49 inches and the weight by 'y' with origin at 56 lbs.

2. With the arbitrary origin at 49 inches, the raw moments for the height frequencies are:

$$\begin{aligned} \bar{x} = p'_x &= -0{\cdot}511861^*, \\ p'_{x^2} &= 3{\cdot}453699, \\ p'_{x^3} &= -4{\cdot}938324, \\ p'_{x^4} &= 30{\cdot}678791. \end{aligned}$$

Thus the mean height is $49 - 3\,(0{\cdot}511861) = 47{\cdot}46$ inches.

The moments referred to mean are given by the formulae:

$$p_{x^2} = p'_{x^2} - (p'_x)^2 = 3{\cdot}191697,$$

$$p_{x^3} = p'_{x^3} - 3p'_{x^2}p'_x + 2p'^3_x = 0{\cdot}096908,$$

$$p_{x^4} = p'_{x^4} - 4p'_{x^3}p'_x + 6p'_{x^2}p'^2_x - 3p'^4_x = 25{\cdot}7912.$$

From these $${}_x\beta_1 = \frac{p_{x^3}^2}{p_{x^2}^3} = {\cdot}0003,$$

$${}_x\beta_2 = \frac{p_{x^4}}{p_{x^2}^2} = 2{\cdot}5318.$$

The standard deviation is $\sigma_x = 1{\cdot}78653$ in 3 inch units, or 5·36 inches.

For weights the arbitrary origin is at 56 lbs., and the raw moments are:

$$\begin{aligned} \bar{y} = p'_y &= -0{\cdot}205412, \\ p'_{y^2} &= 6{\cdot}772096, \\ p'_{y^3} &= 5{\cdot}979090, \\ p'_{y^4} &= 134{\cdot}637146. \end{aligned}$$

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* For the sake of uniformity the constants in all *intermediate* results are given to six places of decimals.

So that the mean weight is 56 – 5 (0·205412) = 55·97 lbs., and the moments referred to the mean are:

$$\begin{aligned} p_{y^2} &= 6{\cdot}729902, \\ p_{y^3} &= 10{\cdot}134965, \\ p_{y^4} &= 141{\cdot}258877, \end{aligned}$$

giving $\quad {}_y\beta_1 = {\cdot}3370, \qquad {}_y\beta_2 = 3{\cdot}1189.$

The standard deviation is $\sigma_y = 2{\cdot}594205$ in 5 lb. units, or 12·97 lbs.

With arbitrary origin at 10 years, the raw moments of the ages are:

$$\begin{aligned} \bar{z} = p'_z &= -0{\cdot}568617, \\ p'_{z^2} &= 6{\cdot}402214, \\ p'_{z^3} &= -9{\cdot}423124, \\ p'_{z^4} &= 80{\cdot}916710. \end{aligned}$$

The mean age is therefore 10 – 0·568617 = 9·43 years, and the moments about the mean age are:

$$\begin{aligned} p_{z^2} &= 6{\cdot}078889, \\ p_{z^3} &= 1{\cdot}130401, \\ p_{z^4} &= 71{\cdot}591474, \end{aligned}$$

leading to $\quad {}_z\beta_1 = {\cdot}0057, \qquad {}_z\beta_2 = 1{\cdot}9374,$

while the standard deviation is $\sigma_z = 2{\cdot}47$ years.

3. The mixed moments p_{xy}, p_{xy^2}, etc., can be found from the correlation tables XI–XIII. The values of the raw moments required are as follows:

$$\begin{aligned} p'_{xy} &= 4{\cdot}311017, & p'_{xz} &= 3{\cdot}949042, \\ p'_{xy^2} &= -1{\cdot}066245, & p'_{xz^2} &= -7{\cdot}108065, \\ p'_{xy^3} &= 74{\cdot}045335, & p'_{xz^3} &= 50{\cdot}313120, \\ p'_{x^2y} &= -3{\cdot}712880, & p'_{x^2z} &= -5{\cdot}488842, \\ p'_{x^3y} &= 35{\cdot}657353, & p'_{x^3z} &= 24{\cdot}225619, \end{aligned}$$

$$p'_{zy} = 5{\cdot}430241, \quad p'_{x^2y^2} = 47{\cdot}927956, \quad p'_{xyz} = -4{\cdot}341768, \quad p'_{x^2z^2} = 37{\cdot}510631.$$

The corresponding moments about the mean are:

$$\begin{aligned} p_{xy} &= p'_{xy} - \bar{x}\bar{y} = 4{\cdot}205875, \\ p_{yz} &= 5{\cdot}313440, \\ p_{zx} &= 3{\cdot}657989, \\ p_{xy^2} &= p'_{xy^2} - 2\bar{y}p'_{xy} - \bar{x}p'_{y^2} + 2\bar{x}\bar{y}^2 = 4{\cdot}128088, \\ p_{xy^3} &= p'_{xy^3} - 3\bar{y}p'_{xy^2} + 3\bar{y}^2p'_{xy} - \bar{x}p'_{y^3} + 3\bar{x}\bar{y}p'_{y^2} = 79{\cdot}144020, \\ p_{x^2y^2} &= p'_{x^2y^2} - 2\bar{y}p'_{x^2y} - 2\bar{x}p'_{xy^2} + \bar{y}^2p'_{x^2} + \bar{x}^2p'_{y^2} + 4\bar{x}\bar{y}p'_{xy} - 3\bar{x}^2\bar{y}^2 \\ &= 49{\cdot}011115, \\ p_{xyz} &= p'_{xyz} - \bar{x}\bar{y}\bar{z} - \bar{x}p_{yz} - \bar{y}p_{zx} - \bar{z}p_{xy} = 1{\cdot}580733. \end{aligned}$$

Similar formulae give:

$$\begin{aligned} p_{x^2y} &= 1{\cdot}302279, & p_{x^2z} &= {\cdot}219749, \\ p_{x^3y} &= 33{\cdot}336761, & p_{x^3z} &= 19{\cdot}108627, \\ p_{xz^2} &= {\cdot}328967, & p_{xz^3} &= 42{\cdot}785163. \end{aligned}$$

4. Further we take: $$q_{x^l y^m z^n} = \frac{p_{x^l y^m z^n}}{\sigma_{x^l}\sigma_{y^m}\sigma_{z^n}}.$$

We now collect into a single table the values of the reduced moments, the correlation coefficients and the correlation ratios.

TABLE XIV.

q_{x^2y}	·157282	r_{xy}	·907489	—	—
q_{xy^2}	·343344	${}_y\eta_x$	·918157	${}_y\eta^2_x - r^2_{xy}$	·019478
$q_{x^2y^2}$	2·281729	${}_x\eta_y$	·915430	${}_x\eta^2_y - r^2_{xy}$	·014476
q_{x^3y}	2·253653	—	—	—	—
q_{xy^3}	2·537432	r_{yz}	·830729	—	—
q_{x^3}	·016995	${}_y\eta_z$	·851196	${}_y\eta^2_z - r^2_{zy}$	·034424
q_{x^4}	2·531793	${}_z\eta_y$	·833060	${}_z\eta^2_y - r^2_{yz}$	·003878
q_{y^3}	·580509	—	—	—	—
q_{y^4}	3·118877	r_{zx}	·830463	—	—
q_{x^2z}	·027925	${}_x\eta_z$	·844792	${}_x\eta^2_z - r^2_{zx}$	·024005
q_{xz^2}	·030294	${}_z\eta_x$	·831447	${}_z\eta^2_x - r^2_{zx}$	·001635
q_{xz^3}	1·597889	—	—	—	—
q_{x^3z}	1·359207	—	—	—	—
$q_{x^2z^2}$	1·605274	—	—	—	—
q_{xyz}	·138335	—	—	—	—

It will be observed that the differences ${}_z\eta^2_y - r^2_{yz}$ and ${}_z\eta^2_x - r^2_{xz}$ are very small, showing that the regression lines of weight on age, and of height on age are nearly straight. The six empirical regression lines are given in Figs. 1–6.

FIG. 1. Regression of Age on Weight.

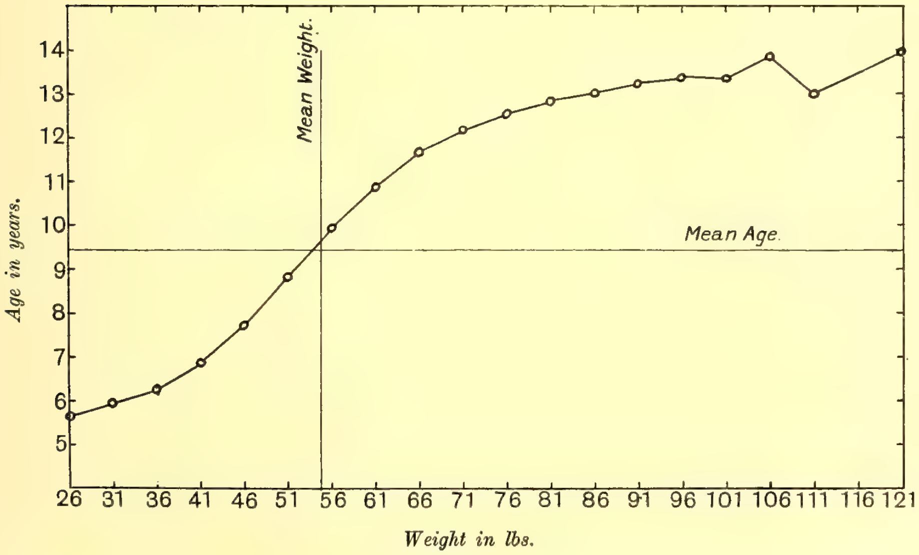


Fig. 2. Regression of Weight on Age.

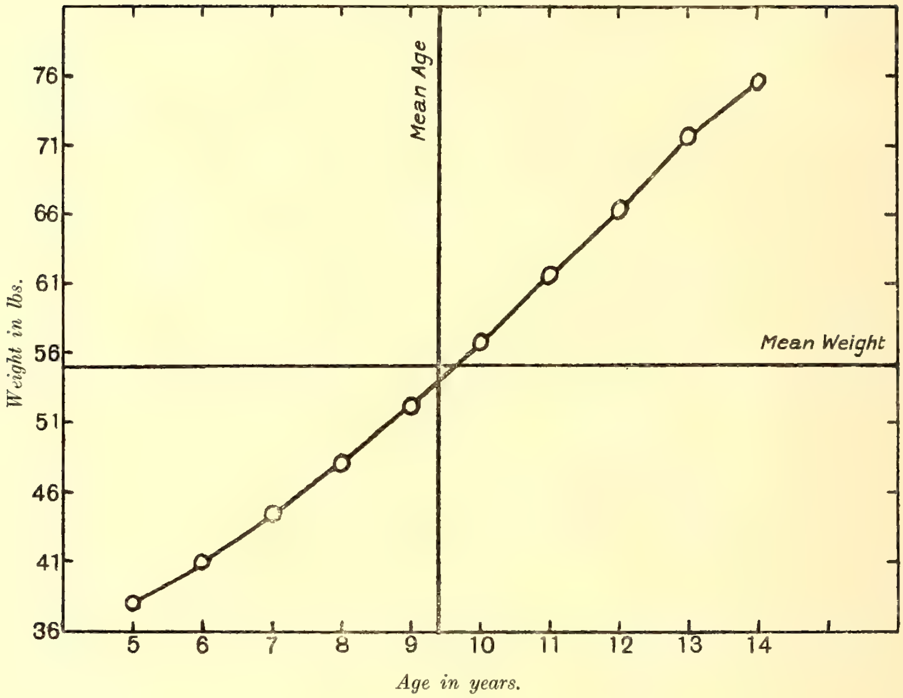


Fig. 3. Regression of Height on Age.

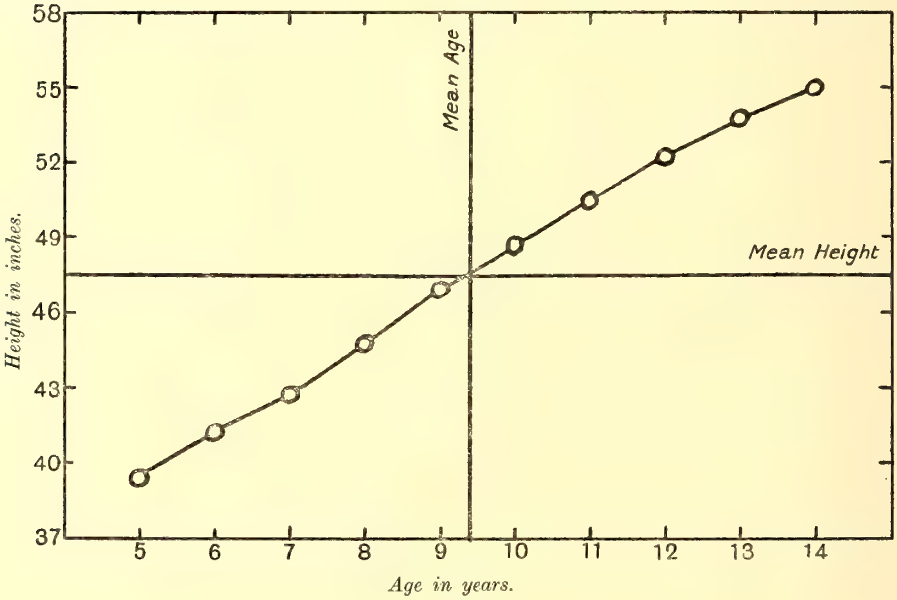


Fig. 4. Regression of Age on Height.

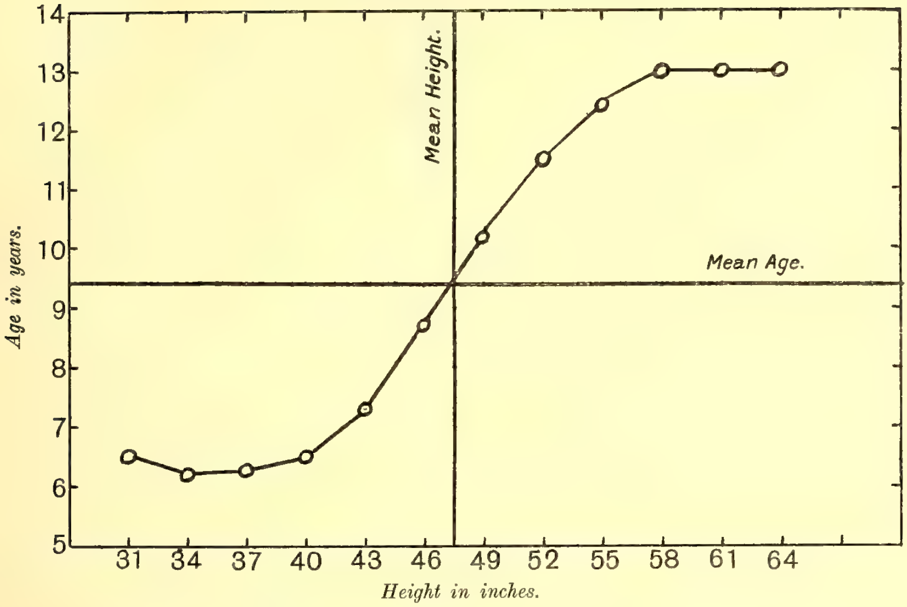


Fig. 5. Regression of Weight on Height.

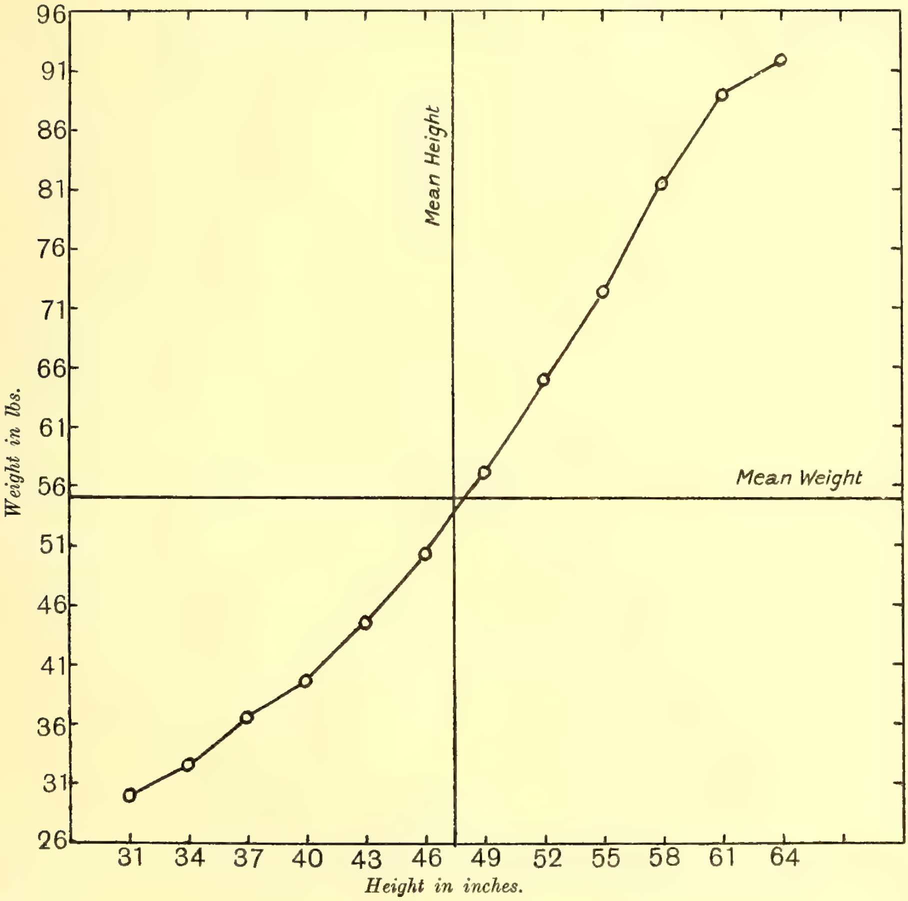
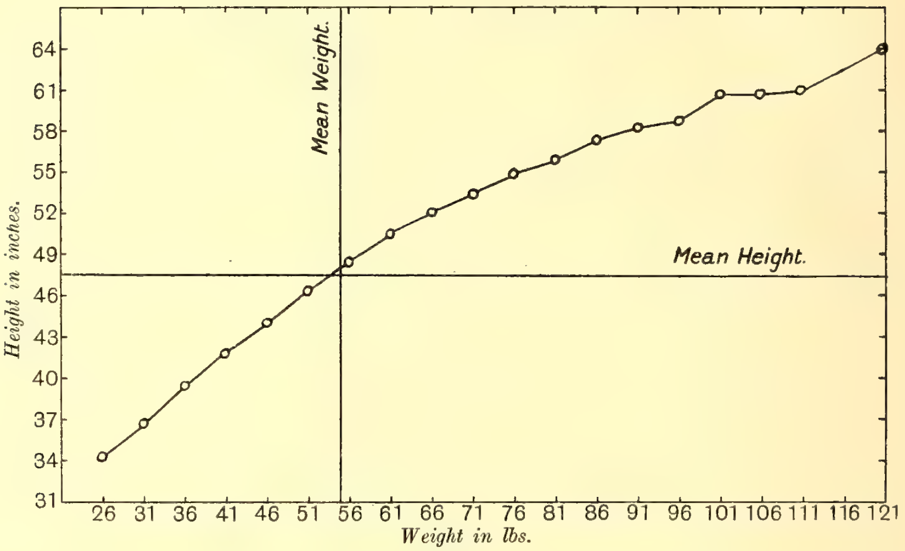


Fig. 6. Regression of Height on Weight.



5. The mean ages of the x, y arrays are given in Table XV. From this table ${}_{xy}H^2{}_z$ is calculated directly by the formula:

$${}_{xy}H^2{}_z = SS\frac{\{n_{xy}(\bar{z} - z_{xy})^2\}}{N\sigma_z{}^2}$$
$$= {\cdot}7519,$$

or

$${}_{xy}H_z = {\cdot}8671.$$

Also

$${}_{xy}R^2{}_z = \frac{r^2{}_{yz} + r^2{}_{zx} - 2r_{yz}r_{zx}r_{xy}}{1 - r^2{}_{xy}}$$
$$= {\cdot}7233,$$

and

$${}_{xy}R_z = {\cdot}8505.$$

We see that the regression of age on height and weight is very far from planar. The difference

$${}_{xy}H^2{}_z - {}_{xy}R^2{}_z = {\cdot}0286.$$

We have at present* no formula for the probable error of the multiple correlation ratio, but some idea of its magnitude may be obtained from Blakeman's formula† for the probable error of ${}_x\eta^2{}_y - r^2{}_{xy}$:

$$\frac{{}_x\eta^2{}_y - r^2{}_{xy}}{\text{Probable error of } ({}_x\eta^2{}_y - r^2{}_{xy})} = \frac{\sqrt{N} \cdot \frac{1}{2} \cdot \sqrt{\eta^2 - r^2}}{67449\sqrt{1 + (1 - \eta^2)^2 - (1 - r^2)^2}}.$$

---

* Since writing the above, the author has obtained a formula for the p.e. of ${}_{xy}H_z$. It is very similar to Pearson's well known formula for the p.e. of the simple η.

† J. Blakeman. *Biometrika*, Vol. IV, p. 332.

With the above values of ${}_{xy}H^2{}_z$ and ${}_{xy}R^2{}_z$, the right-hand side of Blakeman's formula reduces to 13·47, so that on this assumption ${}_{xy}H^2{}_z - {}_{xy}R^2{}_z$ is 13 or 14 times greater than its probable error.

If we examine the regression of weight on height and age, we find in the same way that:

$$ {}_{zx}H^2{}_y = \cdot 8634, \qquad {}_{zx}H_y = \cdot 9292, $$
$$ {}_{zx}R^2{}_y = \cdot 8427, \qquad {}_{zx}R_y = \cdot 9180, $$
$$ {}_{zx}H^2{}_y - {}_{zx}R^2{}_y = \cdot 0207. $$

In this case too the regression is significantly non-planar, Blakeman's formula reducing to 11·43 so that ${}_{zx}H^2{}_y - {}_{zx}R^2{}_y$ is more than 11 times greater than its probable error.

Physically ${}_{zx}H_y$ is more interesting than ${}_{xy}H_z$. A knowledge of the former enables us to construct by means of equation (53) of my Part I, a formula predicting the mean weight of a group of boys of given age and height. Such a formula would be an alternative solution to the one given by Table α on p. 300 of the paper by Miss Elderton referred to above.

6. We may consider now equation (70) of Part I which gives the approximate age of ${}_{xy}H^2{}_z - {}_{xy}R^2{}_z$ in terms of constants *all derivable* from Tables XI, XII, XIII, i.e. constants whose determination does not require a detailed knowledge of the distribution. The equation is:

$$ ({}_{xy}H^2{}_z - {}_{xy}R^2{}_z)\,\frac{q_{x^2y^2} - 1}{q_{x^2y^2} - r^2{}_{xy}} $$
$$ = \frac{r_{xy}q_{x^2y} - q_{xy^2}}{q_{x^2y}r_{xz} - q_{xy^2}r_{yz}} \cdot \frac{r_{yz} - r_{xz}r_{xy}}{1 - r^2{}_{xy}} \left[({}_{y}\eta^2{}_z - r^2{}_{zy}) - \left(\frac{r_{xz} - r_{yz}r_{xy}}{1 - r^2{}_{xy}}\right)^2 ({}_{y}\eta^2{}_x - r^2{}_{xy}) \right] $$
$$ + \frac{r_{xy}q_{xy^2} - q_{x^2y}}{q_{xy^2}r_{yz} - q_{x^2y}r_{xz}} \cdot \frac{r_{xz} - r_{yz}r_{xy}}{1 - r^2{}_{xy}} \left[({}_{x}\eta^2{}_z - r^2{}_{zx}) - \left(\frac{r_{yz} - r_{xz}r_{xy}}{1 - r^2{}_{xy}}\right)^2 ({}_{x}\eta^2{}_y - r^2{}_{xy}) \right] \quad \text{.........(70).} $$

When ${}_{x}\beta_1$, ${}_{y}\beta_1$* are very small and ${}_{y}\eta_x$, ${}_{x}\eta_y$ and r_{xy} are nearly equal, we use instead of (70) the following equations:

$$ {}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \frac{q_{x^2y^2} - r^2{}_{xy}}{q_{x^2y^2} - 1}\,({}_{y}\eta^2{}_z - r^2{}_{zy}) \quad \text{..................(71),} $$

$$ {}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \frac{q_{x^2y^2} - r^2{}_{xy}}{q_{x^2y^2} - 1}\,({}_{x}\eta^2{}_z - r^2{}_{zx}) \quad \text{..................(72),} $$

or approximately

$$ {}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \frac{1 + r^2{}_{xy}}{2r^2{}_{xy}}\,({}_{y}\eta^2{}_z - r^2{}_{zy}) \quad \text{.....................(71)}', $$

$$ {}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \frac{1 + r^2{}_{xy}}{2r^2{}_{xy}}\,({}_{x}\eta^2{}_z - r^2{}_{zx}) \quad \text{.....................(72)}'. $$

---

* ${}_{x}\beta_1$ is used to denote the β_1 of the frequencies of the x character.

For the regression of age on height and weight, the values of ${}_{xy}H^2_z - {}_{xy}R^2_z$ and of ${}_{xy}H^2_z$, given by the preceding equations, are tabulated below:

TABLE XVI.

Source	${}_{xy}H^2_z - {}_{xy}R^2_z$	${}_{xy}H^2_z$
(70)	·0303	·7537
(71)	·0392	·7625
(71)′	·0381	·7615
(72)	·0273	·7507
(72)′	·0266	·7499
Direct Calculation	·0286	·7519

We notice that (72) and (72)′ give much better values than (71) and (71)′. This is to be expected for ${}_{x}\beta_1$ is very small with a value of ·0003, while ${}_{y}\beta_1$ is quite appreciable with a value of ·3370, and since r_{xy}, ${}_{y}\eta_x$, ${}_{x}\eta_y$ are pretty nearly equal (cf. Table XIV), equations (72) and (72)′ may be used to replace (70).

Another point of interest is that there is very little to choose between the values given by the cumbersome formula (70) and those given by (72) and (72)′.

If we now consider the regression of weight on height and age, we find that equation (70) is unsuitable for the calculation of ${}_{zx}H^2_y - {}_{zx}R^2_y$.

For in this case

$$\begin{aligned} {}_{x}\beta_1 &= \cdot 0003, \\ {}_{z}\beta_1 &= \cdot 0057, \\ {}_{x}\eta_z &= \cdot 8448, \\ {}_{z}\eta_x &= \cdot 8314, \\ r_{zx} &= \cdot 8305, \end{aligned}$$

and we expect the right-hand side of (70) to be indeterminate (cf. Part I, p. 406). We find

$$\begin{aligned} r_{zx} q_{zx^2} - q_{z^2x} &= - \cdot 007104, \\ r_{zx} q_{z^2x} - q_{zx^2} &= - \cdot 002767, \end{aligned}$$

and

$$r_{zy} q_{z^2x} - r_{xy} q_{zx^2} = - \cdot 000175.$$

Thus the right-hand side of equation (70) depends on the ratio of quantities which practically vanish when their probable errors are taken into account.

If, nevertheless, (70) is employed, it gives ${}_{zx}H^2_y - {}_{zx}R^2_y = \cdot 1580$, and since ${}_{zx}R^2_y = \cdot 8427$, we get ${}_{zx}H^2_y = 1\cdot 0007$!, i.e. a value just greater than unity*.

The values of ${}_{zx}H^2_y$ obtained from (71) and (72) are tabulated below:

TABLE XVII.

Source	${}_{zx}H^2_y - {}_{zx}R^2_y$	${}_{zx}H^2_y$
(71)	·0054	·8481
(72)	·0203	·8630
(71)′	·0048	·8474
(72)′	·0177	·8604
Direct Calculation	·0207	·8634

---

* Of course by an amount $<$ p.e. of ${}_{zx}R_y$.

In this example also as ${}_x\beta_1$ is much nearer to zero than ${}_z\beta_1$ we get better values from (72) and (72)$'$ than from (71) and (71)$'$.

We may conclude from these numerical cases that, if

(i) ${}_x\beta_1$ and ${}_y\beta_1$ are fairly small, and

(ii) the regressions of y on x and that of x on y do not depart widely from linearity, we get very fair values for ${}_{xy}H^2{}_z$ from an equation of type

$$ {}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \frac{1 + r^2{}_{xy}}{2r^2{}_{xy}} ({}_y\eta^2{}_z - r^2{}_{zy}), $$

and a very good value from one of type

$$ {}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \frac{q_{x^2y^2} - r^2{}_{xy}}{q_{x^2y^2} - 1} ({}_y\eta^2{}_z - r^2{}_{zy}), $$

and that it is better to use the equation which corresponds to the smaller β_1 than to use the mean of the values given by the two equations of each type, i.e. if ${}_y\beta_1$ is smaller than ${}_x\beta_1$ we use the formula in terms of ${}_y\eta^2{}_z - r^2{}_{zy}$ in preference to the one involving ${}_x\eta^2{}_z - r^2{}_{zx}$ and in preference to the mean of the two.

If the conditions (i) and (ii) are not satisfied, we use equation (70), which requires more labour than (71) or (72), but is much easier than a direct calculation.

7*. It is a matter of interest to see whether the regression surface of age on height is of the type considered in Part I of this paper. The regression surface being

$$ \frac{\bar{z}_{xy} - \bar{z}}{\sigma_z} = d + \frac{a\,(x - \bar{x})}{\sigma_x} + \frac{b\,(y - \bar{y})}{\sigma_y} + \frac{c\,(x - \bar{x})\,(y - \bar{y})}{\sigma_x \sigma_y} + \frac{e\,(x - \bar{x})^2}{\sigma_x{}^2} + \frac{f\,(y - \bar{y})^2}{\sigma_y{}^2} \quad \text{...............(28).} $$

The solution of equations (29)–(34) is:

$d = \cdot 060374$, $c = - \cdot 0940467$,
$a = \cdot 360742$, $e = \cdot 072020$,
$b = \cdot 547960$, $f = - \cdot 040709$.

Here c is decidedly smaller than a and b, but e and f are of the same order as c. This seems to suggest that our formulae have a wider range than the type of surface by which they were suggested. If we solve equations (36) to (39) (Part I) in which the coefficients e and f are assumed to be zero, we obtain

$d = \cdot 060172$, $b = \cdot 512257$,
$a = \cdot 376024$, $c = - \cdot 066306$.

On substituting these values in (59) and (60), we find

$$ {}_{xy}H^2{}_z = \cdot 7470, $$

or

$$ {}_{xy}H^2{}_z = \cdot 7534. $$

Now equations (59) and (60) were based on the assumption that the arrays of y's for constant x and the arrays of x's for constant y are homoscedastic.

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* This section differs from the preceding—it does not deal with genuine approximation formulae—for in the determination of the constants a, b, c, d we use q_{xyz}, i.e. the *detailed table*. It illustrates some other points in Part I.

It is easy to remove this restriction. As on p. 402, we write

$$
\begin{aligned}
{}_{xy}H^2_z - {}_{y}\eta^2_z &= S_y \left\{\left(a + \frac{cy}{\sigma_y}\right)^2 \frac{\sigma^2_{x_y}}{\sigma^2_x} \frac{n_y}{N}\right\} \\
&= a^2 (1 - {}_{y}\eta^2_x) + 2ac\, S_y \left(\frac{y}{\sigma_y} \frac{\sigma^2_{x_y}}{\sigma^2_x} \frac{n_y}{N}\right) + c^2 S_y \left(\frac{y^2}{\sigma^2_y} \frac{\sigma^2_{x_y}}{\sigma^2_x} \frac{n_y}{N}\right) \quad \text{......(59)}'.
\end{aligned}
$$

If the x arrays are homoscedastic $\dfrac{\sigma^2_{x_y}}{\sigma^2_x} = 1 - {}_{x}\eta^2_y$ and we get (59).

Similarly equation (60), corrected for heteroscedasticity, becomes

$$
{}_{xy}H^2_z - {}_{x}\eta^2_z = b^2 (1 - {}_{x}\eta^2_y) + 2bc S_x \left(\frac{x}{\sigma_x} \frac{\sigma^2_{y_x}}{\sigma^2_y} \frac{n_x}{N}\right) + c^2 S_x \left(\frac{x^2}{\sigma^2_x} \frac{\sigma^2_{y_x}}{\sigma^2_y} \frac{n_x}{N}\right) \quad \text{.........(60)}'.
$$

Applying these corrections to the case of ${}_{xy}H^2_z$, we find

$$
{}_{xy}H^2_z - \cdot 724535 = \cdot 0221969 + \cdot 0003345 + \cdot 0006875 \quad \text{............(59)}',
$$

or

$$
{}_{xy}H^2_z = \cdot 7478
$$

and

$$
{}_{xy}H^2_z - \cdot 713674 = \cdot 0424845 - \cdot 0053862 + \cdot 0010073 \quad \text{.........(60)}',
$$

or

$$
{}_{xy}H^2_z = \cdot 7518,
$$

showing the separate contributions of the various terms.

The agreement of (60)′ with the empirical value ·7519 is remarkably close.

For the regression of weight on height and age, equations (36) to (39) give the values

$$
a = \cdot 24570, \qquad b = \cdot 70004, \qquad c = \cdot 12160,
$$

if the regression hyperboloid be written in the form:

$$
\frac{\bar{y}_{zx} - y}{\sigma_y} = d + a \frac{z - \bar{z}}{\sigma_z} + b \frac{x - \bar{x}}{\sigma_x} + c \frac{(x - \bar{x})}{\sigma_x} \frac{(z - \bar{z})}{\sigma_z} .
$$

The values given by the equations corresponding to (59) and (60) are

$$
{}_{zx}H^2_y = {}_{x}\eta^2_y + (a^2 + c^2)(1 - {}_{x}\eta^2_z) = \cdot 838012 + \cdot 021519 = \cdot 8595,
$$

$$
{}_{zx}H^2_y = {}_{z}\eta^2_y + (b^2 + c^2)(1 - {}_{z}\eta^2_x) = \cdot 693989 + \cdot 15580 = \cdot 8498,
$$

while the equations corresponding to (59)′ and (60)′ give

$$
{}_{zx}H^2_y = \cdot 838012 + \cdot 0172780 + \cdot 00011322 + \cdot 0034254 = \cdot 8588,
$$

and

$$
{}_{zx}H^2_y = \cdot 6939891 + \cdot 15128 + \cdot 0048971 + \cdot 0091002 = \cdot 8593,
$$

the empirical value of ${}_{zx}H^2_y$ being ·8634.

These approximations are not quite so good; they are bound up rather closely with the assumed hyperboloidal form of the regression surface, while Mr Soper has shown that in the case of weight on age and height, a good fit is obtained by a surface of the form:

$$
\bar{y}_{zx} = a_0 + a_1 z + a_2 z^2 + (b_0 + b_1 z + b_2 z^2)\, x^*.
$$

8. Mr M. Greenwood, Jr, has kindly drawn my attention to a joint paper, entitled "A Study of Index Correlations," published by Mr J. W. Brown, Miss Frances Wood and himself in the *Journal of the Royal Statistical Society*†.

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* Elderton's paper, *loc. cit.* p. 294.

† Vol. LXXVII, Part III, February, 1914, pp. 317–346.

The authors give the value of a generalized H, obtained by direct calculation in two or three cases. The material they deal with is the distribution of births, deaths and populations in 1000 English registration sub-districts, and also of the birth- and death-rates and populations in these districts. The material is highly heterogeneous and exceedingly high values of β_1 occur. Thus, β_1 for deaths is 16·0205 and for death rates is 61·5392.

Unfortunately in their table on page 341 of the values of the simple η's, the authors fail to discriminate between ${}_y\eta_x$ and ${}_x\eta_y$.

Mr Greenwood, at my request, was good enough to consult his MS. notes and sent me an identification table of the η's, but a recalculation of the η of populations on births and the η of populations on deaths has shown that errors have crept into this identification table.

When amended this table is as follows:

$$\begin{aligned}{}_x\eta_y &= \cdot 9501,\\ {}_y\eta_x &= \cdot 9358,\\ {}_x\eta_z &= \cdot 9241,\\ {}_z\eta_x &= \cdot 7855,\\ {}_z\eta_y &= \cdot 7527,\\ {}_y\eta_z &= \cdot 8871,\end{aligned}$$

where $x =$ population, $y =$ births, $z =$ deaths.

With these values our equation (70) gives

$$_{xy}H^2_z - {}_{xy}R^2_z = \cdot 2691,$$

or

$$_{xy}H^2_z = \cdot 8814,$$

and

$$_{xy}H_z = \cdot 9388.$$

The authors give as the empirical value $_{xy}H_z = \cdot 9384$.

As regards the use of the simpler formulae, the only β_1 of reasonably small value is β_1 of population which equals ·0886.

We put

$$_{xy}H^2_z - {}_{xy}R^2_z = \frac{1 + r^2_{xy}}{2r^2_{xy}} ({}_x\eta^2_z - r^2_{zx}),$$

or

$$_{xy}H^2_z - (\cdot 7825)^2 = \frac{1 + (\cdot 9345)^2}{2\,(\cdot 9345)^2} [(\cdot 9241)^2 - (\cdot 7822)^2],$$

or

$$_{xy}H^2_z = \cdot 87200,$$

$$_{xy}H_z = \cdot 9338.$$

Considering the wide divergence in the results obtained by the authors when dealing with the 1000 sub-districts and with 999 of them* it is evident that the use of the above simple formula would have been sufficient for their purpose.

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* *l.c.*, p. 343: $_{\text{population and births}}H_{\text{deaths}} = \cdot 9384$ for 1000 sub-districts, and $= \cdot 9213$ for 999 sub-districts.

GLASGOW SCHOOL BOYS—GROUP A.

TABLE I. *Age* 4·5 *to* 5·5 *years.*

Weight in lbs.

Height in inches	*24—28*	*29—33*	*34—38*	*39—43*	*44—48*	*49—53*	Totals
30—32	2	—	—	—	—	—	2
33—35	5	15	5	—	—	—	25
36—38	1	18	72	8	—	—	99
39—41	—	5	87	90	7	1	190
42—44	—	—	4	35	21	5	65
45—47	—	—	1	—	2	—	3
Totals	8	38	169	133	30	6	384

TABLE II. *Age* 5·5 *to* 6·5 *years.*

Weight in lbs.

Height in inches	*24—28*	*29—33*	*34—38*	*39—43*	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	Totals
30—32	1	1	—	—	—	—	—	—	—	2
33—35	4	17	2	—	—	—	—	—	—	23
36—38	1	36	92	14	1	1	1	—	—	146
39—41	—	16	191	258	38	1	—	—	1	505
42—44	—	2	27	232	178	28	2	—	—	469
45—47	—	—	1	12	39	37	6	—	—	95
48—51	—	—	—	1	1	—	2	—	—	4
Totals	6	72	313	517	257	67	11	—	1	1244

TABLE III. *Age* 6·5 *to* 7·5 *years.*

Weight in lbs.

Height in inches	*24—28*	*29—33*	*34—38*	*39—43*	*44—48*	*49—53*	*54—58*	*59—63*	Totals
30—32	1	1	—	1	—	—	—	—	3
33—35	—	8	6	2	—	—	—	—	16
36—38	—	6	37	10	2	—	—	—	55
39—41	—	4	93	178	35	2	—	—	312
42—44	—	—	22	220	294	61	3	1	601
45—47	1	1	—	21	133	138	38	—	332
48—50	—	—	—	2	—	13	12	4	31
51—53	—	—	—	—	—	1	—	1	2
Totals	2	20	158	434	464	215	53	6	1352

TABLE IV. *Age* 7·5 *to* 8·5 *years.*

Weight in lbs.

Height in inches	29—33	34—38	39—43	44—48	49—53	54—58	59—63	64—68	69—73	74—78	Totals
33—35	2	3	1	—	—	—	—	—	—	—	6
36—38	—	13	15	3	1	—	—	—	—	—	32
39—41	—	38	63	18	2	—	—	—	—	—	121
42—44	1	13	145	257	74	1	—	—	—	—	491
45—47	—	2	36	204	283	89	4	1	—	—	619
48—50	—	—	1	13	82	86	40	7	—	—	229
51—53	—	—	—	6	9	4	9	1	1	1	31
54—56	—	—	—	—	—	2	—	—	—	—	2
57—59	—	—	—	—	2	1	1	—	—	—	4
Totals	3	69	261	501	453	183	54	9	1	1	1535

TABLE V. *Age* 8·5 *to* 9·5 *years.*

Weight in lbs.

Height in inches	29—33	34—38	39—43	44—48	49—53	54—58	59—63	64—68	69—73	74—78	79—83	Totals
30—32	1	—	—	—	—	—	—	—	—	—	—	1
33—35	—	—	—	—	—	—	—	—	—	—	—	—
36—38	2	2	1	1	1	1	—	—	—	—	—	8
39—41	—	4	21	15	2	1	—	—	—	—	—	43
42—44	—	3	48	90	41	5	—	—	—	—	—	187
45—47	—	2	18	150	286	118	9	—	—	—	—	583
48—50	—	—	3	10	118	217	98	11	1	—	—	458
51—53	—	—	—	1	5	20	41	23	9	—	—	99
54—56	—	—	—	1	5	2	1	3	4	1	1	18
57—59	—	—	—	—	1	—	—	—	—	1	—	2
60—62	—	—	—	—	—	1	1	—	1	—	—	3
Totals	3	11	91	268	459	365	150	37	15	2	1	1402

TABLE VI. *Age* 9·5 *to* 10·5 *years.*

Weight in lbs.

Height in inches	34—38	39—43	44—48	49—53	54—58	59—63	64—68	69—73	74—78	79—83	Totals
33—35	—	—	1	—	—	—	—	—	—	—	1
36—38	2	3	—	1	1	1	—	—	—	—	8
39—41	1	7	6	2	—	—	—	—	—	—	16
42—44	2	14	27	23	4	—	—	—	—	—	70
45—47	—	5	54	148	93	10	1	—	—	—	311
48—50	—	3	3	127	281	159	35	5	—	—	613
51—53	—	1	—	7	42	130	108	27	7	1	323
54—56	—	—	—	—	—	3	11	10	5	1	30
57—59	—	—	—	—	1	2	—	—	1	—	4
60—62	—	—	—	—	2	—	—	—	—	—	2
63—65	—	—	—	—	—	—	1	—	—	—	1
Totals	5	33	91	308	424	305	156	42	13	2	1379

TABLE VII. *Age* 10·5 *to* 11·5 *years.*

Weight in lbs.

Height in inches	*34—38*	*39—43*	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	*69—73*	*74—78*	*79—83*	*84—88*	Totals
33—35	—	—	—	—	1	—	—	—	—	—	—	1
36—38	1	—	—	—	—	—	—	—	—	—	—	1
39—41	—	—	3	1	—	—	—	1	—	—	—	5
42—44	1	2	8	12	1	—	—	—	—	—	—	24
45—47	—	3	16	65	55	10	2	—	—	—	—	151
48—50	—	1	5	40	184	161	45	11	—	—	—	447
51—53	—	—	—	7	49	166	148	81	12	2	—	465
54—56	—	—	—	—	1	13	42	56	45	8	3	168
57—59	—	—	—	1	—	2	—	2	1	3	5	14
60—62	—	—	—	—	—	—	2	—	—	—	—	2
Totals	2	6	32	126	291	352	239	151	58	13	8	1278

TABLE VIII. *Age* 11·5 *to* 12·5 *years.*

Weight in lbs.

Height in inches	*34—38*	*39—43*	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	*69—73*	*74—78*	*79—83*	*84—88*	*89—93*	*94—98*	*99—103*	Totals
36—38	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
39—41	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
42—44	—	1	—	3	1	—	—	—	—	—	—	—	—	—	5
45—47	—	—	3	23	22	2	4	—	—	—	—	—	—	—	54
48—50	—	—	3	22	83	101	36	11	—	2	—	—	—	—	258
51—53	—	—	—	—	41	125	168	108	26	4	2	—	—	—	474
54—56	—	—	—	—	5	14	64	100	97	33	6	2	—	—	321
57—59	—	—	—	—	—	1	—	3	16	20	10	5	1	1	57
60—62	—	—	—	—	—	—	—	1	—	1	1	—	—	1	4
63—65	—	—	—	—	—	—	1	—	—	1	1	—	—	—	3
Totals	1	1	6	49	152	243	273	223	139	61	20	7	1	2	1178

TABLE IX. *Age* 12·5 *to* 13·5 *years.*

Weight in lbs.

Height in inches	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	*69—73*	*74—78*	*79—83*	*84—88*	*89—93*	*94—98*	*99—103*	*104—108*	*109—113*	Totals
42—44	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
45—47	2	6	12	3	—	—	—	—	1	—	—	—	—	—	24
48—50	2	8	35	60	20	6	3	—	—	—	—	—	—	—	134
51—53	1	2	12	80	142	112	51	14	6	1	—	—	—	—	421
54—56	—	—	1	13	59	129	138	50	17	6	1	—	1	—	415
57—59	—	—	1	—	3	12	38	42	44	15	15	1	—	—	171
60—62	—	—	—	—	—	—	4	2	2	14	5	5	—	1	33
63—65	—	—	—	—	—	—	—	—	—	1	—	2	—	—	3
Totals	5	17	61	156	224	259	234	108	70	37	21	8	1	1	1202

TABLE X. *Age 13·5 to 14·5 years.*

Weight in lbs.

Height in inches	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	*69—73*	*74—78*	*79—83*	*84—88*	*89—93*	*94—98*	*99—103*	*104—108*	*109—113*	*114—118*	*119—123*	Totals
42—44	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
45—47	—	2	3	1	—	—	—	—	—	—	—	—	—	—	—	—	6
48—50	1	1	5	7	7	1	—	—	—	—	—	—	—	—	—	—	22
51—53	—	2	3	16	31	36	9	1	—	—	—	—	—	—	—	—	98
54—56	—	—	—	7	27	41	49	27	6	2	2	—	—	—	—	—	161
57—59	—	—	1	—	—	7	19	28	28	16	7	3	—	—	—	—	109
60—62	—	—	—	—	—	—	1	1	5	4	4	5	5	—	—	—	25
63—65	—	—	—	—	—	—	—	—	1	—	2	1	1	—	—	1	6
Totals	1	6	12	31	65	85	78	57	40	22	15	9	6	—	—	1	428

TABLE XI. *Correlation Table of Heights and Weights.*

Height in inches.

Weight in lbs.	*30—32*	*33—35*	*36—38*	*39—41*	*42—44*	*45—47*	*48—50*	*51—53*	*54—56*	*57—59*	*60—62*	*63—65*	Totals
24—28	4	9	2	—	—	1	—	—	—	—	—	—	16
29—33	3	42	62	25	3	1	—	—	—	—	—	—	136
34—38	—	16	220	414	72	6	—	—	—	—	—	—	728
39—43	1	3	51	617	697	95	11	1	—	—	—	—	1476
44—48	—	1	7	122	875	603	38	8	1	—	—	—	1655
49—53	—	—	4	12	249	988	411	33	5	4	—	—	1706
54—58	—	1	3	1	17	436	905	171	11	4	3	—	1552
59—63	—	—	1	—	1	39	630	568	51	6	1	—	1297
64—68	—	—	—	1	—	8	161	621	206	3	2	2	1004
69—73	—	—	—	1	—	—	35	374	340	24	2	—	776
74—78	—	—	—	—	—	—	3	106	335	76	5	—	525
79—83	—	—	—	—	—	—	2	22	120	93	4	1	242
84—88	—	—	—	—	—	1	—	8	32	87	8	2	138
89—93	—	—	—	—	—	—	—	1	10	36	18	1	66
94—98	—	—	—	—	—	—	—	—	3	23	9	2	37
99—103	—	—	—	—	—	—	—	—	—	5	11	3	19
104—108	—	—	—	—	—	—	—	—	1	—	5	1	7
109—113	—	—	—	—	—	—	—	—	—	—	1	—	1
114—118	—	—	—	—	—	—	—	—	—	—	—	—	—
119—123	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	8	72	350	1193	1914	2178	2196	1913	1115	361	69	13	11382

TABLE XII. *Correlation Table of Height and Age.*

Age in years

Height in inches	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	Totals
30—32	2	2	3	—	1	—	—	—	—	—	8
33—35	25	23	16	6	—	1	1	—	—	—	72
36—38	99	146	55	32	8	8	1	1	—	—	350
39—41	190	505	312	121	43	16	5	1	—	—	1193
42—44	65	469	601	491	187	70	24	5	1	1	1914
45—47	3	95	332	619	583	311	151	54	24	6	2178
48—50	—	4	31	229	458	613	447	258	134	22	2196
51—53	—	—	2	31	99	323	465	474	421	98	1913
54—56	—	—	—	2	18	30	168	321	415	161	1115
57—59	—	—	—	4	2	4	14	57	171	109	361
60—62	—	—	—	—	3	2	2	4	33	25	69
63—65	—	—	—	—	—	1	—	3	3	6	13
Totals	384	1244	1352	1535	1402	1379	1278	1178	1202	428	11382

TABLE XIII. *Correlation Table of Age and Weight.*

Weight in lbs.	Age in years										
	5	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	Totals
24—28	8	6	2	—	—	—	—	—	—	—	16
29—33	38	72	20	3	3	—	—	—	—	—	136
34—38	169	313	158	69	11	5	2	1	—	—	728
39—43	133	517	434	261	91	33	6	1	—	—	1476
44—48	30	257	464	501	268	91	32	6	5	1	1655
49—53	6	17	215	453	459	308	126	49	17	6	1706
54—58	—	11	53	183	365	424	291	152	61	12	1552
59—63	—	—	6	54	150	305	352	243	156	31	1297
64—68	—	1	—	9	37	156	239	273	224	65	1004
69—73	—	—	—	1	15	42	151	223	259	85	776
74—78	—	—	—	1	2	13	58	139	234	78	525
79—83	—	—	—	—	1	2	13	61	108	57	242
84—88	—	—	—	—	—	—	8	20	70	40	138
89—93	—	—	—	—	—	—	—	7	37	22	66
94—98	—	—	—	—	—	—	—	1	21	15	37
99—103	—	—	—	—	—	—	—	2	8	9	19
104—108	—	—	—	—	—	—	—	—	1	6	7
109—113	—	—	—	—	—	—	—	—	1	—	1
114—118	—	—	—	—	—	—	—	—	—	—	—
119—123	—	—	—	—	—	—	—	—	—	1	1
Totals	384	1244	1352	1535	1402	1379	1278	1178	1202	428	11382

TABLE XV. *Mean Ages of Height and Weight Arrays.*

x	y	z_{xy}	x	y	z_{xy}	x	y	z_{xy}	x	y	z_{xy}	x	y	z_{xy}
-6	-6	-4·25	-3	0	-1	0	-1	-0·508516	2	2	2·194174	4	3	0·5
	-5	-2·666667		2	-4		0	0·046409		3	2·361764		4	3·2
	-3	-3		3	1		1	0·604762		4	2·531343		5	3
-5	-6	-4·555556	-2	-5	-3·333333		2	1·118012		5	2·758333		6	3·5
	-5	-4·071428		-4	-3·083333		3	1·542857		6	2·812500		7	3·222222
	-4	-3·5625		-3	-3·008608		4	3		7	3		8	3·444444
	-3	-2·666667		-2	-2·6222857		5	2		8	3·666667		9	3·363636
	-2	0		-1	-1·943775	1	-3	0		10	3		10	4
	0	1		0	-1·235294		-2	-1·25	3	-1	-1		11	3
-4	-6	-4·5		1	-3		-1	-0·151515		0	1·25	5	2	1
	-5	-4·096774	-1	-6	-3		0	0·883041		1	0·333333		5	2
	-4	-3·927273		-5	-3		1	1·158450		2	3		6	3
	-3	-3·078431		-4	-2·5		2	1·624798		3	3		7	3
	-2	-2·428571		-3	-2·084210		3	2·048128		4	2·921052		8	4
	-1	-1·75		-2	-1·815920		4	2·367924		5	3·021505		9	3·333333
	0	-1·666667		-1	-1·292510		5	2·545455		6	3·091954		10	4
	1	0		0	-0·658257		6	2·75		7	3·305555		13	4
-3	-5	-4·04		1	0·256410		7	3		8	3·260869			
	-4	-3·763285		2	1	2	-2	-1		9	3·4			
	-3	-3·505673		6	3		-1	-1	4	0	-0·333333			
	-2	-2·786885	0	-3	-1·272727		0	0·727273		1	-1			
	-1	-1·5		-2	-0·5		1	2·098039		2	1			

CORRELATION OF ANTHROPOMETRICAL MEASUREMENTS IN CAIRO-BORN NATIVES*

By MYER M. ORENSTEEN

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1. *The Material.* Since 1910, owing to the completion of an anthropometrical study undertaken by the Egyptian Government under the direction of Mr J. I. Craig, now Controller-General of the Statistical Department, a series of measurements of about ten thousand Egyptian natives has been filed in the Survey Department. It has been thought of interest to take this opportunity to re-examine the material in order to determine the correlation between the different organs of the native body available.

Referring to the nature of the characters considered in the paper, it will be necessary to remember here that the data belong to anthropometrical slips of prisoners, taken from the Cairo Anthropometrical Bureau. The slips have been drawn absolutely at random without any particular character of selection.

To Mr J. I. Craig, the writer addresses his best thanks for kindly allowing him full access to the material, and for valuable advice.

2. *The Data.* The measurements available and dealt with in the paper are:

(*L.*) *Length of the Head.* This is the shortest distance between the glabella and the occipital point as measured with callipers.

(*B.*) *Breadth of the Head.* This is the maximum breadth of the head. It is measured with callipers just above the back of the ears.

(*L.M.F.*) *Length of the left middle Finger.*

(*L.F.*) *Length of the left Foot.*

(*L.C.*) *Length of the left Cubit.*

(*S.*) *Stature.* This is the total height standing without boots.

From the ten thousand individuals in hand, who are natives of all parts of Egypt up to Aswan, 1191 persons have been selected as being exclusively males born in Cairo. From this selected group a further number of 389 has been excluded for being, at the date when measured, youths of under 20 years of age. The remaining 802 individuals providing the final group of measurements dealt with throughout the paper are, therefore, adult male Egyptians born in Cairo. Of this final group 76 per cent. were measured during 1904–1905, the remaining 24 per cent. fall within the years 1901 and 1906.

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* As the Editor was unable to consult the author of this paper, he has considered it best to alter as little as possible the literary expression.

3. *Social class to which the Natives dealt with belong.* From the fact that these natives have undergone police examination, objection may arise as to whether the measurements of such persons can represent the ordinary native body, or if really they belong to a selected group of individuals, namely the criminal class of natives.

TABLE I.

Movement of the natives measured, their distribution as regards length of imprisonment, and their comparison with migration of ordinary inhabitants.

Length of Imprisonment	Place where migrated to			Total
	Lower Egypt	Cairo Town	Upper Egypt	
One year and less	8·8	71·2	6·8	86·8
Over one year and up to three	0·8	10·5	1·0	12·3
Over three years	0·0	0·8	0·1	0·9
Total	9·6	82·5	7·9	100·0
Ordinary Egyptian inhabitants	6·3	91·1	2·5	100·0

Mr J. I. Craig, writing on the same material, says* "it may be objected that criminality in itself is a determinating factor of selection, but the objection does not hold in Egypt. Here it cannot be said that there exists a definite criminal class, and criminals are rather amateurs than professionals. This state of things is in all probability due to the easy conditions under which the lower class live."

In support of this, I may mention some recent conclusions which Dr Charles Goring comes to†. "Criminals as criminals are not a physically differentiated class of the general community....The physical and mental constitution of both criminal and law-abiding persons of same age, stature, class and intelligence are identical....There is no such thing as an anthropological type."

* "Anthropometry of Modern Egyptians," *Biometrika*, July, 1911.

† *The English Convict: A Statistical Study,* by Charles Goring, M.D.

These statements are suggestive for our material, and some investigation has been thought desirable in order to ascertain how far the former is justified for the Cairo group when considered alone. This investigation was carried out in two directions:

(1) The seriousness of the offence or crime committed. This of course is proportionate to the length of imprisonment.

(2) The social class to which the natives belong. This is traced by looking into the daily occupation of each individual.

TABLE II.

Daily occupation of men measured.

	%		%
Coffee-house keeper ...	9·4	Tailor	2·4
Pedlar	9·2	Cultivator	2·2
Servant	7·9	Baker	2·0
Driver	7·4	Butcher	1·7
Workman	6·9	Cobbler	1·6
Blacksmith	6·4	Barber...	1·3
Vagabond	6·2	Washerman	1·2
Carpenter	5·5	Coal-seller	1·1
Painter	4·8	Water-carrier ...	0·8
Cook	4·0	Shoe-black	0·8
Tobacconist	4·0	Bricklayer	0·6
Clerk	3·1	Student	0·6
Fruiterer	2·7	Printer	0·5
Weaver	2·6	Milkman	0·3
Confectioner	2·5	Watchman	0·3

Table I gives in per cent. the different lengths of imprisonment. This table also exhibits the stability of the group measured when compared with the total mass of the ordinary native male population. The figures of the latter group were taken from the *Census of Egypt in* 1907, where 221,185 individuals have been classified as being male Egyptians born in Cairo. From Table I we observe that most of the men have been condemned to very short punishments. On the whole only nine natives have been registered for murder (manslaughter); 86·8 per cent. have been kept in custody for a period varying between a few days and one year; 12·3 per cent. have been kept in for a length varying from one year and up to three. The remaining 0·9 per cent. have been imprisoned for a period over three years. This state of things is surely due to slight offences done without premeditation.

The figures under the heading "migration" show clearly that most of the Cairenes settle in their native town (as is also the case in the whole country). Out of the Cairo migrating element, ten per cent. prefer to settle down in Lower Egypt while eight per cent. move into Upper Egypt. We further remark that the ordinary native migrates less than the group measured by the police. The natives of the former selection remaining in the town exceed by 10 per cent. those of the latter. This in all probability is to be explained by the fact that the census figures include children and boys under 20, up to which age there is less tendency to migration than afterwards.

Table II gives a fairly exact idea of the daily occupation or social class to which the men measured belong. From this table we first remark that practically all men have their daily occupation, and that the greater part of them deal directly with the public. The proportion of these is 87 per cent.; the ordinary native element is represented by 52 per cent. Vagabonds and men without distinct occupation come in with 6 per cent.

As a result of the analysis of Tables I and II, we may come to the conclusion that the people dealt with in this paper are not necessarily of the poorest class of the total mass of the Cairo natives. And although they have undergone the police examination they are not necessarily of a selected class of natives, nor are they under special conditions of health.

4. *Age of the People.* As we have already said, all youths under twenty years of age have been excluded. Most of the natives are ignorant of their right age and could easily deviate by ten or twenty years from the truth though with apparent good faith. The age was estimated by the police officer on duty.

The percentage frequency of men per group of age is as follows:

		Census
From 20 to 25 years ...	52 %	25 %
„ 26 to 30 „ ...	28 %	18 %
„ 31 to 40 „ ...	16 %	26 %
„ 41 to 70 „ ...	4 %	32 %

It will be seen that the above grouping eliminates the greater part of accidental errors due to personal equation of the observer. The figures given by the census are said to be very unreliable.

TABLE III.

Means, Standard-deviations and their respective Probable Errors.

Character	$M \pm E_M$	$\sigma^* \pm E_\sigma$
Length of Head, in mm. ...	190·52±0·14	5·90±0·10
Breadth of Head, in mm. ...	144·45±0·11	4·67±0·08
Left Middle Finger, in mm.	114·09±0·15	6·14±0·10
Left Foot, in mm.	257·64±0·30	12·36±0·21
Left Cubit, in mm.	457·51±0·52	21·21±0·36
Stature, in cm.	165·82±0·14	5·93±0·10
Cephalic Index, in % ...	75·82±0·07	2·91±0·05

* Sheppard's corrections have been applied.

5. *Frequency Distributions.* The total number of individuals measured being 802, the measurements were grouped so as to save time and labour.

The grouping of the data has been arranged so as to give approximately eleven sub-ranges of grouping in each case with limits as follows:

Length of the head ...	every	3	millimetres
Breadth of the head ...	,,	3	,,
Left middle finger ...	,,	4	,,
Left foot	,,	7	,,
Left cubit	,,	15	,,
Stature	,,	4	centimetres

Table III gives the mean, the standard deviation corrected by Sheppard's adjustments, and the corresponding probable error for each character observed. The cephalic index is computed as usual from the ratio of the breadth to the length multiplied by a hundred; this gives a measure as to whether the head is relatively long or broad.

The frequency distributions were further analysed. Sheppard's corrections were applied to all the moments, and the criterion κ for the classification of the curves determined. κ in all cases was less than unity; β_1 and β_2 computed from μ_2, μ_3, μ_4 (the second, third and fourth moments) were practically 0 and 3 respectively for all distributions.

The normal curve of the form $Y = y_0 e^{-x^2/2\sigma^2}$ was therefore chosen.

The theoretical frequencies for every sub-group of each distribution of measurements have been computed from the following formulae:

Length of the head: $f = 162{\cdot}73e^{-x^2/69{\cdot}6132}$,

Breadth of the head: $f = 205{\cdot}50e^{-x^2/43{\cdot}6194}$,

Left middle finger: $f = 208{\cdot}43e^{-x^2/75{\cdot}3952}$,

Left foot: $f = 181{\cdot}20e^{-x^2/305{\cdot}5150}$,

Left cubit: $f = 226{\cdot}30e^{-x^2/899{\cdot}3250}$,

Stature: $f = 215{\cdot}77e^{-x^2/70{\cdot}3072}$,

where f is the frequency for every unit of grouping,

x is the distance from the origin (in these cases the origin and the mean coincide) to the foot of the ordinate to be calculated,

e is the base of the Napierian system of logarithms.

The comparison between the observed frequencies and the theoretical frequencies computed from the above formulae is given in Table IV. χ^2 is calculated from the sum of the squares of the departures of the theoretical values from the observed ones, divided by the corresponding theoretical values. This is shown by the formula

$$\chi^2 = \Sigma\,[(y_O - y_C)^2/y_C].$$

P is the probability that, in another random sample of the same size, a worse fit would be obtained.

TABLE IV.

(*a*) Length of Head

Centre of Group mm.	Observed frequency O	Computed frequency C	$\Delta = O - C$
174	4	3	+ 1
177	8	12	− 4
180	41	33	+ 8
183	69	72	− 3
186	114	122	− 8
189	177	158	+ 19
192	160	158	+ 2
195	98	122	− 24
198	75	73	+ 2
201	39	34	+ 5
204	7	12	− 5
207	10	3	+ 7

$\chi^2 = \Sigma\Delta^2/C = 30{\cdot}3$ $P = {\cdot}0015$

(*b*) Breadth of Head

Centre of Group mm.	Observed frequency O	Computed frequency C	$\Delta = O - C$
131	6	3	+ 3
134	8	17	− 9
137	75	58	+ 17
140	121	130	− 9
143	190	196	− 6
146	202	195	+ 7
149	125	128	− 3
152	55	56	− 1
155	11	16	− 5
158	8	3	+ 3
161	1	0	+ 1

$\chi^2 = \Sigma\Delta^2/C = 19{\cdot}2$ $P = {\cdot}0378$

(*c*) Left Middle Finger

Centre of Group mm.	Observed frequency O	Computed frequency C	$\Delta = O - C$
94·5	1	1	0
98·5	6	8	− 2
102·5	32	35	− 3
106·5	111	97	+ 14
110·5	176	176	0
114·5	215	208	+ 7
118·5	137	161	− 24
122·5	80	82	− 2
126·5	36	27	+ 9
130·5	5	6	− 1
134·5	3	1	+ 2

$\chi^2 = \Sigma\Delta^2/C = 9{\cdot}8$ $P = {\cdot}4590$

(*d*) Left Foot

Centre of Group mm.	Observed frequency O	Computed frequency C	$\Delta = O - C$
225	5	6	− 1
232	19	21	− 2
239	70	58	+ 12
246	111	117	− 6
253	178	169	+ 9
260	167	178	− 11
267	122	136	− 14
274	88	76	+ 12
281	28	30	− 2
288	11	9	+ 2
295	3	2	+ 1

$\chi^2 = \Sigma\Delta^2/C = 8{\cdot}7$ $P = {\cdot}5677$

(*e*) Left Cubit

Centre of Group mm.	Observed frequency O	Computed frequency C	$\Delta = O - C$
395	3	4	− 1
410	20	18	+ 2
425	72	70	+ 2
440	160	161	− 1
455	224	225	− 1
470	186	190	− 4
485	100	97	+ 3
500	26	30	− 4
515	10	6	+ 4
530	0	1	− 1
545	1	0	+ 1

$\chi^2 = \Sigma\Delta^2/C = 3{\cdot}9$ $P = {\cdot}9439$

(*f*) Stature

Centre of Group cm.	Observed frequency O	Computed frequency C	$\Delta = O - C$
149·5	4	5	− 1
153·5	28	25	+ 3
157·5	73	81	− 8
161·5	185	166	+ 19
165·5	212	215	− 3
169·5	167	178	− 11
173·5	87	93	− 6
177·5	31	31	0
181·5	14	7	+ 7
185·5	1	1	0

$\chi^2 = \Sigma\Delta^2/C = 11{\cdot}7$ $P = {\cdot}2341$

FIG. 1. Length of Head. $(y=202{\cdot}90\times 2{\cdot}71828^{-x^2/69{\cdot}6132})$

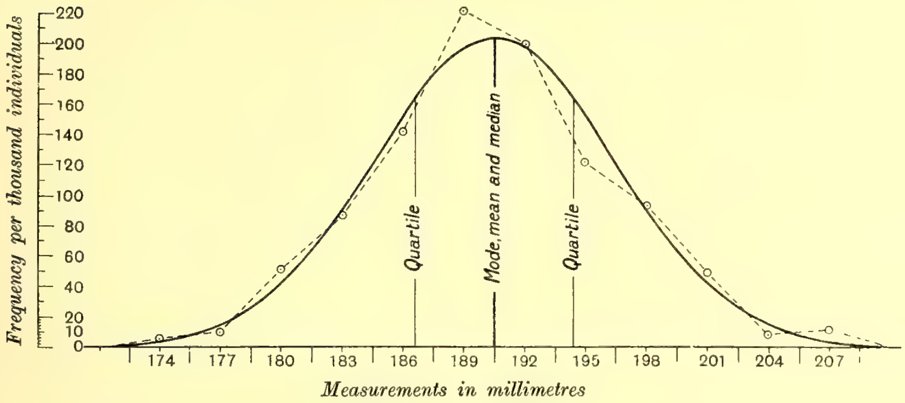


FIG. 2. Breadth of Head. $(y=256{\cdot}23\times 2{\cdot}71828^{-x^2/43{\cdot}6194})$

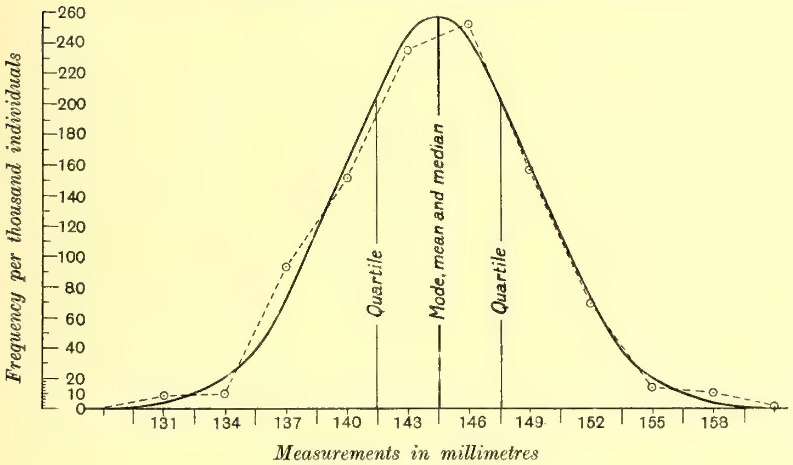


FIG. 3. Left Middle Finger. $(y=259{\cdot}89\times 2{\cdot}71828^{-x^2/75{\cdot}3952})$

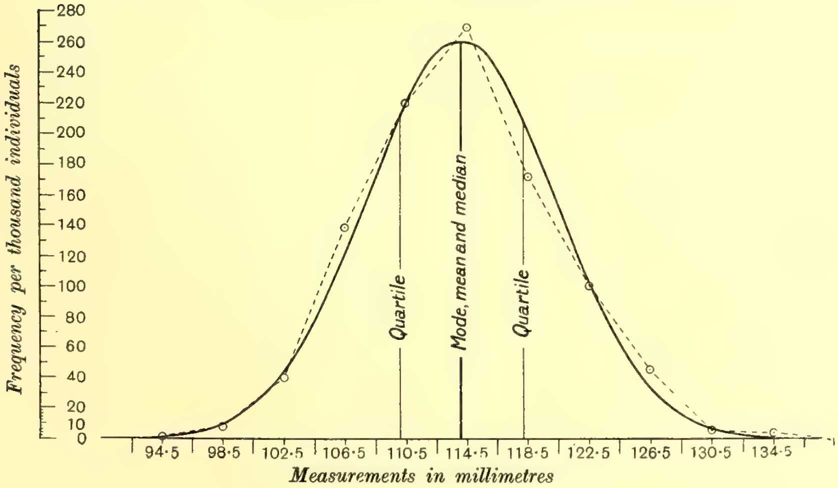


FIG. 4. Left Foot. ($y=225\cdot93\times2\cdot71828^{-x^2/305\cdot515}$)

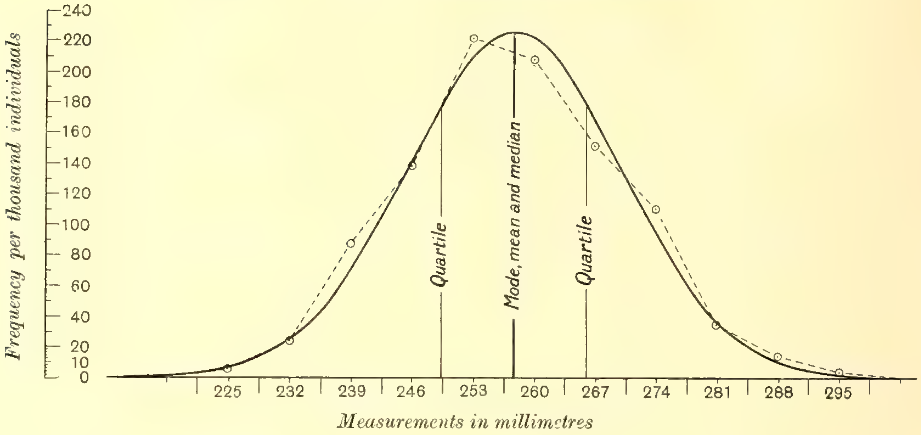


FIG. 5. Left Cubit. ($y=282\cdot17\times2\cdot71828^{-x^2/899\cdot3250}$)

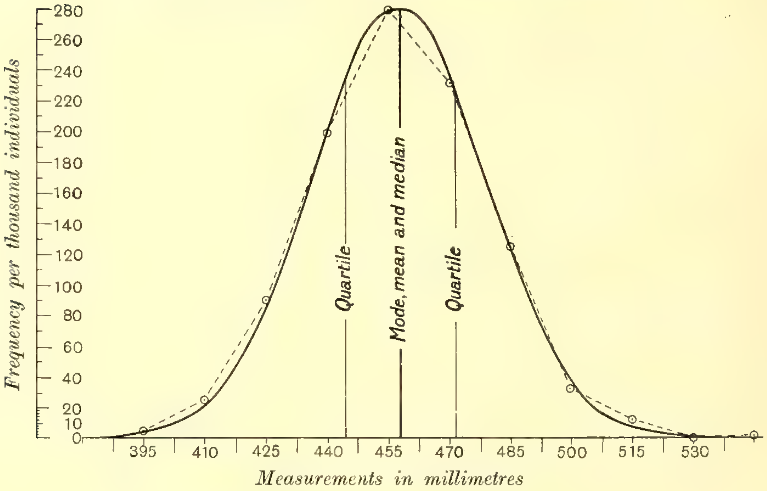
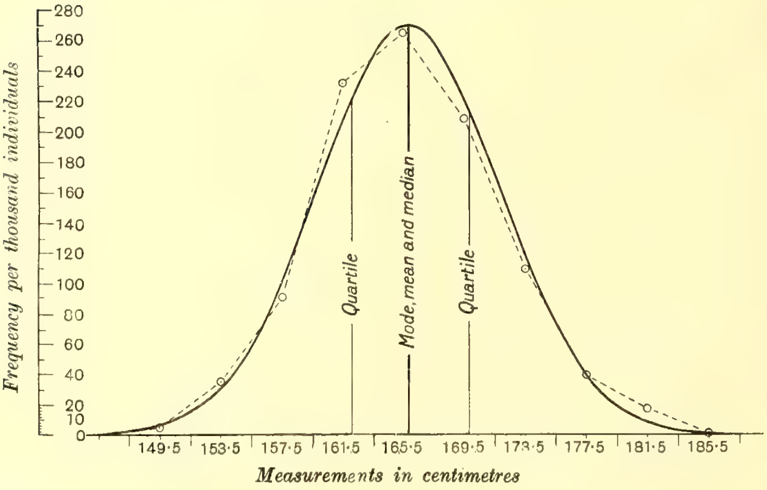


FIG. 6. Stature. ($y=269\cdot04\times2\cdot71828^{-x^2/70\cdot3872}$)



Figures 1 to 6 give the calculated frequency curves. The lengths of the ordinates represent the frequency per thousand of observations in a corresponding group where the ordinate considered is the mean for the unit of grouping. The dotted curves have been drawn from the observed frequencies reduced to the same base. A glance at these figures is sufficient to show that normal curves fit pretty well our statistics. The quartiles have been drawn with the aid of an integraph.

6. *Coefficients of Correlation.* With the same grouping of the data I have constructed correlation tables. These are given in Tables VIII to XIV.

The coefficient of correlation for the length and the breadth of the head was found to be $r = +0{\cdot}244 \pm 0{\cdot}020$. This coefficient is as we should have expected lower than the results from the remaining pairs of characters. Attention may be drawn to the fact that this particular coefficient varies very much from race to race. A comparison with similar results for different races may be of interest here:

♂	
Smith Sound Eskimos ...	$r = +0{\cdot}47 \pm 0{\cdot}08$
Ainos	$r = +0{\cdot}43 \pm 0{\cdot}06$
Naqada Race	$r = +0{\cdot}34 \pm 0{\cdot}05$
Germans	$r = +0{\cdot}29 \pm 0{\cdot}06$
Bavarian Peasants	$r = +0{\cdot}28 \pm 0{\cdot}06$
Cairo Natives	$r = +0{\cdot}24 \pm 0{\cdot}02$
French Peasants	$r = +0{\cdot}13 \pm 0{\cdot}09$
British Columbian Indians ...	$r = +0{\cdot}08 \pm$?

Taking into consideration the great variation of this coefficient from race to race, the writer is greatly inclined to believe that a correlation between the length and the breadth of the head really exists, but that its variation in value is very probably due to special factors, probably working independently and tending to modify the real measure of relationship. This is being further investigated, and a closer study of such factors is being undertaken.

The results of the computation of Tables VIII to XIV are grouped in Table V. From this table we may deduce the following conclusions: Were all human bodies perfectly similar all the coefficients of correlation between the different organs would be unity; if on the contrary there should exist a complete independency of the organs, the coefficients would be all nil. We may therefore conclude from Table V that we are to expect a tall man to have more often long feet than long fingers and short men will more often have long fingers than long arms. Further, men with long arms will very rarely have short fingers, and such people will more often be men with greater stature than men with long feet, and so on.

Table VI gives the probable error for each coefficient of correlation shown in Table V. They are in all cases very small, which proves our results to be very reliable.

The results shown in Table V have been compared with the like data for European countries, and no serious deviation in the coefficients has been noticed. This tends to confirm the statement made above for the head characters.

TABLE V.

Coefficients of Correlation.

Character	*L.M.F.*	*L.F.*	*L.C.*	*S.*
L.M.F.	1·000	0·763	0·832	0·661
L.F.	0·763	1·000	0·757	0·715
L.C.	0·832	0·757	1·000	0·803
S.	0·661	0·715	0·803	1·000

TABLE VI.

Probable Errors of Coefficients of Correlation.

Character	*L.M.F.*	*L.F.*	*L.C.*	*S.*
L.M.F.	0·000	±0·010	±0·007	±0·013
L.F.	±0·010	0·000	±0·010	±0·012
L.C.	±0·007	±0·010	0·000	±0·008
S.	±0·013	±0·012	±0·008	0·000

7. *Regression Equations.* Considering the significancy of all the coefficients of correlation shown above, we may conclude that the variability between the human organs is not chaotic, but governed by very definite laws. We are therefore able to predict an unknown measurement in terms of the given length of a known organ.

Table VII results from this investigation. This table shows us how and with what accuracy human characters can be calculated one from another. The

TABLE VII.

Table for Application of linear Regression Equations.

	Character	Constants	Computed from			
			L.M.F. in mm.	*L.F.* in mm.	*L.C.* in mm.	*S.* in mm.
Calculated measurements	*L.M.F.* in mm.	*m*	—	0·3790	0·2409	0·0684
		c	—	+16·4	+3·9	+0·7
		p.e.	—	±2·7	±2·3	±3·1
	L.F. in mm.	*m*	1·5359	—	0·4411	0·1490
		c	+82·4	—	+55·8	+10·6
		p.e.	± 5·4	—	± 5·5	± 5·8
	L.C. in mm.	*m*	2·8741	1·2990	—	0·2872
		c	+129·6	+122·8	—	−18·7
		p.e.	± 7·9	± 9·4	—	± 8·5
	S. in mm.	*m*	6·3839	3·4304	2·2451	—
		c	+929·9	+774·4	+631·0	—
		p.e.	±29·5	±27·5	±23·5	—

regression equations given in this table are of course linear, and are to be put into the form

$$y = mx + c,$$

where y is the measurement looked for,

x is the known measurement,

m is the change of the dependent measurement for a unit change of the independent variable, *p.e.* is the probable error for each calculated measurement, and is computed from

$$0{\cdot}6745\sigma_y n^{\frac{1}{2}}(1-r^2)^{\frac{1}{2}}/(n-2)^{\frac{1}{2}},$$

and c is a constant.

All characters, stature included, are in millimetres.

Besides the formulae given above, the following multiple regression equations for the determination of the stature may be of interest:

Reconstruction of stature from a knowledge of one measurement:

$$\begin{aligned}\text{Stature} &= 6{\cdot}3839\ (\text{Finger}) + 929{\cdot}9,\\ \text{Stature} &= 3{\cdot}4304\ (\text{Foot}) + 774{\cdot}4,\\ \text{Stature} &= 2{\cdot}2451\ (\text{Cubit}) + 631{\cdot}0.\end{aligned}$$

Reconstruction of stature from a knowledge of two measurements:

$$\begin{aligned}\text{Stature} &= -0{\cdot}2228\ (\text{Finger}) + 2{\cdot}2981\ (\text{Cubit}) + 632{\cdot}2,\\ \text{Stature} &= 2{\cdot}6699\ (\text{Finger}) + 2{\cdot}4195\ (\text{Foot}) + 730{\cdot}2,\\ \text{Stature} &= 1{\cdot}7136\ (\text{Foot}) + 1{\cdot}2034\ (\text{Cubit}) + 564{\cdot}2.\end{aligned}$$

Reconstruction of stature from a knowledge of three measurements:

$$\text{Stature} = 1{\cdot}4309\ (\text{Foot}) + 1{\cdot}9866\ (\text{Cubit}) - 1{\cdot}0166\ (\text{Finger}) + 496{\cdot}6,$$

where all the values, stature included, are in millimetres.

8. *Summary of Conclusions.*

(*a*) The raw material, though taken from the anthropometrical slips of prisoners, is a fair sample of the ordinary native community.

(*b*) The natives dealt with are not of the poorest nor of the lowest class of the Cairo natives.

(*c*) The records are homogeneous, and are sufficient in number for a good statistical study.

(*d*) The distributions of stature, cubit, finger, etc., are normal.

(*e*) The coefficients of correlation between all organs are significant, no matter to what race the records belong.

(*f*) The variability between the human organs is not chaotic, but governed by definite laws which enable us to predict measurements one from another.

(*g*) The coefficients of correlation for the human organs are practically identical from one race to another, only the coefficients resulting from the head measurements vary to any great extent.

(*h*) The great variation which the coefficient of correlation for the head measurements undergoes from race to race, is to be traced to special factors not yet determined, which influence this measure of relationship. I hope to investigate this further.

TABLE VIII. *Correlation Table of Length and Breadth of Head.*

Length of Head in mm.

Breadth of Head in mm.

Groups	173—175	176—178	179—181	182—184	185—187	188—190	191—193	194—196	197—199	200—202	203—205	206—208	Total
130—132	—	1	—	1	1	1	1	—	1	—	—	—	6
133—135	—	—	—	2	2	2	2	—	—	—	—	—	8
136—138	3	3	10	9	9	10	15	8	8	—	—	—	75
139—141	—	1	5	14	17	29	35	8	4	4	3	—	120
142—144	—	2	9	16	24	54	31	24	19	8	1	2	190
145—147	1	1	9	17	39	40	41	29	12	11	1	1	202
148—150	—	—	3	7	19	32	22	13	18	8	1	2	125
151—153	—	—	5	1	2	6	11	13	7	5	1	4	55
154—156	—	—	—	2	1	1	2	2	2	1	—	1	12
157—159	—	—	—	—	—	2	—	1	4	1	—	—	8
160—162	—		—	—	—	—	—	—	—	1	—	—	1
Total	4	8	41	69	114	177	160	98	75	39	7	10	802

TABLE IX. *Correlation Table of Left Foot with Left Middle Finger.*

Left Foot in mm.

Left Middle Finger in mm.

Groups	222—228	229—235	236—242	243—249	250—256	257—263	264—270	271—277	278—284	285—291	292—298	Total
93—96	—	—	—	1	—	—	—	—	—	—	—	1
97—100	1	3	2	—	—	—	—	—	—	—	—	6
101—104	1	5	14	11	1	—	—	—	—	—	—	32
105—108	3	9	28	31	32	7	1	—	—	—	—	111
109—112	—	2	17	42	62	36	15	2	—	—	—	176
113—116	—	—	9	22	63	67	36	14	4	—	—	215
117—120	—	—	—	4	13	46	34	35	2	3	—	137
121—124	—	—	—	—	7	9	27	24	8	4	1	80
125—128	—	—	—	—	—	2	9	12	11	—	2	36
129—132	—	—	—	—	—	—	—	1	—	4	—	5
133—136	—	—	—	—	—	—	—	—	3	—	—	3
Total	5	19	70	111	178	167	122	88	28	11	3	802

TABLE X. *Correlation Table of Left Cubit with Left Middle Finger.*

Left Cubit in mm.

Left Middle Finger in mm.

Groups	388—402	403—417	418—432	433—447	448—462	463—477	478—492	493—507	508—522	523—537	538—552	Total
93—96	—	—	1	—	—	—	—	—	—	—	—	1
97—100	2	1	1	1	1	—	—	—	—	—	—	6
101—104	1	9	17	4	1	—	—	—	—	—	—	32
105—108	—	6	33	59	13	—	—	—	—	—	—	111
109—112	—	4	18	62	75	11	6	—	—	—	—	176
113—116	—	—	1	29	100	68	17	—	—	—	—	215
117—120	—	—	1	5	30	75	21	5	—	—	—	137
121—124	—	—	—	—	4	27	36	11	2	—	—	80
125—128	—	—	—	—	—	5	19	8	4	—	—	36
129—132	—	—	—	—	—	—	1	2	2	—	—	5
133—136	—	—	—	—	—	—	—	—	2	—	1	3
Total	3	20	72	160	224	186	100	26	10	0	1	802

TABLE XI.

Correlation Table of Stature with Left Middle Finger.

Stature in cm.

Left Middle Finger in mm.

Groups	*148—151*	*152—155*	*156—159*	*160—163*	*164—167*	*168—171*	*172—175*	*176—179*	*180—183*	*184—187*	Total
93—96	—	—	—	1	—	—	—	—	—	—	1
97—100	2	1	2	—	1	—	—	—	—	—	6
101—104	1	3	17	9	2	—	—	—	—	—	32
105—108	—	11	24	41	24	10	1	—	—	—	111
109—112	1	9	21	64	49	29	3	—	—	—	176
113—116	—	1	9	50	78	51	19	7	—	—	215
117—120	—	3	—	15	43	43	22	7	4	—	137
121—124	—	—	—	5	11	26	25	8	5	—	80
125—128	—	—	—	—	4	6	16	8	2	—	36
129—132	—	—	—	—	—	1	—	1	3	—	5
133—136	—	—	—	—	—	1	1	—	—	1	3
Total	4	28	73	185	212	167	87	31	14	1	802

TABLE XII.

Correlation Table of Left Foot with Left Cubit.

Left Foot in mm.

Left Cubit in mm.

Groups	*222—228*	*229—235*	*236—242*	*243—249*	*250—256*	*257—263*	*264—270*	*271—277*	*278—287*	*288—291*	*292—298*	Total
388—402	1	2	—	—	—	—	—	—	—	—	—	3
403—417	—	4	10	6	—	—	—	—	—	—	—	20
418—432	4	8	20	22	16	2	—	—	—	—	—	72
433—447	—	5	24	43	56	28	3	—	1	—	—	160
448—462	—	—	13	32	68	70	32	7	2	—	—	224
463—477	—	—	3	8	28	47	61	37	1	1	—	186
478—492	—	—	—	—	9	19	19	33	17	3	—	100
493—507	—	—	—	—	1	—	5	7	5	5	3	26
508—522	—	—	—	—	—	1	2	4	1	2	—	10
523—537	—	—	—	—	—	—	—	—	—	—	—	0
538—552	—	—	—	—	—	—	—	—	1	—	—	1
Total	5	19	70	111	178	167	122	88	28	11	3	802

TABLE XIII.

Correlation Table of Stature with Left Foot.

Stature in cm.

Left Foot in mm.

Groups	*148—151*	*152—155*	*156—159*	*160—163*	*164—167*	*168—171*	*172—175*	*176—179*	*180—183*	*184—187*	Total
222—228	1	1	1	2	—	—	—	—	—	—	5
229—235	2	3	8	6	—	—	—	—	—	—	19
236—242	—	10	23	24	10	3	—	—	—	—	70
243—249	1	9	16	41	35	8	—	1	—	—	111
250—256	—	5	21	65	53	30	4	—	—	—	178
257—263	—	—	4	36	57	51	18	1	—	—	167
264—270	—	—	—	9	37	41	24	10	1	—	122
271—277	—	—	—	1	17	29	22	14	5	—	88
278—284	—	—	—	1	3	4	12	4	3	1	28
285—291	—	—	—	—	—	1	4	1	5	—	11
292—298	—	—	—	—	—	—	3	—	—	—	3
Total	4	28	73	185	212	167	87	31	14	1	802

TABLE XIV.

Correlation Table of Stature with Left Cubit.

Stature in cm.

Left Cubit in mm.

Groups	*148—151*	*152—155*	*156—159*	*160—163*	*164—167*	*168—171*	*172—175*	*176—179*	*180—183*	*184—187*	Total
388—402	3	—	—	—	—	—	—	—	—	—	3
403—417	1	15	3	1	—	—	—	—	—	—	20
418—432	—	6	33	27	5	1	—	—	—	—	72
433—447	—	4	27	76	44	8	1	—	—	—	160
448—462	—	3	9	60	91	54	7	—	—	—	224
463—477	—	—	1	17	54	77	29	6	2	—	186
478—492	—	—	—	3	17	26	36	16	2	—	100
493—507	—	—	—	1	1	1	12	6	5	—	26
508—522	—	—	—	—	—	—	2	3	5	—	10
523—537	—	—	—	—	—	—	—	—	—	—	0
538—552	—	—	—	—	—	—	—	—	—	1	1
Total	4	28	73	185	212	167	87	31	14	1	802

A STUDY OF THE CRANIA OF THE MORIORI, OR ABORIGINES OF THE CHATHAM ISLANDS, NOW IN THE MUSEUM OF THE ROYAL COLLEGE OF SURGEONS.

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1. *Introductory.* The fact that pure bred Morioris are now very rare if not extinct renders of special value any study of such crania of them as still exist. In papers referred to below Dr Scott* deals with 50 and Dr Duckworth† with 10 Moriori crania. Professor Thane possesses two specimens and there are a few scattered through various British and Continental museums. Hence the 63 crania in the museum of the Royal College of Surgeons almost double the available number. Yet if we pool the present measurements with such of Dr Scott's series as are homogeneous, our results are still too slender to warrant the publication of more than the measurements themselves and their mean values. As one of the best uses of methods of treatment which do not extend beyond the mean the type sections of the Moriori crania have been reconstructed. By most kind permission of Professor Arthur Keith I have been able to study the collection of Moriori skulls in his charge, and he most readily provided facilities for the photographing of a considerable number of the crania. Although the collection has been for a considerable time in the Royal College of Surgeons no complete set of measurements appears to have been published‡. Dr Duckworth in the paper just cited makes some remarks on a few of the abnormal skulls. Forty-four crania were purchased by the Royal College as "from the Chatham Islands" in 1892, i.e. 765^1 to 765^{44}, six, 765^{45} to 765^{50}, appear to be later additions, while thirteen, 758–760, 760^1, 760^2, 761–765, seem to have an earlier origin, as well as the three 1015, 1015^A and 1016 from the Barnard Davis collection. The very remarkable character of the Moriori skull is not fully accounted for by the outlying character of their island, although anthropologists naturally look to such islands for the survival of primitive types. Chatham

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* Contribution to the Osteology of the Aborigines of New Zealand and of the Chatham Islands. *Transactions of New Zealand Institute,* Vol. XXVI. pp. 1—64. Wellington, 1894.

† On a collection of Crania of the Moriori, *Journal of the Anthropological Institute,* Vol. XXX. p. 141 *et seq.* London, 1900.

‡ Eight have certain measurements given in Flower's *Catalogue of the Royal College of Surgeons.* Barnard Davis gives measurements of three others in his *Thesaurus Craniorum.*

Island lies to the east of New Zealand. According to their own traditions the Moriori reached their present home direct from Hawaiiki, so that they are not degraded Maoris but form a special branch of the Polynesian race. Never numerous, they were mostly slaughtered in the year 1838—two years previous to the first British colonisation of New Zealand—by a party of Ngatilawa landed on the island by a whaler. This party enslaved the few survivors. Some years ago the remaining Moriori were stated to be reduced to fifty, and these were probably not of pure race*. There was nothing of special note in their occupations, habits or beliefs. Most of their life was utilised in procuring food, making garments and sleeping mats, or in shaping stone implements in which they showed considerable ingenuity. Fish hooks, fish spears and gaffs in constant use were cleverly shaped out of bone. The dwellings were ordinary conical huts constructed out of fern tree and thatched with toi-toi grass; they were often sufficiently large to accommodate twenty or thirty persons. Some of them were ornamented with rude carvings, but most of these places have been demolished by the Maori, or perished from age, so that no specimens of their art are now obtainable. They lived peaceably, chiefly along the coast, spending their time as a rule in procuring supplies of fish, fern roots, nuts, wild duck, etc. Periodically they put off in rafts or large canoes for the hunting of seal and albatross. They are said to have had a confused notion of good and evil spirits and to have invoked numerous deities prior to any undertaking of importance†. In short, such accounts as we have of them do not seem to differentiate them markedly from the inhabitants of many other Polynesian islands. They stand out differentiated rather by their cranial than by their cultural characters.

2. *Measurements and Methods of Measurement.* The measurements I have taken are similar to those adopted by previous biometric craniologists. They were made in the manner described by Fawcett (*Biometrika*, Vol. I. p. 416) and Macdonell (*Ibid.* Vol. III. p. 200). The measurements are as follows:

C = capacity taken with mustard seed, packing in skull and in measuring glass, not by our usual laboratory method of weighing as no fine chemical balance was available, the work being done away from the Laboratory. In the tables it will be seen that I have given two figures for each capacity. The seed was tightly packed into the skull and then tightly packed into the measuring glass. This is the standard method. It involves the great labour of double packing; the second packing is avoided in our laboratory use of the balance. It occurred to me to be worth while trying loose packing in the measuring glass and correlating the results for tight packing and loose packing with the view of saving the time employed in the second tight packing on another occasion. The loose packing consisted in taking the seed from the skull, gently pouring it into the measuring glass and then giving a slight shake sufficient to obtain a level surface for reading. The results are given in the footnote below*; it suffices to remark here that for practical purposes it appears approximate enough if we deduct one-twentieth from the volume of the loosely packed seed in order to obtain the volume of the tightly packed. My corrected results from loose packing are given in brackets in the tables. The means are:

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* R. Lydekker, *Living Races of Mankind*, Vol. I. p. 72.

† Extracted from Notes on the Chatham Islands by J. W. Williams, of Waitangi West, Chatham Island, *Journal of the Anthropological Institute*, Vol. XXVII. p. 343. London, 1897.

Tight packing, ♂ 1422·07; Loose packing, ♂ 1423·03,
„ „ ♀ 1310·73; „ „ ♀ 1308·47.

The mean difference (regardless of sign) of the results obtained from loose packing from those obtained from tight packing is, however, 8 cm.^{3} How much of this is due to variation in the tight packing in the measuring glass, how much to fortuitous settlements in the loose packing it is not possible to say. The differences suggest at any rate that a method may well be devised by which tight packing in the measuring cylinder may be avoided.

Further measurements made are:

F = Flower's Ophryo-occipital length. L' = glabellar horizontal length. L = maximum length from glabella to occiput. B = maximum horizontal breadth. B' = least forehead breadth from one temporal crest to the other. H = Basio-bregmatic height. OH = auricular height as measured on the

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* Let C_t be the capacity from tight packing, C_l from loose packing; let σ be a standard deviation, V a coefficient of variation and r_{tl} the correlation of C_t and C_l, then in cubic centimetres:

For male crania.		*For female crania.*	
Mean $C_t = 1422{\cdot}07$	Mean $C_l = 1498{\cdot}11$	Mean $C_t = 1310{\cdot}74$	Mean $C_l = 1377{\cdot}37$
$\sigma_t = 110{\cdot}185$	$\sigma_l = 113{\cdot}263$	$\sigma_t = 90{\cdot}265$	$\sigma_l = 90{\cdot}122$
$V_t = 7{\cdot}75$	$V_l = 7{\cdot}56$	$V_t = 6{\cdot}89$	$V_l = 6{\cdot}81$
$r_{tl} = {\cdot}9935$		$r_{tl} = {\cdot}9969$	

It will be seen that the correlations are very high. The regression lines giving C_t for C_l are:

$$C_t = {\cdot}96654\ C_l - 25{\cdot}91 \text{ for males,}$$
$$C_t = {\cdot}99847\ C_l - 64{\cdot}53 \text{ for females.}$$

We should have expected the same line to have resulted from the two series, as it does not it is clear that the coefficient of C_l is a function of C_l. To determine the nature of loose packing a little further we took the regression lines of $(C_l - C_t)/C_l$ and C_l or equations of the form $(C_l - C_t)/C_l = AC_l + B$. There resulted:

$$(C_l - C_t)/C_l = {\cdot}06843 - {\cdot}0000{,}1173\ C_l \text{ for males,}$$
$$(C_l - C_t)/C_l = {\cdot}056939 - {\cdot}0000{,}0619\ C_l \text{ for females.}$$

These may be written:

$$C_t = C_l\ ({\cdot}93157 + {\cdot}0000{,}1173\ C_l) \text{ for males,}$$
$$C_t = C_l\ ({\cdot}94306 + {\cdot}0000{,}0619\ C_l) \text{ for females.}$$

The terms depending on C_l^2 are so small that we may insert in them for C_l, the mean values in each case, when we find:

$$C_t = C_l\ ({\cdot}94914) \text{ for males,}$$
$$C_t = C_l\ ({\cdot}95159) \text{ for females.}$$

The differences are now not unreasonable considering the paucity of the data, and we see that very approximately it suffices to take for both sexes, $C_t = C_l\ (1 - \frac{1}{20})$, the rule given above.

The whole matter is to be reconsidered on more ample material. It is not clear at present how far it depends on personal equation in tight and loose packing of different amounts and how far on the diameter of the measuring glass, i.e. the pressure varying with height or with C'_l and so affecting the looseness of the loose packing.

craniophor, that is the height perpendicular to the horizontal plane above the line joining the upper points of the margins of the auricular orifices. LB = length from basion to nasion. $\angle P$ = profile angle, measured with goniometer, the skull being on the craniophor. Q = transverse arc from upper margin of one auricular passage to upper margin of the other, the arc being taken perpendicular to the horizontal plane with the skull on the craniophor. This perpendicular arc goes generally behind the bregma through a point, the "apex," which is marked on the skull by the sliding bar of the craniophor, i.e. the bar by which OH is obtained. Q' is the same measure taken when the skull is off the craniophor*. S = sagittal arc from nasion to opisthion. S_1 = arc from nasion to bregma, S_2 = arc from bregma to lambda, S_3 = arc from lambda to opisthion. S'_3 = chord from lambda to opisthion. U = horizontal circumference measured directly above the superciliary ridges and round the most projecting part of occiput. $G'H$ = upper face height from nasion to alveolar point. GB = face breadth from the lower end of one zygomatic-maxillary suture to that of the other. J = zygomatic breadth, from the most lateral point of one zygomatic arch to that of the other. NH = nasal height from nasion to the lowest edge of left pyriform aperture (in cases where this is damaged the lowest edge of right aperture is taken; if there be a marked difference between the heights of the two apertures, both heights are given). NB = nasal breadth, greatest breadth of nasal aperture, wherever it may be. O_1 = breadth of orbit, R and L, the greatest breadth from side to side†. O_2 = height of orbit, R and L, taken perpendicular to O_1. G_1 = length of palate, from the point of the spina nasalis posterior to an imaginary surface tangential to the inner alveolar surfaces of the mesial incisors. G'_1 = length from base of spine. G_2 = breadth of palate between the inner alveolar walls at the second molars. GL = profile, or basi-alveolar length, from basion to alveolar point. fml = length of foramen magnum; fmb = greatest breadth of foramen magnum. GH = face height.

Weight of skull in grams. In this series the actual weight of the skull (measured without mandible) was taken, as the crania being so abnormally heavy, it was thought the individual weights of the skulls might be of some interest.

The indices calculated are: the cephalic indices $100\,B/L'$, $100\,H/L'$, $100\,B/L$, $100\,H/L$ and $100\,B/H$; the upper facial index $100\,G'H/GB$; the nasal index $100\,NB/NH$; the right and left orbital indices $100\,O_2/O_1$; the palate index $100\,G_2/G_1$; the occipital index $Oc.\ I. = 100\dfrac{S_3}{S'_3}\sqrt{\dfrac{S_3}{24(S_3-S'_3)}}$; the foraminal index $100\dfrac{fmb}{fml}$. Finally the gnathic index has been obtained from mean values.

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* Q is taken with the tape over the skull and measured from the top of one ear rod to the top of the other; it is somewhat troublesome to take. Q' is easier, but owing to the recessing of the margins of the auricular passages is not quite the same as Q. The differences in Q and Q' are partly due to this, partly to observational error. The mean values: Q = 316·11 for ♂s, = 310·00 for ♀s, and Q' = 315·94 for ♂s, = 310·20 for ♀s, show that there is close concordance of both methods in determining type.

† On the difficulties of measurement of O_1: see *Biometrika*, Vol. III. p. 201.

The twelve mandibles available were measured: w_1 = condylar width, or greatest width from outside one condyle to outside of second; w_2 = greatest width at angles from outside one angle to outside of the other; h_1 = greatest height, from lowest median projection to top of alveolar margin between mesial incisors; and zz = distance between foramina mentalia. The mandibles were too few in number to justify a more elaborate system of measurements. Also the following angles were taken in the manner described by Cicely Fawcett*: (*a*) profile angle ($\angle P$) as already mentioned measured with Ranke's goniometer; (*b*) angle between upper face length and profile length, the alveolar angle ($\angle A$), determined from the length measurements GL, $G'H$ and LB by aid of Pearson's trigonometer; (*c*) angle between skull basis, and upper face length, the nasal angle ($\angle N$), determined as in (*b*); (*d*) angle between skull basis and profile length, the basilar angle ($\angle B$), determined as in (*b*); (*e*) angle between skull basis and the standard horizontal plane (θ_1), the basio-nasal horizontal angle, determined as in (*b*) with the use of profile angle as found by goniometer. (*f*) Angle between profile length and the standard horizontal plane (θ_2), the basio-alveolar horizontal angle, determined as in (*e*).

Sex. I first recorded my own opinion as to the sex of the skulls, and then Professor Pearson gave me his judgment on them. In one or two cases, where it was most difficult to determine the sex, and we could not agree, appeal was made to Professor Keith, and he kindly gave his opinion also.

3. *Contours.* A study of the three chief contours of these crania was made. These contours were drawn by aid of a Klaatsch contour tracer, in the same manner as the contours traced by Dr Crewdson Benington in his paper in *Biometrika*, Vol. VIII. p. 123, on Cranial Type Contours. From these contours were obtained the *average* or *type* contours of the Moriori crania. The construction of the type contours was as follows:

(*a*) *The transverse, vertical, or auricular coronal section.* The skull being placed on the craniophor, and adjusted in the usual manner to the standard horizontal plane, the horizontal rod of the craniophor determines the "apex" of the skull. The plane through the apex of the skull (which is marked with a pencil stroke) and the "auricular points" is the transverse vertical plane of the skull. The skull being now transferred to the Klaatsch apparatus, the vertical plane is made horizontal and the contour traced from "auricular point" to "auricular point" through the apex. The line joining auricular point to auricular point is the auricular line of the contour, and the line perpendicular to it through its mid-point is the vertical axis MA of the transverse vertical contour. This vertical axis is divided into ten equal parts by aid of the proportional compasses, and lines parallel to the auricular line are drawn through the points of division; these lines will not necessarily be bisected by the vertical axis owing to the asymmetry of the skull. They are numbered from below upwards 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, where 1 represents the lengths on the auricular axis. An eleventh line is taken one-quarter of the last section from the apex, the distances to R and L of the vertical axis up to the contour are measured off along these eleven horizontals. Also the length of the vertical axis MA is measured. Since the skull is placed with the occiput downwards on the Klaatsch, the right side of the skull is to the observer's left when viewing the contour, and the left of the skull to his right. This is reversed in plotting type-contours in order to obtain the *norma occipitalis**.

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* *Biometrika*, Vol. I. p. 418.

The following measurements are now entered in the columns of the record for each contour—the sum of $1R$ and $1L$ being the interauricular diameter:

$M.A.$	$1R$	$1L$	$2R$	$2L$	$3R$	$3L$	$4R$	$4L$	$5R$	$5L$	$6R$	$6L$

$7R$	$7L$	$8R$	$8L$	$9R$	$9L$	$10R$	$10L$	$A\frac{1}{4}R$	$A\frac{1}{4}L$

The means of the columns give the corresponding lengths for the type skull and from these we can plot the points on the type section; this section is then drawn between the points by aid of a spline, and we thus reach the transverse type contour. See Figs. III and IV for male and female respectively.

(*b*) *The glabellar horizontal section.* This is obtained by placing the skull on the craniophor, and by aid of the scriber marking points on the same horizontal plane as the nasion and the gamma, in the region above both auricular passages. The skull is now adjusted in the Klaatsch holder, apex downwards, until these four points are shown to be in the same horizontal plane. The horizontal contour is then drawn parallel to this plane but through the glabella; it will fall below the lambda, and does not pass far from the occipital point. The points in the median plane immediately above the nasion and below the lambda on this contour are especially marked with the Klaatsch tracer. They will be spoken of as F and O, the glabellar and occipital points. FO is the fronto-occipital line of the horizontal section. This fronto-occipital line being found, it is divided into ten equal parts by the proportional compasses, and lines perpendicular to the fronto-occipital line are drawn through the points of division, and the lengths up to the contour measured right and left. To obtain the frontal and occipital curvatures, parallel lines at $F\frac{1}{4}$ and $O\frac{1}{4}$ were taken as in the other cases.

In the case of the Moriori contour there were such sharp projections at the *linea temporalis* that ordinates perpendicular to the fronto-occipital line both right and left were drawn to the most projecting points. These are the ordinates

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* Benington (*Biometrika*, Vol. VIII. p. 151 etc.) gives the transverse section as *norma facialis*, and his tracings must be reversed in superposition on mine.

T_R and T_L with abscissae x_R and x_L. To complete this contour in the neighbourhood of the glabella extra lines were measured at quarter and half the distance from F to 2; these lines are spoken of as the $F\frac{1}{4}$ and $F\frac{1}{2}$ lines. All these points being recorded they were found sufficient by aid of a spline to reconstruct the horizontal section effectively.

When we look at any horizontal contour with the frontal point towards us, the right of the skull is on our right, as it would be if we were facing the actual skull the right way up. The ordinates are reversed in plotting the type contour in order to obtain the *norma verticalis*.

We have the following measurements entered in our recording book:

FO	$F\frac{1}{4}R$	$F\frac{1}{4}L$	$F\frac{1}{2}R$	$F\frac{1}{2}L$	$2R$	$2L$	$3R$	$3L$	$4R$	$4L$	$5R$	$5L$	$6R$	$6L$

$7R$	$7L$	$8R$	$8L$	$9R$	$9L$	$10R$	$10L$	$O\frac{1}{4}R$	$O\frac{1}{4}L$	T_R	T_L	x_R	x_L

The means are again found for all the columns. The results are plotted, splined in, and we obtain the type horizontal section for ♂ and ♀. See Figs. V and VI.

(*c*) *The sagittal or median section.* The skull being again placed on the craniophor and adjusted to the Frankfurt horizontal plane, the point γ on the same horizontal level as the nasion is marked off on the median plane of the skull by means of the scriber. The skull is now removed to the Klaatsch, and adjusted so that the nasion, bregma and lambda lie in the same horizontal plane. The contour is then drawn, starting from the alveolar point. The alveolar point, the nasion, the bregma, the apex, the vertex, the lambda, the gamma, the inion, the opisthion, the basion, and the auricular point are all marked on this contour drawing. We now take the horizontal line through the nasion, i.e. the line linking the nasion to the gamma, as our "horizontal axis" and divide it into ten equal pieces by aid of the proportional compasses. The nasion counting as 0, and the other divisions from nasion to gamma as 1, 2, 3, 4...9, ordinates are drawn through these ten points perpendicular to the horizontal axis. Lines 8 and 9 are produced, and continued on the other side of the horizontal axis; a further upright is taken near the gamma at $\frac{1}{4}$ the distance from the gamma to 9; this for brevity is termed the $\gamma\frac{1}{4}$ line; this line is also produced and continued below the horizontal axis. At 0, i.e. vertically through the nasion, there will in most male skulls be a sensible ordinate roughly of about 20 mm. In order to get the curvature in the neighbourhood of the glabella a *vertical* tangent was drawn by aid of the set-square to the most projecting part of the glabella; the point in which the tangent meets the prolonged gamma-nasion line being T and the point of contact of the tangent being G, the ordinate GT and the abscissa TN represented by the letters G_y and G_x respectively were measured, they are practically the coordinates of the glabella. To complete the drawing of the frontal part of the sagittal contour, an extra ordinate was measured at $\frac{1}{4}$ the distance from 0 to 1; this line is spoken of as the $N\frac{1}{4}$ line. Perpendiculars were also dropped from the opisthion, auricular point and suborbital point to $N\gamma$. Also a perpendicular from the most projecting point of the contour just above the inion was drawn to $N\gamma$. These points will now be found amply sufficient to reproduce the individual contour as closely as it could be redrawn by the Klaatsch.

In our record book, we have accordingly the following entries, as well as the number and sex of the cranium:

$N\gamma$	$N\beta$	β-sub. to $N\lambda$	β-sub. to GI	V-sub. to GI	G_x	G_y	Ordinates above $N\gamma$												
							0	$N\frac{1}{4}$	1	2	3	4	5	6	7	8	9	$\gamma\frac{1}{4}$	$\gamma\frac{1}{8}$	γ

Ordinates below $N\gamma$				Occ.		λ		Suborb.		Aur.		Op.		Lengths					Angles					
8	9	$\gamma\frac{1}{2}$	$\gamma\frac{1}{4}$	x	y	x	y	x from N	y	x from γ	y	x from γ	y	GI	NI	γ to Bas.	N to Bas.	γI	$\phi = \angle\beta GI$	$\phi' = \angle\beta NI$	$A\angle$	$N\angle$	$B\angle$	$P\angle$

The type contours are constructed as before. See Figs. I and II.

(*d*) *General remarks on Contours.* Illustrations of the type contour of the Moriori are shown in Figs. I to VI. For the sagittal contour:

N = Nasion, *Gl.* = Glabellar point, B = basion, β = Bregma, V = vertex, λ = Lambda, γ = Gamma, I = Inion, *Op.* = Opisthion, *Aur.* = Auricular point, *Bas.* = Basion, *Alv.* = Alveolar point, *Suborb.* = Infra-orbital point. *Suborb.* to *Aur.* if joined by a line gives the standard horizontal plane. $N\gamma$ is the horizontal plane through the nasion. The angles N, B and A are the nasal, basilar and alveolar angles of *N B Alv.* the fundamental triangle. P = the profile angle measured from the Frankfurt horizontal, ϕ = the angle between the glabella-inion and glabella-bregma lines, ϕ' = the angle between the nasion-inion and nasion-bregma lines. There are three interesting subtenses, s, x and y. y is the maximum subtense or height of the calvaria measured from the glabella-inion line, s is the corresponding bregma subtense and x is the bregma subtense to nasio-lambda line.

The most marked characteristics of the Moriori skull are the depressed and retreating forehead, the bulging glabella, and the high sagittal crest. On looking at Figs. I and II, it will be seen that these points are well demonstrated in the type sagittal contours. The glabella is not of course so pronounced in the female contour, but it is pronounced for a female skull. Figs. III and IV, illustrating the transverse contours of these skulls, show nothing particularly characteristic; the sagittal crest can be seen, but it is more marked further back than the transverse section. Figs. V and VI, showing the horizontal contours, demonstrate very markedly that the Moriori skull is of a low type. There is a very substantial difference between the type horizontal contour of the Moriori skull and that of the English skull illustrated in Dr Crewdson Benington's paper on Cranial Type Contours: see *Biometrika,* Vol. VIII. p. 123. It may also be noted that in these Moriori horizontal contours, there is a decided flattening at the occipital.

Before we discuss the quantitative value of these contours it is well to examine the arithmetical results obtained for the measured characters.

TABLE I. *Mean Values.*

Character	♂	♀	Character	♂	♀
Capacity	1422·07	1310·73	G_1	53·12	50·46
	(1423·03)*	(1308·47)*	G_2	38·85	37·10
Weight	764·27	622·23	GL	100·93	97·36
F	182·00	176·09	fml	35·29	34·41
L'	185·64	177·77	fmb	30·78	30·15
L	186·87	178·60	GH	121·8	108·57
B	141·40	138·41	w_1	123·6	114·0
B'	95·31	94·01	w_2	106·4	101·87
H	135·91	132·81	h_1	33·17	30·27
OH	117·05	115·08	zz	51·48	48·5
LB	105·60	102·31	$Oc\ I$	62·17	64·15
Q	316·11	310·00	B/L'	76·51	77·90
Q'	315·94	310·20	H/L'	73·23	74·76
S	368·77	354·40	B/L	76·07	77·52
S_1	126·03	121·39	H/L	72·78	74·52
S_2	122·40	117·36	B/H	104·53	103·64
S_3	121·10	115·95	$G'H/GB$	74·13	72·09
S'_3	102·34	99·60	NB/NH	43·91	48·22
U	522·79	502·59	O_2/O_1L	84·03	84·30
$G'H$	76·40	70·01	O_2/O_1R	84·35	84·59
GB	103·15	96·63	G_2/G_1	73·20	73·71
J	137·43	128·93	$\frac{fml}{fmb}$	87·43	87·78
NH	57·33	52·53			
NB	25·30	25·27	$N\angle$	64·84	65·67
O_1R	44·61	42·67	$A\angle$	71·75	73·64
O_1L	44·42	42·36	$B\angle$	43·39	40·67
O_2R	37·59	36·01	θ_2	12·88	10·84
O_2L	37·29	35·52	θ_1	30·33	29·83
G'_1	50·10	47·38	$P\angle$	84°·69	84°·50

* Values obtained by "loose packing."

4. *Mean Values of Characters.* To judge of the degree of accuracy possible in reaching a Moriori type, we must note that the means were based for males on 33 to 35 crania and for females on 20 to 23 crania, according to the character measured; the one exception being capacity, for which only 28 male and 19 female crania were available. The non-adult crania have been omitted from the mean value determinations. The values are given in Table I. The question now arises: how far can we consider these values as adequately describing the Moriori skull? We may first compare our results with those obtained by Dr Duckworth for the Moriori crania at Cambridge*. Owing to the smallness of his series (7 male and 3 female) he has pooled his sexes to obtain his means. We have pooled ours in the same proportions; the following values result:

	Thomson	Duckworth
Maximum Length	184·4	182·5
Maximum Breadth	140·5	142·5
Ophryo-Iniac Length ...	170·5†	173·0
Basi-bregmatic Height ...	135·0	132·5
Nasal Height	55·9	54·2
Nasal Width	25·3	24·2
Basi-nasal Length	104·6	101·2
Basi-alveolar Length	99·9	97·2
Horizontal Circumference ...	516·7	513·6

With the exception of the breadth and our doubtful ophryo-iniac length, Dr Duckworth's crania appear slightly smaller than those at the Royal College of Surgeons. A somewhat closer result is obtained if we pool our males and females without regard to Duckworth's proportions. His numbers are, however, too few for us to do more than say that his results agree not unreasonably with ours‡.

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[*] *loc. cit.* p. 150. No. 4 is given ? ♀. No. 2 is given as sex doubtful on p. 142, but as definite ♂ in table on p. 150.

[†] Obtained only from contours.

[‡] The German anthropological catalogue (*Die Anthropologische Sammlungen Deutschlands*, Bd. I. 1877—1902, Berlin v. S. 51—2, Bresike Catalogue, 1880) provides measurements of three Moriori crania from the Chatham Islands in the Anatomical Museum of Berlin University. Two of the skulls are of children and the third of an aged person. They are thus of little use for comparative purposes. The measurements given of one of the young skulls which retains one milk tooth and is metopic are:

$$C = 1300,\ L = 170,\ B = 137,\ H = 126,\ S_1 = 121,\ S_2 = 124,\ S_3 = 108,\ GH = 97.$$

The second child skull is metopic, has no 3rd molars and the palate is not yet closed; further it is deformed, a large area on right side of temporal and left side of parietal being much depressed. The measurements given are:

$$C = 1140,\ L = 164,\ B = 133,\ H = 126,\ S_1 = 116,\ S_2 = 104,\ S_3 = 125,\ GH = 93.$$

The aged skull has depression of the parietals, alveolar rim much absorbed after loss of incisors and the three sutures completely synostosed. The measurements given are:

$$C = 1450,\ L = 183,\ B = 136,\ H = 136,\ S_1 = 130,\ S_2 = 125,\ S_3 = 120.$$

The greatest length, assuming it a male, is somewhat below the average, H is about normal, S_3 and B somewhat below average, but the other characters including the capacity above the average, but not greatly. It appears from the description—namely roof shaped, with slightly retreating forehead, More valuable comparative material is that of Scott*. I have placed in parallel columns in Table II those measurements of Dr Scott which in any degree correspond with mine. We agree so closely in the main straightforward measurements, that where a really substantial difference is apparent, I think it must be attributable to differences in method of measuring rather than to differences of material. I ought to say that I am responsible for the determination of the means from Dr Scott's tables of measurements. While the sex ratio in my material is roughly as 3 : 2 that of Dr Scott's is 3 : 1, so that he has only about half my number of females, although his males are about the same in number as mine. I think it probable that we have followed a different appreciation of sex, because, while his male measurements are on the whole very close to mine, practically all his absolute female measurements are less; thus, the difference he gets between the sexes is far greater than my difference, and rather greater than is usual among uncivilised races, where there is a greater tendency to equality in the measurements than among civilised races. I would venture with all deference to suggest that possibly size has weighed too much with him in sexing†. The first difference between our results to be noted is 33 cm.^{3} in the male capacity, but he used shot and I used mustard seed and a difference of this order may well arise with such difference of method. It is more or less confirmed by the result for female crania, because although his skulls are distinctly less than mine in size, he gets only a difference of 19 cm.^{3} in the capacity. Our F, L, B, B', H and (?) LB‡ values are in good agreement for males; so also are our U, $G'H$, J, NH, NB, O_2L, G_1, GL, fml, fmb, and all the indices except the orbital index for the same sex. Our difference in the orbital index is due to measurement of the breadth O_1L, but this is precisely a measurement, which, without *absolutely* identical method, often leads to great divergence. Turning to our differences we first note Q, the transverse arc; the considerable difference here is repeated in the females.

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prognathous, and with the teeth very much ground down—to have the usual Moriori characteristics. Four crania, one a child's, two male and one female, were collected during the *Challenger* Expedition. They are described and measured in the *Challenger Reports, Zoology*, Vol. x. p. 73, Report on the Human Crania. The two males are clearly fine crania, exceeding our average, the female is below average. Three crania are described by Zuckerkandl in the *Reise der Novara*; *Anthropologischer Theil*, 1 Abth. S. 104; but the few measurements given are of small use for comparative purposes. Finally five at Paris are considered by de Quatrefages and Hamy in the *Crania Ethnica*, p. 461, Paris, 1882. The three male crania appear to be very large as compared with our series, giving a mean $C = 1600$ cm.^{3} (!), $U = 548$, $L = 194$, $B = 146$, $H = 137$ and $J = 142$, while the two females have a mean $C = 1565$ cm.^{3} (!), $U = 524$, $L = 185$, $B = 144$, $H = 141$, $J = 132$.

Either the measurements are not according to our standards or the crania have been selected as specially remarkable skulls.

* *loc. cit.* on our p. 82.

† We have included Scott's three doubtful crania with his females, not only because he himself in his table puts them in that section, but because, being the rather larger crania of the section, they somewhat reduce the differences between his and our female mean characters.

‡ I think it possible that Dr Scott and I have not determined the alveolar point in precisely the same manner, and I have accordingly bracketed this measurement and the resulting Gnathic Index in Table II.

TABLE II*.

Mean Characters of Moriori Crania, Thomson and Scott.

	Males			Females		
	Thomson	Scott	Combined	Thomson	Scott	Combined
Capacity	1422·07 (28)	1454·97 (29)	1438·81 (57)	1310·73 (19)	1291·22 (9)	1304·46 (28)
F	182·00 (35)	181·08 (32)	181·56 (67)	176·09 (22)	172·60 (10)	175·00 (32)
L	186·87 (35)	186·84 (31)	186·86 (66)	178·60 (22)	175·60 (10)	177·66 (32)
B	141·40 (34)	142·10 (31)	141·74 (65)	138·41 (22)	134·20 (10)	137·09 (32)
B'	95·31 (35)	94·09 (32)	94·73 (67)	94·02 (22)	91·20 (10)	93·14 (32)
H	135·91 (34)	135·59 (32)	135·75 (66)	132·81 (21)	128·10 (10)	131·29 (31)
LB	105·60 (34)	103·75 (32)	[104·70] (66)	102·31 (21)	98·70 (10)	[101·15] (31)
Q	316·11 (34)	307·12 (32)	[310·81] (66)	310·00 (21)	289·00 (10)	[303·23] (31)
S_1	126·03 (34)	130·25 (32)	[128·09] (66)	121·39 (23)	120·40 (10)	121·09 (33)
S_2	122·40 (34)	122·56 (32)	122·48 (66)	117·36 (23)	117·00 (10)	117·25 (33)
S_3	121·10 (34)	122·84 (32)	121·94 (66)	115·95 (22)	114·00 (10)	115·34 (32)
S'	368·77 (34)	375·75 (32)	[372·15] (66)	354·40 (22)	351·40 (10)	353·46 (32)
U	522·79 (34)	521·97 (31)	522·40 (65)	502·59 (22)	493·80 (10)	499·74 (32)
$G'H$	76·40 (35)	75·86 (28)	76·16 (63)	70·01 (21)	67·25 (8)	69·25 (29)
J	137·43 (34)	138·89 (28)	138·09 (62)	128·93 (20)	127·10 (10)	128·32 (30)
NH	57·33 (35)	57·50 (32)	57·41 (67)	52·53 (21)	50·90 (10)	52·00 (31)
NB	25·30 (35)	26·47 (32)	25·86 (67)	25·27 (21)	24·80 (10)	25·12 (31)
O_1L	44·42 (35)	41·53 (32)	[43·04] (67)	42·36 (21)	39·10 (10)	[41·31] (31)
O_2L	37·29 (34)	36·75 (32)	37·03 (66)	35·52 (21)	35·30 (10)	35·45 (31)
G_1	53·12 (33)	54·10 (30)	53·59 (63)	50·46 (21)	49·00 (10)	49·99 (31)
GL	100·93 (34)	101·42 (31)	101·16 (65)	97·36 (21)	96·50 (10)	97·08 (31)
fml	35·29 (33)	35·53 (32)	35·41 (65)	34·41 (20)	34·00 (10)	34·27 (30)
fmb	30·78 (34)	30·94 (32)	30·86 (66)	30·15 (22)	29·70 (10)	30·01 (32)
B/L	76·07 (34)	76·29 (30)	76·17 (64)	77·52 (22)	76·44 (10)	77·18 (32)
H/L	72·78 (34)	72·60 (30)	72·70 (64)	74·52 (21)	72·96 (10)	74·02 (31)
NB/NH	43·92 (34)	46·14 (32)	44·99 (66)	48·22 (21)	48·75 (10)	48·39 (31)
O_2/O_1	84·03 (33)	88·64 (32)	[87·65] (65)	84·30 (21)	90·32 (10)	[86·24] (31)
fml/fmb	87·43 (32)	87·20 (32)	87·32 (64)	87·78 (20)	87·44 (10)	87·67 (30)
Gnathic Index	95·58† (34)	97·81 (31)	[96·64] (65)	95·16† (21)	97·18 (10)	[95·81] (31)

* Figures in round brackets give the number of skulls measured. Means in square brackets are based on possibly non-comparable methods of measurement.

† Found from mean lengths only.

I think it is due to difference in method. Assuming Scott to have measured from auricular margin to auricular margin, he has probably measured the least (or geodesic) arc, and not through the "apex"*. Our difference in the frontal part of the sagittal arc, S_1, is very possibly due to his more careful pressing of the tape round the glabella, especially as it is less sensible in the female results. The upper face height is a measurement subject to some variation and doubt if the alveolar ridge be at all worn; our value of $G'H$ seems reasonable, but Scott's to be in excess of any comparative material. His values for the female differ in upper face height in exactly the opposite direction. Thus in the 27 measurements compared in the case of the male crania, our differences in four, the capacity, the transverse arc, the frontal section of sagittal arc and the orbital breadth (involving a fifth, the orbital index) are, I believe, due to differences in the methods of measurement. In the case of the female measurements the comparison is not so satisfactory; not only are my numbers small, but Scott's are still smaller. This fact, however, would not account for his absolute lengths being almost invariably smaller than mine. As I have said I think he may have laid too great stress on mere smallness of the skull in sexing. Notwithstanding the combined measurements, transverse arc, orbital breadth and orbital index excepted, give even in the case of the female crania by no means unsatisfactory results. Unless —which seems now very improbable—further long series of Moriori crania are found, Table II, combining Scott's results with my new measurements, must stand as the best available system of Moriori cranial data. It involves measurements of nearly 100 skulls of a race now practically extinct, and should be helpful in determining the racial affinities of the Moriori.

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* He measured the entire transverse circumference and divided it into "supra-" and "infra-" auricular parts, not further defined.

Before leaving Scott's important paper, I should like to note some of the peculiarities of the Moriori crania emphasised by him. He states that typical Moriori crania are usually large and heavy; our table of comparative measurements (Table III) amply verifies this statement as to size. As to the weights we find:

Mean Weight of Crania.

	Moriori (Thomson)	Fuegian (Sergi and Martin)	Egyptian (Davin)
Males	764·3 grs.	872·3 grs.	623·6 grs.
Females	622·2 grs.	722·5 grs.	482·4 grs.

It is possible that the Fuegian crania were weighed, unlike ours, with mandible attached. If so this would account for some, if not all the difference observed. Clearly the Fuegians have very heavy crania, and the Moriori heavy crania, but the latter are not absolutely unique in this respect. The average weight of an Egyptian mandible is 73·1 grs., but of course in the case of these crania the material is in a very different state to the Moriori crania. Still I do not think that the observed difference, 108 in males and 100 in females, can be wholly due to mandibles being included in Sergi and Martin's weighings of Fuegian and not included in our weighings of Moriori. I am inclined to think that while the Moriori crania are heavy, they are not as heavy as the Fuegian. The contours also bring out the same point of great size.

Scott further draws attention to the prominent parietal eminences, the roof-like vertices and the pentagonal character of the *norma occipitalis.* Our Plates XIV, XV, IV, and VI will amply demonstrate to the reader these Moriori features. Further he remarks on the low flattened retreating frontal region as a most striking feature (cf. our Plates III, VII and XV). This character is well measured by the *frontal index* = 100 × subtense of nasio-bregmatic arc/nasio-bregmatic chord. The value of this index for the Moriori crania is ♂ 19·39, ♀ 21·14. Comparative numbers are:

Cranium of the Murderer Thurtell	♂ 21·65
Whitechapel English of 17th century	♂ 22·21
Guanche Crania	♂ 22·50
Egyptians of XXVI–XXX Dynasty	♂ 22·77
Eskimo Crania	♂ 22·78
Negroes (Congo)	♂ 22·80
Cro-Magnon Skull	♂ 24·27

It will be seen that the male Moriori stands lowest on the list of racial values available.

Of further points noted by Scott the statement* that "the excess of width over height is generally well-marked; indeed in the most typical members of the group the brain case is distinctly flattened," needs, perhaps, further consideration. The feature in question may be measured by the use of Pearson's index $100\,(B-H)/L$. We have then the following racial table:

Values of $100\,(B-H)/L$.

Würtemberg Women ...	+ 9·6	Aino Women	+ 0·9
Würtemberg Men	+ 9·5	Aino Men	+ 0·9
Bavarian Women	+ 9·2	Gaboon Men (1880)	+ 0·9
Bavarian Men	+ 9·0	Maori Men	+ 0·7
English Women (Moorfields) ...	+ 7·9	Maori Women	+ 0·5
French Men	+ 7·2	Coptic Women	+ 0·5
English Men (Moorfields) ...	+ 7·1	Gaboon Women (1864) ...	+ 0·5
English Women (Whitechapel)	+ 5·6	Theban Mummies, Men ...	+ 0·4
English Men (Whitechapel) ...	+ 4·2	Gaboon Men (1864)	+ 0·0
Moriori Men	+ 3·5	Kaffir Men	+ 0·0
Moriori Women	+ 3·2	Australian Men	− 0·2
Congo Men	+ 2·6	Coptic Men	− 0·2
Australian Women	+ 2·4	Prehistoric Egyptian (Naqada) Men	− 0·3
Long Barrow Women ...	+ 2·3	Gaboon Women (1880) ...	− 0·4
Long Barrow Men	+ 2·2	Zulu Men	− 0·7
Fuegian Men	+ 2·0	Angoni Men	− 1·2
Congo Women†	+ 1·9	Negro Men (North Africa) ...	− 1·3
Theban Mummies, Women ...	+ 1·8		
Prehistoric Egyptian (Naqada) Women	+ 1·0		

Judged by this index the Moriori stand for excess of breadth over height per unit of length at the top of all the early and primitive races and nearest to modern Europeans, and this is a feature which differentiates them from Maori and

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* *loc. cit.* p. 25.

† Erroneously placed as − 1·9 in *Biometrika*, Vol. VIII. p. 307.

Australians and markedly from the full-blooded negro races. Scott continues: "the orbits are as a rule high, and the appearance of height is increased by the form of the superciliary ridges; while the nasal opening is narrow*." Table III (p. 98) will show that the orbits (O_2L) are higher than any of the comparative material provided, although the Fuegians come fairly close and the Maori follow them, but they are distinctly higher than modern European races and the Negroes. On the other hand the orbits are distinctly broader than those of any other race, and the result is that relatively to the breadth they about equal the Fuegians in orbit, fall behind the Maori and about equal the Negroes and Bavarians. It would probably be better to state that Moriori orbits are large rather than high. Again the statement that the Moriori nasal opening is narrow requires qualification. Its absolute breadth is about the same as Maori, Fuegian and Aino, and in excess of modern Europeans. It will be found that it is the nasal height of the Moriori which is the noteworthy absolute feature of their noses, and the result is that their nasal index falls below that of any of the races placed in Table III for comparison. This is markedly true of the males.

Of general descriptive properties Dr Scott says there were no metopic crania in his series, that the coronal and sagittal sutures were simple, but the lambdoid often very complicated; these remarks would apply well to the Royal College of Surgeons' collection, although two of the skulls of Moriori children at Berlin are, as might well be, metopic. The Moriori cranium does, indeed, strike one as "primitive," but in a different sense to the Australian cranium. A comparison of the following table will show how very little resemblance there is between the Australian and Moriori crania:

Character	Australians (Robertson†)		Moriori (Thomson)	
	♂ (78)	♀ (22)	♂ (34)	♀ (22)
Maximum Length (L) ...	183·56	175·59	186·87	178·60
Maximum Breadth (B)...	130·60	128·73	141·40	138·41
Basio-apical Height (H)	131·13	124·50	135·91	132·81
100 B/L	71·29	73·36	76·07	77·52
100 H/L	71·50	70·95	72·78	74·52
100 H/B	100·50	96·64	96·12	95·95

It is clear that the Moriori is a much larger skull than the aboriginal Australian, and of wholly different proportions. And yet those who have handled the Moriori skull cannot doubt that in its special manner it is as primitive as the Australian. The very bulging glabella, the low receding forehead, the considerable flattening on either side of the sagittal suture giving rise to the sagittal crest, combined with great weight strike one at once. Anatomical peculiarities as will be noticed from my "Remarks" are frequent. Thus flattening of the obelion, parietal bulging, ossicles of the lambda are common; epipteric ossicles were noted in 17 out of the 63 crania. There were four cases of asymmetric foramina of which the most pronounced is photographed in Plate XIII. Marked inions or inionic ridges occur in several cases, while of rarer anomalies I noted an *incisura crotaphitico-buccinatorus* in one female, an ossicle of the coronal suture (Plate XV), and para-occipital processes articulating with the axis in other cases. Of non-congenital deformities, perhaps the most remarkable is a healed fracture of the mandible on the left side (Plates XX—XXII). Extreme attrition of teeth* was also noticed in a large proportion of the crania in this collection, a feature on which Dr Duckworth has already laid great stress in his paper. The general asymmetry of the skulls will be well realised, not only from a perusal of the remarks on the individual crania, but also by examining the measurements right and left given on the horizontal and transverse type contours. To sum up in the language of Flower and Turner, the Moriori are a megacephalic, mesatricephalic, metriocephalic, mesoseme (although close to the borders of microseme), leptorhine in male, mesorhine in female, orthognathous, brachyuranic† race. They are thus mediocre in most cranial characters, but their non-mediocre features, those of megacephaly, orthognathy and brachyurany, associate them rather with the more highly civilised than with the low races. As we have already said, the "primitive" character of the Moriori cranium does not lie in the same characters as differentiate the Australian aborigines or even the Negro from the European. It is a type of primitiveness hardly described by such characters as we have studied above. It is, perhaps, better considered in reference to the contours.

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* *loc. cit.* p. 25. It should be noted that Scott means these remarks to apply to his first type, but he admits the continuity of his first with his second type (p. 26).

† *Proceedings R. S. Edinburgh*, 1910—1911, Vol. XXXI. pp. 1—16. Craniological Observations on the Length, Breadth and Height of 100 Australian Aboriginal Crania.

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* A similar attrition of the teeth was noticed in the prehistoric Egyptians and has been attributed by some to the use of stone mortars for grinding the grain.

† Scott (*loc. cit.* p. 4) defines palato-maxillary index as 100 × palato-maxillary length/palato-maxillary breadth. But his lengths are always less than his breadths, just as are Turner's corresponding values for the four Moriori skulls acquired on the *Challenger* Expedition. Scott has accordingly, since his indices are over 100, really used, like Turner, the ratio of breadth to length. Turner following Flower measures the length as the distance from the alveolar point to the line drawn between the most posterior parts of the maxillary tuberosities. This is not exactly our palate length, but appears close to it in value. On the other hand Scott and Turner include in their palate breadth the alveolar arch and thus get a breadth very largely in excess of ours, and, we consider, no true palate breadth at all. To allow for the breadth of the alveolar arch, we have supposed it proportional to the palate breadth as measured in our manner. But we can only compare our breadth with Scott's breadth for our two series as we have not the two measurements in either case. There results for the mean breadth of the (double) alveolar arch at the second molars:

Moriori males, 26·47, Moriori females, 22·79.

We thus deduce for our Moriori the palato-maxillary index of Turner as 123 for males and 119 for females corresponding to the brachyuranic group. If we reverse the process, altering the above values in the ratio of Scott's palatal breadth for Maori to his palatal breadth for Moriori, i.e. subtract 26·47 × 62·77/65·32 from 62·77 and 22·79 × 59·40/59·89 from 59·40 for males and females respectively, we obtain the values for the palatal breadths on our plan, i.e. 37·33 and 36·80 recorded in brackets under Maori in Table III, and the corresponding values of the palatal index.

TABLE III. *Comparison of Moriori and other Races in Cranial Characters.*

Character	Moriori		Maori		Aino		Fuegian		Egyptian (Naqadas)		Northern Negroes
	♂ (c. 34)	♀ (c. 22)	♂ (c. 43)	♀ (c. 20)	♂ (c. 88)	♀ (c. 63)	♂ (c. 34)	♀ (c. 24)	♂ (c. 98)	♀ (c. 123)	♂ (c. 39)
C	1498	1378	1476	1288	1462	1308	[1474]	[1290]	1381	1288	1348
L	186·9	178·6	185·5	175·6	185·8	177·2	192·0	185·0	185·1	177·5	182·9
B	141·4	138·4	140·1	134·8	141·2	136·8	145·0	141·0	134·9	131·5	133·2
B'	95·3	94·0	95·7	91·2	96·2	92·4	97·1	91·0	91·1	88·2	95·9
H	135·9	132·8	137·6	133·0	139·5	135·1	141·0	137·0	135·2	129·5	135·3
OH	117·1	115·1	—	—	119·3	115·0	—	—	115·5	113·1	115·0
LB	105·6	102·3	103·9	100·5	105·4	100·3	{104·5}	{96·3}	99·3	94·9	103·0
$P\angle$	84°·7	84°·5	—	—	82°·0	81°·0	{82°·0	Sergi}	84°·4	84°·5	82°·0
Q	316·1	310·0	307·5	293·8	328·5	317·1	{316·5}	{298·6}	304·2	296·5	306·8
S	368·5	354·4	378·1	361·8	372·8	360·7	{390·0}	{370·0}	373·0	363·6	367·7
U	522·8	502·6	518·0	496·8	522·5	501·7	531·0	502·0	511·0	493·7	508·5
$G'H$	70·6	69·6	71·4	64·4	69·8	65·5	{71·3}	{69·7}	67·6	65·8	68·0
GB	103·2	96·6	—	—	102·1	96·7	[110·0?]	[103·2?]	95·9	92·5	96·6
J	137·4	128·9	136·8	127·9	137·3	129·7	143·6	132·1	125·6	117·0	129·5
NH	57·3	52·5	53·8	49·4	50·5	47·4	{54·0}	{49·6}	48·9	46·7	49·5
NB	25·3	25·3	25·6	23·4	25·6	24·7	{26·3}	{23·8}	25·1	24·3	27·3
O_1L	44·4	42·4	40·5	39·2	40·9	39·8	{42·5}	{40·3}	38·1	37·2	39·6
O_2L	37·3	35·5	35·0	33·6	34·9	33·9	{36·0}	{34·7}	32·3	32·0	34·2
G_1	53·1	50·5	52·4	48·5	53·0	51·4	[{56·0}]	[{54·0}]	55·8	53·9	49·4
G_2	38·9	37·1	[37·3]	[36·8]	38·2	37·4	[{43·0}]	[{39·0}]	40·3	38·9	39·7
GL	100·9	97·4	100·8	96·9	104·9	100·1	{100·0}	{96·0}	94·7	91·0	105·3
fml	35·3	34·4	35·2	33·1	—	—	{36·0}	{37·3}	—	—	—
fmb	30·8	30·2	30·8	29·0	—	—	{31·0}	{31·7}	—	—	—
$100\ B/L$ ...	76·1	77·5	75·4	74·7	76·5	77·6	76·6	77·7	73·0	74·2	72·9
$100\ H/L$ ...	72·8	74·5	74·7	74·2	75·6	76·7	73·4	72·9	73·3	73·2	74·2
$100\ B/H$ ...	104·5	103·6	[101·8]	[101·4]	101·24	101·24	[102·8]	[102·9]	99·8	101·6	98·3
$100\ NB/NH$ ...	43·9	48·2	47·9	49·1	50·7	50·5	{48·5}	{47·9}	51·1	52·3	55·4
$100\ O_2/O_1$ ...	84·0	84·3	86·1	86·2	85·3	85·2	{84·6}	{85·0}	84·8	86·0	87·2
$100\ G_2/G_1$ ...	74·1	72·1	[71·2]	[75·9]	72·1	72·8	[{76·8}]	[{72·2]}	71·9	72·3	81·5
$100\ fmb/fml$ ...	87·4	87·8	87·4	88·6	—	—	89·0	84·2	—	—	—
Gnathic Index	[95·6]	[95·2]	97·0	96·4	[99·5]	[99·8]	96·2	99·3	[95·4]	[95·9]	[102·3]

Character	Congo Bantus		Gaboon Bantus		English (Whitechapel)		Long Barrow		Bavarian (Alt-bayerische)	
	♂ (c. 50)	♀ (c. 27)	♂ (c. 50)	♀ (c. 44)	♂ (c. 72)	♀ (c. 80)	♂ (c. 12)	♀ (c. 8)	♂ (c. 100)	♀ (c. 100)
C	1344	1206	1381	1232	1477	1300	—	—	1504	1336
L	177·8	171·4	179·5	171·6	189·1	180·4	190·6	182·6	180·6	173·5
B	138·5	130·9	135·5	130·6	140·7	134·7	142·4	138·6	150·5	144·0
B'	97·5	92·6	96·5	92·8	98·0	93·1	98·9	94·1	103·7	96·3
H	133·8	127·5	135·4	130·1	132·0	124·6	137·8	135·1	133·8	128·0
OH	113·9	109·0	—	—	114·6	109·2	120·7	118·0	[111·5?]	[109·0?]
LB	98·2	93·3	100·3	95·8	101·6	95·3	102·0	96·8	100·3	96·3
$P\angle$	82°·8	81°·1	—	—	86°·1	87°·1	83°·0	?	89°·1	88°·8
Q	310·7	298·2	310·5	296·3	307·9	294·0	321·8	312·0	329·7	318·7
S	361·8	349·8	364·3	350·4	377·1	362·8	384·8	382·0	365·1	353·4
U	504·3	489·3	501·2	481·2	524·3	503·8	534·9	518·7	524·4	501·4
$G'H$	63·4	59·3	66·4	62·7	70·2	65·9	69·9	66·8	70·8	66·8
GB	94·8	92·2	95·6	90·9	90·9	84·9	95·9	92·8	95·1	89·7
J	126·5	122·1	129·0	120·5	130·1	120·3	134·0	132·5	135·0	126·3
NH	47·2	45·0	48·2	45·1	51·2	48·7	49·4	47·0	50·9	48·2
NB	26·0	25·0	26·6	24·7	24·3	23·2	24·1	22·8	24·8	23·7
O_1L	40·4	40·5	41·5	39·6	43·1	41·2	38·1	36·9	39·9	38·3
O_2L	34·6	33·4	35·0	33·7	33·5	33·6	31·8	30·6	33·7	33·2
G_1	50·3	50·2	52·1	50·1	48·3	45·1	55·3	57·0	[44·3?]	[42·2?]
G_2	38·9	35·9	39·5	37·4	36·8	35·2	43·7	{36·0}	[33·2?]	[32·1?]
GL	96·5	93·9	100·5	98·5	95·9	90·4	95·3	92·6	—	—
fml	35·9	35·3	35·9	34·0	35·4*	34·3*	35·7	34·5	—	—
fmb	30·0	28·8	30·0	28·1	29·7*	29·0*	27·7	30·3	—	—
$100\ B/L$ ...	78·0	76·5	75·5	76·2	74·3	74·7	74·9	76·3	83·2	83·1
$100\ H/L$ ...	75·4	74·6	75·5	75·7	70·0	69·1	72·7	74·0	74·2	73·9
$100\ B/H$ ...	103·6	102·6	100·1	100·7	106·3	108·5	[103·3]	[102·6]	112·5	112·5
$100\ NB/NH$ ...	55·2	55·8	55·3	54·9	47·6	47·8	49·0	49·1	48·7	49·2
$100\ O_2/O_1$ ...	85·7	82·5	84·6	85·2	77·9	81·7	83·4	81·7	84·5	86·6
$100\ G_2/G_1$ ...	76·8	70·8	76·1	74·8	76·3	77·7	[79·0]	[{63·2}]	[74·4]	[76·0]
$100\ fmb/fml$ ...	83·5	81·8	83·9	81·2	84·2*	84·5*	[77·7]	[87·7]	—	—
Gnathic Index	[98·2]	[100·6]	[100·4]	[101·8]	[94·4]	[94·8]	[99·4]	[99·9]	—	—

* Values for English (Moorfields).

5. *Comparison of Moriori cranial Characters with those of other Races.* In Table III I have gathered together such material as seemed of interest for comparative purposes. Unfortunately I have not been able to obtain a good series of Australian measurements for comparison, but the values of the few characters given by Robertson for fairly long series, show that there is little relationship between the two races. In the first place we put Scott's Maori values. We follow these by data for the Aino, the means being calculated from the measurements in Koganei's* Tables; then such slender results as we have been able to obtain for the Fuegians†. Lastly we place three Negro races, North Africans‡ and Benington's Congo and Gaboon measurements§, to complete the series of non-European primitive races. For European races we give English|| and Bavarians¶ as modern representatives, and the Long Barrow** as an ancient race, unfortunately with very slender numbers. Finally the Egyptians find representation in Fawcett's Naqada measurements††.

In Table III we have been obliged to make various assumptions to deduce certain of the values provided. They are either not provided by the authorities cited, or have been differently measured. The difficulties are greatest when we come to deal with orbital and palate measurements, where most divergence of method has existed. We have in certain cases had to allow for length of spine or breadth of alveolar arch as obtained from other cranial series. Again some authorities do not record the index‡‡ 100 B/H or the gnathic index, and these have had to be supplied from the ratio of mean values. All such measurements are enclosed in square brackets, and when especially doubtful are marked by queries. In other cases where the numbers dealt with are quite inadequate the values are enclosed in curled brackets. We cannot in the present paper deal at length with the basis of these corrections and modifications but a few words as to our treatment of Fuegian data may make clear certain points. The fundamental memoir here is that of Rudolf Martin. He had only five Fuegian crania, but before his date,

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* "Beiträge zur physischen Anthropologie der Aino." *Mittheilungen aus der medicinischen Facultät der k. j. Universität.* Tokio, 1894, Bd. II. Reduced in 1900 in the Biometric Laboratory.

† Rudolph Martin, "Zur physischen Anthropologie der Feuerländer," *Zeitschrift für Anthropologie,* Bd. XXII. S. 155—218, 1893. J. V. Hultkrantz, "Zur Osteologie der Ona und Yahgan Indianer des Feuerlandes," *Wissenschaftliche Ergebnisse der schwedischen Expedition nach den Magellanländern 1895—97 unter Leitung von O. Nordenskjoeld.* Stockholm, 1900.

‡ Reduced in the Biometric Laboratory in 1900 from data in the German Anthropological Catalogues.

§ R. Crewdson Benington, "A Study of the Negro skull with special reference to the Congo and Gaboon Crania." *Biometrika,* Vol. VIII. pp. 292—337, 1912.

|| Results for Whitechapel 17th century English used; see W. R. Macdonell, "A Study of the Variation and Correlation of the Human Skull, with special reference to English Crania." *Biometrika,* Vol. III. pp. 191—244, 1904.

¶ Reduced in Biometric Laboratory in 1900 from the data in J. Ranke's paper, "Beiträge zur physischen Anthropologie der Baiern," Bd. I.

** Edgar H. J. Schuster, "Long Barrow and Round Barrow Skulls in the Oxford Museum." *Biometrika,* Vol. IV. pp. 351—362, 1905.

†† C. D. Fawcett, "A Second Study of the Variation and Correlation of the Human Skull with special reference to the Naqada Crania." *Biometrika,* Vol. I. pp. 408—465, 1902.

‡‡ Unfortunately in the tables (*Biometrika,* Vol. I. pp. 426—7) Fawcett has for the two races Bavarian and Aino (♂ and ♀) the values of 100 H/B put under 100 B/H.

1893, Mantegazza had dealt with 16, Sergi with 14, Garson with 8, Hyades with 8, Sir William Turner with 4 and Quatrefages with 2 skulls. Since his date Hultkrantz has had six additional specimens to measure. Thus there are really between 60 and 70 Fuegian crania certainly available and no doubt if further search were made we might run the value up to nearly 100, and get substantial and important results. Unfortunately some of the writers referred to made very inadequate series of measurements, even on the few crania they had at their command. Martin puts together a good deal of their measurements, but apparently thinks it sufficient to average their averages. This is somewhat dangerous considering the large differences in the numbers they have dealt with. He does not also, we think, sufficiently emphasise the wide divergencies to be expected when means are based on 2 to 10 crania. We have endeavoured to weight the results of the various recorders and such are the numbers given without *curled* brackets in the Fuegian column. The values in curled brackets are based solely on Martin's 2 ♂ and 3 ♀ crania. The statistically trained mind would say that they are worthless as a determination of type. With this statement we should not fail to concur, did not the means provided by Hultkrantz for his six skulls combined accord so closely with our results for male Fuegians as to force us to the conclusion that his crania were practically all males and that by a lucky chance we have reached even with such miserably slender material results closely approximating to a true Fuegian type*.

Character	Table III ♂	Hultkrantz	Character	Table III ♂	Hultkrantz
Capacity	1474	1443	Nasal Height ...	54·0	54·6
Maximum Length ...	192·0	191·5	Nasal Breadth ...	26·3	25·3
Maximum Breadth ...	145·0	146·1	Orbital Breadth ...	42·5	40·0
Basio-bregmatic Height	141·0	140·6	Orbital Height ...	36·0	35·3
Minimum Frontal Breadth	97·1	98·0	Orbital Index ...	84·6	87·4
Profile Angle	82°·0	83°·6	Cephalic Index ...	76·6	76·3
Horizontal Circumference	531·0	531·0	Height Index ...	73·4	72·1
Sagittal Circumference ...	390·0	381·0	Breadth-Height Index	102·8	103·9
Transverse Circumference	316·5	316·1	Foraminal Length ...	36·0	35·3
Bizygomatic Breadth ...	143·6	143·5	Foraminal Breadth ...	31·0	31·0
Profile Length	100·0	106·0	Foraminal Index ...	89·0	87·9

We see how remarkably alike the results are even for such short series, and there is sufficient indication that with uniform methods of measurement the existing Fuegian crania would suffice to provide a quite good typical series of values.

Turning to examine Table III a little more closely, we see that we can dismiss any relation to the Negro race, either in Sudanese or Bantu form. Egyptians and central Europeans show no affinities. It is only with the English or the Long Barrow crania that there is any approach, and the latter are so slender in number that it seems idle to deal in more detail with them. On the other hand there is undoubted affinity to the Maori; and rather surprising resemblances to the Aino and to the Fuegians are worth taking into consideration. The full discussion of the proper method of representing racial affinity by a single coefficient must remain undiscussed at present. A rough measure will only be used here; we take the mean percentage difference of each race, for males only, on the basis of the Moriori means, and then consider the mean of these percentages as a working coefficient.

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* A more thorough collection and examination of all available Fuegian data will shortly be undertaken.

Table IV gives these percentage differences to a single decimal place, and we note at once the following points:

(i) While the Maori are nearest of the races considered to the Moriori, both Aino and Fuegian are surprisingly close.

(ii) Seventeenth century English are more distant than the Fuegians, but have very far from the deviation of a race like the Sudanese Negroes (i.e. 3·92 as compared with 5·80).

(iii) The current view that the Moriori are like the Maori an intermixture of Polynesian and Melanesian stock scarcely receives confirmation from Table IV; the Moriori stand nearer to the Londoner of the 17th century than to the Negro.

(iv) The Maori themselves are still more closely related to the Fuegian (2·87 as against 3·13) than the Moriori.

Thus it is difficult to believe that there has not at some time been a race-link between the present denizens of extreme South America and the denizens of extreme Polynesia.

It is very regrettable that our data for the Long Barrow inhabitants of Britain are so slender. Macdonell has already pointed out their relative closeness to 17th century Londoners and the work of Crewdson Benington* seems to show that the English commonalty of to-day is one with the 17th century Londoners. We have to look to the bulk of the English population to-day as being non-Teutonic in origin, and much talk of our Germanic relationship may ultimately prove to be idle. Craniologically the English commonalty belong in bulk to an earlier and probably more primitive race. It would be unprofitable to speculate until we have larger series and more complete measurements—especially of Polynesian, Melanesian and American Indian crania—on the meaning of the results suggested in the present comparison between the denizens of the extreme fringes of enormous areas. They give one at least a suggestion of vast folk-wanderings in a world possibly of different conformation. Practically they warn us at least to consider the problem of the relationship of the American Indian to either a north-Asiatic or a Polynesian stock as still unsettled†. Even the English relationship shows that the European origin of the American is still a workable hypothesis. We have confidence that if our craniological series consisted of hundreds instead of often units, statistical methods would enable us to construct a racial human pedigree which would throw immense additional light on human descent. The first desideratum at the present time is the collection of crania from what we may call "fringe" races, and the second desideratum the construction of type crania, both by mean measurements and by contours, of the material thus collected. Only then will the science of craniology become a really useful guide to the anthropologist. At present arguments based on half-a-dozen crania—unaccompanied by any previous study of cranial variability—are of the most slender weight and should simply be disregarded by the anthropologist*.

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* *Biometrika*, Vol. VIII. p. 131.

† It is in our opinion impossible to accept the views of either Brinton or Hale as final.

TABLE IV. *Comparison of Male Crania of Moriori with those of other Races.*

Character	Percentage excess value of Moriori character over:				Northern negro
	Maori	Aino	Fuegian	English	
Capacity	+ 1·5	+ 2·4	+ 1·6	+ 1·4	+10·0
Maximum Length ...	+ 0·7	+ 0·6	− 2·7	− 1·2	+ 2·1
Maximum Breadth ...	+ 0·9	+ 0·1	− 2·5	+ 0·5	+ 5·8
Minimum Frontal Breadth	− 0·4	− 0·9	− 1·9	− 2·8	− 0·6
Basio-bregmatic Height ...	− 1·3	− 2·6	− 3·8	+ 2·9	− 0·4
Auricular Height ...	—	− 1·9	—	+ 2·1	+ 1·7
Cranial Base	+1·6	+ 0·2	+ 1·0	+ 3·8	+ 2·5
Profile angle	—	+ 3·2	+ 3·2	− 1·7	+ 3·2
Transverse Arc	+ 2·7	− 3·9	− 0·1	+ 2·6	+ 2·9
Sagittal Arc	− 2·6	− 1·2	− 5·8	− 2·3	+ 2·2
Horizontal Circumference	+ 0·9	+ 0·1	− 1·6	− 0·3	+ 2·7
Upper Face Height ...	− 1·1	+ 1·1	− 1·3	+ 0·6	+ 3·7
Face Breadth	—	+ 1·1	− 6·6	+11·9	+ 6·3
Bizygomatic Breadth ...	+ 0·4	+ 0·1	− 4·5	+ 5·4	+ 5·1
Nasal Height	+ 6·1	+11·9	+ 5·8	+10·6	+13·6
Nasal Breadth	− 1·2	− 1·2	− 4·0	+ 4·0	− 7·9
Orbital Length	+ 8·8	+ 7·8	+ 4·3	+ 2·9	+11·0
Orbital Breadth	+ 6·2	+ 6·4	+ 3·5	+10·2	+ 8·3
Palate Length	+ 1·3	+ 0·2	− 5·5	+ 9·0	+ 6·8
Palate Breadth	+ 3·9	+ 1·8	−10·6	+ 5·4	− 2·3
Profile Length	+ 0·1	− 4·0	+ 0·9	+ 5·0	− 4·4
Foraminal Length ...	+ 0·0	—	− 2·0	− 0·0	—
Foraminal Breadth ...	− 0·0	—	− 0·6	+ 3·6	—
Cephalic Index	+ 0·8	− 0·5	− 0·7	+ 2·4	+ 4·2
Vertical Index	− 2·6	− 3·8	− 0·8	+ 3·8	− 1·9
Breadth-Height Index ...	+ 2·6	+ 3·2	+ 1·5	− 1·6	+ 5·9
Nasal Index	− 9·1	−15·5	−10·5	− 8·2	−26·2
Orbital Index	− 2·5	− 1·5	− 0·7	+ 7·3	+ 3·8
Palatal Index	+ 3·9	+ 2·7	− 3·6	− 3·0	− 9·8
Foraminal Index	+ 0·1	—	− 1·8	+ 3·7	—
Gnathic Index	− 1·5	− 4·1	− 0·6	+ 1·3	+ 7·0
Number of Characters ...	28	28	30	31	28
Mean Percentage Deviation	**2·31**	**3·00**	**3·13**	**3·92**	**5·80**

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* It is astounding to note what wide reaching results even the examination of a mere half-dozen Moriori, Fijian, Maori or Eskimo crania led the older craniologists to propound!

6. *Study of the Moriori Type Contours.* The purposes served by these contours are (i) the comparison with the like contours of other races, and (ii) their use as a method for actually determining the mean values of certain characters. We shall consider them from both aspects.

For the first purpose the sole available material consists in the contours for Bantu, Egyptian (XXVI—XXX Dyn.), English, Eskimo and Guanche crania drawn to scale and published with tracings in Crewdson Benington's paper*. Some attempts to obtain type contours by other processes are either based on inadequate numbers, are deduced from far too few plotted points by an unsuitable method, or are reproduced to no standard, or at any rate to no easily comparable scale. For a type contour to be really valuable it should be drawn to full-size and published not only on ordinary but also on transparent paper. It is only in this form that it can be of real value for comparative purposes. Further the contour in the case of the sagittal section should contain the basal triangle and the occipital region. These were included in Dr Benington's individual contours, but unfortunately he omitted to add their mean values to his type contour. We have included them in the Moriori contours.

Starting with the transverse section, we note that the length of the interauricular line is relatively large as compared with the Bantu or Egyptian crania; it is more nearly approached by those of the Guanche or 17th century English, which, however, still fall short of it. The Eskimo transverse section, with a somewhat larger interauricular diameter, has near the "apex" a more marked sagittal crest; this crest so marked in the Moriori crania is, however, thrown as a rule further back in their case. If we consider a primitive form like the Cro-Magnon which has an interauricular length nearly as large as the Moriori type skull, its auricular height is so great that the Moriori transverse section falls entirely inside it. The following table suggests the nature of these results, and confirms the conclusion

Measurements on the Transverse Type Section†, ♂s.

Race	Interauricular Length	Auricular Height	Index
Moriori	125·2	109·5	87·5
Eskimo	128·0	114·2	89·2
English	122·4	111·5	91·1
Guanche	119·6	109·8	91·8
Egyptian	117·4	110·2	93·9
Congo, Bantu‡ ...	116·8	113·6	97·3
Cro-Magnon... ...	124·0	121·0	97·6

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* *Biometrika*, Vol. VIII. p. 123.

† These measurements are from the type sections; it must be carefully borne in mind that they may differ from measurements of the same name on the crania themselves.

‡ Fernand Vaz, 1864 Series.

previously reached, i.e. that the Moriori cranium approaches to a certain European type represented by modern English and by prehistoric Guanche* far more closely than it does to the Negro skull.

As measured on the actual skull Scott obtains values of the interauricular length which give a male mean = 126·8, and we have obtained directly for the auricular height of our series of males 117·1; the reasons for these divergencies will be considered later. It would have been of the greatest value had we possessed type sections of the Aino and Fuegian crania for further comparison.

Let us now consider the horizontal section. This as we have stated is taken through the glabella, while the cranium is arranged so that the plane of the drawing is parallel to the standard horizontal plane. Now the glabella is not a well-defined point, but rather a small area, and a little personal equation will make a considerable change in appearance of the horizontal section according as the plane of section recedes very little from the zygomatic and approaches slightly to the stephanic region. But as the horizontal sections discussed in the present paper are all drawn by members of the same school we are fairly certain that the concavity in the contour behind the temporal line is far more marked in the Moriori than in other crania we have drawn. But it has seemed to us—in view of the importance of the temporal lines from the evolutionary standpoint—desirable in future to take additional coordinates for the points, T_R and T_L†, where the section cuts the temporal line on the frontal bone. Otherwise these important points are lost in the general averaging out of the contours. Allowing, however, for some difference of treatment our contours as well as our plates (see especially Plates II, III, VII, IX, etc.) indicate how marked is the *linea temporalis* of the Moriori skull, and we think that this is an essential factor of our appreciation of its primitive character. If the reader will compare our Moriori horizontal contour with those of the Eskimo, Bantu and English crania, he will realise how relatively great is the depression of the *facies temporalis* in the Moriori frontal bone. A similar opinion will be formed by comparing the photographs of English crania on Plates II and V of Macdonell's memoir‡ with the corresponding photographs of Moriori crania on Plates IX, XV, XVI and XXI, where the shadows indicate the massive character of the *linea temporalis*. This depression of the *facies temporalis* so noteworthy in the Moriori cranium is recognisable in the Cro-Magnon contour, but scarcely more than indicated in Egyptian, Congo and English crania. Now if the reader will superpose the English type horizontal section§ on the Moriori so that the point V falls on F and O on O, he will perceive that it is at ordinates 3 and 4 that the maximum difference takes place; it is the depression of the *facies temporalis* which is the marked differential feature of the two contours. If the Guanche contour (Fig. XXVI) be superposed in the same manner, the depression is substantial, but not so marked. When we turn to the Egyptian and Congo contours (Figs. XX and XI), we must adopt a somewhat different superposition. We place *O* on *O* and *OF* in the direction *OF*. We find at once that the posterior portions of these contours fit surprisingly close to the Moriori contour, but that the massive frontal development of the Moriori is completely lacking. To bring out this special feature of the Moriori we may form a *Temporal Index* by measuring the percentage that Ordinate 3 is of the length of the section. We have the following results:

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* The resemblance of 17th century English to the Guanche type has already been insisted on: see *Biometrika*, Vol. VIII. p. 135.

† The general asymmetry of the Moriori cranium is well illustrated by the fact that T_R lies posteriorly to T_L in the type cranium: anteriorly the ordinates show that the left side, posteriorly the right side of the Moriori skull is larger.

‡ *Biometrika*, Vol. III. p. 191 *et seq.*

§ Tracing Fig. XXIX of Crewdson Benington's paper, to which the other figures when attached to tracings refer.

Race	Temporal Index	$100 \cdot \frac{\text{Ordinate 3}}{\text{Minimum Frontal Breadth}}$
Moriori	50·9	99·1
Cro-Magnon	55·7	—
Egyptian	56·2	112·0
Eskimo	57·2	—
Congo, Bantu*	57·5	104·8
Guanche	58·3	—
English	58·8	111·6

There are clearly somewhat mixed factors at work here, but the essentially differentiated character of the Moriori cranium is well-marked. It is the development of the fronto-temporal regions which especially distinguishes the crania of the English commonalty and the Guanche crania from the Moriori.

We now turn to the sagittal contours, where, as we have already remarked, we have not unfortunately the basal triangle and occipital portion of the Benington contours for full comparison. The first problem is: Which standard line is to be superposed? We have actually the inion, the lambda, the bregma and the nasion as definite points, and the glabella and occipital as less definite localities. Schwalbe takes the glabella-inion line, Klaatsch the glabella-lambda line; we may also take the nasio-lambda line, or the nasio-gamma line, which is a line through the nasion parallel to the Frankfurt standard horizontal plane, meeting the occipital in γ. Personally we doubt the value of any line drawn to either glabella or inion, as applied to the *individual* cranium; the terminals are too vague, although doubtless when we deal with a type contour, these so-called points average out to fairly reliable positions. The glabella-lambda line has been asserted to be nearly parallel to the standard horizontal plane†, but we know of no properly drawn type contour in which the height of the lambda is not two to three times as great as the height of the glabella above the nasio-gamma line. In default of other points on defective portions of crania either glabella or inion may be used. About the standard horizontal plane three statements may be made in the case of type crania:

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* Fernand Vaz, 1864 Series.

† Quain's *Elements of Anatomy*, Vol. IV. Part I. p. 132.

(i) It passes very nearly through, but a little below the inion.

(ii) It is approximately parallel to the basio-opisthion line, i.e. parallel to the plane of the *foramen magnum*.

(iii) It is approximately parallel to the maximum length, i.e. the glabella-occipital line.

None of these statements are of course true of the individual skull, but on an average or in type cranial contours they are nearly true as the reader will observe if he examines the Moriori type contours.

If we put aside the glabella as an indefinite "point," and the inion as not available in our comparative contours, we are left with three available lines for superposition: (i) $N\beta$, the nasio-bregmatic line; (ii) $N\lambda$, the nasio-lambda line; and (iii) $N\gamma$, the nasio-gamma or standard horizontal. We will examine the comparison by each of these lines as superposition lines. Let us first, however, recall to mind the essential features of the Moriori skull as revealed by measurement, simple inspection and photograph. These are the flattening of the frontal region and the sagittal ridge or crest behind the apex. We superpose first the English typical sagittal contour (Fig. XXVII). Nasion on nasion and the lines $N\gamma$ in the same direction. We have at once brought out the Moriori characters, the flattening of the frontal and the sagittal crest starting at the bregma or before, eaching behind the apex and extending towards the lambda. There is no question of these striking features. Rotate the tracing until $N\lambda$ coincides in direction with $N\lambda$, all trace of the crest disappears, there is increased frontal flattening, but it is the English skull which now might be thought crested! Now make $N\beta$ coincide with $N\beta$. Here we get a really true measure of the flattening of the frontal bone, but practically the English and Moriori contours now coincide from bregma to lambda. In our opinion there is not the least doubt that comparison by the nasio-gamma line, not the nasio-lambda line, brings out the characteristic features of the Moriori cranium in a manner which fails entirely when we use the nasio-lambda line. Even in the question of the appreciation of frontal flattening, it is by no means certain that the flattening of the frontal bone is the sole source of the physiognomic impression; the rotation of the frontal bone as a whole (which would not alter the ratio of subtense to nasio-bregmatic chord (see p. 110)) may have equal physiognomic value. In the present case this physiognomic factor gives an aspect of flatness to the English skull, although there is a high value of the nasio-bregmatic frontal index. Anyone looking at the photographs in Macdonell's memoirs on the English skull* would assert that there is physiognomically a flattened frontal. It is largely produced by rotation of the frontal bone and not by its flattening.

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* See *Biometrika*, Vol. v. p. 104, Plates XII, XVI, etc.

All these points are well brought out when we compare by the standard horizontal planes and have no suggestiveness at all when we superpose the nasio-lambda lines, which give the English skull a protrusion of the frontal *both* by rotation of $N\beta$ and curving of the frontal bone, while actually the two factors act in opposed directions.

Another point which tells considerably against the use of the nasio-lambda line as reference line in the case of two type contours is the experience that this superposition usually throws out very much the two basal triangles (i.e. nasion, basion, alveolar point). As a matter of fact when the basal triangles are as nearly as possible superposed, then the nasio-bregma lines will be closest together, next the nasio-gamma lines and last the nasio-lambda lines, but sufficient material is not yet available to study properly this point, and accordingly no stress is laid on it here.

If the Guanche tracing (Fig. XXIV) be superposed on the Moriori, N on N and $N\gamma$ along $N\gamma$, we have the same result, the frontal flattening and the post-bregmatic crest; this is equally so if $N\beta$ be set to $N\beta$, but the crest disappears entirely, if we set $N\lambda$ to $N\lambda$. When we compare the Negro contour (Congo Bantu, Fig. IX) with the Moriori we have still, for $N\gamma$ to $N\gamma$, the flattening of the frontal ($N\beta$ almost coincides with $N\beta$) and the emphasis of the crest, but a new feature comes in, namely the dwarfing of the occipital portion of the Negro cranium. To a less marked extent this occurs with the Egyptian type contour (Fig. XVIII), but the emphatic flattening of the Moriori frontal bones appears reduced in this case by the physiognomic flattening of the Egyptian due to rotation of the frontal bone.

From other aspects there are considerable objections to the use of $N\lambda$ as a reference line especially in the case of *individual* crania. While the suborbital and auricular points are capable of fairly accurate determination when the skull is on the craniophor and the bregma likewise as the coronal suture is not usually deeply serrated, the lambdoid suture is often most markedly serrated and the actual position of the lambda may be the result of a random serration, which appears to have little bearing on the average sweep of the lambdoid suture, and the intersection of that sweep with the sagittal suture.

It is not possible to compare the constants of the type sagittal section of the Moriori as found for Schwalbe's glabella-inion line with the like constants for the races dealt with in Benington's paper, for the position of the inion has not been averaged off from his individual contours. We shall accordingly content ourselves with comparison of the indices obtained from bregmatic subtense and maximum subtense on the nasio-gamma, the nasio-lambda and Klaatsch's glabella-inion lines as base, giving further the angles which the nasio-bregmatic or glabella-bregmatic line as the case may be makes with these base lines. We shall still have to confine our attention to male crania as the Benington paper has not provided the contours for female crania.

Table V results:

TABLE V.

Values of Constants of Sagittal Contours.

A. Nasio-gamma Base Line.

Race	Base Line	Max. Subtense		β Subtense		Index Max. Sub.		Index β Sub.		Δ (Ind. Max. Sub. – Ind. β Sub.)		Bregmatic Angle
	B_0	y	x	y	x	y/B_0	x/B_0	y/B_0	x/B_0	y/B_0	x/B_0	
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)
Eskimo	186·1	93·9	104·1	89·3	72·1	50·5	55·9	48·0	38·7	2·5	17·2	51°·1
Moriori	180·2	90·0	103·8	84·9	73·0	49·9	57·6	47·1	40·5	2·8	17·1	49°·3
English	183·5	84·9	103·7	83·0	77·2	46·3	56·5	45·2	42·1	1·0	14·4	47°·1
Guanche ...	183·4	84·8	99·0	83·0	74·4	46·2	54·0	45·3	40·6	0·9	13·4	48°·1
Egyptian* ...	181·9	84·4	106·2	82·5	79·0	46·4	59·5	45·4	43·4	1·0	16·1	46°·2
Congo, Bantu† ...	175·6	85·4	97·7	83·5	70·6	48·6	55·6	47·6	40·2	1·0	15·4	49°·8
B. Nasio-lambda Base Line.												
Eskimo	183·1	71·2	89·5	71·2	88·5	38·9	48·9	38·9	48·3	0	0·6	38°·8
Moriori	180·0	66·0	100·2	66·0	91·0	36·7	55·7	36·7	50·6	0	5·1	35°·9
English	181·2	68·7	90·0	68·7	90·0	37·9	49·7	37·9	49·7	0	0	37°·4
Guanche... ...	181·5	68·5	79·8	68·1	88·0	37·7	44·0	37·5	48·5	0·2	–4·5	37°·7
Egyptian* ...	180·1	70·2	85·2	70·0	89·6	39·0	46·8	38·9	49·8	0·1	–3·0	38°·0
Congo, Bantu† ...	176·3	69·8	90·1	69·8	84·0	39·6	51·1	39·6	47·6	0	3·5	39°·7
C. Glabella-lambda Base Line.												
Eskimo	184·5	66·0	100·4	65·2	87·1	35·8	54·4	35·3	47·2	0·5	7·2	36°·8
Moriori	181·0	61·2	102·0	60·6	88·7	33·8	56·4	33·5	49·0	0·3	7·4	34°·3
English	182·8	65·7	92·5	65·6	88·6	35·9	50·6	35·9	48·5	0	2·1	36°·5
Guanche... ...	183·2	64·6	82·3	64·6	82·3	35·3	44·9	35·3	44·9	0	0	38°·1
Egyptian* ...	180·9	66·8	91·5	66·8	88·9	36·9	50·6	36·9	49·1	0	1·5	36°·9
Congo, Bantu† ...	176·6	64·7	91·7	64·3	80·9	36·6	51·9	36·4	45·8	0·2	6·1	38°·5

* XXVI—XXX Dynasties. † Fernand Vaz, 1864 Series.

In the above table columns (i)—(v) give absolute measurements, (vi)—(xi) provide indices and (xii) the "bregmatic angle," or angle the base line makes with the line joining nasion or glabella to bregma. It has been suggested that this angle measures in some manner the flattening of the frontal. It seems to us incapable of really determining the grade of primitiveness in the cranium. Other things being equal this angle will merely measure the extent to which the bregma is thrown back on the cranial vault, a relatively large frontal bone will in itself produce a small bregmatic angle. This is seen at once by comparing the relative position of the bregma as measured by the index x/B_0 for the bregma (column (ix)):

Nasio-gamma Base Line.

Race	Angle	x/B_0	y/B_0	Position as to size
Egyptian	46°·2	43·4	45·4	Fourth
English	47°·1	42·1	45·2	Second
Guanche	48°·1	40·6	45·3	Third
Moriori	49°·3	40·5	47·1	Fifth
Bantu	49°·8	40·2	47·6	Sixth
Eskimo	51°·0	38·7	48·0	First

We see at once from this table that the bregmatic angle has little if any relation to size of the skull. It gives absolutely the same order as the relative horizontal position of the bregma, and (except for the anomalous Egyptian, which is, however, greater than the English and Guanche by a slender difference) the same order as the bregmatic height difference. In biometric language we should say that the bregmatic angle is highly and negatively correlated with the horizontal position of the bregma, and is highly and positively correlated with the vertical position of the bregma. These results might be anticipated from its functional form $\tan^{-1}(y/x)$, but we shall learn little as to racial order from the bregmatic angle. Indeed we might even state the more primitive the race the bigger the bregmatic angle. This conclusion is amply confirmed by the Cro-Magnon skull which gives:

Angle	x/B_0	y/B_0
55°·2	34·6	49·6

Thus we must conclude that a high bregmatic angle is the result of a relatively forward bregma and is a sign of primitive rather than advanced cranial type. The reader must carefully distinguish between a "forward" bregma and a relatively large frontal bone. If we take as measure of the latter, the base $N\beta$ of the frontal bone in proportion to $N\gamma$, the order is:

Congo, Bantu ...	63·1	English ...	61·5
Egyptian ...	62·5	Cro-Magnon ...	61·0
Moriori ...	62·1	Guanche ...	60·5
Eskimo... ...	61·5		

an order, to which it would be difficult to give any interpretation of value.

When we turn to either the nasio-lambda or the glabella-lambda as base line we find none of the above identical order of vertical index, horizontal index and bregmatic angle. We do now find that the Moriori have a low instead of a high bregmatic angle, but if we were to associate this with a racially primitive condition, then the Congo Bantus would stand as the most advanced race in our list judged from either base line. As a matter of fact we see the explanation lies in the fact that the Moriori have from the nasio-lambda base the relatively least forward, and the Bantus the most forward position of the bregma, while the same is nearly true for the glabella-lambda base. The results found for these two base lines whether we consider the bregmatic angle or the horizontal or vertical bregmatic index are not in any accordance, and although these base lines must be used when the subject consists merely of a skull cap, or even lacks the nasion, we doubt whether for racial inter-relations they are nearly as valuable as the nasio-gamma base. The general measure of the flattening of the frontal bone is best read off as on our p. 95, and this gives the Moriori the premier position on the list. If we desire to measure the total physiognomic flatness we might, I think, combine the bregmatic angle for the nasio-gamma line with the angle the line joining the nasion to the arc end of the nasio-bregmatic subtense makes with the nasio-bregmatic line. The following table brings out the contributions to frontal flattening due (i) to flattening of the frontal bone and (ii) rotation of the frontal bone as a whole.

Race	Frontal Bone Flatness	Rotation of base of Frontal Bone	Physiognomic Angle of Flatness
Cro-Magnon	24°·7	55°·2	79°·9
Eskimo	28°·0	51°·1	79°·3
Bantu	27°·3	49°·8	77°·1
Guanche	26°·4	48°·1	74°·5
English	26°·6	47°·1	73°·7
Moriori	22°·3	49°·3	71°·6
Egyptian	24°·9	46°·2	71°·1

This table seems of some interest. As we have before noted the physiognomic flatness consists of two factors; of these the rotation is roughly twice as important as the actual flattening of the frontal bone. Thus the English low physiognomic flatness is largely due to rotation; the Cro-Magnon skull with much frontal flattening is physiognomically at the top of the list owing to the excess of rotation, and in appearance has consequently a well-developed frontal. The Moriori stands at the bottom of the list for frontal bone flattening, but has less physiognomic flatness than the Egyptian, although the difference is very minute. It is clearly impossible to judge any grade of "primitiveness" by the mere smallness of the bregmatic angle, and we venture to hold that physiognomic flatness must in future be distinguished from flatness of frontal bone.

Returning to our Table V, we note that in the case of the nasio-lambda base the maximum subtense for all practical purposes is the bregmatic subtense. In other words if the nasion, but not the lambda, be ascertainable on a skull cap a *rough* approximation can be made to the nasio-lambda line by drawing through the nasion a parallel to the tangent to the sagittal arc at the bregma. The reader will see by examining Δ Index x/B, column (xi), Table V, that the maximum falls sometimes on one side, and sometimes on the other of the bregma, but its position. never more than half-a-dozen millimetres from the bregma, causes the two subtenses to be equal owing to the general flatness in the region of the bregma. The maximum difference between the two subtenses is 0·2 for the Guanche type contour, a difference well within our probable error of determination. If we use the glabella-lambda base line very little difference can be made between the maximum and bregmatic subtenses, although the maximum subtense now always falls behind the bregma, but by something less than 8 mm. As a matter of fact if we set off on the contour βf anteriorly and βl posteriorly to bregma both equal to about 15 mm., then the parallel to fl will give *approximately* $N\lambda$ and the parallel to βl the glabella-lambda line. While these constructions are merely approximate for type contours, they will be only very rough for individual crania, yet in default of better rules they will give some idea of the position of the λ, if only the bregma and either the nasion or glabella are known. If we want the actual position of the lambda we can only note that the foot of the bregmatic subtense is at an average distance ·491 of the nasio-lambda line from the nasion, and at an average distance ·474 from the glabella in the glabella-lambda line, but these distances vary much for individual races, and *a fortiori* will vary more for individual crania*.

Now the point just discussed, i.e. the practical equality of the bregmatic and maximum subtenses for both the nasio-lambda and glabella-lambda cases, suggests how important it is to use a base line which will determine the extent to which the vertex of the cranium rises above and recedes behind the bregma. For the rising of the parietals above the frontal bone, throwing the vertex far back, is one of the marked features of primitive man. Whatever may be said against the Frankfurt standard horizontal plane—and many of the criticisms arise from insufficient experience of its value—it can hardly be disputed that on the *average*, i.e. in type contours, it is a fair approximation to the "horizontal," remembering that even for the same individual the "horizontal" is only itself a sort of average position. Some plane must be taken to express the marked physiognomic characters conveyed by the idea of a retreating vertex and an elevated vertex. This no nasio-lambda or glabella-lambda line can accomplish. As far as we can find no very definite statement as to how the vertex is to be determined has yet been accepted†. We define the "apex" to be the point of the sagittal suture vertically above the auricular line when the skull is adjusted to the standard horizontal position on the craniophor. The "vertex" is the highest point of the sagittal or medial plane when the skull is in the same position. It is the point of contact of the tangent to the sagittal contour drawn parallel to the nasio-gamma line. From this standpoint we are able to give quantitative measures to the elevation and recession of the vertex relative to the bregma. All we need do is to subtract the indices y/B

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[*] Thus we attempted to reconstruct the Cro-Magnon skull from a knowledge of glabella and bregma. A very poor value of λ was found from ·474, but an excellent one when the skull was assumed to be Guanche. In any such reconstructions it is probably desirable to use any suggestion of racial affinity.

[†] See Quain's *Anatomy*, Vol. IV. Part I. p. 124 (1915), for example, where the vertex is defined to be the "highest point of the arch of the sagittal section." In the diagram, Fig. 104 of this section, the nasio-lambda line appears to slope downwards instead of upwards and the "vertex" for this position appears to be in front of the bregma. In Fig. 112 the nasio-lambda line is made nearly horizontal and this adds much to the prognathous physiognomic effect aimed at!

and x/B of the vertical and bregmatic subtenses. There results the following table:

Race	Elevation	Recession
Moriori	2·8	17·1
Eskimo	2·5	17·2
Egyptian, XXVI–XXX Dyn.	1·0	16·1
Congo, Bantu	1·0	15·4
English	1·0	14·4
Guanche	0·9	13·4

The Cro-Magnon cranium stands between the Eskimo and Egyptian with a recession of 17·2 and an elevation of 1·5. Owing to the flatness of the skull in the neighbourhood of the vertex, the values of the recession can only be looked upon as approximations. Far more racial determinations are desirable before conclusions can be drawn, but undoubtedly these measures of the elevation and recession of the vertex suffice to differentiate those races like Moriori and Eskimo in which a receding sagittal crest is a marked feature.

As a last feature of the sagittal section we may consider the angle between the nasio-lambda line and the standard horizontal. We have found no case of the depression of this line below the nasio-gamma line; the racial elevation is always about 9° to 13°. We have:

Race	Slope of Nasio-lambda Line
Moriori	13°·5
Cro-Magnon	12°·6
Eskimo	12°·3
English	10°·5
Guanche	10°·3
Congo, Ferdinand Vaz, 1864	9°·9
Congo, Batetelu	9°·9
Egyptian, XXVI–XXX Dyn.	8°·4

Thus we again see the Moriori and Eskimo at the top. It might be thought accordingly that the elevation and recession of the vertex in Moriori and Eskimo was due to a mere tilt of the cranium. The reply to this is that there is an actual physical ridge, there being actual concavity in some cases of the parietals on either side of the sagittal suture.

A useful piece of work would be to add the basion, the inion, the opisthion and the suborbital and auricular points to Crewdson Benington's contours—they are given on his individual tracings; so that racial comparisons might be made of the lower portion of the cranium from his sagittal type contours.

7. *On Mean Values found from Type Contours.* We now turn to the second problem relating to the type contours: How far can we be certain of mean values obtained from them? We may note several general points before we proceed to

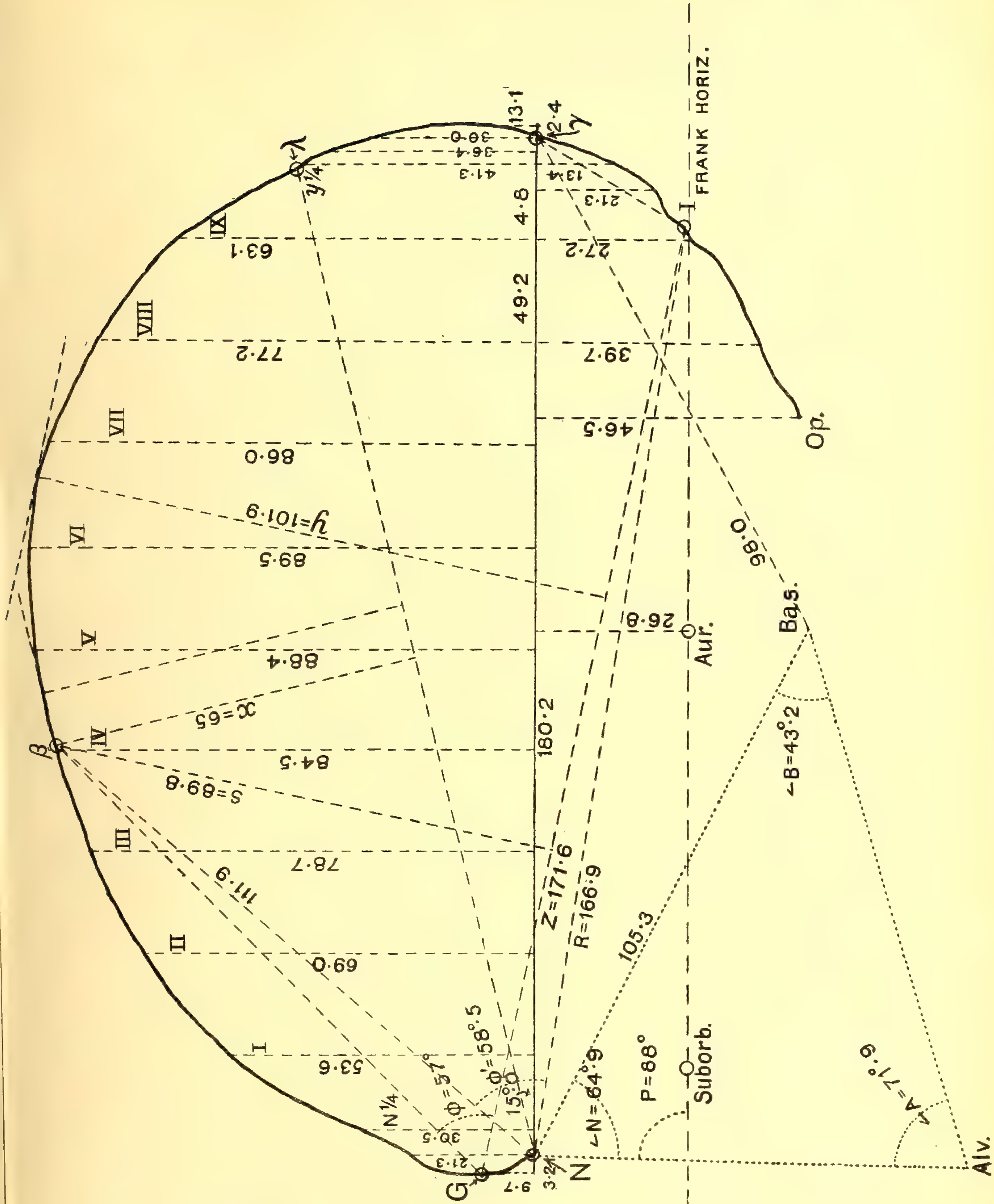
FRANK HORIZ.
λ
y¼
γ
13·1
12·4
30·0
36·4
41·3
13·4
21·3
4·8
I
27·2
63·1
IX
VIII
49·2
77·2
39·7
VII
46·5
Op.
86·0
y=101·9
VI
89·5
98·0
Bas.
V
26·8
Aur.
88·4
x=65
IV
β
180·2
84·5
S=89·8
∠B=43°·2
III
78·7
111·9
Z=171·6
R=166·9
II
69·0
105·3
I
53·6
φ'=58°·5
φ=57°
15°·0
N¼
∠N=64°·9
P=88°
Suborb.
30·5
21·3
9·7
3·2
G
N
∠A=71°·9
Alv.

FIG. I. Moriori Type Sagittal Contour. ♂. (From 33 skulls.)

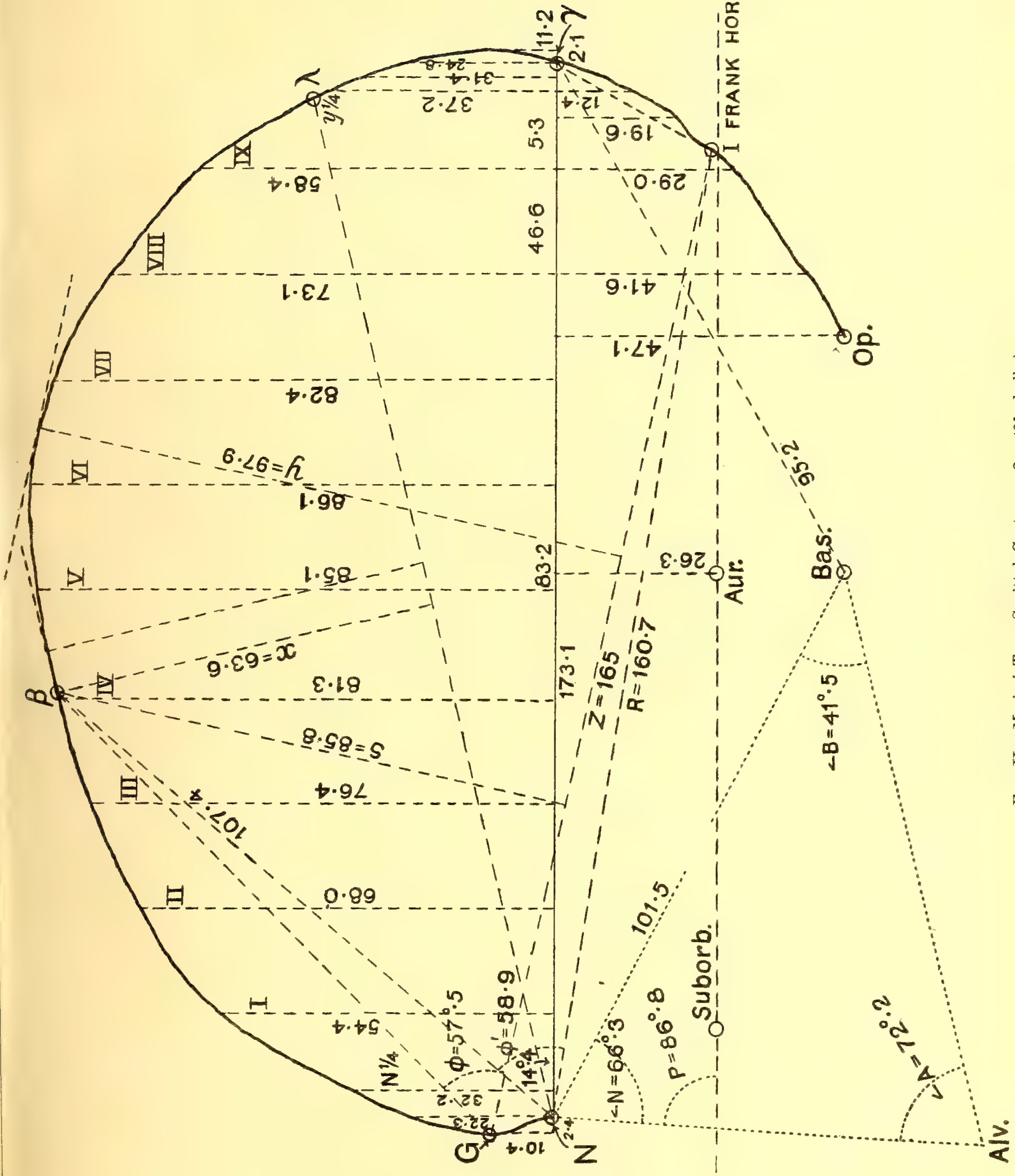
I FRANK HORIZ
γ
11·2
2·1
λ
y/4
24·8
31·4
37·2
12·4
5·3
19·6
29·0
IX
58·4
46·6
VIII
73·1
41·6
Op.
47·1
VII
82·4
95·2
y=97·9
VI
86·1
83·2
26·3
Aur.
Bas.
V
85·1
x=63·6
β
IV
81·3
173·1
Z=165
R=160·7
∠B=41°·5
S=85·8
III
76·4
107·4
II
68·0
101·5
Suborb.
I
54·4
φ=57°·5
φ′=58·9
14°·4
N¼
32·2
22·3
∠N=66°·3
P=86°·8
∠A=72°·2
G
10·4
N
2·4
Alv.

FIG. II. Moriori Type Sagittal Contour. ♀. (21 skulls.)

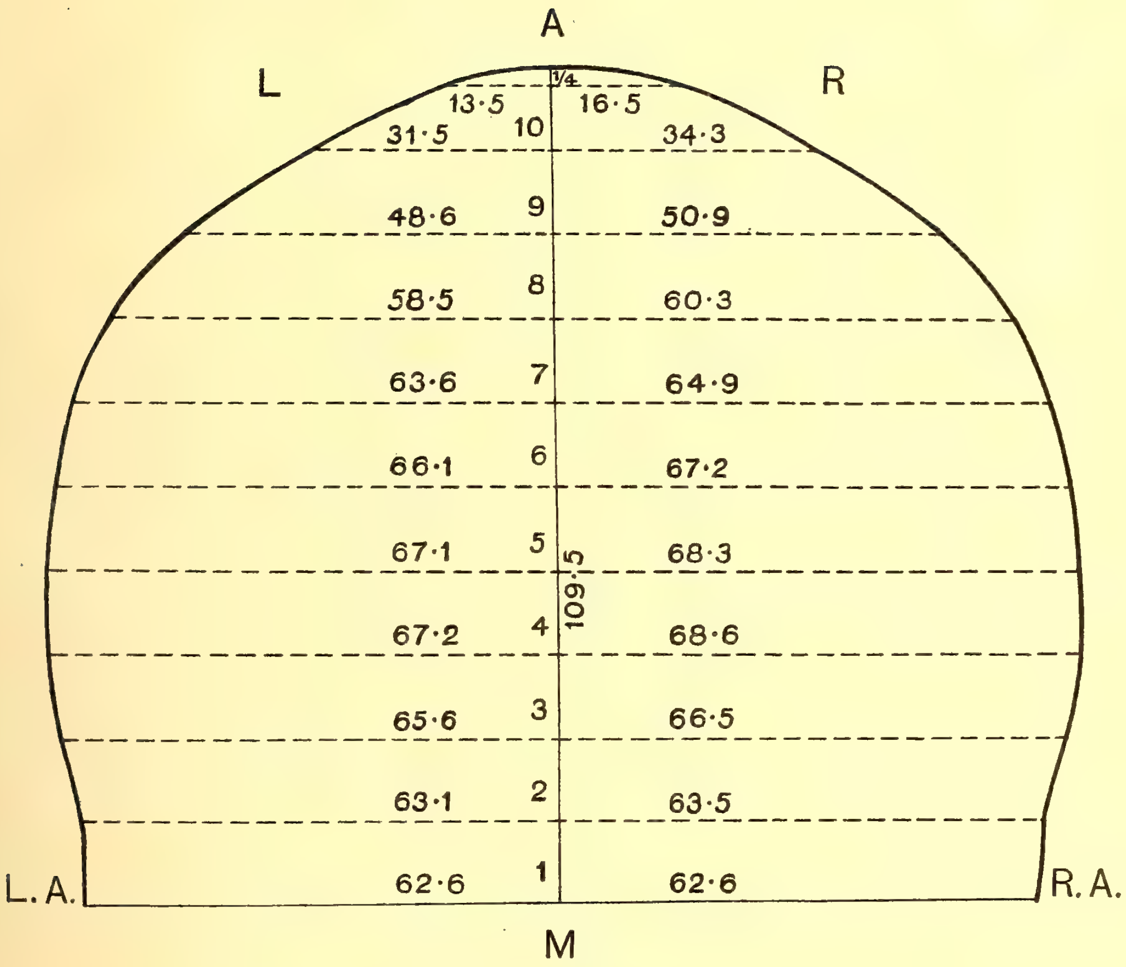
A
L
R
¼
13·5
16·5
31·5
10
34·3
48·6
9
50·9
58·5
8
60·3
63·6
7
64·9
66·1
6
67·2
67·1
5
68·3
109·5
67·2
4
68·6
65·6
3
66·5
63·1
2
63·5
L. A.
62·6
1
62·6
R. A.
M

FIG. III. Moriori Type Vertical Contour. ♂. (33 skulls.)

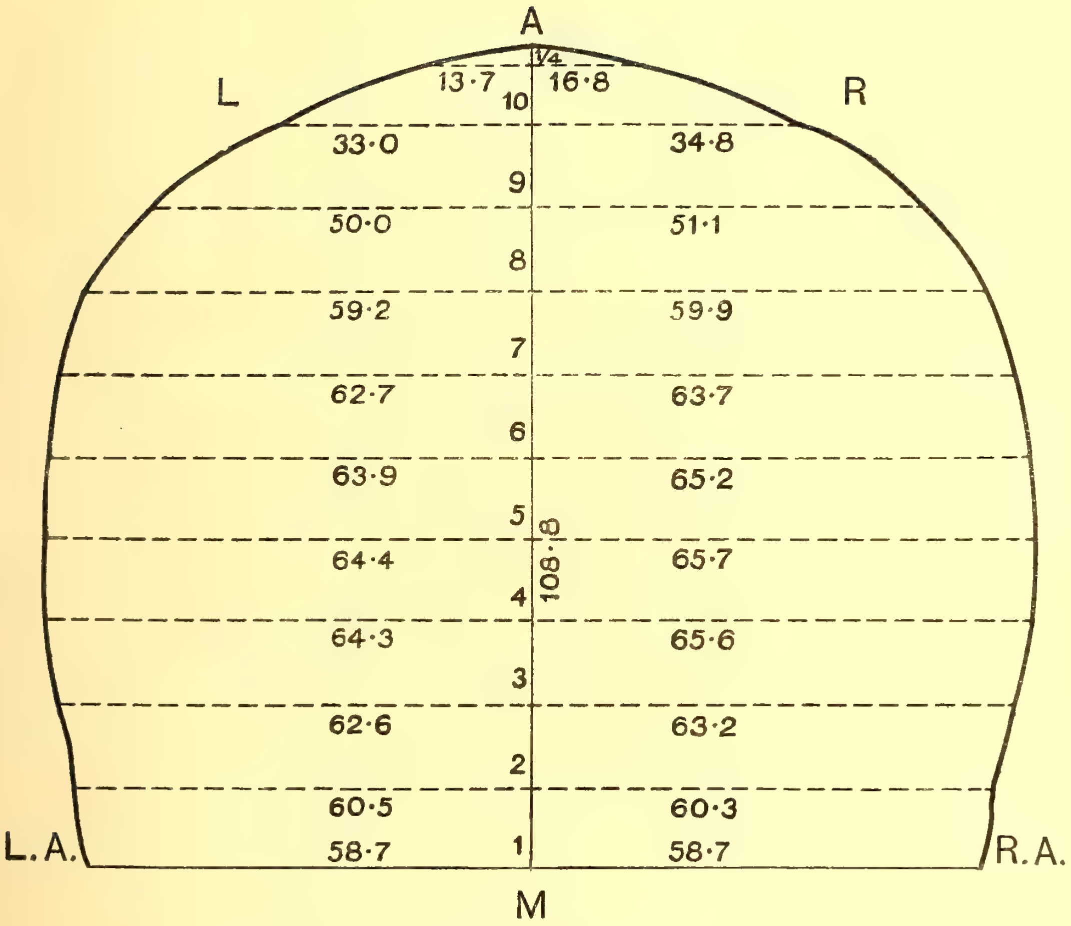
A
L
R
1/4
13·7
16·8
10
33·0
34·8
9
50·0
51·1
8
59·2
59·9
7
62·7
63·7
6
63·9
65·2
5
108·8
64·4
65·7
4
64·3
65·6
3
62·6
63·2
2
60·5
60·3
58·7
1
58·7
L.A.
R.A.
M

Fig. IV. Moriori Type Vertical Contour. ♀. (21 skulls.)

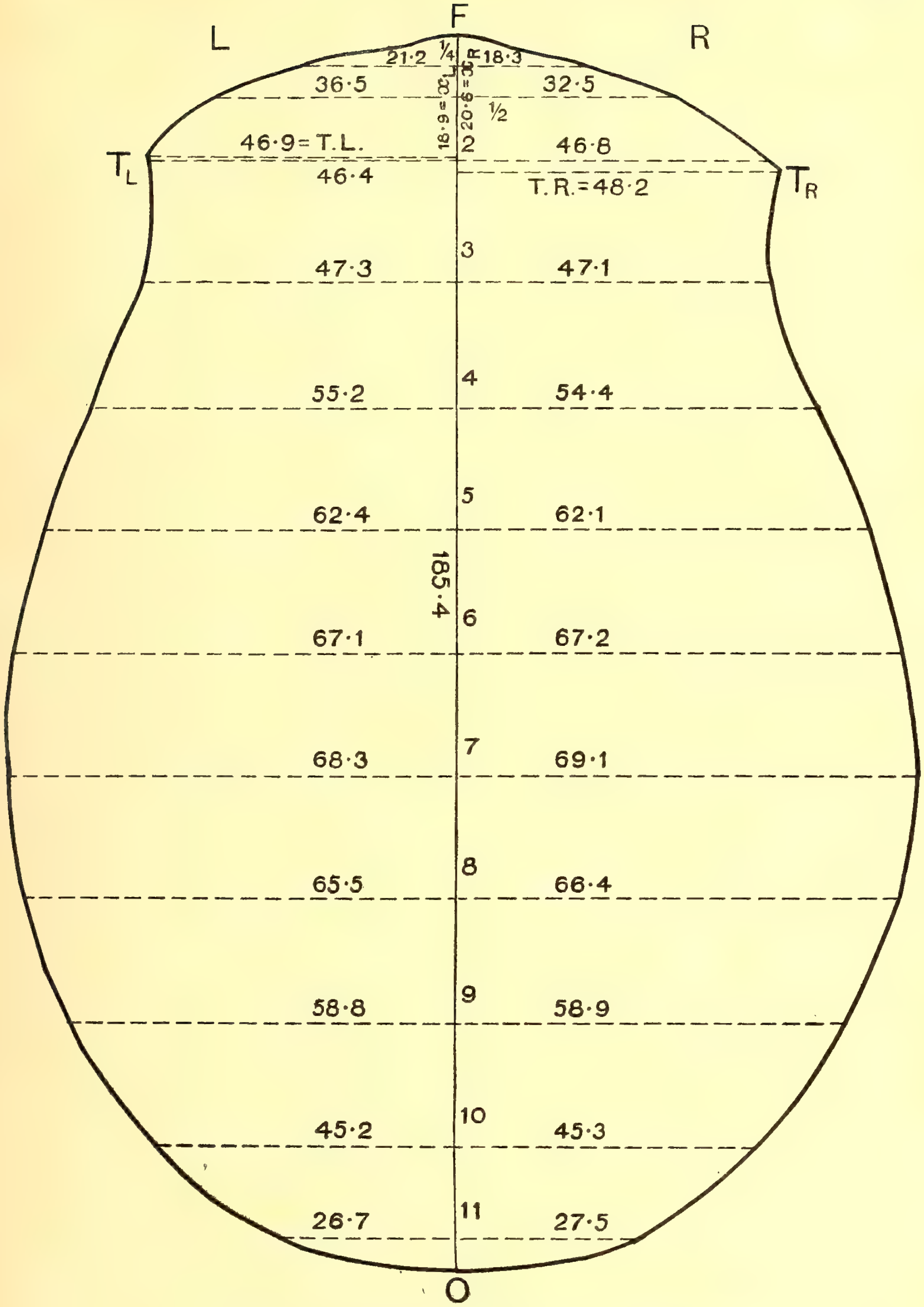
F
L
R
21·2
¼
18·3
36·5
32·5
18·9 = 3L
20·6 = 3R
½
46·9 = T.L.
2
46·8
T_L
T_R
46·4
T.R. = 48·2
3
47·3
47·1
4
55·2
54·4
5
62·4
62·1
185·4
6
67·1
67·2
7
68·3
69·1
8
65·5
66·4
9
58·8
58·9
10
45·2
45·3
11
26·7
27·5
O

Fig. V. Moriori Type Horizontal Contour. ♂. (33 skulls.)

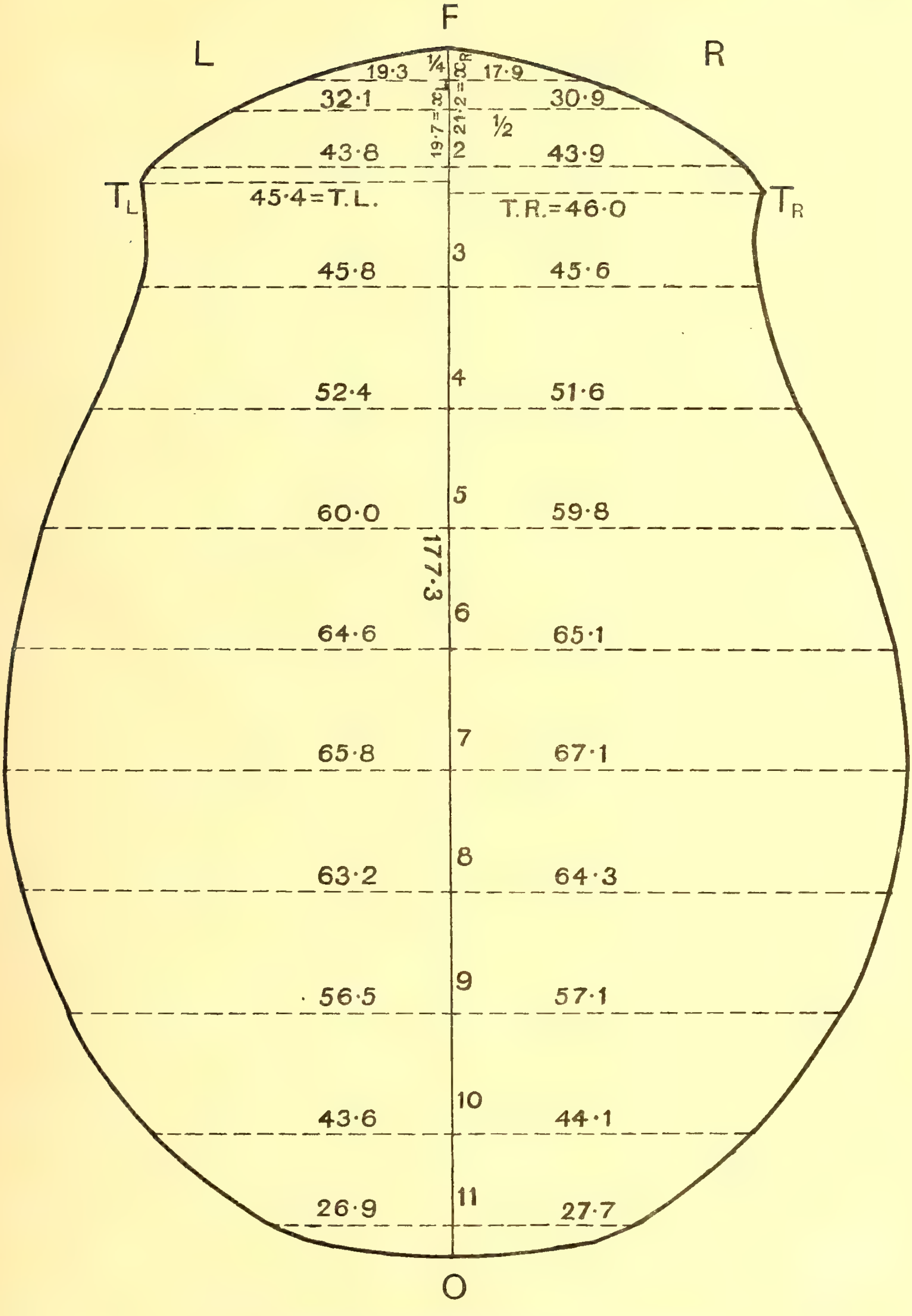
F
L
R
19·3
¼
17·9
32·1
30·9
$19{\cdot}7=\mathfrak{X}_L$
$21{\cdot}2=\mathfrak{X}_R$
½
43·8
2
43·9
T_L
T_R
45·4=T.L.
T.R.=46·0
3
45·8
45·6
4
52·4
51·6
5
60·0
59·8
177·3
6
64·6
65·1
7
65·8
67·1
8
63·2
64·3
9
56·5
57·1
10
43·6
44·1
11
26·9
27·7
O

FIG. VI. Moriori Type Horizontal Contour. ♀. (21 skulls.)

details. After all the values have been averaged, some error is almost invariably introduced in plotting, especially as the contours have to be somewhat thickened up for the engraver; thus readings should be made to the *middle* of the contour boundary. But even then with the greatest care the engraver seems to find it difficult to reproduce to exactly the same size; however strict the injunctions may be we rarely find the engraver's final result to agree absolutely with the plotted contour. There may indeed be as much as a millimetre difference. In deducing new indices therefore it is well to read off the diagrams, disregarding the numbers placed upon them and take the ratio of lengths thus read. We will now turn to the type contours. Let us consider first the glabella-occipital length. We ought to be able to get this exactly from the sagittal mean sections, and approximately from the horizontal sections. There results by measurement:

Mean glabella-occipital Length L.

	Males		Females	
	No.	Value	No.	Value
Direct Measurement ...	35	186·9	22	178·6
From Sagittal Contour ...	33	186·5	21	178·3
From Horizontal Contour	33	185·4	21	177·3

That the horizontal contour does not give the same result as the sagittal is to be anticipated, for the horizontal line through the glabella does not necessarily pass through the exact occipital point, but the measurement on the sagittal section should theoretically agree with direct measurement. Is the slight difference due in this case to the fact that one or two skulls could not be used for type contours? Unfortunately the type crania in the earlier cases were not finished so fully in the occipital region as the present samples, but I give the following examples:

Mean glabella-occipital Length L for Males.

Race...	English		Egyptian, XXVI –XXX Dyn.		Congo, Batetelu		Gaboon, Bantu	
	No.	Value	No.	Value	No.	Value	No.	Value
Direct Measurement ...	137	186·9	100*	184·1	50	177·8	50	179·5
From Sagittal Contour ...	100	186·8	100	184·4	41	178·4	50	177·9
From Horizontal Contour	100	185·9	100	183·5	41	177·5	50	177·6

* The 100 ♂ crania to which direct measurement was applied were not absolutely the same as the 100 selected for contouring. Fawcett gives 185·1, MacIver 184·5 for prehistoric Egyptians.

I think we may conclude from these results that:

(i) It is needful to add about 1 mm. to make the measurement on the horizontal contour agree with that on the sagittal contour.

(ii) Unless the occipital and glabella are reconstructed with great accuracy, we may have an average error of something like 0·6 mm. in determining the glabella-occipital length from the sagittal section.

Of course the maximum breadth *B* cannot be read off on the horizontal section; the temporal breadth should be something *greater* than the minimum frontal breadth; it is for ♂s 95·5 and for ♀s 92·3 against minimum frontal breadth, ♂s 95·3 and ♀s 94·0. I am unable to account for the defect of 1·7 in the females except by the difference of errors of reading with callipers and reading with a scale on the diagrams*. Probably it would be best in future contour drawings with the Klaatsch to mark off specially the temporal ridges, as one is apt in turning a sharp point on the contour to bring the side of the needle rather than its point into contact with the ridge, but I do not think this would *lessen* the dimensions; such a lessening might arise from shear of the plasticine on which the skull is bedded, if too great pressure were applied. Whatever the source of the error, the lesson must be to test each individual contour against the available individual measurement before proceeding to the type contour.

I now pass to the transverse section. In drawing this we pass from "auricular point" to "auricular point," the "auricular line" being defined as the tangent to the upper borders of the external meatus. We have been forced to the conclusion that when the skull rests on the Ranke craniophor, the "auricular line" corresponding to the top of the plugs (wedge type, not conical type) does not coincide with the auricular line determined by joining the upper borders of the external meati as marked on the transverse contour. The ear wedges have to be thrust some distance into the auditory meatus† and thus their horizontal tops are considerably below the line joining the upper borders. They are of course not as low as the line joining the "centres" of the external meatus. Thus it comes about that we can get three auricular heights. The one from the centre of the auricular passages is that usually determined on the living, and by field workers on craniological material; the one given by the contour tracer is that which we ought to find in strict accordance with the Frankfurt Concordat, and the one actually found in laboratory work with a craniophor is only strictly in accordance with the Concordat, when the external meatus has an upper surface, actually horizontal and accurately passing through the uppermost point of the border of the auditory external meatus.

We now turn to the type transverse contours and note in the light of the above remarks the auricular heights:

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* The cranium which it was not possible to contour had a large frontal breadth, but its omission only reduces the mean to 93·8 mm.

† In order that the skull may be stable on the craniophor.

Moriori Auricular Height OH.

	Male		Female	
	No.	Value	No.	Value
Measured on Craniophor	34	117·5	22	115·1
Measured on Transverse Contour ...	33	109·5	21	108·8
Measured on Sagittal Contour ...	33	116·5	21	112·4

Let us examine similar data for other races:

Male Auricular Height OH.

Race...	English		Egyptian, XXVI–XXX Dyn.		Congo, Batetelu		Gaboon, Bantu	
	No.	Value	No.	Value	No.	Value	No.	Value
Measured on Craniophor...	135	114·6	100	113·6	47	113·9	?	?
Measured on Transverse Contour	100	111·5	100	110·2	41	112·0	50	113·6

It will be seen that there is not much comparative material, but in all cases the craniophor auricular height is in excess of the transverse contour height. Dr Benington was more successful than I have been in making his type transverse contour approach the auricular height of the craniophor measurement*, but even he may be 2 to 5 mm. in defect. My determinations from the sagittal contour are in better agreement with my craniophor measurements, but may be 2 to 3 mm. divergent, while the transverse contour differs by 6 to 8 mm. I attribute these big differences to the very different manner in which the auricular point is determined (i) when the skull is on the craniophor, (ii) when it is orientated by auricular points and apex, (iii) when it is orientated by nasion, bregma and lambda. In the latter case in particular there is no certainty that the auricular line is perpendicular to the plane of the paper. In (ii) the auricular point is reached by a stile arriving from the apex, in (iii) by a stile adjusted from the basal side of the orientated skull, and there is far more personal equation in its determination than may be at once realised. It is clear, I think, that determinations of the auricular height from the transverse type contour are of small value. But I doubt very much whether the auricular height as found on the Ranke craniophor accords well with the definition of the Frankfurt concordat, or indeed whether the "auricular line" is capable of sufficiently close definition*. At any rate the transverse contours need clearer determination at their lower terminals. In further work it might be valuable to place upon them the projections of the basion, suborbital and alveolar points.

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* It is possible that the top of the auricular passage is less nearly horizontal in the case of the Moriori than in the races which Dr Benington dealt with.

I have not taken directly the interauricular breadth of my Moriori skulls. Read off from the transverse contours, I find: ♂s, 125·2, and ♀s, 117·6, which agree fairly well with Scott's measurements, namely ♂s, 126·8, and ♀s, 118·5, although mine for both sexes are consistently less.

I made an attempt to ascertain the circumferences from the contours and found:

Circumference	Male		Female	
	By Tape	From Contour	By Tape	From Contour
Transverse: Q	316·1	300·1 (316·1)	310·0	297·3 (309·6)
Horizontal: U	522·8	522·0	502·6	501·0
Sagittal: S_1	126·0	125·7	121·4	121·6
Sagittal: S_2	122·4	122·9	117·4	117·4
Sagittal: S_3	121·0	118·9	116·0	110·0
Sagittal: $S = S_1 + S_2 + S_3$	369·4	367·5	354·8	349·0

After the contour measurements for Q are placed in brackets the results which would be reached by adding to the contour arc *twice* the difference in the auricular heights. The agreement is then very close, showing that the divergence is solely due to difference in position of the auricular line and not to shape of contour. The results on the whole are fairly close; the occipital arc is the worst, and I have possibly not accentuated sufficiently the ridges of the occipital and inionic regions†.

I now give (Table VI) a number of other characters which can be read off the type sagittal contour with comparison of their measured values. On the whole the agreement is not bad. Indeed it is sufficiently good to suggest that with the experience now gained an improved technique would enable the sagittal type section to be directly used as a means of determining the customary racial characters of the skull, or that at all events this section would serve as a valuable control method for verifying the mean values determined by direct measurement and arithmetic. The measurements of greatest divergence are:

(i) The foraminal length. The type contour exaggerates it by 2 mm. about for men and 4 mm. for women. But the draughtsman has plotted in the case of the female skull the opisthion with an abscissa of 44·6 instead of 46·6 mm. from the gamma. Clearly the error observed in the occipital arcs is largely due to this displacement of the opisthion 2 mm. in the male and 4 mm. (2 of which are by

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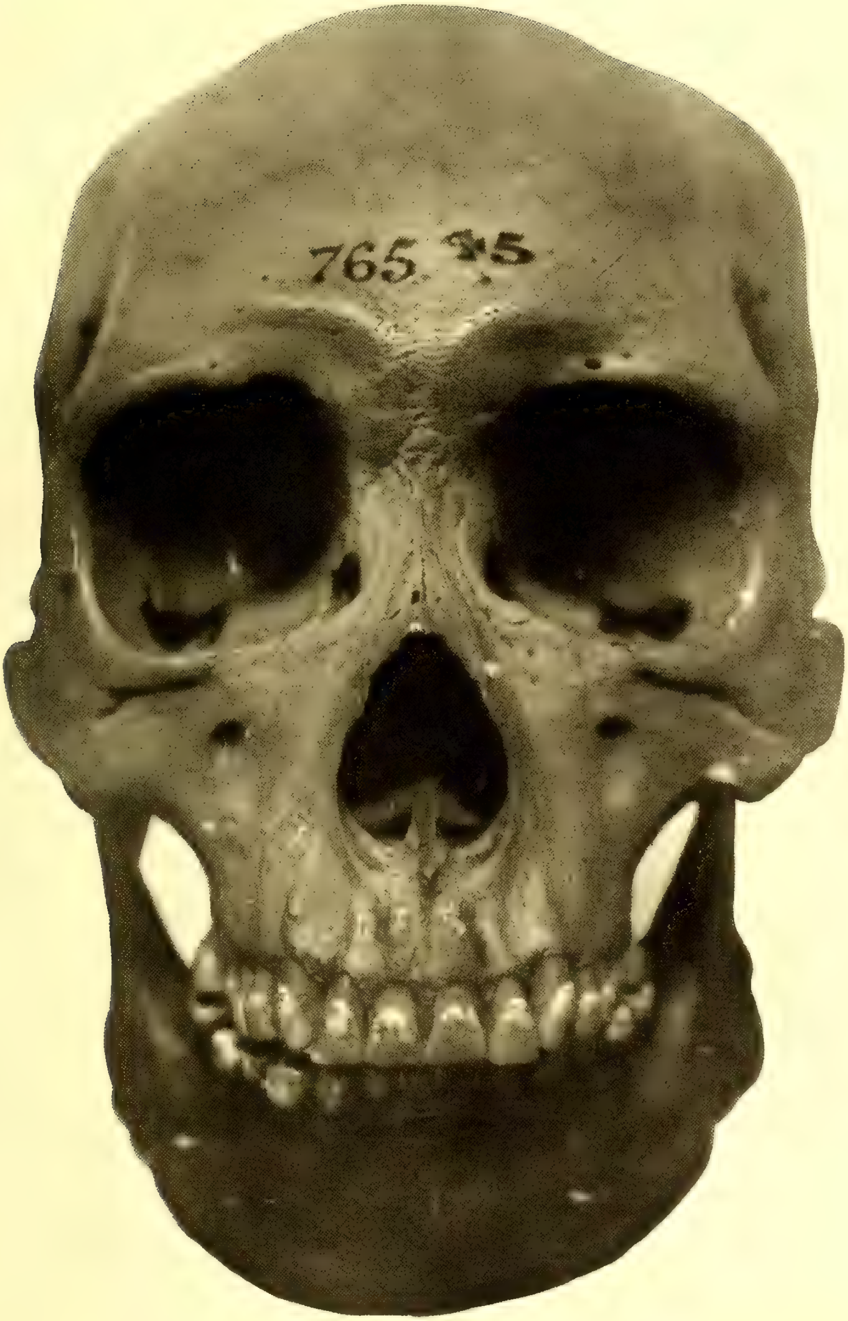
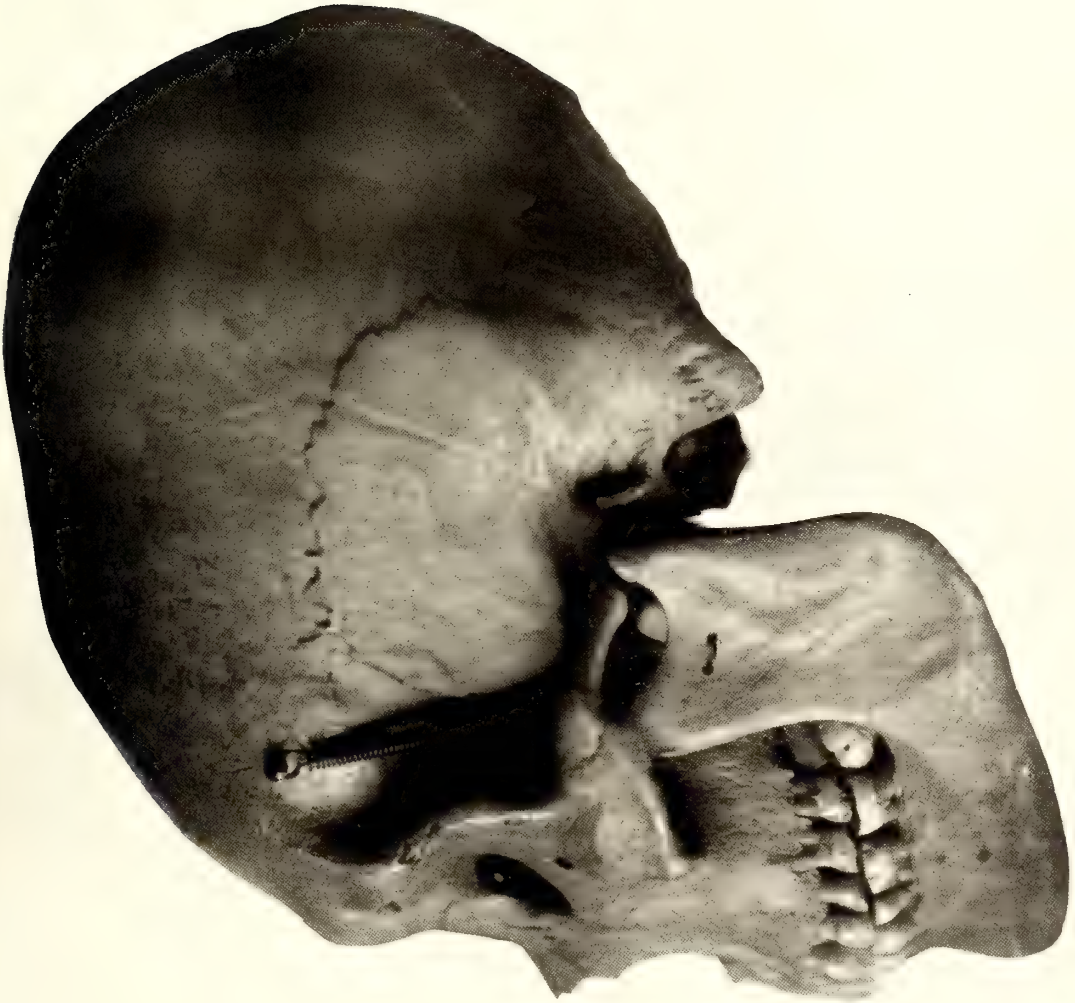
* It is proposed to follow this matter up by actually continuing the tracing of the transverse section into the auditory meatus to the limit of the Klaatsch tracer's possibilities and comparing with the craniophor auricular heights.

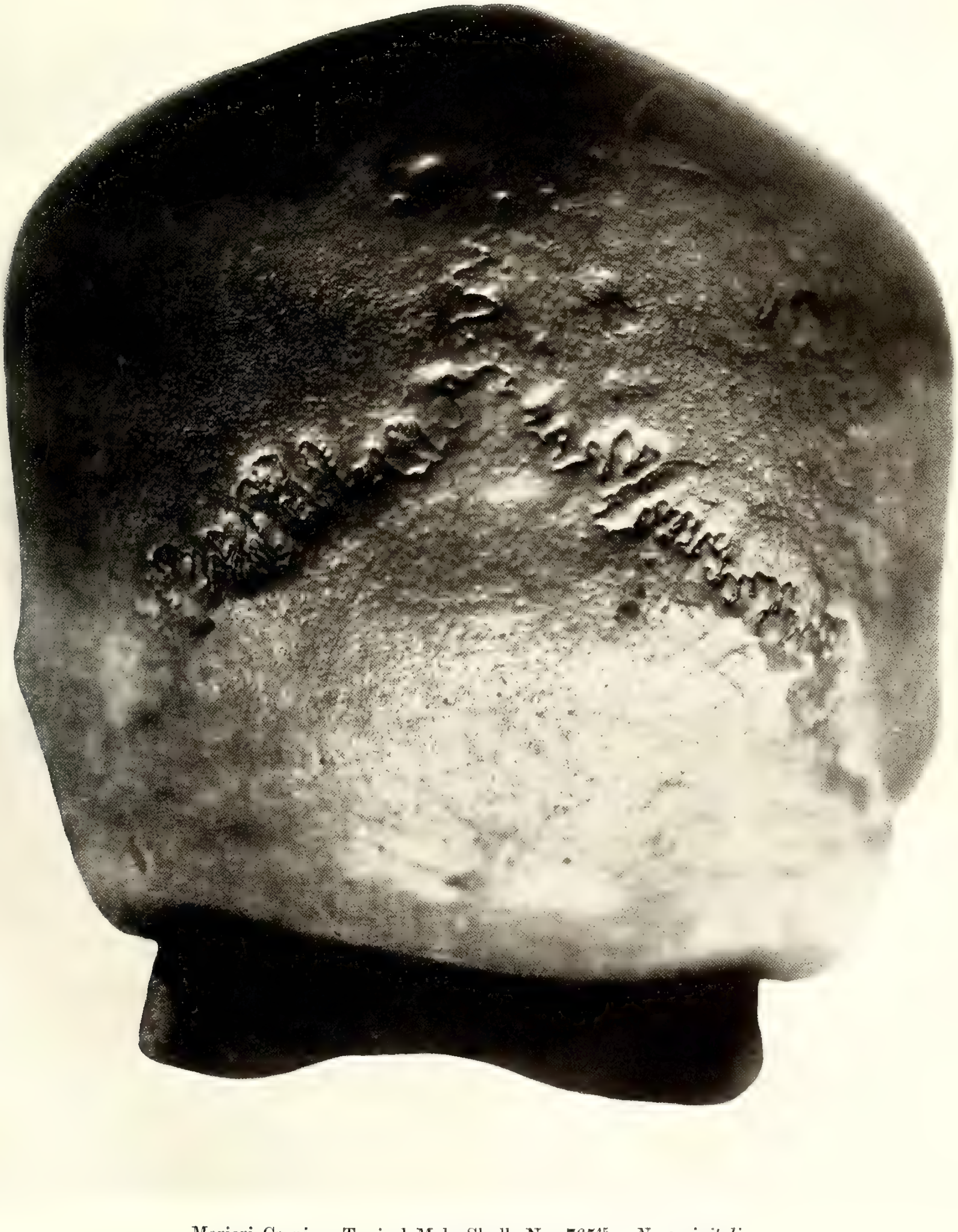
† See, however, the remarks later on this point.



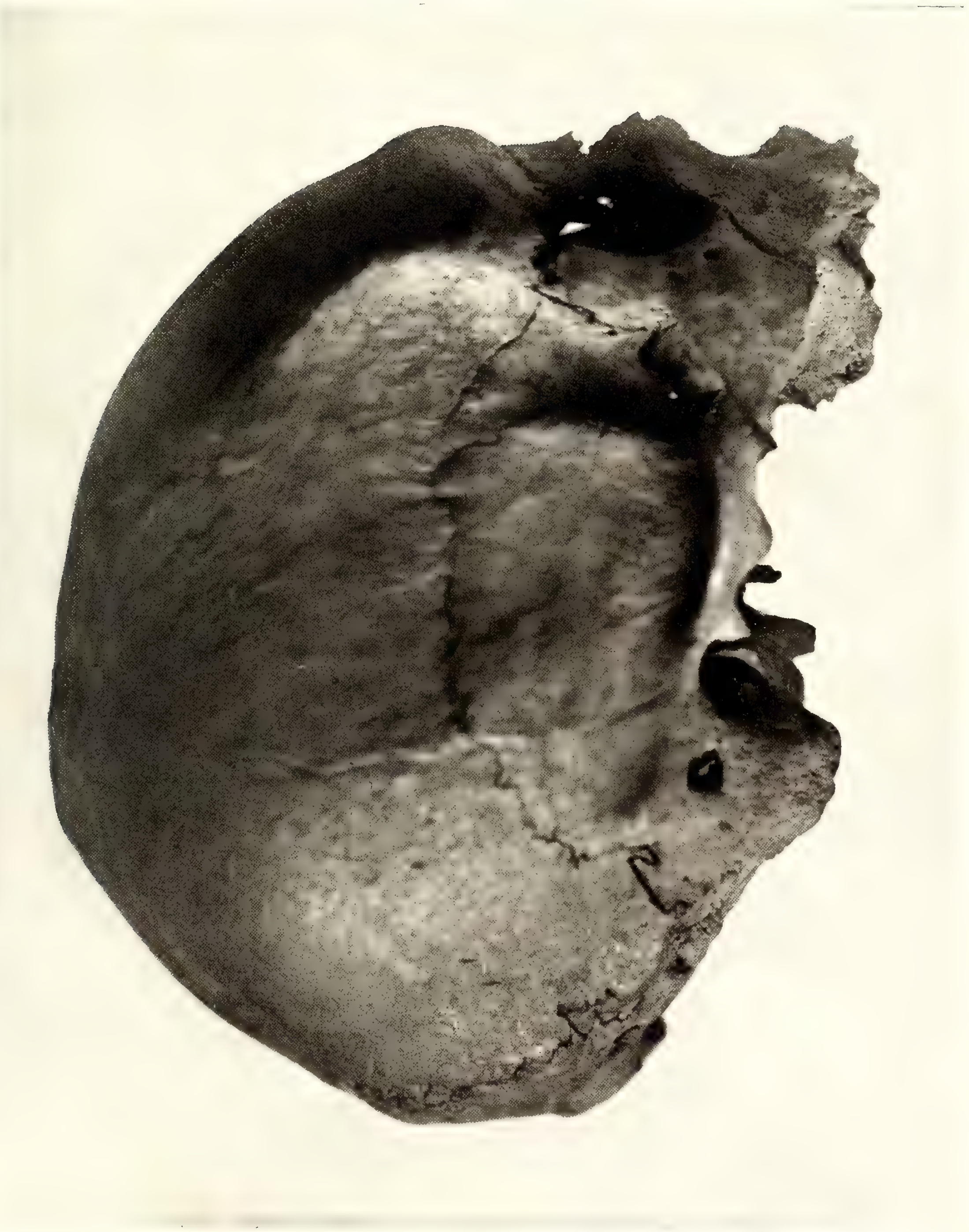
Moriori Crania. Typical Male Skull, No. 765^{45}. *N. facialis.*



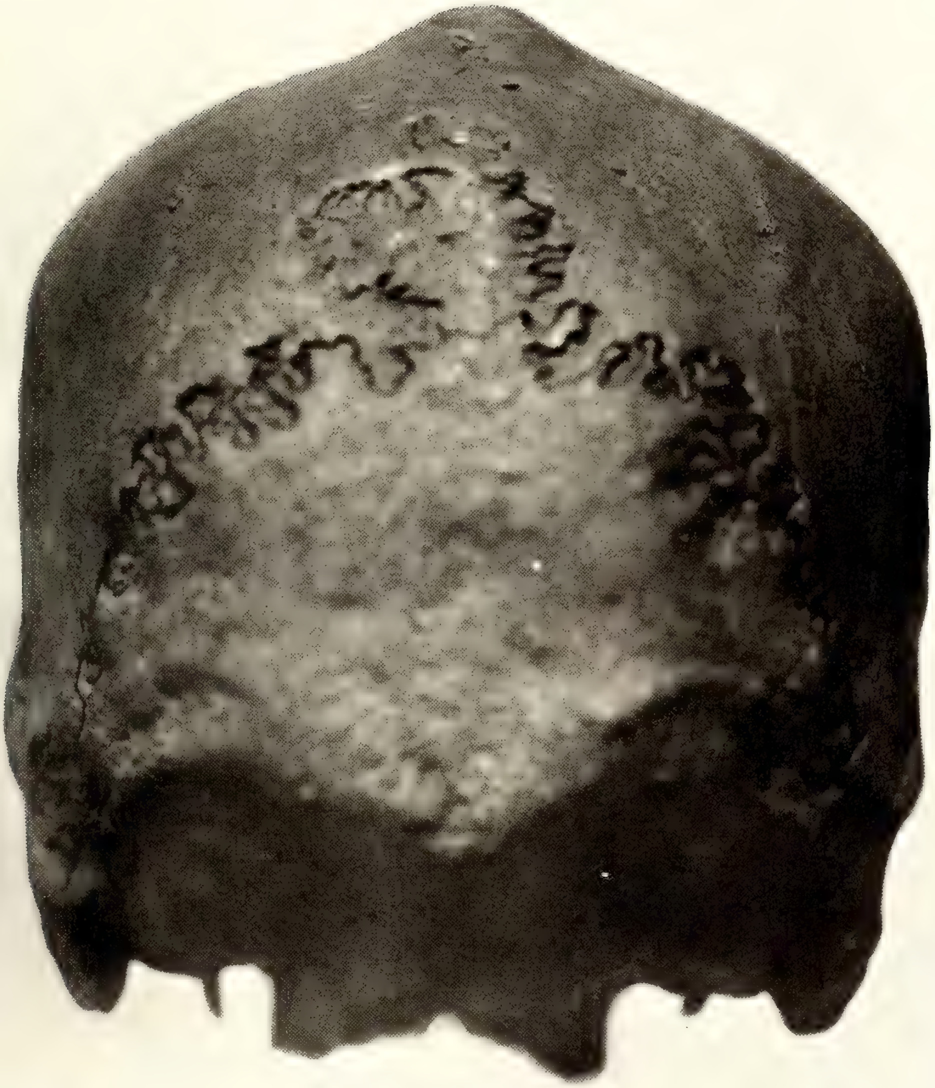
Moriori Crania. Typical Male Skull, No. 765^{45}. *N. lateralis*



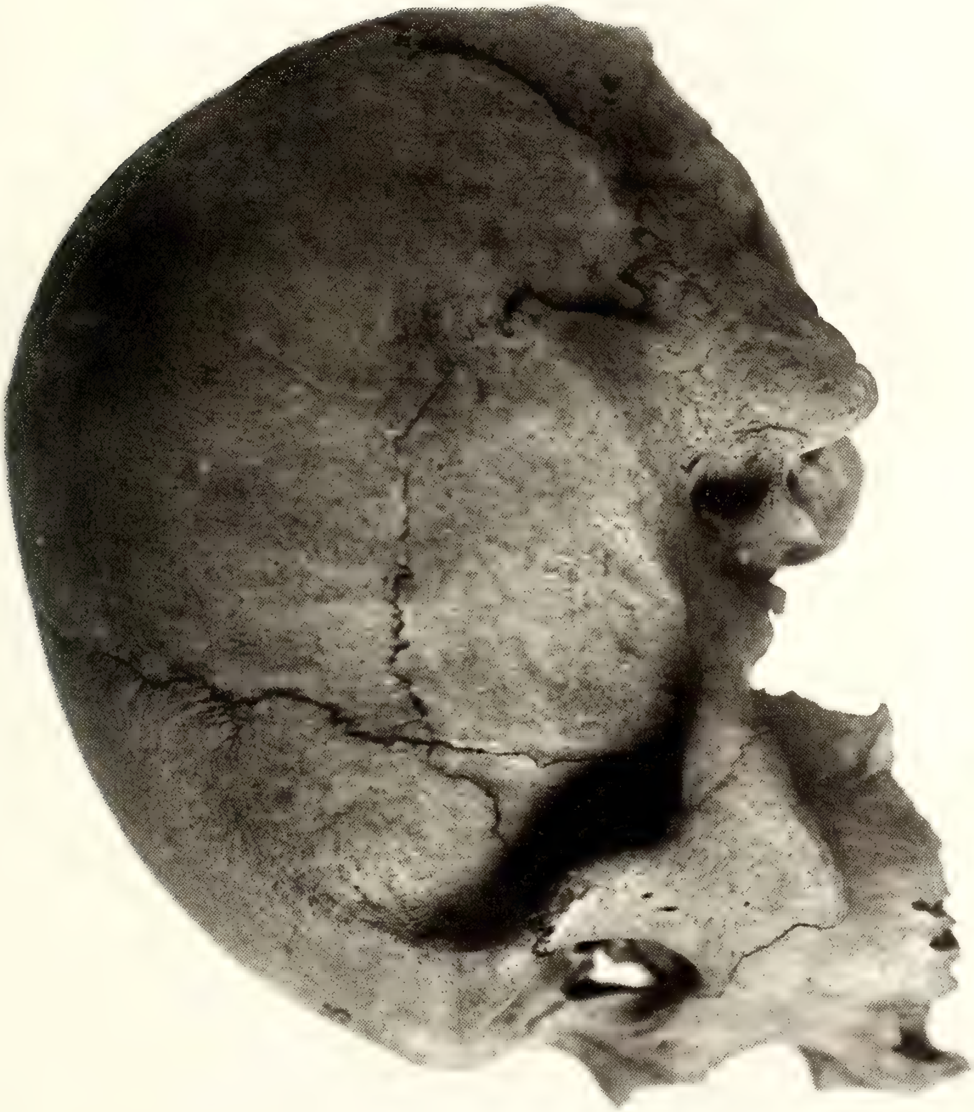
Moriori Crania. Typical Male Skull, No. 765^{45}. *N. occipitalis.*



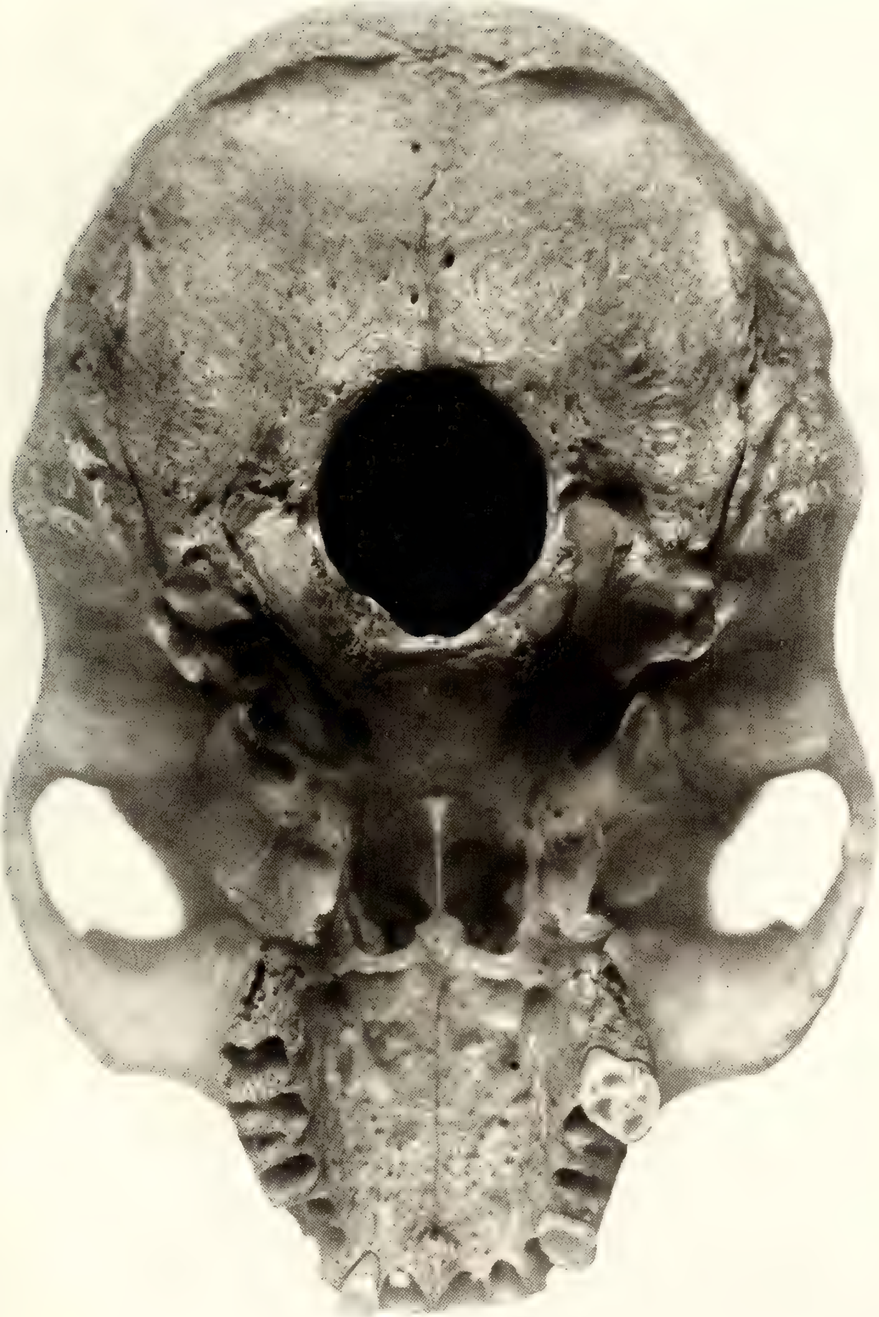
Moriori Crania. Typical Male Skull, No. 765^{10}. *N. lateralis.*



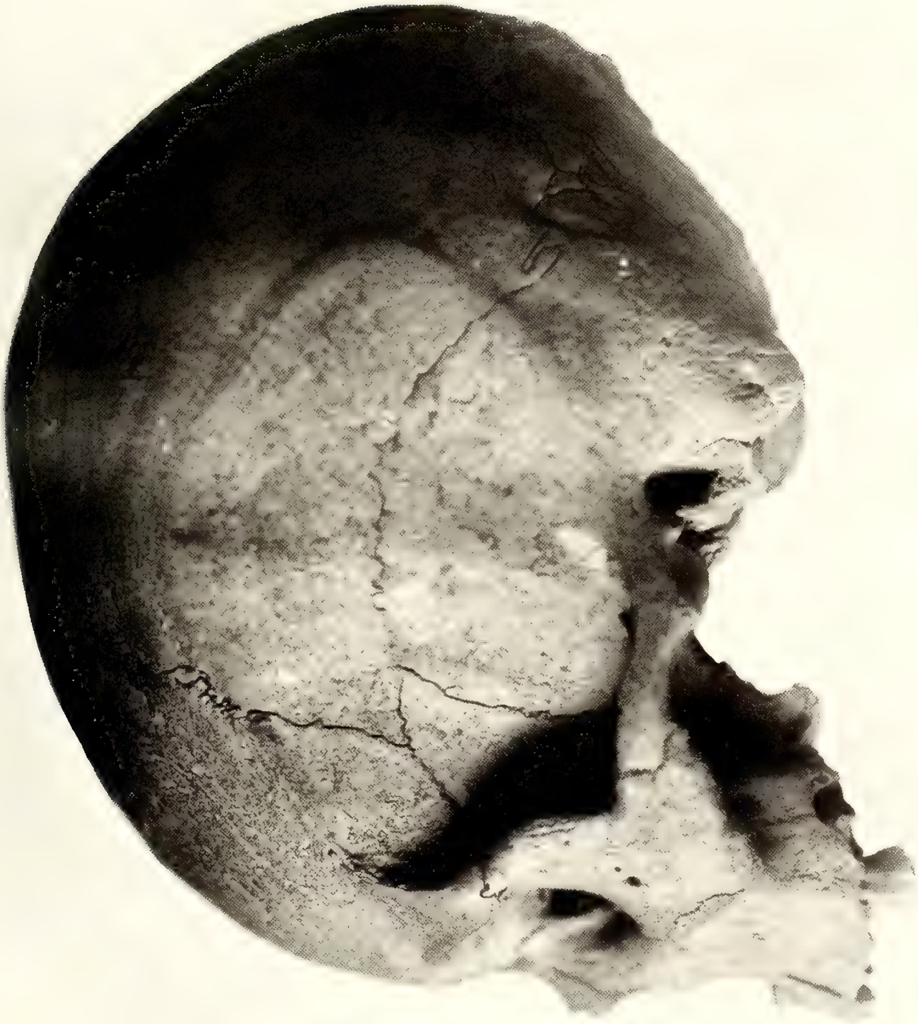
Moriori Crania. Typical Male Skull, No. 765^{19}. *N. occipitalis.*



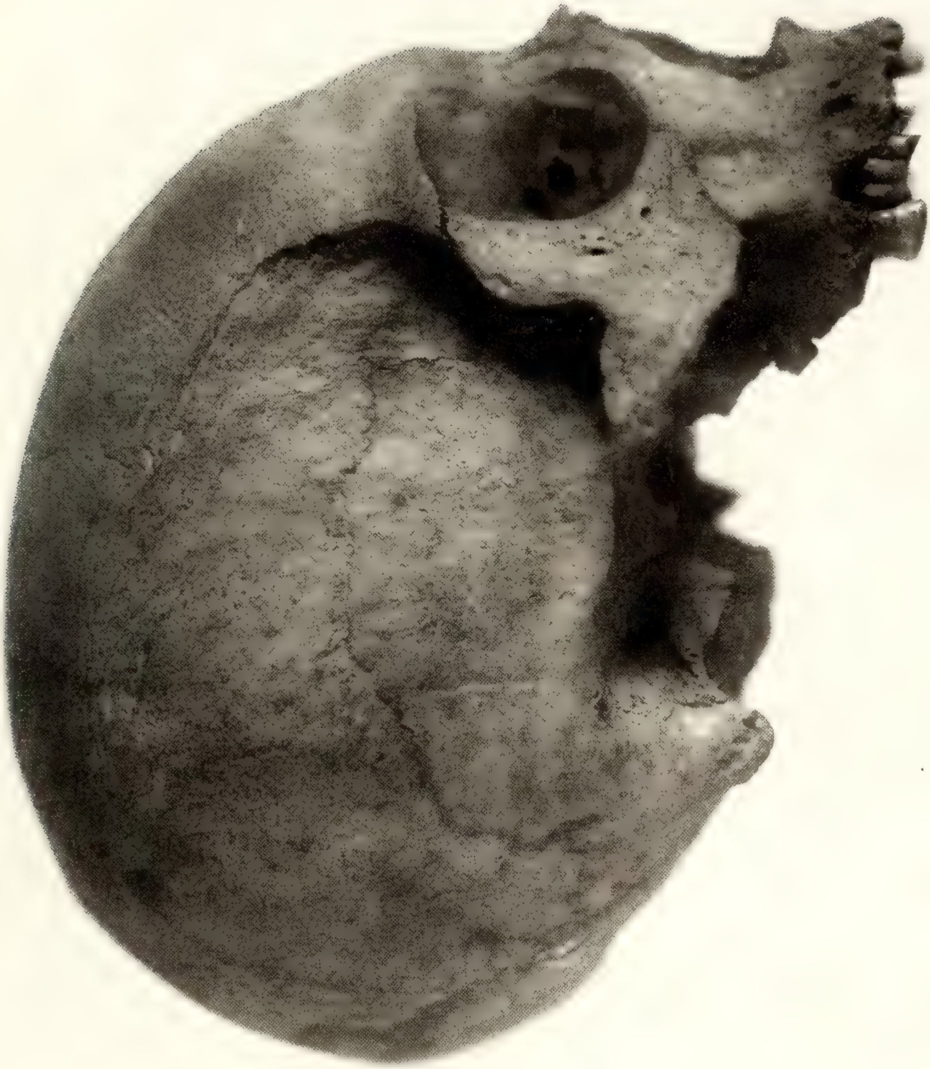
Moriori Crania. Typical Male Skull, No. 765^{25}. *N. lateralis.*



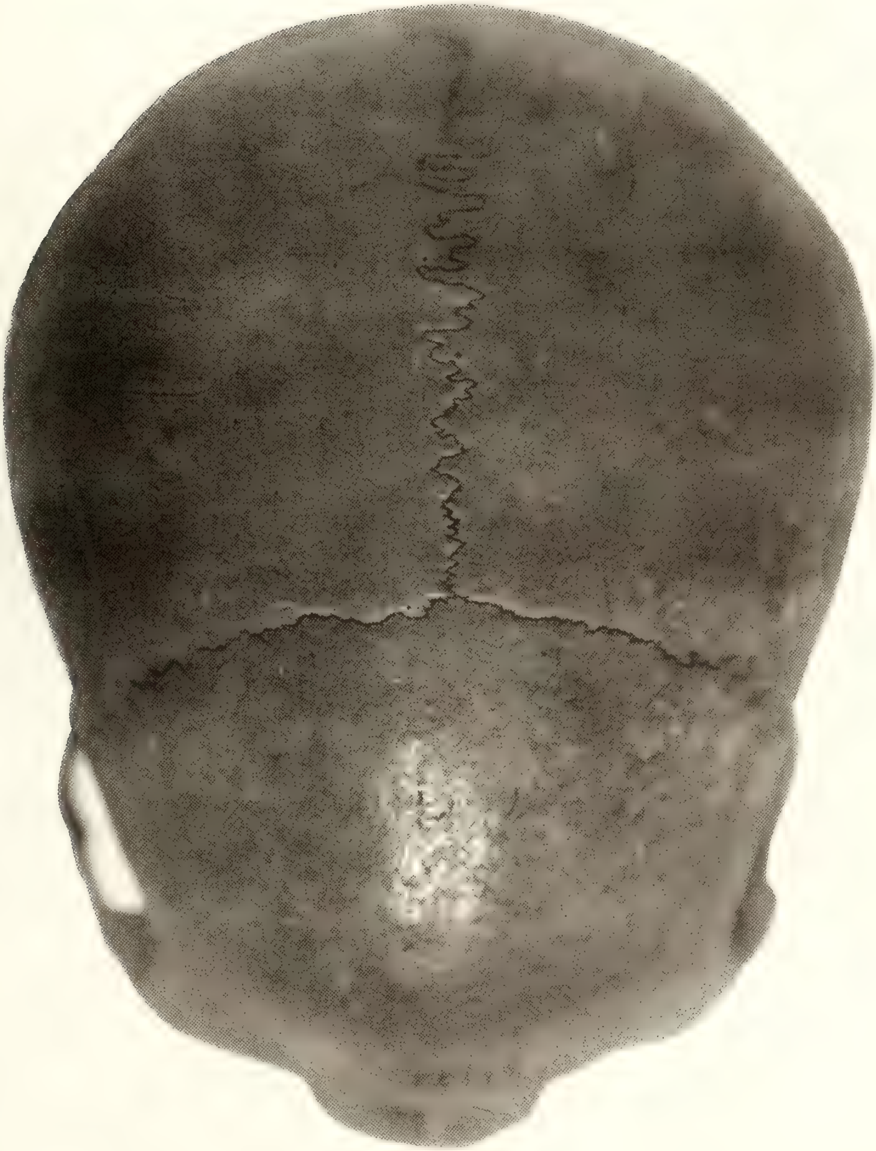
Moriori Crania. Typical Male Skull, No. 765[25]. *N. basalis.*



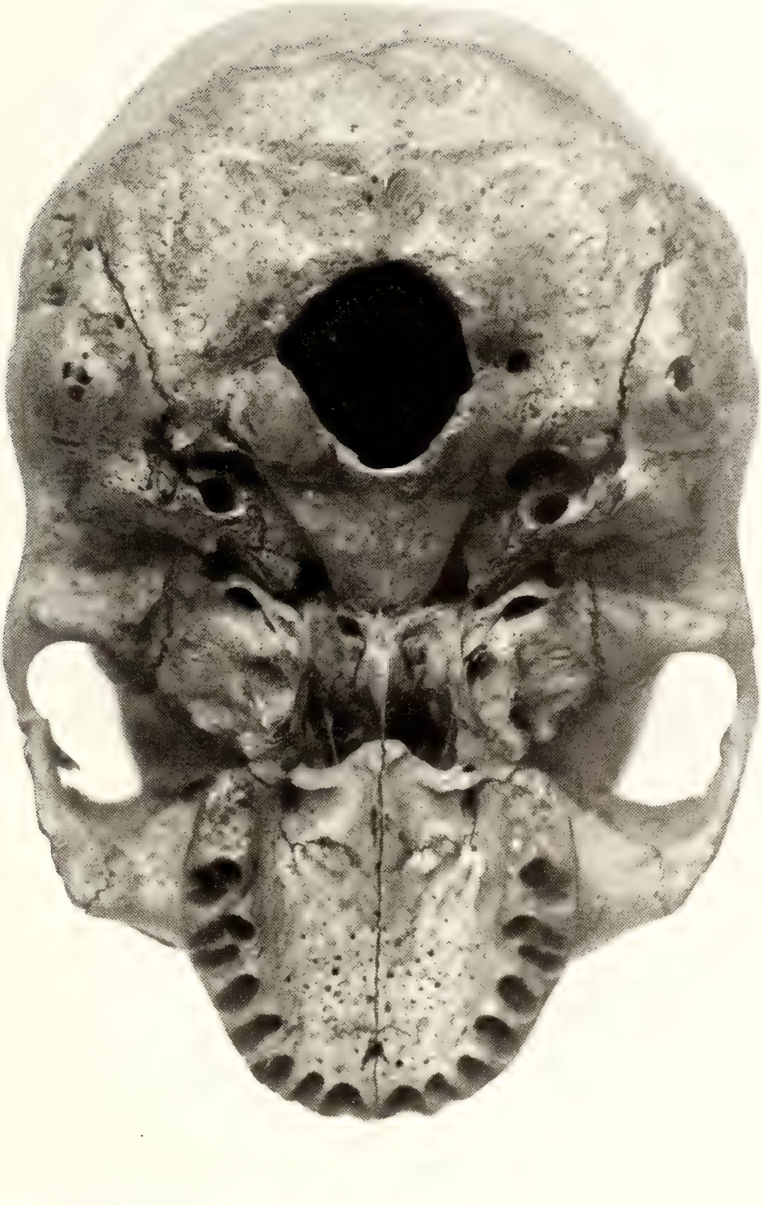
Moriori Crania. Typical Female Skull, No. 765. *N. lateralis.*



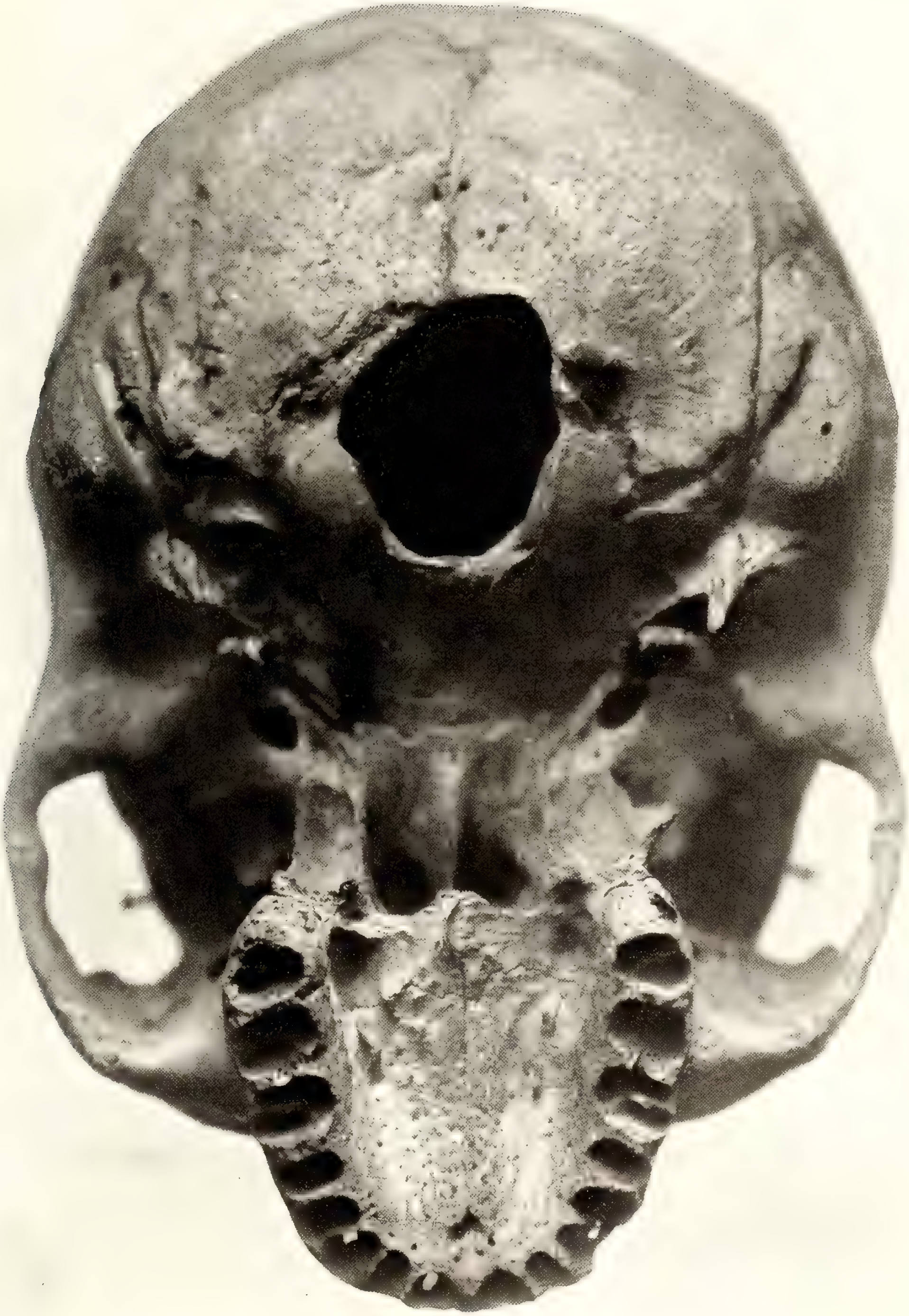
Moriori Crania. Typical Female Skull, No. 765^{32}. *N. lateralis.*



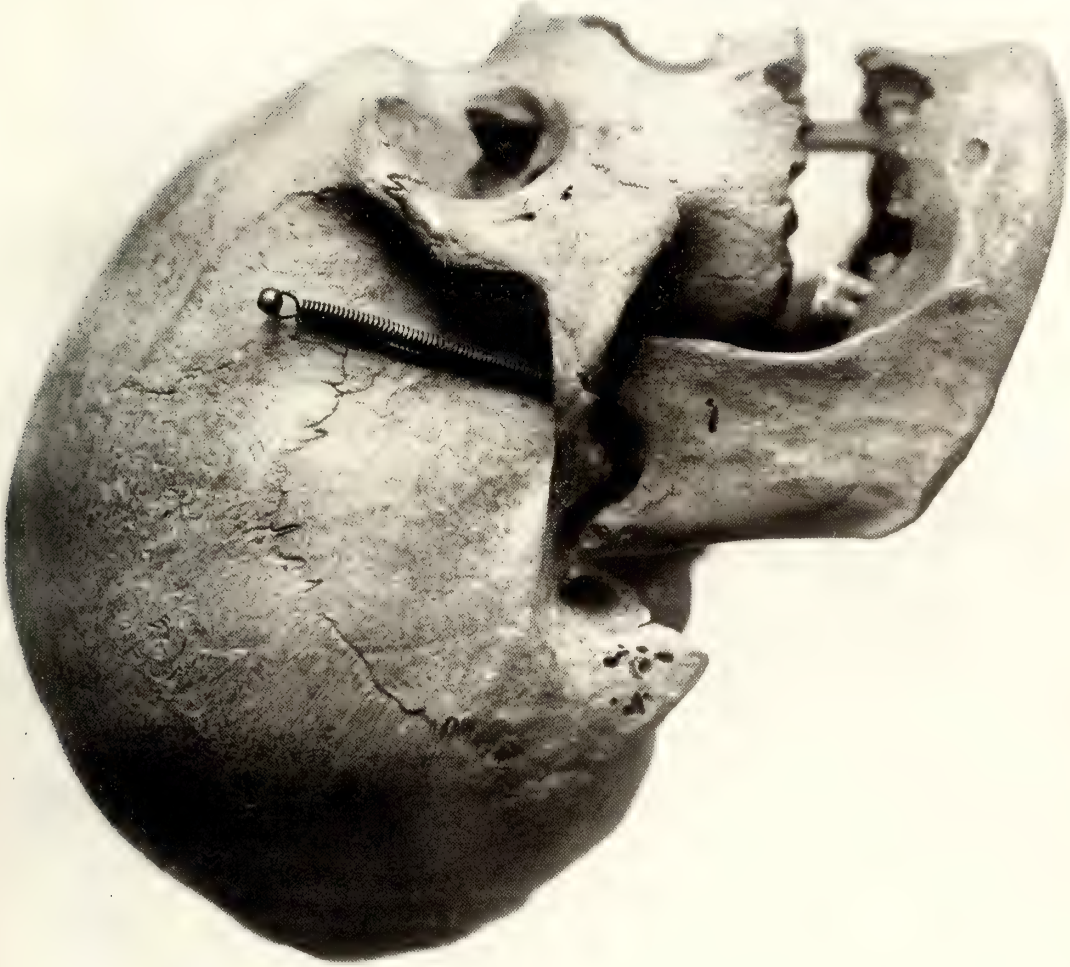
Moriori Crania. Typical Female Skull, No. 763. *N. verticalis.*



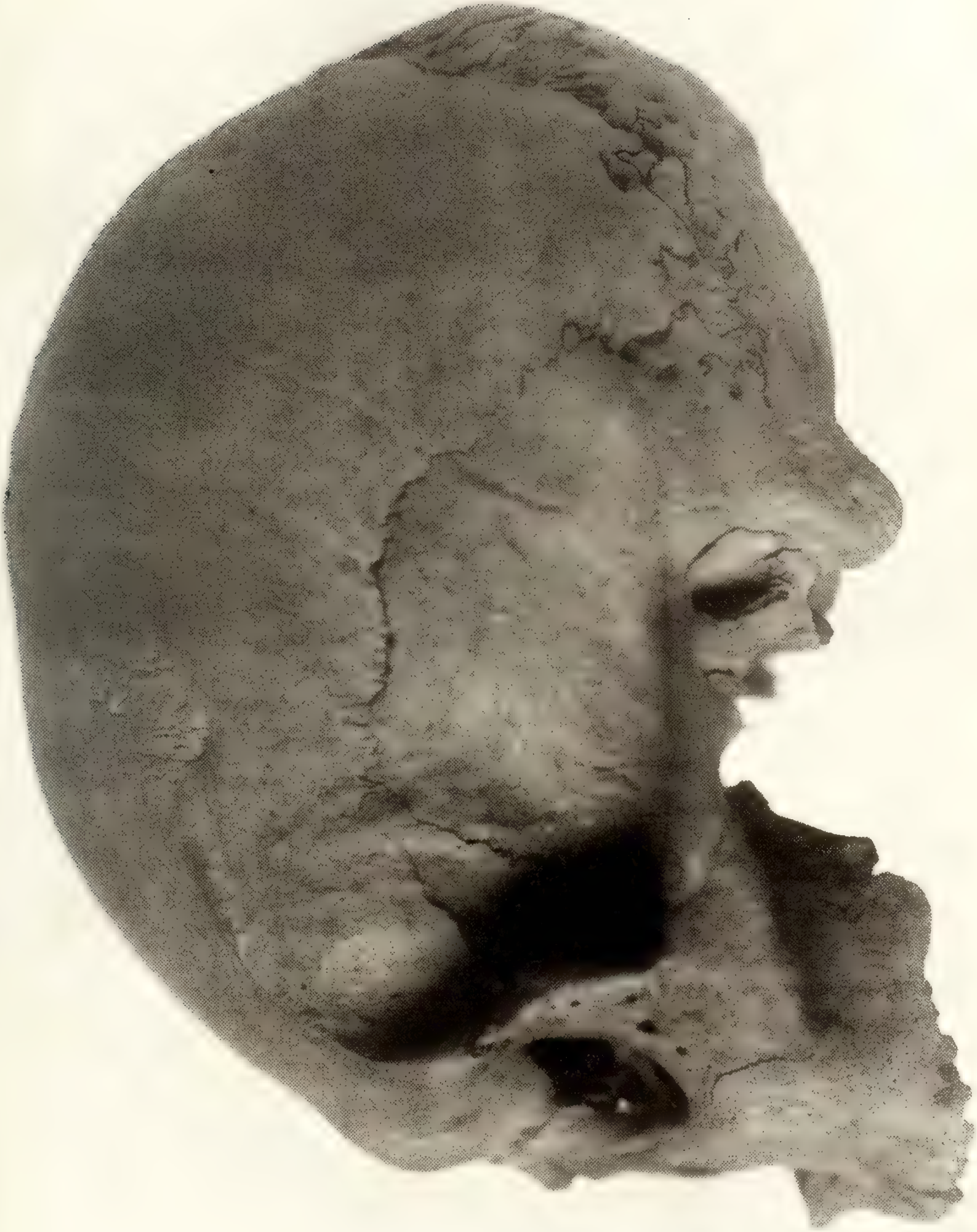
Moriori Crania. Typical Female Skull, No. 763. *N. basalis.*



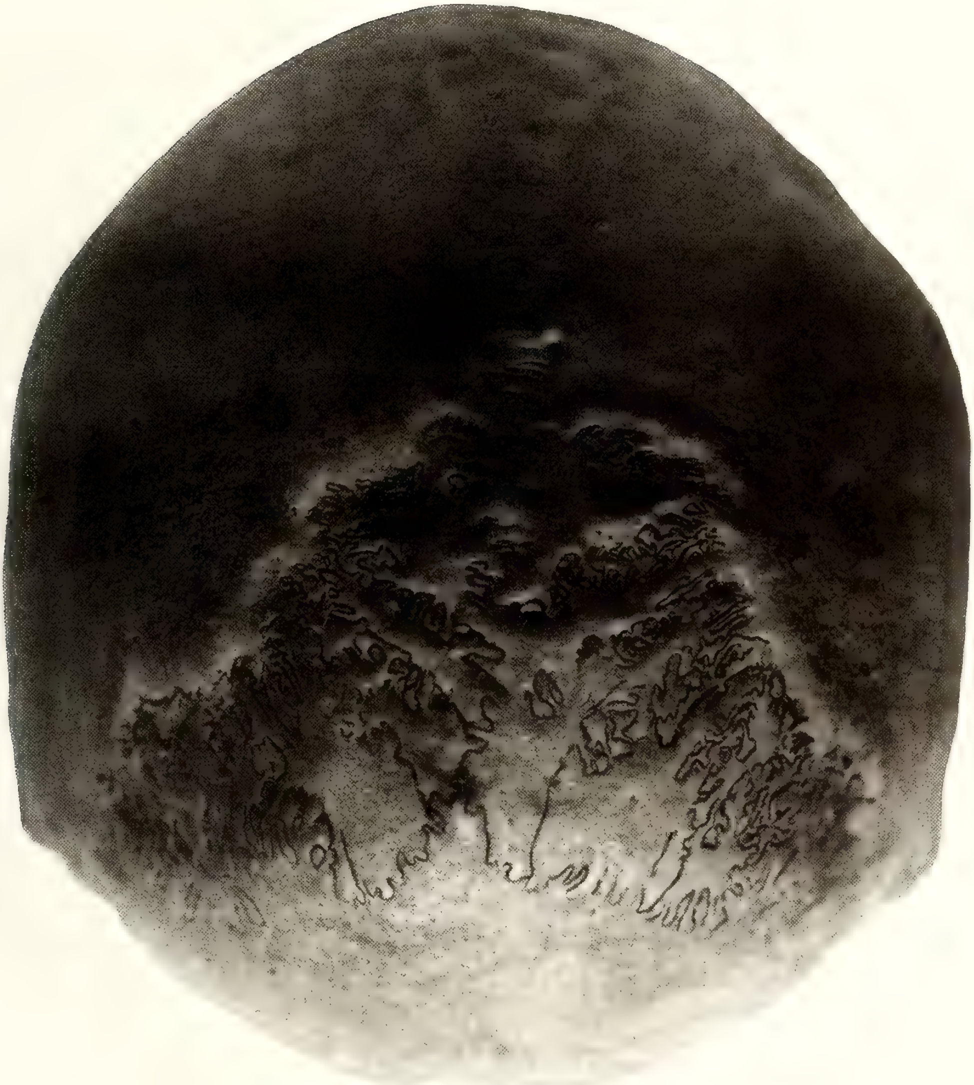
Moriori Crania. Male Skull, No. 765^5. Marked asymmetry of foramen magnum. *N. occipitalis.*



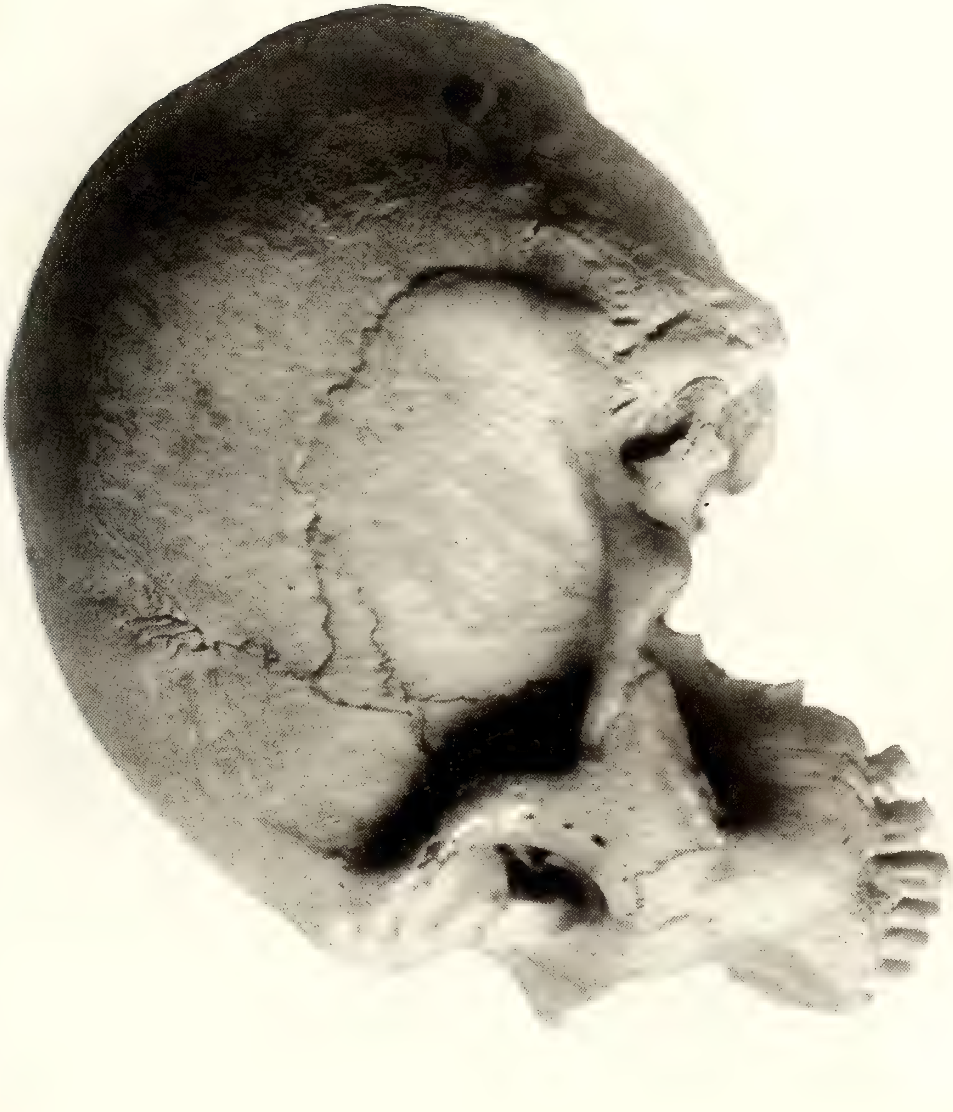
Moriori Crania. Male Skull, No. 765^5. *N. lateralis.*



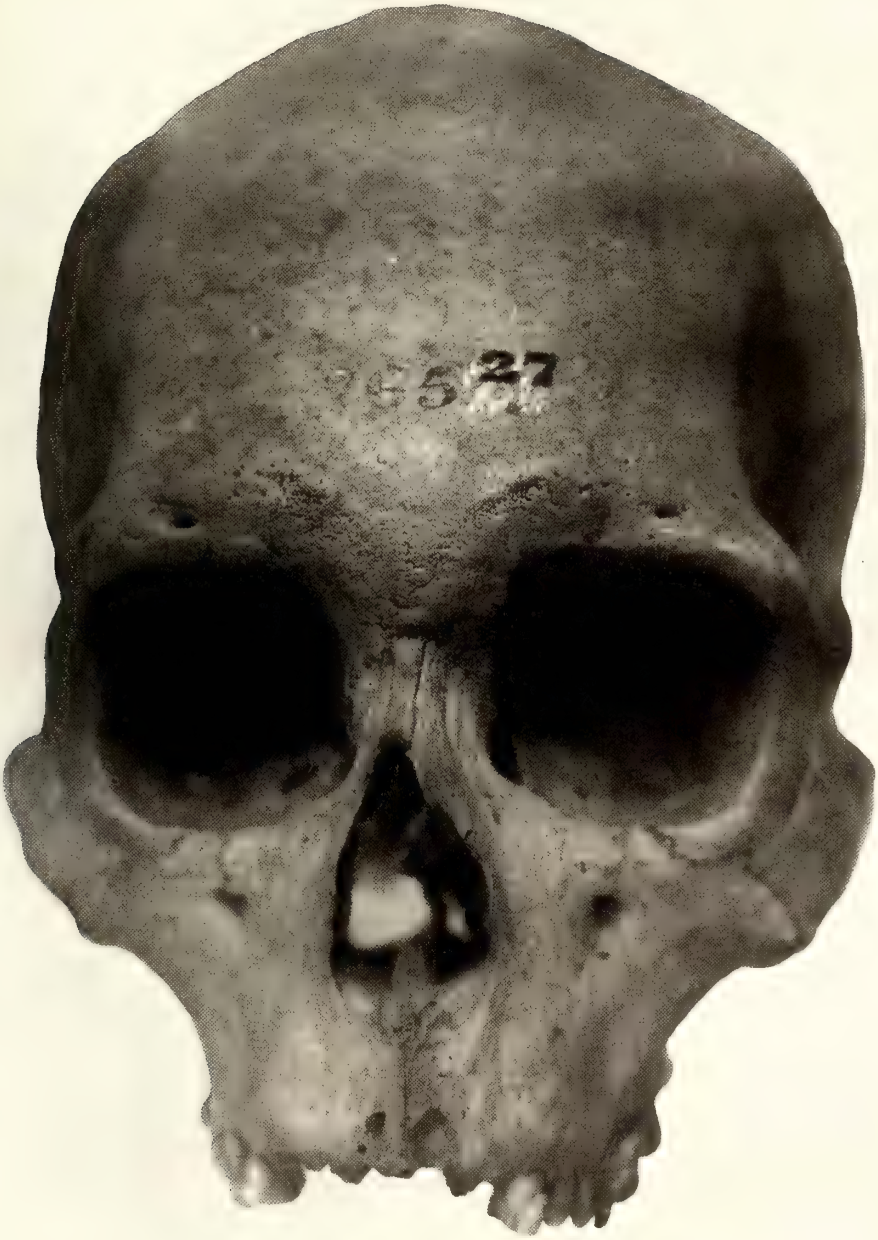
Moriori Crania. Male Skull, No. 762. Ossicle of coronal suture; occipital bathrocephalic in character. *N. lateralis.*



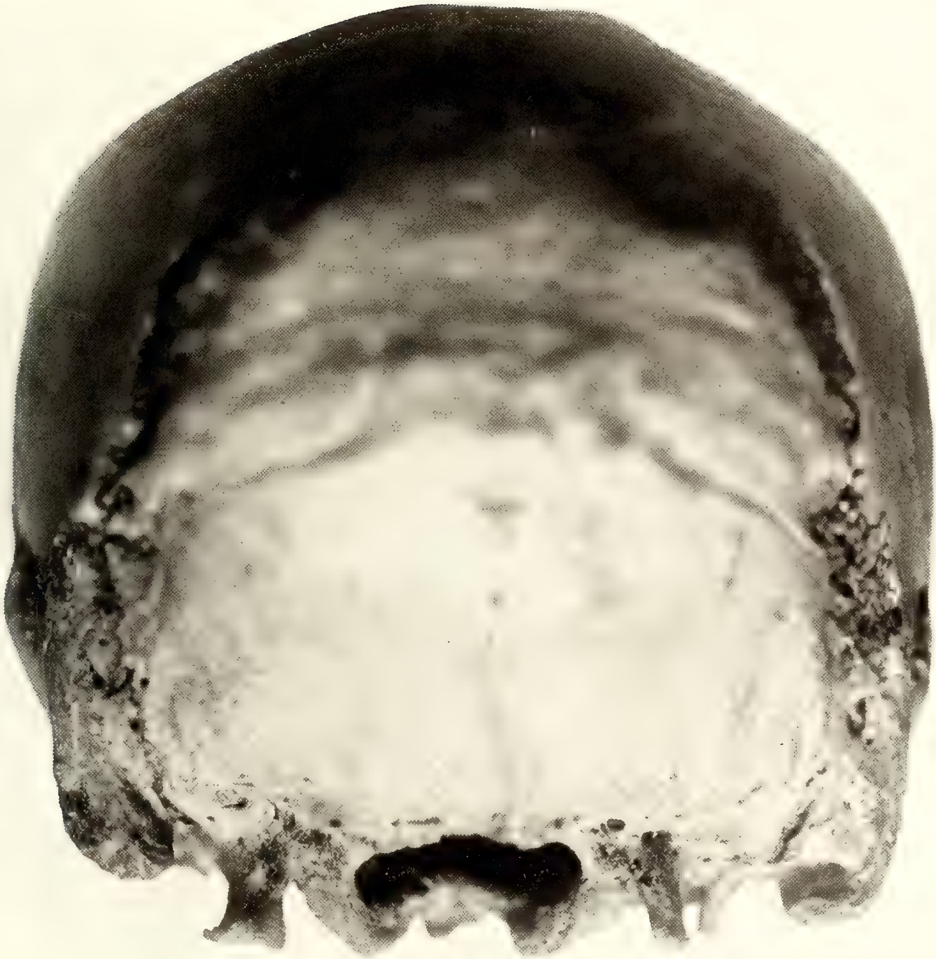
Moriori Crania. Male Skull, No. 762. Ossicles of the lambdoid suture. *N. occipitalis.*



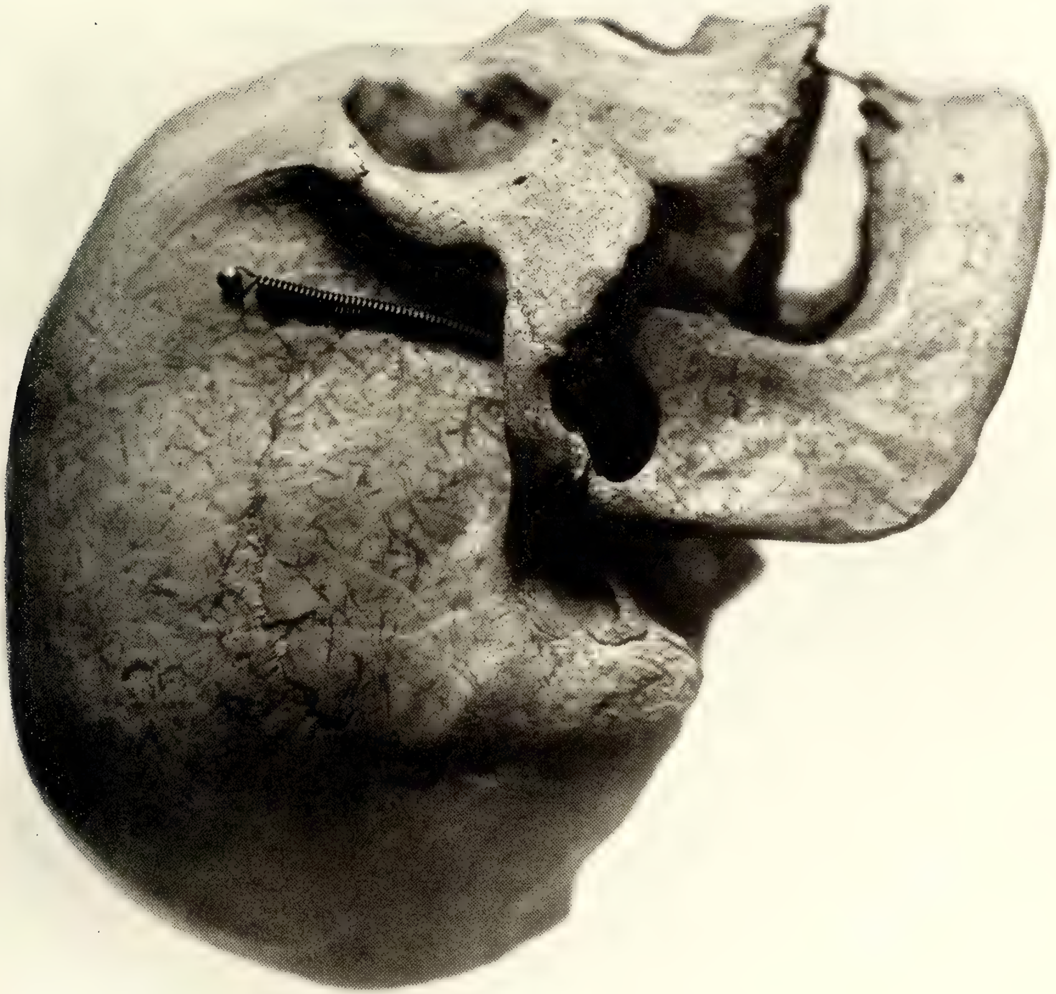
Moriori Crania. Male Skull, No. 765^{37}, with epipteric. *N. lateralis.*



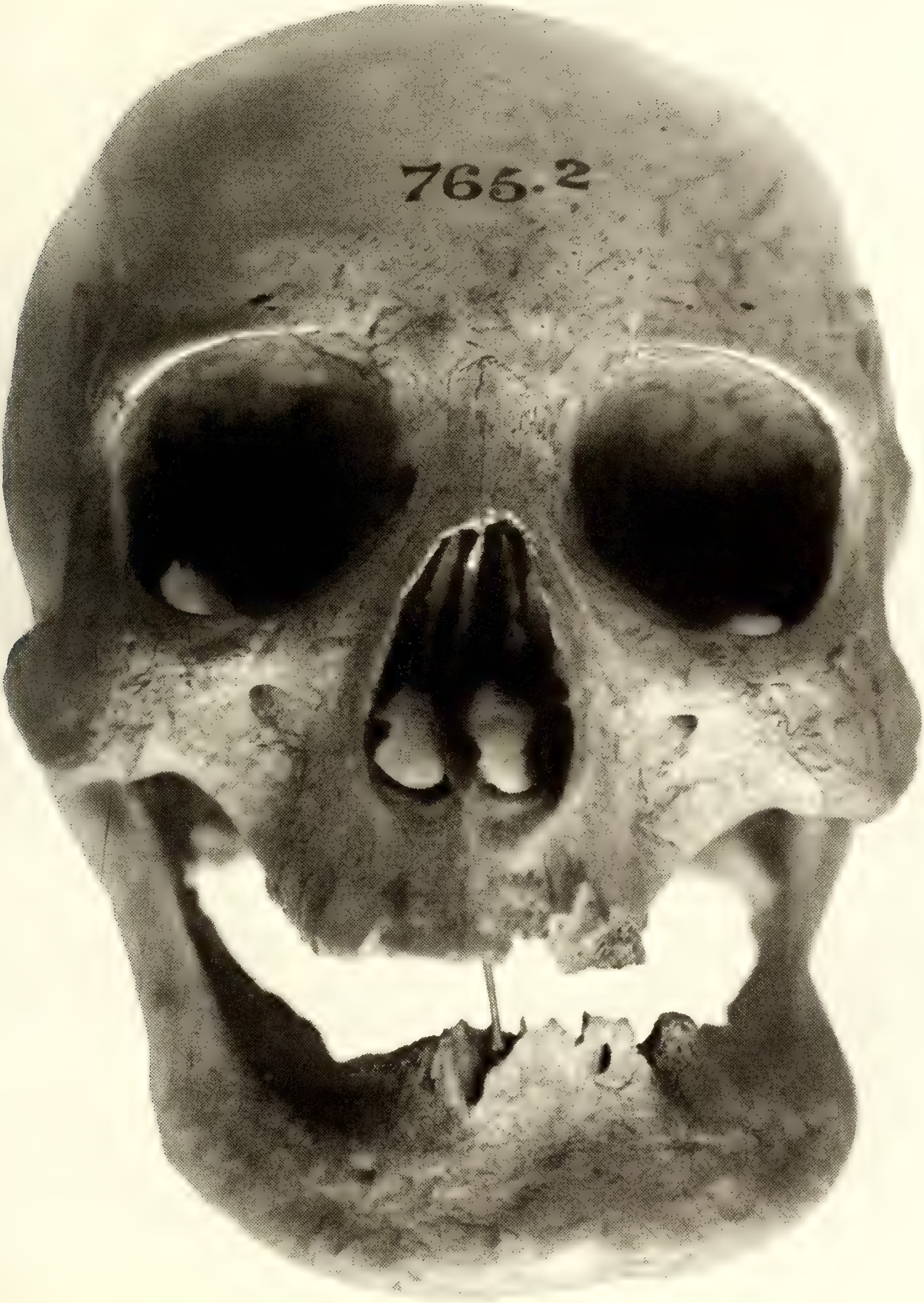
Moriori Crania. Male Skull, No. 765²⁷. Massive glabella. *N. facialis.*



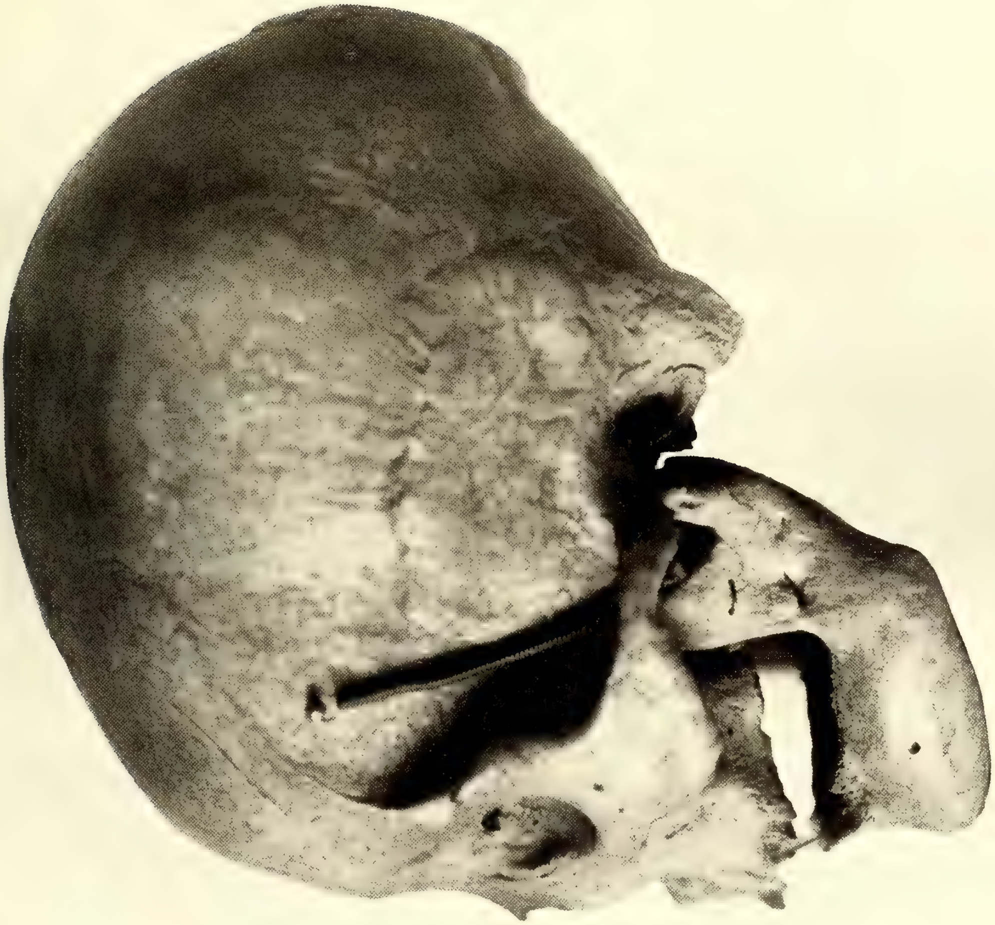
Moriori Crania. Skull, No. 765^{17}, with marked bilateral paraoccipital processes articulating with atlas.
For a unilateral case: see *Biometrika*, Vol. v. p. 104, Plate X.



Moriori Crania. Male Skull, No. 765^2, illustrating fracture of left ramus of mandible. *N. lateralis*, R. side with sound ramus.



Moriori Crania. Male Skull, No. 765², illustrating fracture of left ramus of mandible. *N. facialis*, showing asymmetry produced by the injury.



Moriori Crania. Male Skull, No. 765^2, illustrating fracture of left ramus of mandible. *N. lateralis*, L. side with healed fracture and dwarfing of ramus and perigonal region.

TABLE VI. *Measurements from Sagittal Type Contours.*

Character	Male		Female	
	By Cranial Average	By Type Contour	By Cranial Average	By Type Contour
F	182·0	182·2	176·1	175·8
L'	185·6	185·6	177·8	178·2
H	135·9	135·4	132·8	131·4
LB	105·6	106·0	102·3	101·5
$G'H$	76·4	77·0	70·0	71·1
GB	103·2	101·0	96·6	97·5
fml	35·3	37·2	34·4	38·5
$Oc\ I$	62·2	62·1	64·2	64·2
Frontal Index	21·6	19·4	21·9	21·1
$N\angle$	64°·9	64°·8	65°·7	66°·3
$A\angle$	71°·8	71°·9	73°·6	72°·2
$B\angle$	43°·4	43°·2	40°·7	41°·5
$P\angle$	84°·7	88°·0	84°·5	86°·8

draughtsman's error) in the female. I take it that in the measurement of the foramen with the dividers the tendency is to measure the inside border of the foramen, but with the Klaatsch contour tracer to stop just short of the edge. It seems to me reasonable to suppose that if the basion and opisthion were first marked off as points and the foraminal length then compared with actual measurement, we should insure not only the exactitude of the foraminal length, but of the occipital arc.

(ii) The profile angle. Here the difference is 3°·3 in the males and 2°·3 in the females; this is serious. I take it to arise from a divergence in the determination of the alveolar point (i) with the sagittal contour vertical and using the goniometer, and (ii) with the sagittal contour horizontal and using the Klaatsch. But I must reserve the discussion of the point until further experiments have been made. The fundamental triangles are in quite good accordance, and accordingly the evil may lie in the determination of the auricular point, when the skull is off the craniophor. We note how closely the occipital indices deduced from the contours approach those obtained from the measurement of occipital arc and chord. Indices from the contours were taken as i = occipital subtense/occipital chord. Then if I be the occipital index as defined by

$$I = 100 \frac{S_3}{S'_3} \sqrt{\frac{S_3}{24\,(S_3 - S'_3)}},$$

it is easy to deduce that

$$I = 100 \left(\frac{1}{2} i + \frac{1}{8i}\right).$$

The occipital indices so far obtained are:

	Male		Female	
English (Moorfields)	58·5		59·2	
Moriori	62·2		64·2	
Gaboon, Bantus	68·3	68·5	69·2	68·1
Congo, Batetelus	68·8		67·0	

The smaller the index the smaller is the radius of curvature of the occipital arc in relation to its chord, i.e. the occipital is more rounded. The order shows that the English have most, the Negro least occipital development, and that the Moriori male stands nearer to the European than to the Negro.

The Frontal Index is on another footing. If the nasio-bregmatic chord and arc be given, i.e. S'_1 and S_1, then

$$I_f = 100\frac{S_1}{S'_1}\sqrt{\frac{S_1}{24(S_1 - S'_1)}}$$

gives the frontal index in the form of the ratio of the radius of curvature of the frontal arc to the frontal chord. To obtain this index the distance from nasion to bregma along the chord should be measured with the callipers. Not having measured this, I took the values from the contours which should be extremely close to the measured values as the terminals are very definite points. The results were:

Males, 68·7; Females, 67·9.

If now we wish to replace the index by $i_f = 100 \times$ subtense/chord, we must deduce i_f from

$$i_f = 100\left(\frac{I_f}{100} - \sqrt{\left(\frac{I_f}{100}\right)^2 - \cdot 25}\right).$$

There resulted the values 21·6 and 21·9 to compare with the direct contour values of i_f, 19·4 and 21·1. The reason for the excess is clear: it is due to the exaggeration of the length of the frontal arc (treated as a circular arc) owing to the massive glabella, more divergent in the case of the male than the female*. On the other hand if we take English males, we find from arc and chord measurements of the frontal $I_f = 70{\cdot}1$ leading to $i_f = 21{\cdot}0$, which is in closer accord with 22·2, the direct value of the subtense to chord index, thus indicating that with less glabellar prominence, the results of the two methods would be more nearly equal and would more closely justify the assumption of the approximate circularity of the frontal arc†. On the whole if the frontal curvature is to be determined by direct measurement, i.e. not by contour drawing, it will be desirable to adopt a modified form of Merejkowsky's simometer‡. It would not be hard to devise, but it would have to be considerably larger, and the subtense would have to be obtained by contact—it would not necessarily be at the bisector of the chord.

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* Had we supposed the arc to be catenary rather than circular the values (see *Tables for Biometricians and Statisticians*, p. 64) would have been 22·7 and 23·1, still more divergent. A parabolic arc was found to be even less successful.

† Tasmanian aborigines (Berry and Robertson, *R. S. Edin. Proc.* Vol. XXXI. Table p. 60) give for combined sexes, from I_f (64·91), $i_f = 22{\cdot}33$.

‡ *Biometrika*, Vol. VIII. p. 317.

To sum up: the comparison we have made suffices to show that cranial type contours can be of great service; they are justified by the generally good concordance they give when tested against average measurements of the skull made in the ordinary manner. But if they are to serve for the purpose of accurate measurement and for more than a rough comparison of race with race, we think a considerably improved technique will be essential and if, as we believe, it is desirable to compare the contours of the individual crania step by step with their actual measurements, then the contour drawing will not relieve us from the usual craniometric processes, but at first serve only as a control. In particular the difficulties which have arisen in our own case centre especially round: (*a*) a better agreement in the position of the auricular point as determined (i) by the craniophor and (ii) on the transverse and sagittal contours; (*b*) in the better location of the opisthion and alveolar point on the sagittal contour.

These do not seem substantial difficulties, but they need practice and caution. The basion can always be inserted from the nasio-basionic length and basio-bregmatic lengths (LB and H); the opisthion from the foraminal length and the lambda opisthionic chord (fml and S'_3). The alveolar point is then determined by the upper face height (GH') and the profile length (GL). To avoid the contour determination of the auricular point, we might mark the apex (A) of the individual skull when on the craniophor as well as the gamma. Then on the type contour by drawing a line through the average apex perpendicular to the average $N\gamma$, and plotting off along this line the mean auricular height (OH), we should obtain an auricular point which would agree with the cranial measurements. The parallel to $N\gamma$ through the auricular point thus determined would give the standard horizontal plane, and the profile angles by contour and measurement should now coincide.

A marked feature of the type Moriori sagittal contours for both male and female is evident on inspection—the auricular point is vertically above the basion. It is not true for individual crania, but it may be true for type crania (i.e. on the average) for other races than the Moriori. If so the vertical through the basion would give the apex and the auricular point could be determined from the mean auricular height, and control would be easier to establish. In general the basi-auricular line as an approximate or average vertical of the head deserves consideration*.

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* It is proposed shortly to observe skull by skull the difference between the contour and the direct measurements with a view to improving technique. The English male sagittal type contour when the basion and alveolar points were added as above suggested gave a profile angle of 86°·3, according reasonably with the measured angle 86°·1, but the same method applied to the Congo Batetelu cross-section only gave a profile angle of 81°·2 as against 82°·8 measured directly on the crania. The actual marking of apex and gamma when the skull is on the craniophor and the transfer of these marked points to the contour will undoubtedly aid in ascertaining what is needful to bring contour measurement and direct measurement systems into full unison. At the same time it must be remembered that the characters of a mean or type contour cannot be demonstrated theoretically to coincide absolutely with the means of characters not used in constructing the contour.

8. *On the Moriori Mandible.* A difficulty arises in treating mandibles in general from the fact that they are usually very few in number, often separated from the skulls, and then difficult to sex, and further that as many writers neglect them altogether there are few data for comparative purposes*. The present material contains only eleven mandibles and of these one certainly did not belong to the skull with which it was associated. Four were female and seven were male. Scott has fourteen mandibles, twelve male and two female. In the case of the Maori he has 24 male and seven female mandibles. In the following table I have added Scott's data to mine for the Moriori and compare results with Macdonell's for English, Fawcett's for Naqada Egyptians, Koganei's for the Aino, and a few scattered results for Negro races.

TABLE VII. *Measurements of the Mandible.*

Race...	Moriori			Maori			Aino		
Sex ...	♂	♀	♂+♀	♂	♀	♂+♀	♂	♀	♂+♀
Bicondylar Width... ...	124·1	114·8	122·3	127·2	120·9	125·8	102·0	95·6	99·5
Bigonal Width	106·4	101·6	105·3	104·5	97·3	102·8	—	—	—
Height at Middle Incisors	35·3	30·7	34·2	34·5	29·7	33·4	33·8	32·6	33·4
Mental Distance	51·6	49·1	51·0	50·5	49·1	50·2	—	—	—
Total Numbers (round) ...	19	6	25	24	7	31	60	35	95

Race...	English	Gaboon, Bantus			Northern Negro	Fuegian	Egyptian, Naqada		
Sex ...	♂+♀	♂	♀	♂+♀	♂	♂+♀	♂	♀	♂+♀
Bicondylar Width... ...	113·2	112·6	110·0	111·6	117·0	120·6	110·5	106·4	108·2
Bigonal Width	95·4	94·3	90·1	94·4	97·2	103·4	93·6	87·6	90·4
Height at Middle Incisors	30·8	32·7	30·5	31·8	—	35·4	32·9	31·6	32·2
Mental Distance	43·7	45·4	44·3	44·9	—	—	44·4	43·1	43·7
Total Numbers (round) ...	140	29	20	49	16	5	50	60	110

Now these results, slender as are the numbers, seem of extreme interest. All comparison of the English and Aino mandible with that of the Moriori is now out of place. The Moriori, Maori and Fuegian are the only mandibles with a bicondylar width over 120 mm. and a bigonal width over 100. But the most marked differentiation of the Moriori and Maori from those of other races is the distance apart of the *foramina mentalia* which is over 50 cms. It is a misfortune that we have not this distance for the Fuegian mandibles, in order that we might ascertain whether it is usual for heavy large mandibles to have a mental distance of these dimensions, or whether it is a special racial character in Moriori and Maori. Judged by the general massiveness of mandible the racial order is:

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* The Laboratory has in hand a series of many hundred Egyptian mandibles on which a much fuller system of measurements has been made. It is hoped to reach more definite criteria for sexing from this material.

Moriori and Maori—Fuegian—Northern Negro—English—Gaboon Bantu—Egyptian—Aino.

Taken in order of depth of chin, we have:

Fuegian—Moriori—Maori—Aino—Egyptian—Gaboon Bantu—English.

Lastly, if we class these races by the index 100 × bigonal width/bicondylar width, which to some extent measures the verticality of the ramus, we have the order:

Moriori (86·1)—Fuegian (85·7)—Gaboon Bantu (84·6)—English (84·3)—Egyptian (83·6)—Northern Negro (83·1)—Maori (81·7)—Kaffir (81·0).

These orders are not very illuminating except as grouping the primitive races of the southern hemisphere together by massiveness of mandible, and showing the Moriori with the most vertical, the Kaffir with the most sloping ramus. It is clear that two racial types of skull may approach each other fairly closely, while there can at the same time be a wide difference between their mandibles.

9. *Description of and Remarks on the Photographic Plates of Moriori Crania.* Plates II, III and IV show a typical Moriori male cranium, No. 765^{45}. Plate II gives the *norma facialis* and indicates the high nose, the massive glabella, the flattened frontal and receding temporals. Plate III, the *norma lateralis*, left profile, indicates not only the same last three points, but the post-bregmatic depression followed by the sagittal crest, and further the massive mandible with vertical ramus. Plate IV, the *norma occipitalis*, marks the pentagonal form of this aspect as well as the generally massive character of the Moriori skull. The apex of the "pentagon" is the result of the sagittal crest.

Plates V and VI, No. 765^{10}, give another typical male cranium. Plate V, the *norma lateralis*, right profile, shows well the characteristic frontal region and the marked sagittal crest, also general massiveness in the occipital, inion and mastoid regions. Plate VI, the *norma occipitalis*, brings out, in a still more marked manner than even Plate IV, the sagittal crest and indicates well the flattening of the parietals on either side of the sagittal suture.

Plates VII and VIII are again photographs of a male skull, No. 765^{25}. Plate VII, the *norma lateralis*, left profile, is an endeavour to catch the marked temporal line and the recession of the temporal bone behind it, which gives the characteristic aspect to the horizontal contour. Plate VIII gives the *norma basalis* and indicates well the large and rotund character of the *foramen magnum*, although the foraminal index of this cranium does not reach the mean value. The heavy inion ridge is to be noted.

Plate IX, No. 765, and Plate X, No. 765^{32}, give the *norma lateralis*, left and right profiles respectively, of typical female crania. We see the same features as in the male, the massive frontal, marked temporal lines, receding *facies temporalis*, and the elevation and recession of the vertex produced by the sagittal crest. These features are less emphasised than in the male, but are in striking contrast to the female characters of less primitive races.

Plates XI and XII provide the *norma verticalis* and *norma basalis* of a typical female, No. 763. In Plate XI we see the pear-shaped vertical aspect*, and in Plate XII the characteristic palate, and the asymmetrical foramen.

Plates XIII and XIV further illustrate in a male cranium, No. 765^{5}, the racial characteristics. Plate XIII shows in the *norma basalis* a marked asymmetry of the foramen as well as the general massiveness of the base. Plate XIV, the *norma lateralis*, right profile, gives the usual primitive factors of massive frontal sinuses, receding forehead, sagittal crest and vertical ramus; but the vertex in this case while markedly elevated does not recede; as the sagittal contour of this cranium indicates it is practically at the apex.

Plates XV and XVI give *norma lateralis*, left profile, and *norma occipitalis* of a male skull, No. 762. Besides all the usual racial features of which the depression of the *facies temporalis* is especially marked, we see a rarely occurring ossicle of the coronal suture, and a protrusion of the occipital almost amounting to bathrocephaly. Plate XVI indicates the source of this in the conglomeration of ossicles in the lambdoid suture.

Plates XVII and XVIII reproduce another male skull, No. 765^{27}, with massive frontal, receding forehead, depressed *facies temporalis* and sagittal crest. Plate XVII, the *norma lateralis*, left profile, shows an epipteric †. Plate XVIII, the *norma facialis*, indicates how much of the frontal bulge is really due to glabellar protrusion, and how little nasal resemblance the Moriori has to the Negro.

We conclude this series of photographs with reproductions of two crania of some interest. Plate XIX, No. 765^{17}, shows a cranium with marked para-occipital processes articulating with the atlas. Macdonell has figured in Plate X of his memoir on the Liverpool Street English crania ‡ a case of unilateral para-occipital process of a like character, the present Moriori instance is bilateral.

Plates XX, XXI and XXII of a male skull, No. 765^{2}, illustrate how without surgical aid recovery from a serious injury is possible in a primitive people. The

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* The pear-shaped cranium of Plate XI should be compared with the pear-shaped cranium Plate XI of Macdonell's paper, *Biometrika*, Vol. v. p. 104. The reader will understand at once what we mean by the heavy temporal lines and the depression of the *facies temporalis* in the Moriori. Indeed, if the reader will compare the photographs of the *norma verticalis* in Macdonell's two papers (*Biometrika*, Vol. III. p. 216, and Vol. v. p. 104) on the English skull, with our Plate XI, he will appreciate why the Moriori horizontal type contour differs so essentially from the English horizontal type contour. I regret that I have not provided more Moriori photographs illustrating this essential difference. In the Moriori crania the *norma verticalis* shows the temporal ridges, in the English they rarely appear at all.

† The curved dental arch is also worthy of note.

‡ *Biometrika*, Vol. v. p, 104.

IENTS

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GL	Remarks
87?	Basilar synchondrosis. 3rd molars still in alveolus. Small epipterics on R. Infra-orbital suture meets cillary suture R. and L. Flattening on both sides of sagittal suture. Bulging of parietals. Flattening of
92	th. Sutures open. Flattening of obelion. Sagittal crest.
104·5	ondrosis. Sutures open. Malar marginal processes R. and L. Small ossicles in lambdoid suture. Infra- ures R. and L. Sagittal crest. Flattening of obelion. Ossicle near R. asterion.
103	Sutures open. Infra-orbital sutures R. and L. Sagittal crest. Flattening of obelion. Narrow head with rietals.
100	ar synchondrosis. 3rd molars still in alveolus. Very asymmetric foramen. Sutures open. Bulging parietals.
97·5	als. Sutures open; remaining teeth good. Curious bony bridge on R. of foramen. Many wormians in lamb- e. Bulging parietals. Flattening of obelion.
94·5	ninence with highly receding forehead. Sagittal crest. Ossicle in coronal suture. Great complication of lambdoid suture. Infra-orbital suture meets malar maxillary suture R. and L. Asymmetric foramen. Large ight torus palatinus.
103	utures R. and L. Sutures open. No 3rd molars. Large shallow palate. Torus palatinus. Small ossicle in suture. Bulging glabella. Marked malar marginal processes. Marked inion ridge. Incisura crotaphitico- us on R.
98	crest. Bulging parietals. Marked inion. Sagittal suture almost entirely obliterated, coronal suture obliterated. Bulging glabella. Marked malar marginal processes.
104	Flattening of obelion. Bulging parietals. Marked malar marginal process on R.
99·	stoids. Teeth good. Sagittal suture almost entirely obliterated. Lambdoid suture becoming obliterated. lar marginal processes R. and L. Ridge on parietals R. and L. Deep palate.
103	forehead with bulging glabella. Asymmetric flattening of frontal bone. Bulging parietals. Numerous in lambdoid suture. Symmetric flattening of parietals each side of lambdoid suture.
d	, especially face and base. Very flat head with very bulging parietals. Sutures very open.
98	neets malar maxillary suture R. and L. Teeth very worn. Small epipterics R. and L. Sutures very open. at R. pterion. Bulging at mid-sagittal.
103	ng, with slight depression. Coronal and sagittal sutures much obliterated. Considerable absorption of n. Heavy inion ridge. Mandible fractured.
102	ull. Sutures open. Teeth good, but bad abscess of bone round R. canine. Anterior supra-mastoid tubercles Bulging parietals. Flattening at obelion.
108	head. Infra-orbital suture meets malar maxillary suture on R. Sagittal suture considerably obliterated. inion. Marked malar marginal processes. Mid-sagittal crest.
10	forehead, and high sagittal crest. Sutures open. Some absorption of alveolus. Anterior sagittal bulging. lar marginal process on R. Asymmetric foramen.
9	ead. Sagittal crest; bulging parietals with some depression. Bulging occipital.
10	obliterated. Infra-orbital meet malar maxillary sutures R. and L. Epipteric on R. 21 mm. long. Receding ulging parietals. Marked malar marginal processes R. and L.
10	la. Receding forehead with sagittal crest. Depression at obelion. Sutures very open. Some absorption Infra-orbital suture on L. Ossicle at lambda (18 × 18). Small epipteric on R.
10	forehead. Infra-orbital suture on R. Teeth good. Sutures becoming obliterated at pterion and obelion. lambdoid suture. Heavy inion ridge.
	receding forehead. Curious high mid-sagittal crest. Bulging occipital with heavily marked inion ridge. coming obliterated. Ossicle at lambda. Some depression of parietals.
1	tures R. and L. Marked malar marginal process on R. Flattening at obelion. Bulging parietals. Sutures lightly obliterated. Epipteric on R. (25 mm. long).
1	uture meets malar maxillary suture R. and L. Marked malar marginal processes. Bulging parietals; high l crest. Sutures open. Teeth worn.
1	al bulging. Teeth much worn. Curious ridge on occipital base. Flattening at obelion. Aural exostosis
	oped superciliary ridges. Low receding forehead. Infra-orbital sutures R. and L. Coronal suture open; ure becoming obliterated. Many ossicles at lambda, and in lambdoid suture. Occipital bulging. Heavy y marked supra-mastoid tubercles R. and L.
	astoids. Flattening of obelion. Very heavy skull. Receding forehead. Malar marginal processes R. and s open. Small epipteric on R.
	lla. Receding forehead.
	high sagittal crest. Asymmetric occipital bulging. Marked malar marginal processes. Small epipteric R. and L. mbdoid suture. Teeth good. Para-occipitals, especially the left, are large. Two tubercles on the basi-occipital.

lal

TABLE OF MEASUREMENTS OF MORIORI SKULLS IN THE ROYAL COLLEGE OF SURGEONS' MUSEUM

		Capacity and Weight		Lengths								Arcs								Face			Nose		Orbit				Palate and Profile				Foramen		Mandible					Indices												Angles						
No.	Sex	*C*	W. in grs.	*F*	*L′*	*L*	*B*	*B′*	*H*	*OH*	*LB*	*Q*	*Q′*	*S*	S_1	S_2	S_3	S'_3	*U*	*G′H*	*GB*	*J*	*NH*	*NB*	O_1R	O_1L	O_2R	O_2L	G'_1	G_1	G_2	*GL*	*fml*	*fmb*	*GH*	w_1	w_2	h_1	*zz*	*Oc I*	*B/L′*	*H/L′*	*B/L*	*H/L*	*B/H*	*G′H/GB*	*NB/NH*	$\frac{O_2/O_1}{L}$	$\frac{O_2/O_1}{R}$	G_2/G_1	$\frac{fmb}{fml}$	$N\angle$	$A\angle$	$B\angle$	θ_2	θ_1	$P\angle$	Remarks
758	Child	*d*	407	170·5	170·5	170·5	134	89	125	110·5	94·5	299	299	341	117	115	109	92·5	481	59	83·5	115 ?	47·5	25	40	39	35	34	40 ?	42 ?	29	87 ?	35	29	*d*	*d*	*d*	*d*	*d*	62·1	78·59	73·31	78·54	73·27	107·20	71·06	52·85	87·40	87·50	69·52	83·33	64°·1	77°·8	38°·1	10°·2	27°·9	88°	Very young. Basilar synchondrosis. 3rd molars still in alveolus. Small epipterics on R. Infra-orbital suture meets malar maxillary suture R. and L. Flattening on both sides of sagittal suture. Bulging of parietals. Flattening of obelion.
759	♀	1280 (1278)	660 (+ *m*, 775)	174	176	176	135	89	132	110	101	295	295	352	119	122	111	98	493	68	94	126·5 ?	52·5	25	43·5	42·5	37·5	36	45·5	48	36	92	35·5	32·5	108	114	99	27	46	67·2	76·70	75·00	76·70	75·00	102·27	72·34	47·05	85·17	85·35	75·00	91·26	62°·0	76°·9	41°·1	9°·1	32°·0	86°	Very good teeth. Sutures open. Flattening of obelion. Sagittal crest.
760	Young ♂	*d*	608	180	184	184	136	95	132	117	109	318	314	358	120	123	115	100·5	512 ?	73	97	127·5	56	25	45	45	39	39	50	52	40	104·5	37·5	31	*d*	*d*	*d*	*d*	*d*	65·5	73·91	71·74	73·91	71·74	103·03	75·10	44·64	86·28	86·22	76·62	82·57	66°·9	73°·0	40°·1	12°·9	27°·1	86°	Basilar synchondrosis. Sutures open. Malar marginal processes R. and L. Small ossicles in lambdoid suture. Infra-orbital sutures R. and L. Sagittal crest. Flattening of obelion. Ossicle near R. asterion.
760^{1}	♀	1230 (1235)	605	169	172	172	136	86·5	133	114	101	300	300	349	120	119	110	98	480	72	94	127	54·5	22	41	40 ?	38	37·5	49	53	35	103	30	29·5	*d*	*d*	*d*	*d*	*d*	69·4	79·07	77·44	79·07	77·44	102·10	76·80	40·37	93·75	92·68	66·04	97·01	71°·8	66°·6	41°·6	14°·4	27°·2	81°	Teeth good. Sutures open. Infra-orbital sutures R. and L. Sagittal crest. Flattening of obelion. Narrow head with bulging parietals.
760^{2}	♀	1410 (1397)	505 (+ *m*, 590)	175·5	175	176	135	95	132	114·5	100·5	314	311	350	122	117	111	100	500	67	88·5	117	48	25	42	40	37·5	36	49	50·5	36	100	*d*	33	103	105	95·5	30·5	50	71·5	77·14	75·43	76·70	75·00	102·27	75·71	52·08	90·50	89·23	70·89	*d*	68°·2	73°·1	38°·7	8°·9	29°·8	82°	Young. Basilar synchondrosis. 3rd molars still in alveolus. Very asymmetric foramen. Sutures open. Bulging parietals.
761	♀	1420 (1425)	672 (+ *m*, 765)	171	172	173	143	100	139·5	119	103	325	326	353	127	111	115	100	504	73·5	106	136	55·5	23	43	43·5	35	36	48	53	35 ?	97·5	35	30	*d*	*d*	*d*	*d*	*d*	65·0	83·14	81·10	82·66	80·63	102·51	69·34	41·59	90·50	81·01	66·28	85·95	64°·1	72°·8	43°·1	13°·2	29°·9	86°	Bulging parietals. Sutures open; remaining teeth good. Curious bony bridge on R. of foramen. Many wormians in lambdoid suture. Bulging parietals. Flattening of obelion.
762	♂	1360 (1368)	625	182·5	188	189·5	136	89	127	108·5	98	295	299	372	121	129	122	99·5 ?	528	74	99·5	133	57	24	46	43	39	38	53	55	39·5	94·5	33·5	30	*d*	*d*	*d*	*d*	*d*	58·1	72·34	67·44	71·80	66·94	107·25	74·37	42·25	87·50	84·34	71·82	89·57	64°·6	70°·2	45°·2	12°·8	32°·4	83°	Glabellar prominence with highly receding forehead. Sagittal crest. Ossicle in coronal suture. Great complication of ossicles in lambdoid suture. Infra-orbital suture meets malar maxillary suture R. and L. Asymmetric foramen. Large palate. Slight torus palatinus.
763	♀ ?	1160 (1159)	670	172	175	175	133·5	88	128·5	114	100	311	306	346	115	123	108	91	489	67	95	126	50	24·5	41	42·5	34·5	35·5	52·5	56	38	103	33·5	30·5	*d*	*d*	*d*	*d*	*d*	61·1	76·28	73·48	76·28	73·48	103·81	70·53	48·60	83·33	84·39	68·20	90·91	73°·0	68°·3	38°·7	12°·7	26°·0	81°	Infra-orbital sutures R. and L. Sutures open. No 3rd molars. Large shallow palate. Torus palatinus. Small ossicle in lambdoid suture. Bulging glabella. Marked malar marginal processes. Marked inion ridge. Incisura crotaphitico-buccinatorus on R.
764	♂	1545 (1539)	672 (+ *m*. 781)	185	188	189	146·5	93	132·5	116	102	310	312	377	133	124	120	99	530	82·5	108	136	61	26	45	44·5	37·5	38·5	51	54	40	98	38	31	124	122	106	31	52·5	59·0	77·81	70·47	77·40	70·10	110·41	76·48	42·48	86·68	83·55	74·07	82·01	63°·1	68°·1	48°·8	14°·9	33°·9	83°	High sagittal crest. Bulging parietals. Marked inion. Sagittal suture almost entirely obliterated, coronal suture becoming obliterated. Bulging glabella. Marked malar marginal processes.
765	♀	1426 (1425)	837	184	185	185	143	96	142·5	122·5	107	327	327	376	128	130	118	103	519	71·5	95	127	53·5	25	41	41·5	36·5	36	48	51	38	104	38	33·5	*d*	*d*	*d*	*d*	*d*	65·9	77·30	76·91	77·30	76·91	100·49	75·05	46·73	87·16	89·26	74·70	87·63	68°·1	72°·5	39°·4	10°·5	28°·9	83°	Sutures open. Flattening of obelion. Bulging parietals. Marked malar marginal process on R.
1016 (B. D.)	♂	1380 (1368)	677	182	186	187	140	100	129·5	117·5	107	315	318	360	125	120	115	97	517	73·5	98	135	60·5	24	43	44·5	37	38	46	47·5	41	99·5	36·5	31	*d*	*d*	*d*	*d*	*d*	61·4	75·27	69·51	74·87	69·14	108·27	75·05	39·80	85·65	85·81	85·89	85·39	69°·7	66°·3	44°·0	22°·7	21°·3	89°	Very large mastoids. Teeth good. Sagittal suture almost entirely obliterated. Lambdoid suture becoming obliterated. Marked malar marginal processes R. and L. Ridge on parietals R. and L. Deep palate.
1015 (B. D.)	♂	*d*	*d*	176	183·5	183·5	133·5	91	135	114	108	299	296 ?	351	128	106	117	101	504	68	103	137·5	54	25	44	43·5	36	*d*	51	54	37	103	37·5	31·5	*d*	*d*	*d*	*d*	*d*	63·8	72·64	73·67	72·64	73·67	98·59	66·02	46·30	*d*	81·82	68·52	84·45	68°·0	75°·0	37°·0	8°·0	29°·0	83°	Very receding forehead with bulging glabella. Asymmetric flattening of frontal bone. Bulging parietals. Numerous wormians in lambdoid suture. Symmetric flattening of parietals each side of lambdoid suture.
1015*A* (B. D.)	♀ ? ?	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	337	120	107	110	*d*	480	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	32	29	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	90·63	*d*	*d*	*d*	*d*	*d*	*d*	Very damaged, especially face and base. Very flat head with very bulging parietals. Sutures very open.
765^{1}	♀	1320 (1320)	592 (+ *m*, 675)	176	176	177·5	138	94·5	132	115	102	313	310	359	125	119	115	99	504	76	102	129	57	27	44·5	43·5	35	35	46	50	35·5	98	31	30	118·5	116	109	35·5	46	63·6	78·41	75·00	77·75	74·37	104·55	74·31	47·37	80·47	79·10	70·80	96·77	64°·5	71°·0	44°·5	17°·0	27°·5	88°	Infra-orbital meets malar maxillary suture R. and L. Teeth very worn. Small epipterics R. and L. Sutures very open. Depression at R. pterion. Bulging at mid-sagittal.
765^{2}	♂	1348 (1349)	812 (+ *m*, 895)	181·5	183·5	183·5	150	101	136	120	108	330	330	364	135	120	109	97	532	80·5	96	136	59 ?	27	44	44	38	37·5	51	54	34·5	103·5	30·5	30·5	116	126	104	32 ?	51	69·6	81·74	74·11	81·74	74·11	110·29	83·75	45·76	84·38	86·36	64·25	99·34	64°·8	70°·7	44°·5	17°·1	27°·4	88°	Parietal bulging, with slight depression. Coronal and sagittal sutures much obliterated. Considerable absorption of alveolar rim. Heavy inion ridge. Mandible fractured.
765^{3}	♂	1470 (1472)	1050 (+ *m*, 1200)	180	182	183	150	92	140·5	122	106	331	328	373	124	122	127	108	531	77	109	137	55	25	43·5	42·5	37	36	55	58	42	102·5	33	32	126	128·5	108	34·5	52	62·1	82·42	77·08	81·97	76·66	106·91	70·82	45·62	84·50	85·07	72·66	95·78	65°·8	70°·8	43°·4	11°·2	32°·2	82°	Very heavy skull. Sutures open. Teeth good, but bad abscess of bone round R. canine. Anterior supra-mastoid tubercles R. and L. Bulging parietals. Flattening at obelion.
765^{4}	♂	1410 (1406)	712 (+ *m*, 849)	178	182·5	183	142	102	131·5	114	101	310	310	369	131	125	113	96	528	77·5	101·5	144	58·5	27	45	45	37·5	37	53	59	38	108	32·5	31	128 ?	130	114	34	49·5	62·0	77·85	72·09	77·60	71·85	107·98	76·25	46·23	81·77	83·11	64·41	95·09	73°·3	63°·4	43°·3	16°·6	26°·7	80°	Receding forehead. Infra-orbital suture meets malar maxillary suture on R. Sagittal suture considerably obliterated. Projecting inion. Marked malar marginal processes. Mid-sagittal crest.
765^{5}	♂	1410 (1406)	590 (+ *m*, 717)	179	180	182·5	138	92·5	133	117	104·5	313	313	353	123	124	106	95	510	74	101	136	57	27	45	45	36	35	52	53·5	37·5	104·5	*d*	32	*d*	121	103	37	49	70·7	76·67	73·89	75·65	72·91	103·76	73·06	47·37	78·12	80·00	69·81	*d*	69°·3	69°·5	41°·2	16°·5	24°·7	86°	Very receding forehead, and high sagittal crest. Sutures open. Some absorption of alveolus. Anterior sagittal bulging. Marked malar marginal process on R. Asymmetric foramen.
765^{6}	♂	1630 (1625)	727	190	193	194·5	148	96	140	122	106	318	318	387	131	129	127	103	545	78	102	138	60·5	27	45	43	37·5	37·5	48	49·5	39	99	36	29	*d*	*d*	*d*	*d*	*d*	57·9	76·68	72·54	76·17	72·05	105·71	76·47	44·95	87·21	82·80	79·19	81·11	62°·8	72°·3	44°·9	11°·7	33°·2	84°	Receding forehead. Sagittal crest; bulging parietals with some depression. Bulging occipital.
765^{7}	♂	1540 (1520)	882	184	188	190	147	101	135·5	119	107	330	327	376	125	124	127	105·5	538	75	111	143	57	26	46	46	35	34·5	52·5	56	46·5	100·5	36	28·5	*d*	*d*	*d*	*d*	*d*	59·7	78·19	72·07	77·37	71·31	108·48	67·57	45·61	75·00	75·65	82·97	78·61	64°·1	73°·5	42°·4	*d*	*d*	*d*	Sutures much obliterated. Infra-orbital meet malar maxillary sutures R. and L. Epipteric on R. 21 mm. long. Receding forehead, bulging parietals. Marked malar marginal processes R. and L.
765^{8}	♂	1260 (1273)	780	178	183	183	141	92·5	132	116	105	313	315	355	121	120	113	95	505	76	110·5	140·5	57	25·5	46·5	46	38	39	52	56·5	38	100	35·5	28	*d*	*d*	*d*	*d*	*d*	60·8	77·05	72·13	77·13	72·21	106·81	68·95	44·73	84·78	82·55	67·49	78·89	65°·0	71°·3	43°·7	15°·7	28°·0	87°	Bulging glabella. Receding forehead with sagittal crest. Depression at obelion. Sutures very open. Some absorption of alveolus. Infra-orbital suture on L. Ossicle at lambda (18 × 18). Small epipteric on R.
765^{9}	♀ ?	1300 (1301)	729	178·5	182	182·5	133	98	133	115	104	307	308	365	123	111	131	105	506	71	101	132·5	54	27	45	43·5	34	34·5	49·5	52·5	38·5	100	34·5	31	*d*	*d*	*d*	*d*	*d*	57·1	73·08	73·08	72·91	72·91	100·00	70·15	50·00	79·12	75·56	73·08	90·08	66°·6	72°·8	40°·6	11°·2	29°·4	84°	Low receding forehead. Infra-orbital suture on R. Teeth good. Sutures becoming obliterated at pterion and obelion. Ossicles in lambdoid suture. Heavy inion ridge.
765^{10}	♂	*d*	697 ?	185	189	190	134	89	137	115	110	299	300	375	125	134	116	95	520	67	102·5	139	54·5	26	43·5	43 ?	40·5	39	*d*	*d*	37	98	34·5	30·5	*d*	*d*	*d*	*d*	*d*	58·6	70·90	72·49	70·53	72·11	97·81	65·49	47·88	91·16	93·02	*d*	*d*	68°·8	71°·6	39°·6	14°·4	25°·2	86°	Very narrow receding forehead. Curious high mid-sagittal crest. Bulging occipital with heavily marked inion ridge. Sutures becoming obliterated. Ossicle at lambda. Some depression of parietals.
765^{11}	♀ ?	1230 (1244)	667	173	177	178	139·5	94·5	133·5	115	103	317	316	346	120	112	114	98	498	66	105	131	52	27·5	43	43	38	37	48	50	39	100	36·5	30·5	*d*	*d*	*d*	*d*	*d*	63·1	78·81	75·36	78·45	75·02	104·57	62·86	52·50	86·05	88·37	78·00	83·33	68°·1	73°·8	38°·1	14°·2	23°·9	88°	Infra-orbital sutures R. and L. Marked malar marginal process on R. Flattening at obelion. Bulging parietals. Sutures becoming slightly obliterated. Epipteric on R. (25 mm. long).
765^{12}	♂ ?	1364 (1366)	647	175	181	182	140	93	135	118	107	320	318	361	117	122	122	106	513	75	108	144·5	58	25	46	46·5	37	36	51	54	47	101·5	34	29	*d*	*d*	*d*	*d*	*d*	64·9	77·35	74·59	76·92	74·18	103·70	69·44	43·10	77·43	80·00	87·36	85·88	64°·8	73°·0	42°·2	11°·0	31°·2	84°	Infra-orbital suture meets malar maxillary suture R. and L. Marked malar marginal processes. Bulging parietals; high mid-sagittal crest. Sutures open. Teeth worn.
765^{13}	♂	1445 (1425)	625	182	187	187	143	97	132	118	106	318	320	370	125	123	122	102·5	532	80·5	104	143	62	25	44	44·5	37	37	49	52	40	102	35	31	*d*	*d*	*d*	*d*	*d*	61·5	76·47	70·59	76·47	70·59	108·33	77·21	40·19	83·15	84·09	76·53	88·57	64°·3	70°·1	45°·6	17°·4	28°·2	87°·5	Anterior sagittal bulging. Teeth much worn. Curious ridge on occipital base. Flattening at obelion. Aural exostosis on the L.
765^{14}	♂	1420 (1430)	670	180	186	187	148	92	135	115	103·5	318	318	368	124	111	133	110·5	522	83	108	144	62	26	47·5	46	39	38	48	55	42	101	35	29	*d*	*d*	*d*	*d*	*d*	59·6	79·57	72·47	79·14	71·01	109·79	77·03	41·94	82·61	82·11	76·00	82·86	64°·0	67°·8	48°·2	13°·2	35°·0	81°	Heavily developed superciliary ridges. Low receding forehead. Infra-orbital sutures R. and L. Coronal suture open; sagittal suture becoming obliterated. Many ossicles at lambda, and in lambdoid suture. Occipital bulging. Heavy inion. Very marked supra-mastoid tubercles R. and L.
765^{15}	♂	1318 (1335)	845	177	181	181	140	92	137	118	99·5	321	320	365	126	120	119	102	507	77	104	127·5	55·5	24·5	41·5	42·5	35·5	36	46	49	35	96	37	29	*d*	*d*	*d*	*d*	*d*	63·0	77·35	75·69	77·35	75·69	102·19	74·04	43·96	84·63	85·95	71·02	78·80	64°·2	69°·4	46°·4	17°·6	28°·8	87°	Very heavy mastoids. Flattening of obelion. Very heavy skull. Receding forehead. Malar marginal processes R. and L. Sutures open. Small epipteric on R.
765^{16}	♂	1380 (1377)	747	183·5	186	188	137	95	139·5	116·5	106	317	316	369	130	122	117	100	520	79	101	141	55	25	44	44	36	36	49	53	35	100·5	36	30	*d*	*d*	*d*	*d*	*d*	62·7	73·66	75·00	72·94	74·28	98·21	78·22	45·09	81·40	81·82	65·78	83·79	63°·8	71°·0	45°·2	16°·2	29°·0	87°	Bulging glabella. Receding forehead.
765^{17}	♂ ?	1378 (1368)	635	177	183	184·5	143	95·5	133·5	111	100	295	295	363	121	122	120	103	522	74·5	99	138	58	26	43	43	38	38	46	49	38·5	94	35	33	*d*	*d*	*d*	*d*	*d*	63·4	78·14	72·95	77·50	72·28	107·11	75·25	44·83	88·11	87·96	78·16	94·29	63°·0	72°·0	45°·0	12°·0	33°·0	84°	Very marked high sagittal crest. Asymmetric occipital bulging. Marked malar marginal processes. Small epipteric R. and L. Complex lambdoid suture. Teeth good. Para-occipitals, especially the left, are large. Two tubercles on the basi-occipital.

The measurements are tabulated to the nearest 0·5 millimetres; the indices are given for the full measurements, using Ernst's Tables.

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REMARKS
ıtals R. and L. Sutures open. Teeth worn. Palatine bridge on R. Marked occipital ridge. Ossicle at lambda.
epipteric on L. Marked malar marginal processes.
ıtal suture on L. Marked malar marginal processes. Small epipteric on R. Open complex sutures. Small
ıle on occipital base. Slight depression at obelion.
ry skull. Infra-orbital suture on L. meets malar maxillary suture. Coronal and sagittal sutures much obliterated.
ossicles in lambdoid suture. Marked malar marginal process on R. Depression at obelion. Very small mastoids.
ırn. Sutures considerably obliterated. Marked malar marginal processes. Depression at obelion.
ıalar marginal processes. Receding forehead. Sagittal suture considerably obliterated. Bulging occipital.
good. Small epipteric on R. Bulging parietals. Ossicle in lambdoid suture.
ital suture on L. Slightly bulging glabella. Coronal suture becoming slightly obliterated. Inion very pronounced.
absorption of alveolus. Sagittal crest marked. Flattening of obelion.
forehead. Symmetrical flattening on either side of sagittal suture. Flattening of obelion. Sutures open. Teeth
Some parietal bulging.
ding forehead. Sutures open. Heavy inion ridge. Anterior supra-mastoid tubercles R. and L.
vy skull. Sagittal suture almost entirely obliterated; coronal and lambdoid suture becoming obliterated. Marked
ridge. Flattening of obelion. Teeth good but worn. Deep palate.
ling forehead. Infra-orbital sutures R. and L. Anterior supra-mastoid tubercles R. and L. Epipterics R. and L.
palate; worn teeth. Group of ossicles at lambda.
y receding forehead. Considerable parietal absorption. Coronal and sagittal sutures much obliterated. Large
ids. Bulging occipital. Teeth very worn.
malar marginal processes. Sutures open. Ossicle at lambda. Anterior supra-mastoid tubercles R. and L.
ıg occipital. Depression at obelion.
y receding forehead. Sagittal suture much obliterated. Teeth worn.
suture entirely obliterated. Marked mid-sagittal crest. Coronal suture fairly open, lambdoid suture much
rated. Epipteric on R. Anterior supra-mastoid tubercles R. and L. Marked inion.
ry worn. Sutures slightly obliterated.
ital suture meets malar maxillary suture R. and L. Slight post-coronal constriction. Sutures open. Marked
marginal processes. Slight asymmetry of foramen magnum.
ɔpen.
ɔpen. Infra-orbital suture meets malar maxillary suture R. and L. Depression at pterion on R.
Teeth good. No 3rd molars. Sutures open. Flattening at obelion. Epipterics R. and L. Palatine bridge on L.
ɔst-coronal constriction. Bulging glabella. Sutures open. Infra-orbital suture on L. Basilar synchondrosis.
of skull extremely asymmetric. Mid-sagittal crest. Third occipital articular surface.
ital suture meets malar maxillary suture R. and L. Small epipteric on L. Sutures open. Small ossicles in
ɑl, lambdoid and sagittal sutures. Bulging occipita .
very open. 3rd molars not fully developed. Epipterics R. and L., large on L. (36·8 × 11·0 mm.). Ossicle on R.
ıbda (25 × 26·5 mm.). Bulging occipital.
ng. Basilar synchondrosis. Infra-orbital suture meets malar maxillary suture on R.; not on L. Malar marginal
sses R. and L. Small ossicles in lambdoid suture.
Basilar synchondrosis. 3rd molar on L. not yet erupted. Small epipteric on R. Flattening of obelion.
ling forehead. Bulging parietals. Sutures open. Some bulging of occipital.
ng. Basilar synchondrosis. Some of 2nd teeth not yet erupted, still in alveolus. Surface of bone much mutilated.
epipteric on R. and L. Many ossicles in lambdoid suture. Ossicles very open. Infra-orbital sutures R. and L.
ısiderably damaged. Sutures very obliterated.
y damaged. Sutures very open.
eding forehead. Bulging and projecting parietals. Markedly good set of teeth in both lower and upper jaws.
ed malar marginal processes. Coronal and sagittal sutures considerably obliterated. Occipital bone asymmetric.
ng glabella.
becoming obliterated. Parietal depression. Teeth extremely worn. Some absorption of alveolus. Bad abscess
ne on L. maxillary. Mid-sagittal projection.
ınd coronal suture much obliterated. Teeth very worn. Low receding forehead. Bulging parietals. Marked inion.
ital suture meets malar maxillary suture R. and L. Coronal and sagittal sutures becoming obliterated. Some
ning of obelion. Marked malar marginal processes. Teeth extremely worn and decayed.
ry worn; sutures becoming slightly obliterated. Infra-orbital suture meets malar maxillary suture on L. Marked
marginal processes. Slight bulging at mid-sagittal. Bulging parietals.
sicle on L. occipital, 51 × 31 mm. Heavy projecting inion. Small epipteric on L. Sutures becoming obliterated
erion; and obelion. Teeth very worn.

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TABLE OF MEASUREMENTS OF MORIORI SKULLS (*Continued*)

		Capacity and Weight		Lengths							
No.	Sex	C	W. in grs.	F	L'	L	B	B'	H	OH	LB
765^{18}	♂	1440 (1444)	785	185	187	188	142·5	90	140	119	111
765^{19}	♀	1440 (1425)	605	182·5	185	186	139·5	99	132·5	114	108
765^{20}	♂?	1390 (1404)	899	183	184	185	146	96	140·5	120	106·5
765^{21}	♀?	*d*	630	180·5	185	185	139	102	127	118	103
765^{22}	♂	1510 (1525)	734	186	189	191	140	93	142	119	112
765^{23}	♀	1218 (1216)	654	178	179	181	133	97	132·5	113	100
765^{24}	♀?	1440 (1425)	662	181	182·5	185	144	95	138	117	101
765^{25}	♂	1440 (1444)	887	185	187	190	140	98	141	112·5	107
765^{26}	♂	1680 (1672)	885	188	191	192	147	96	137·5	121	105
765^{27}	♂	1360 (1368)	830	176	182	183	144	91	137·5	119	104
765^{28}	♂	*d*	872	191	196	197	142	94	136·5	117	108
765^{29}	♂	*d*	904	191	192	193·5	150	99	142·5	119	111·5
765^{30}	♂	*d*	701	177	183	184	137	97·5	139	120	112
765^{31}	♂	1640 (1654)	982	194	198·5	199	150	104	137	120	109
765^{32}	♀	1300 (1292)	602	175·5	178	179	139	93	130	114	103
765^{33}	♀	1180 (1178)	565	168	170	170	135	92	131	112	101·5
765^{34}	♀	1360 (1349)	635	180	180	180	141	88	131·5	114	101
765^{35}	♀	1200 (1216)	417	172·5	173·5	175	133	89·5	127·5	111	98·5
765^{36}	♀	1280 (1273)	619	175	174	175	143	95	136	117	105
765^{37}	♀	1400 (1392)	632	173	177·5	177·5	144·5	93	135	119	105·5
765^{38}	Child	1360 (1349)	504	173	173	173	140	90·5	130	115	92
765^{39}	Young ♀	*d*	510	185	185	186	143	95	126·5	113	96
765^{40}	Child	1320 (1311)	475	174	173·5	173	137	89	130	115	92?
765^{41}	Child	1390 (1387)	485	180	182	183	144	93	125	114	99·5
765^{42}	Child	*d*	422	169	170	170	133	90	128	112	87
765^{43}	♂??	*d*	*d*	188	*d*	192	*d*	103	*d*	*d*	*d*
765^{44}	♀?	*d*	*d*	177?	*d*	180?	137·5	98	*d*	*d*	*d*
765^{45}	♂	1370 (1387)	686	174	180	181	139	94	132·5	112	99·5
765^{46}	♂	1340 (1330)	715 (+ *m*, 819)	179·5	181	183	139	92·5	140	117	104
765^{47}	♂	1160 (1168)	807 (+ *m*, 932)	167·5	172	173	136	93	126	110	101·5
765^{48}	♀?	1310 (1311)	599 (+ *m*, 695)	173	174	175	138	91·5	136	116	105
765^{49}	♂	1370 (1368)	814	183	187	189	140	93	141·5	121	108
765^{50}	♂	1460 (1454)	719	187	188	189	143	103·5	135	119	101

	Arcs								Face			Nose		Orbit			
No.	Q	Q'	S	S_1	S_2	S_3	S'_3	U	$G'H$	GB	J	NH	NB	O_1R	O_1L	O_2R	O_2L
765^{18}	322	322	376	126	120	130	104	522	80	98·5	138	61	25·5	47	43·5	37	37
765^{19}	315	313	358	127	115	116	103	522	71	103	133	55·5	29	44	43	34·5	34
765^{20}	325	325	374	127	123	124	103	526	74	100	140	55	23	44	43	38	38
765^{21}	317	317	363	123	122	117	93	517	70·5	94	137·5	52·5	26	43	43	36	35
765^{22}	311	311	370	126	121	123	109	528	78·5	109	137·5	58·5	29	46	46	38·5	37·5
765^{23}	297	300	358	123	119	116	96	508	70	95·5	126?	52	28	42·5	42	35	36
765^{24}	313	313	364	117	130	117	100	521	71	99	132	50	26·5	42	41·5	33·5	33·5
765^{25}	305	307	374	130	125	119	100	528	76·5	104	143	57·5	27	43	45	37	36·5
765^{26}	324	324	389	130	123	136	115	542	80·5	102	135·5	63·5	24	44	44	38	38
765^{27}	322	324	358	125	104	129	109·5	516	75·5	108·5	137	55	23	46·5	46·5	39	39
765^{28}	311	312	378	126	122	130	103·5	531	73·5	105	140	59	26	47	46	37·5	38
765^{29}	340	339	384	135	118	131	96·5	544	81	102	138·5	58·5	23	43	44	39	38·5
765^{30}	324	323	352	113	132	106	93·5	508	73	99·5	143·5	56·5	28	44	45	39	38·5
765^{31}	333	333	392	132	145	114	98	555	77	100	139	58	24·5	43	44	38	41
765^{32}	304	304	350	120	120	110	96	505	73	96	134	54	25	46	44	38	38
765^{33}	300	300	340	115	116	109	94·5	486	69	96	124	52	23	43	44	37·5	36·5
765^{34}	304	304	364	124	118	122	103·5	508	65	93	125	49	26	39	39	33	32·5
765^{35}	298	298	342	116	109	117	99	491	70	93	125	54	24	43	43	36	35
765^{36}	322	322	345	115	116	114	98·5	500	72	97·5	130	55	23·5	44·5	43	39·5	39
765^{37}	320	320	362	121	126	115	106	512	70	94	126	53	23·5	43·5	44	37·5	36·5
765^{38}	312	312	352	119	121	112	93	487	61	82	*d*	45	20	37	37	35	35
765^{39}	309	310	367	124	106	137	111·5	517	61	87·5	*d*	47	26	37	39	36·5	34
765^{40}	310	310	361	123	126	112	98	487	58	86	114·5	44	22·5	39	37	32	32
765^{41}	306	307	356	123	120	113	95	514	68·5	89	*d*	51	24	40	39	35	34
765^{42}	305	304	350	118	125	107	94	480?	58	88·5	114	43	21	38·5	38	35	34
765^{43}	*d*	311?	*d*	128	147	*d*	*d*	*d*	60·5?	*d*	*d*	47·5?	26	*d*	42	*d*	34
765^{44}	*d*	*d*	*d*	126	121	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*
765^{45}	303	303	360	124	111	124	107·5	506	78	107	135	58·5	24·5	46	45	37	36
765^{46}	312	311	368	124	120	124	108	509	78·5	103	138	57·5	26	46	47	40	39·5
765^{47}	307	307	344	117	108	119	101·5	492	76	100	127·5	57	22	42	42	37	35
765^{48}	315	315	351	122	111	118	100	497	67	100	135	52	25	45	45	34	33
765^{49}	323	323	373	127	125	121	104	522	78	103	129	56	26	44·5	45·5	38	37
765^{50}	316	316	380	130	120	130	113	530	73	103	132	51·5	22	44	44	36	36

	Palate and Profile				Foramen		Mandible					Indices					
No.	G'_1	G_1	G_2	GL	fml	fmb	GH	w_1	w_2	h_1	zz	$Oc\ I$	B/L'	H/L'	B/L	H/L	B/H
765^{18}	51·5	56	36	106·5	30·5	29	*d*	*d*	*d*	*d*	*d*	56·9	76·09	74·87	75·73	74·49	101·64
765^{19}	51	54·5	40	107	36·5	32	*d*	*d*	*d*	*d*	*d*	68·7	75·41	71·56	75·00	71·18	105·36
765^{20}	50	52	36	101	33	31	*d*	*d*	*d*	*d*	*d*	59·7	79·35	76·36	78·92	75·95	103·91
765^{21}	47	49	41	96	*d*	27	*d*	*d*	*d*	*d*	*d*	56·6	75·14	68·65	75·14	68·65	109·45
765^{22}	52·5	56·5	38	107	40·5	32	*d*	*d*	*d*	*d*	*d*	67·9	74·07	75·13	73·30	74·35	98·59
765^{23}	47	50	35·5	97	37	30	*d*	*d*	*d*	*d*	*d*	59·4	74·30	73·96	73·48	73·14	100·45
765^{24}	48	52	39	101·5	36·5	31	*d*	*d*	*d*	*d*	*d*	62·4	78·85	75·57	77·84	74·59	104·35
765^{25}	49	54·5	42	100	41	33·5	*d*	*d*	*d*	*d*	*d*	60·8	74·87	75·40	73·68	74·21	99·29
765^{26}	50	52·5	37	96	33	30	*d*	*d*	*d*	*d*	*d*	61·4	76·96	71·93	76·64	71·63	106·98
765^{27}	51·5	53	41	105·5	40	32	*d*	*d*	*d*	*d*	*d*	61·6	79·01	75·54	78·57	75·13	104·58
765^{28}	51	54·5	41	107	37	33	*d*	*d*	*d*	*d*	*d*	56·8	72·45	69·64	72·08	69·29	104·03
765^{29}	47·5	49	36	103	35·5	33·5	*d*	*d*	*d*	*d*	*d*	54·0	78·13	74·11	77·55	73·57	105·41
765^{30}	50	55	39·5	113	33·5	31·5	*d*	*d*	*d*	*d*	*d*	67·4	74·86	75·96	74·46	75·54	98·56
765^{31}	49	52	41·5	102·5	34	30	*d*	*d*	*d*	*d*	*d*	63·4	75·57	69·01	75·38	68·84	109·49
765^{32}	47	50	39	93	32·5	28	*d*	*d*	*d*	*d*	*d*	65·6	78·09	73·03	77·65	72·63	93·53
765^{33}	45	49	34	92	33	27	*d*	*d*	*d*	*d*	*d*	64·6	79·29	77·06	79·29	77·06	102·90
765^{34}	43·5	44·5	35·5	95	34	30·5	*d*	*d*	*d*	*d*	*d*	61·8	78·22	72·94	78·22	72·94	107·23
765^{35}	50	53·5	34	92	35	29	*d*	*d*	*d*	*d*	*d*	61·5	76·66	73·54	76·00	73·48	104·23
765^{36}	44·5	47·5	40	91	32·5	27	*d*	*d*	*d*	*d*	*d*	63·8	82·18	78·16	81·71	77·71	105·14
765^{37}	49	53·5	42	101	33	29·5	*d*	*d*	*d*	*d*	*d*	79·2	81·36	76·05	81·36	76·05	107·03
765^{38}	42	44	30·5	89	34	29	*d*	*d*	*d*	*d*	*d*	59·7	81·01	75·23	81·01	75·23	107·69
765^{39}	39	41	32	85	38·5	33	*d*	*d*	*d*	*d*	*d*	58·1	77·30	68·38	76·88	68·51	113·04
765^{40}	38·5	41	31·5	80	29·5	27	*d*	*d*	*d*	*d*	*d*	66·0	78·96	74·92	79·19	75·14	105·38
765^{41}	47	49	34·5	94	35·5	32	*d*	*d*	*d*	*d*	*d*	60·6	79·12	68·79	78·69	68·41	115·01
765^{42}	38	41	34	78	39	29	*d*	*d*	*d*	*d*	*d*	66·3	78·11	75·29	78·11	75·29	103·75
765^{43}	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*
765^{44}	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	76·39	*d*	*d*
765^{45}	50·5	52·5	42	95·5	37·5	30	*d*	121	110	32	57	64·3	77·22	73·50	76·80	73·09	105·06
765^{46}	48	49·5	33	91	36	34·5	117·5	123·5	104	34·5	50	65·2	76·80	77·35	75·96	76·50	99·29
765^{47}	51·5	53	40	98	30	30	119·5	117	102	31	51·5	62·2	79·07	73·26	78·61	72·83	107·94
765^{48}	48·5	52	37	98	35	30	104·5	121	104	28	51·5	61·7	79·31	78·27	78·86	78·33	101·32
765^{49}	50	53	36	102·5	34	29	*d*	*d*	*d*	*d*	*d*	63·4	74·87	75·72	74·07	74·91	98·87
765^{50}	48	51	37	94	37·5	31	*d*	*d*	*d*	*d*	*d*	64·7	76·06	71·81	75·66	71·43	105·92

	Indices						Angles						
No.	$G'H/GB$	NB/NH	$\frac{O_2/O_1}{L}$	$\frac{O_2/O_1}{R}$	G_2/G_1	$\frac{fmb}{fml}$	$N\angle$	$A\angle$	$B\angle$	θ_2	θ_1	$P\angle$	Remarks
765^{18}	81·01	41·94	85·51	79·14	64·29	95·70	65°·2	71°·5	43°·3	18°·8	24°·5	90°	Infra-orbitals R. and L. Sutures open. Teeth worn. Palatine bridge on R. Marked occipital ridge. Ossicle at lambda. Small epipteric on L. Marked malar marginal processes.
765^{19}	68·93	51·89	79·53	77·95	73·40	87·91	70°·2	71°·3	38°·5	10°·7	27°·8	82°	Infra-orbital suture on L. Marked malar marginal processes. Small epipteric on R. Open complex sutures. Small tubercle on occipital base. Slight depression at obelion.
765^{20}	73·85	41·82	87·50	86·66	64·49	93·94	65°·0	73°·3	41°·7	10°·7	31°·0	84°	Very heavy skull. Infra-orbital suture on L. meets malar maxillary suture. Coronal and sagittal sutures much obliterated. Many ossicles in lambdoid suture. Marked malar marginal process on R. Depression at obelion. Very small mastoids.
765^{21}	74·78	49·42	81·40	83·72	83·26	*d*	64°·0	74°·9	41°·1	15°·1	26°·0	90°	Teeth worn. Sutures considerably obliterated. Marked malar marginal processes. Depression at obelion.
765^{22}	71·87	49·57	81·08	84·06	67·14	79·02	65°·5	72°·5	42°·0	10°·5	31°·5	84°	Heavy malar marginal processes. Receding forehead. Sagittal suture considerably obliterated. Bulging occipital. Teeth good. Small epipteric on R. Bulging parietals. Ossicle in lambdoid suture.
765^{23}	73·22	53·85	86·11	82·36	70·60	81·08	67°·0	71°·2	41°·8	11°·8	30°·0	83°	Infra-orbital suture on L. Slightly bulging glabella. Coronal suture becoming slightly obliterated. Inion very pronounced. Some absorption of alveolus. Sagittal crest marked. Flattening of obelion.
765^{24}	71·57	53·00	80·48	79·90	75·00	85·39	69°·9	69°·1	41°·0	10°·9	30°·1	80°	Receding forehead. Symmetrical flattening on either side of sagittal suture. Flattening of obelion. Sutures open. Teeth good. Some parietal bulging.
765^{25}	73·55	46·96	81·02	85·58	76·69	81·71	63°·4	73°·3	43°·3	9°·7	33°·6	83°	Very receding forehead. Sutures open. Heavy inion ridge. Anterior supra-mastoid tubercles R. and L.
765^{26}	78·86	37·80	86·36	85·90	70·36	90·91	60°·4	72°·4	47°·2	14°·4	32°·8	87°	Large heavy skull. Sagittal suture almost entirely obliterated; coronal and lambdoid suture becoming obliterated. Marked inion ridge. Flattening of obelion. Teeth good but worn. Deep palate.
765^{27}	69·44	41·82	84·23	84·01	77·65	80·00	70°·0	67°·8	42°·2	16°·8	25°·4	85°	Low receding forehead. Infra-orbital sutures R. and L. Anterior supra-mastoid tubercles R. and L. Epipterics R. and L. Large palate; worn teeth. Group of ossicles at lambda.
765^{28}	69·97	44·07	83·04	80·00	75·56	89·19	69°·0	71°·0	40°·0	13°·0	27°·0	84°	Extremely receding forehead. Considerable parietal absorption. Coronal and sagittal sutures much obliterated. Large mastoids. Bulging occipital. Teeth very worn.
765^{29}	79·41	39·32	87·50	90·70	73·47	94·29	62°·2	73°·3	44°·3	13°·5	30°·8	87°	Marked malar marginal processes. Sutures open. Ossicle at lambda. Anterior supra-mastoid tubercles R. and L. Bulging occipital. Depression at obelion.
765^{30}	73·44	50·09	85·61	87·78	71·45	93·94	72°·0	70°·0	38°·0	12°·7	25°·3	83°	Extremely receding forehead. Sagittal suture much obliterated. Teeth worn.
765^{31}	76·84	41·89	92·76	88·78	79·42	87·64	64°·0	73°·3	42°·7	11°·7	31°·0	87°	Sagittal suture entirely obliterated. Marked mid-sagittal crest. Coronal suture fairly open, lambdoid suture much obliterated. Epipteric on R. Anterior supra-mastoid tubercles R. and L. Marked inion.
765^{32}	76·25	46·38	85·90	82·17	78·20	87·30	61°·1	75°·4	43°·5	9°·0	34°·5	84°·5	Teeth very worn. Sutures slightly obliterated.
765^{33}	71·51	44·23	82·95	86·97	69·39	82·92	61°·7	77°·0	41°·3	9°·0	32°·3	86°	Infra-orbital suture meets malar maxillary suture R. and L. Slight post-coronal constriction. Sutures open. Marked malar marginal processes. Slight asymmetry of foramen magnum.
765^{34}	69·89	52·84	83·07	84·31	79·55	89·71	65°·2	76°·0	38°·8	7°·0	31°·8	83°	Sutures open.
765^{35}	75·27	44·81	80·93	83·72	63·17	82·86	63°·4	73°·4	43°·1	8°·5	34°·6	82°	Sutures open. Infra-orbital suture meets malar maxillary suture R. and L. Depression at pterion on R.
765^{36}	73·99	42·36	90·70	89·16	84·22	84·21	58°·3	79°·4	42°·3	10°·6	31°·7	90°	Young. Teeth good. No 3rd molars. Sutures open. Flattening at obelion. Epipterics R. and L. Palatine bridge on L.
765^{37}	74·57	44·34	82·50	86·40	78·42	89·39	66°·1	74°·0	39°·9	7°·0	32°·9	81°	Slight post-coronal constriction. Bulging glabella. Sutures open. Infra-orbital suture on L. Basilar synchondrosis. Base of skull extremely asymmetric. Mid-sagittal crest. Third occipital articular surface.
765^{38}	74·20	44·41	94·59	94·05	68·77	85·79	67°·1	73°·4	39°·5	10 ·1	29°·4	83°·5	Infra-orbital suture meets malar maxillary suture R. and L. Small epipteric on L. Sutures open. Small ossicles in coronal, lambdoid and sagittal sutures. Bulging occipital.
765^{39}	69·87	55·32	87·40	98·92	78·05	85·73	60°·8	80°·3	38°·9	6°·7	32°·2	87°	Sutures very open. 3rd molars not fully developed. Epipterics R. and L., large on L. (36·8 × 11·0 mm.). Ossicle on R. of lambda (25 × 26·5 mm.). Bulging occipital.
765^{40}	67·05	51·13	86·49	82·05	76·34	92·15	59°·0	82°·3	38°·7	8°·7	30°·0	91°	Very young. Basilar synchondrosis. Infra-orbital suture meets malar maxillary suture on R.; not on L. Malar marginal processes R. and L. Small ossicles in lambdoid suture.
765^{41}	76·96	47·24	87·18	87·18	70·00	90·65	65°·0	73°·7	41°·3	10°·3	31°·0	84°	Young. Basilar synchondrosis. 3rd molar on L. not yet erupted. Small epipteric on R. Flattening of obelion. Receding forehead. Bulging parietals. Sutures open. Some bulging of occipital.
765^{42}	65·91	48·60	88·94	90·38	82·43	74·36	61°·2	77°·8	41°·0	9°·2	31°·8	87°	Very young. Basilar synchondrosis. Some of 2nd teeth not yet erupted, still in alveolus. Surface of bone much mutilated. Small epipteric on R. and L. Many ossicles in lambdoid suture. Ossicles very open. Infra-orbital sutures R. and L.
765^{43}	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	Skull considerably damaged. Sutures very obliterated.
765^{44}	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	Skull very damaged. Sutures very open.
765^{45}	72·71	41·53	79·55	80·78	80·38	80·42	63°·8	69°·1	47°·1	11°·9	35°·2	81°	Very receding forehead. Bulging and projecting parietals. Markedly good set of teeth in both lower and upper jaws. Marked malar marginal processes. Coronal and sagittal sutures considerably obliterated. Occipital bone asymmetric. Bulging glabella.
765^{46}	76·21	45·22	83·82	86·96	66·73	95·27	57°·8	75°·1	47°·1	9°·4	31°·7	84°·5	Sutures becoming obliterated. Parietal depression. Teeth extremely worn. Some absorption of alveolus. Bad abscess of bone on L. maxillary. Mid-sagittal projection.
765^{47}	76·00	38·94	82·46	88·10	75·00	100·00	65°·0	70°·1	44°·9	12°·9	32°·0	83°	Sagittal and coronal suture much obliterated. Teeth very worn. Low receding forehead. Bulging parietals. Marked inion.
765^{48}	67·00	48·08	73·45	75·89	71·81	85·71	65°·0	76°·8	38°·2	10°·2	28°·0	87°	Infra-orbital suture meets malar maxillary suture R. and L. Coronal and sagittal sutures becoming obliterated. Some flattening of obelion. Marked malar marginal processes. Teeth extremely worn and decayed.
765^{49}	75·87	46·43	80·96	85·40	67·92	85·88	64°·5	72°·2	43°·3	8°·8	34°·5	81°	Teeth very worn; sutures becoming slightly obliterated. Infra-orbital suture meets malar maxillary suture on L. Marked malar marginal processes. Slight bulging at mid-sagittal. Bulging parietals.
765^{50}	71·06	43·10	81·44	81·82	72·55	83·10	63°·0	73°·0	44°·0	11°·0	32°·0	85°	Large ossicle on L. occipital, 51 × 31 mm. Heavy projecting inion. Small epipteric on L. Sutures becoming obliterated at pterion; and obelion. Teeth very worn.

The measurements are tabulated to the nearest 0·5 millimetres; the indices are given for the full measurements, using Ernst's Tables.

left ramus of the mandible has been broken right across about the level of the alveolar ridge, possibly sheared across by a chin or side blow. Plate XX shows the right profile with the ramus and right side of the corpus normal; the only very obvious defect being the shortening of the mandible relative to the upper jaw. Plate XXI, the *norma facialis*, shows the asymmetry produced by the accident, the whole of the lower jaw is tilted. The source of this is explained on examining Plate XXII with the left profile; we see that the ramus has been broken right across, and both corpus and ramus on this side have ceased to develop properly. The pôsterior edge of the ramus is no longer nearly vertical as on the right profile, but slopes forward to a diminutive perigonal region. The whole mandible has, so to speak, been rotated round a vertical axis to allow for the dwarfing of the left side of the corpus, thus causing the upper jaw to protrude as seen in the profiles, and further has been rotated round a medial horizontal axis to allow for the dwarfing of the upper portion of the left ramus; the combination of these two rotations has given the remarkable skew effect produced in the *norma facialis*. The subject must have had a difficult time during life, but appears to have long survived his original trouble.

10. *Conclusions.* While I am by no means fully satisfied with this study of the Moriori skull and see many ways in which, if I could restart this investigation, it might be substantially improved, yet I must plead that what I have done has been sufficiently laborious to occupy fully my year's tenure of the Crewdson Benington Studentship. I can only say that experience has taught me certain difficulties which could be rectified should opportunity again offer, and that the detailed account I have given of these difficulties may aid others who plan cranial contour work. Further I hope to have provided an account of the Moriori skull, based on wider data than have hitherto been available; it will, I trust, in the future assist in settling racial affinities as more and more cranial material is reported on in a standardised form. If the racial affinities I find in the Moriori do not wholly coincide with the views of more authoritative anthropologists, they may at least serve as suggestions for the further examination of the primitive races who still or till recently bordered the Pacific Ocean. I feel confident that in the craniology of these races will be found a definite key to the evolution of man in prehistoric times; there have, I believe, been more extensive folk-wanderings, possibly rendered feasible by geological changes, than we have yet recognised. Craniological material from the outlying islands, and the fringes of the Pacific continents—if feasible of extinct races—is what one must above all things desire.

MISCELLANEA.

Note on the Probable Error of the Coefficient of Correlation in the Variate Difference Correlation Method.

BY A. RITCHIE-SCOTT, B.Sc.

In *Biometrika*, Vol. x. p. 278, Dr Anderson of Petrograd has given the following expression for the probable error of the correlation coefficient between differences of the mth order.

$$\text{Probable error of } {}_m r_{xy} = \cdot 67449\, {}_m\sigma_{r_{xy}}$$

$$= \cdot 67449 \frac{1 - {}_m r^2_{xy}}{\sqrt{n-m}} \left\{ \frac{1}{n-m} \left[n - m + 2(n-m-1)\left(\frac{m}{m+1}\right)^2 \right.\right.$$

$$\left.\left. + 2(n-m-2)\left(\frac{m \,.\, m-1}{m+1 \,.\, m+2}\right)^2 + 2(n-m-3)\left(\frac{m \,.\, m-1 \,.\, m-2}{m+1 \,.\, m+2 \,.\, m+3}\right)^2 + \text{etc.} \right] \right\},$$

where m = order of difference,

n = size of original population (i.e. number of years in series of observations etc.).

In this formula the population is supposed to be reduced by unity at each difference taken. It is, however, found more convenient in practice to keep the population constant by taking in an additional observation at each successive operation of taking the difference. If we denote this constant population by n' the expression within the brackets reduces to

$$\frac{1}{n'}\left[n' + 2(n'-1)\left(\frac{m}{m+1}\right)^2 + 2(n'-2)\left(\frac{m \,.\, m-1}{m+1 \,.\, m+2}\right)^2 + 2(n'-3)\left(\frac{m \,.\, m-1 \,.\, m-2}{m+1 \,.\, m+2 \,.\, m+3}\right)^2 + \text{etc.}\right]$$

$$= 1 + 2\left(1 - \frac{1}{n'}\right)\left(\frac{m}{m+1}\right)^2 + 2\left(1 - \frac{2}{n'}\right)\left(\frac{m \,.\, m-1}{m+1 \,.\, m+2}\right)^2$$

$$+ 2\left(1 - \frac{3}{n'}\right)\left(\frac{m \,.\, m-1 \,.\, m-2}{m+1 \,.\, m+2 \,.\, m+3}\right)^2 + \text{etc.}$$

$$= 2\left\{1 + \left(\frac{m}{m+1}\right)^2 + \left(\frac{m \,.\, m-1}{m+1 \,.\, m+2}\right)^2 + \left(\frac{m \,.\, m-1 \,.\, m-2}{m+1 \,.\, m+2 \,.\, m+3}\right)^2 + \text{etc.}\right\}$$

$$- 1 - \frac{2}{n'}\left\{\left(\frac{m}{m+1}\right)^2 + 2\left(\frac{m \,.\, m-1}{m+1 \,.\, m+2}\right)^2 + 3\left(\frac{m \,.\, m-1 \,.\, m-2}{m+1 \,.\, m+2 \,.\, m+3}\right)^2 + \text{etc.}\right\}$$

$$= 2\phi(m) - 1 - \frac{2f(m)}{n'}.$$

The series $\phi(m)$ and $f(m)$ have been summed as follows by Professor Pearson:

$$\phi(m) = 1 + \left(\frac{m}{m+1}\right)^2 + \left(\frac{m \,.\, m-1}{m+1 \,.\, m+2}\right)^2 + \left(\frac{m \,.\, m-1 \,.\, m-2}{m+1 \,.\, m+2 \,.\, m+3}\right)^2 + \text{etc.}$$

$\therefore$ Multiply both sides by $\left(\frac{\underline{|2m}}{\underline{|m}\,\underline{|m}}\right)^2$

$$\left(\frac{\underline{|2m}}{\underline{|m}\,\underline{|m}}\right)^2\phi(m) = \left(\frac{\underline{|2m}}{\underline{|m}\,\underline{|m}}\right)^2 + \left(\frac{\underline{|2m}}{\underline{|m-1}\,.\,\underline{|m+1}}\right)^2 + \left(\frac{\underline{|2m}}{\underline{|m-2}\,.\,\underline{|m+2}}\right)^2 + \text{etc.}$$

$$= (C^{2m}_{m})^2 + (C^{2m}_{m+1})^2 + (C^{2m}_{m+2})^2 + \dots (C^{2m}_{2m})^2$$

$$= \tfrac{1}{2}\sum_{0}^{2m}(C^{2m}_{r})^2 + \tfrac{1}{2}(C^{2m}_{m})^2$$

(since the number of terms is odd)

$$= \tfrac{1}{2}\{C^{4m}_{2m} + (C^{2m}_{m})^2\}$$

$$= \frac{1}{2}\left\{\frac{\underline{|4m}}{\underline{|2m}\,.\,\underline{|2m}} + \left(\frac{\underline{|2m}}{\underline{|m}\,\underline{|m}}\right)^2\right\},$$

$$\therefore\quad \phi(m) = \frac{1}{2}\left\{\frac{\underline{|4m}}{\underline{|2m}\,\underline{|2m}} \times \left(\frac{\underline{|m}\,\underline{|m}}{\underline{|2m}}\right)^2 + 1\right\}$$

$$= \frac{1}{2}\left\{\underline{|4m}\left(\frac{\underline{|m}}{\underline{|2m}}\right)^4 + 1\right\}.$$

The series

$$f(m) = \left(\frac{m}{m+1}\right)^2 + 2\left(\frac{m\,.\,m-1}{m+1\,.\,m+2}\right)^2 + 3\left(\frac{m\,.\,m-1\,.\,m-2}{m+1\,.\,m+2\,.\,m+3}\right)^2 + \text{etc.}$$

has the value $\frac{1}{4}m$, a discovery I made when starting to table it. It is a particular case of a more general series which may be summed as follows:

Consider the following identity:

$$\frac{(m+\overline{m-1})^2}{4m} = (m-1) + m\,.\left(\frac{m-\overline{m-1}}{m+m}\right)^2.$$

Multiply both sides by $\left(\frac{m-\overline{m-2}}{m+\overline{m-1}}\right)^2.$

Then

$$\frac{(m-\overline{m-2})^2}{4m} = (m-1)\left(\frac{m-\overline{m-2}}{m+\overline{m-1}}\right)^2 + m\left(\frac{m-\overline{m-1}}{m+m}\,.\,\frac{m-\overline{m-2}}{m+\overline{m-1}}\right)^2.$$

The process may be continued, thus

$$\frac{(m+\overline{m-2})^2}{4m} = (m-2) + m\left(\frac{m-\overline{m-2}}{m+m}\right)^2 = (m-2) + \frac{(m-\overline{m-2})^2}{4m}$$

$$= (m-2) + (m-1)\left(\frac{m-\overline{m-2}}{m+\overline{m-1}}\right)^2 + m\left(\frac{m-\overline{m-1}}{m+m}\,.\,\frac{m-\overline{m-2}}{m+\overline{m-1}}\right)^2.$$

Multiply both sides by $\left(\frac{m-\overline{m-3}}{m+\overline{m-2}}\right)^2.$

Then

$$\frac{(m-\overline{m-3})^2}{4m} = (m-2)\left(\frac{m-\overline{m-3}}{m+\overline{m-2}}\right)^2 + (m-1)\left(\frac{m-\overline{m-2}}{m+\overline{m-1}}\,.\,\frac{m-\overline{m-3}}{m+\overline{m-2}}\right)^2$$

$$+ m\,.\left(\frac{m-\overline{m-1}}{m+m}\,.\,\frac{m-\overline{m-2}}{m+\overline{m-1}}\,.\,\frac{m-\overline{m-3}}{m+\overline{m-2}}\right)^2.$$

Assume now that this operation has been performed q times and that as a result the following equation is identically true:

$$\frac{\{m - (m - \overline{q+1})\}^2}{4m} = (m - q)\left\{\frac{m - (m - \overline{q+1})}{m + (m - q)}\right\}^2$$

$$+ (m - \overline{q-1})\left\{\frac{m - (m - \overline{q+1})}{m + (m - q)} \cdot \frac{m - (m - q)}{m + (m - \overline{q-1})}\right\}^2 + \text{etc.}$$

Operating as before, we have

$$\frac{\{m + (m - \overline{q+1})\}^2}{4m} = (m - \overline{q+1}) + \frac{\{m - (m - \overline{q+1})\}^2}{4m}.$$

Multiplying by $\left\{\frac{m - (m - \overline{q+2})}{m + (m - \overline{q+1})}\right\}^2$ and inserting the value of $\frac{\{m - (m - \overline{q+1})\}^2}{4m}$ we have

$$\frac{\{m - (m - \overline{q+2})\}^2}{4m} = (m - \overline{q+1})\left\{\frac{m - (m - \overline{q+2})}{m + (m - \overline{q+1})}\right\}^2$$

$$+ (m - q)\left\{\frac{m - (m - \overline{q+1})}{m + (m - q)} \cdot \frac{m - (m - \overline{q+2})}{m + (m - \overline{q+1})}\right\}^2$$

$$+ (m - \overline{q-1})\left\{\frac{m - (m - \overline{q+1})}{m + (m - q)} \cdot \frac{m - (m - q)}{m + (m - \overline{q-1})} \cdot \frac{m - (m - \overline{q+2})}{m + (m - \overline{q+1})}\right\}^2$$

$$+ \text{etc.}$$

Hence if the expansion be true for $q + 1$ it will be true for $q + 2$. But it has been shown true for 1 and 2 and it is therefore true generally.

If now in the expansion for $\frac{\{m - (m - \overline{q+1})\}^2}{4m}$ we put $q = m - 1$, we get

$$\frac{m}{4} = \left(\frac{m}{m+1}\right)^2 + 2\left(\frac{m}{m+1} \cdot \frac{m-1}{m+2}\right)^2 + 3\left(\frac{m}{m+1} \cdot \frac{m-1}{m+2} \cdot \frac{m-2}{m+3}\right)^2 + \ldots = f(m).$$

Collecting these results together we may now write the probable error of ${}_m r_{xy}$

$$= \cdot 67449 \frac{1 - {}_m r^2_{xy}}{\sqrt{n'}} \left\{2\phi(m) - 1 - \frac{m}{2n'}\right\}^{\frac{1}{2}}.$$

Values of the function $(2\phi(m) - 1)$ have been calculated for values of m from $m = 1$ to $m = 20$ and are given in the table below.

m	$2\phi(m) - 1$	m	$2\phi(m) - 1$
1	1·5000	11	4·2282
2	1·9444	12	4·4100
3	2·3100	13	4·5845
4	2·6265	14	4·7527
5	2·9094	15	4·9151
6	3·1673	16	5·0723
7	3·4059	17	5·2248
8	3·6289	18	5·3730
9	3·8390	19	5·5172
10	4·0383	20	5·6578

After $m = 20$, the approximate value

$$2\phi(m) - 1 = \sqrt{1{\cdot}570796\,(m + \cdot 375)}$$

given by Stirling's Theorem is close enough for practical purposes, e.g. $m = 20$, it gives 5·6573 or 5·6578.

On certain Types of Compound Frequency Distributions in which the Components can be individually described by Binomial Series.

By KARL PEARSON, F.R.S.

Certain difficulties with regard to the interpretation of *negative* binomials, which are of constant occurrence in observational frequency series, have suggested the following investigation.

Consider a number u of binomial series of which the sth is $\nu_s(p_s+q_s)^n$ and let us suppose a frequency series compounded by adding together the rth terms of all these series, such will be the compound frequency series it is proposed to discuss. We can realise its nature a little more concretely by supposing n balls drawn out of a bag containing Np white and Nq black balls, N being very large as compared with n, or else each ball returned before a fresh draw, while the values of p and q change discontinuously at the ν_1+1, $\nu_1+\nu_2+1$, $\nu_1+\nu_2+\nu_3+1$, etc. draws. We shall take as origin the point at which the sum of the first terms of all the binomials may be supposed to be plotted. S will denote summation to u terms.

Let $N=S(\nu_s)$, and $N\mu_1'$, $N\mu_2'$ be the moments of the compound system about this origin, $N\mu_1$, $N\mu_2$ its moments about its mean. Thus

$$N\mu_1' = S(\nu_s n q_s),$$

$$N\mu_2' = S\{\nu_s(np_sq_s+n^2q_s^2)\}.$$

Hence

$$N\mu_2 = nS\{\nu_s q_s(1-q_s)\} + n^2\left(S(\nu_s q_s^2) - \frac{\{S(\nu_s q_s)\}^2}{N}\right)$$

$$= N\mu_1' + \frac{n}{N}[nS\{\nu_s\nu_{s'}(q_s-q_{s'})^2\} - NS(\nu_s q_s^2)],$$

since $N=S(\nu_s)$. Accordingly, if σ be the standard deviation and m the mean of the compound series, i.e. $\mu_2=\sigma^2$, $\mu_1'=m$, then

$$\frac{\sigma^2}{m} - 1 = \frac{1}{N}\left(\frac{nS\{\nu_s\nu_{s'}(q_s-q_{s'})^2\} - NS(\nu_s q_s^2)}{S(\nu_s q_s)}\right). \quad\text{......................(i)}$$

Now suppose we had endeavoured to fit a binomial $N(P+Q)^\kappa$ to the compound series, we should have had

$$m=\kappa Q, \qquad \sigma^2=\kappa PQ,$$

and accordingly have found

$$Q = -\frac{1}{N}\left(\frac{nS\{\nu_s\nu_{s'}(q_s-q_{s'})^2\} - NS(\nu_s q_s^2)}{S(\nu_s q_s)}\right), \quad\text{......................(ii)}$$

$$\kappa = -n\,\frac{\{S(\nu_s q_s)\}^2}{nS\{\nu_s\nu_{s'}(q_s-q_{s'})^2\} - NS(\nu_s q_s^2)}. \quad\text{.........................(iii)}$$

Thus, had we attempted to fit a binomial to the heterogeneous series, we should have found Q negative and P greater than unity provided

$$nS\{\nu_s\nu_{s'}(q_s-q_{s'})^2\} \text{ be } > NS(\nu_s q_s^2),$$

a condition which will frequently be found to be satisfied, especially if q_s be small and n large.

In the limit let us take $nq_s=m_s$ and q_s vanishingly small, i.e. suppose the sth binomial to be replaced by the Poisson series

$$e^{-m_s}\left(1+m_s+\frac{m_s^2}{1\,.\,2}+\ldots+\frac{m_s^i}{\underline{|i}}+\ldots\right),$$

then we have at once

$$\left.\begin{aligned} Q &= -\frac{S\{\nu_s\nu_{s'}(m_s-m_{s'})^2\}}{NS\{\nu_s m_s\}}, \\ P &= 1+\frac{S\{\nu_s\nu_{s'}(m_s-m_{s'})^2\}}{NS(\nu_s m_s)}, \\ \kappa &= -\frac{(S\{\nu_s m_s\})^2}{S\{\nu_s\nu_{s'}(m_s-m_{s'})^2\}}. \end{aligned}\right\} \quad\text{............................(iv)}$$

Thus, if two or more Poisson's series be combined term by term *from the first*, then the compound will always be a negative binomial. This theorem was first pointed out to me by "Student" and suggested by him as a possible explanation of negative binomials occurring in material which theoretically should obey the Law of Small Numbers, e.g. "Student's" own Haemacytometer counts*. Of course the negative binomial may quite conceivably arise from other sources† than heterogeneity, but if this be the source of its origin in the material of Bortkewitsch, Mortara and McKendrick‡, it is certainly most unfortunate that such material should have been selected to illustrate Poisson's limit to the binomial.

Now we know that the values about the mean of the successive moment coefficients of the binomial are

$$\left.\begin{aligned}\mu_2 &= npq,\\ \mu_3 &= npq\,(p-q),\\ \mu_4 &= npq\,(1+3\,(n-2)\,pq).\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{v})$$

Further the mean is at a distance nq from the first term p^n. We shall call this distance m.

Let μ_1', μ_2', μ_3' and μ_4' be the moment coefficients round the start of each binomial. Then

$$\left.\begin{aligned}&\mu_1' = nq, \qquad \mu_2' = npq + n^2q^2,\\ &\mu_3' = npq\,(p-q) + 3npq \times nq + n^3q^3,\\ &\mu_4' = npq\,(1+3\,(n-2)\,pq) + 4npq\,(p-q)\,nq + 6\,(npq)\,n^2q^2 + n^4q^4.\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(\text{vi})$$

From these equations we deduce

$$\left.\begin{aligned}&\mu_1' = nq, \qquad \mu_2' - \mu_1' = n\,(n-1)\,q^2,\\ &\mu_3' - 3\mu_2' + 2\mu_1' = n\,(n-1)\,(n-2)\,q^3,\\ &\mu_4' - 6\mu_3' + 11\mu_2' - 6\mu_1' = n\,(n-1)\,(n-2)\,(n-3)\,q^4.\end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{vii})$$

Now let $a_1 = \mu_1'$ for the combined series,

$a_2 = \mu_2' - \mu_1'$ for the combined series,

$a_3 = \mu_3' - 3\mu_2' - 2\mu_1'$ for the combined series,

$a_4 = \mu_4' - 6\mu_3' + 11\mu_2' - 6\mu_1'$ for the combined series.

Then we have, if $\lambda_1 = \nu_1/N$, $\lambda_2 = \nu_2/N$:

$$\left.\begin{aligned}1 &= \lambda_1 + \lambda_2,\\ \frac{a_1}{n} &= \lambda_1 q_1 + \lambda_2 q_2,\\ \frac{a_2}{n\,(n-1)} &= \lambda_1 q_1^2 + \lambda_2 q_2^2,\\ \frac{a_3}{n\,(n-1)\,(n-2)} &= \lambda_1 q_1^3 + \lambda_2 q_2^3,\\ \frac{a_4}{n\,(n-1)\,(n-2)\,(n-3)} &= \lambda_1 q_1^4 + \lambda_2 q_2^4.\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{viii})$$

Multiply each equation by q_1 and subtract from that below it and we find:

$$\left.\begin{aligned}\frac{a_1}{n} - q_1 &= \lambda_2\,(q_2 - q_1),\\ \frac{a_2}{n\,(n-1)} - \frac{a_1 q_1}{n} &= \lambda_2 q_2\,(q_2 - q_1),\\ \frac{a_3}{n\,(n-1)\,(n-2)} - \frac{a_2 q_1}{n\,(n-1)} &= \lambda_2 q_2^2\,(q_2 - q_1),\\ \frac{a_4}{n\,(n-1)\,(n-2)\,(n-3)} - \frac{a_3 q_1}{n\,(n-1)(n-2)} &= \lambda_2 q_2^3\,(q_2 - q_1).\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(\text{ix})$$

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* *Biometrika*, Vol. v. p. 356, and see also L. Whitaker's examination of these data, Vol. x. p. 48.

† Pearson, *Biometrika*, Vol. IV. p. 209.

‡ For an examination of Bortkewitsch and Mortara's instances see L. Whitaker, *loc. cit.* pp. 49–66. McKendrick has recently reached Poisson's series (*Proceedings of the London Mathematical Society*, Vol. XIII. (1914), pp. 405 *et seq.*) without apparently recognising that he was on familiar ground, and has suggested its application to the frequencies obtained by counts of the bacilli ingested by leucocytes. He has fitted his series not by moments, but from the first two terms, and has failed to recognise that a large proportion of such leucocyte counts give also *negative* binomials.

Hence dividing each equation by the preceding one:

$$\frac{\dfrac{a_2}{n(n-1)} - \dfrac{a_1 q_1}{n}}{\dfrac{a_1}{n} - q_1} = \frac{\dfrac{a_3}{n(n-1)(n-2)} - \dfrac{a_2 q_1}{n(n-1)}}{\dfrac{a_2}{n(n-1)} - \dfrac{a_1 q_1}{n}} = \frac{\dfrac{a_4}{n(n-1)(n-2)(n-3)} - \dfrac{a_3 q_1}{n(n-1)(n-2)}}{\dfrac{a_3}{n(n-1)(n-2)} - \dfrac{a_2 q_1}{n(n-1)}} = q_2. \quad \text{.........................(x)}$$

Writing $q_1 q_2 = P_2$ and $q_1 + q_2 = P_1$, we obtain

$$\left.\begin{aligned} a_2 - (n-1)\,a_1 P_1 + n\,(n-1)\,P_2 &= 0, \\ a_3 - (n-2)\,a_2 P_1 + (n-1)(n-2)\,a_1 P_2 &= 0, \\ a_4 - (n-3)\,a_3 P_1 + (n-2)(n-3)\,a_2 P_2 &= 0, \end{aligned}\right\} \quad \text{.........................(xi)}$$

three equations to determine n, P_1 and P_2.

Eliminating $-P_1$ and P_2 we find:

$$\begin{vmatrix} a_2 & (n-1)\,a_1 & n\,(n-1) \\ a_3 & (n-2)\,a_2 & (n-1)(n-2)\,a_1 \\ a_4 & (n-3)\,a_3 & (n-2)(n-3)\,a_2 \end{vmatrix} = 0, \quad \text{...........................(xii)}$$

which expanded gives us the cubic for n:

$$\begin{aligned} &n^3\,(2a_1a_2a_3 - a_2^3 - a_1^2a_4 + a_2a_4 - a_3^2) + n^2\,(-12a_1a_2a_3 + 7a_2^3 + 4a_1^2a_4 - 3a_2a_4 + 4a_3^2) \\ &\quad + n\,(22a_1a_2a_3 - 16a_2^3 - 5a_1^2a_4 + 2a_2a_4 - 3a_3^2) + (-12a_1a_2a_3 + 12a_2^3 + 2a_1^2a_4) = 0. \ldots\text{(xii)}^{\text{bis}} \end{aligned}$$

A root of this cubic substituted in the first two equations of (xi) will give P_1 and P_2 and then the quadratic

$$p^2 - P_1 p + P_2 = 0 \quad \text{....................................(xiii)}$$

will determine the two values q_1 and q_2 corresponding to the value of n. The first two equations of (viii) then complete the solution by providing λ_1 and λ_2.

Until the roots of the cubic (xii)$^{\text{bis}}$ have been discussed we can only assume that three solutions are possible. As a matter of fact in the examples so far dealt with some of these solutions have usually to be discarded.

For the special case of Poisson's limit to the binomial, we make n indefinitely large, q indefinitely small, and $nq = m$ finite. Hence equations (viii) become

$$\left.\begin{aligned} 1 &= \lambda_1 + \lambda_2, \\ a_1 &= \lambda_1 m_1 + \lambda_2 m_2, \\ a_2 &= \lambda_1 m_1^2 + \lambda_2 m_2^2, \\ a_3 &= \lambda_1 m_1^3 + \lambda_2 m_2^3, \\ a_4 &= \lambda_1 m_1^4 + \lambda_2 m_2^4, \end{aligned}\right\} \quad \text{.......................................(xiv)}$$

leading to

$$\begin{aligned} a_2 - a_1 Q_1 + Q_2 &= 0, \\ a_3 - a_2 Q_1 + a_1 Q_2 &= 0, \\ a_4 - a_3 Q_1 + a_2 Q_2 &= 0, \end{aligned}$$

if

$$Q_1 = m_1 + m_2, \qquad Q_2 = m_1 m_2.$$

Thus we find:

$$\begin{aligned} Q_1 &= (a_3 - a_1 a_2)/(a_2 - a_1^2), \\ Q_2 &= (a_3 a_1 - a_2^2)/(a_2 - a_1^2), \end{aligned}$$

subject to the condition*

$$a_4\,(a_2 - a_1^2) + 2a_1 a_2 a_3 - a_3^2 - a_2^3 = 0. \quad \text{...........................(xv)}$$

Hence m_1 and m_2 are roots of

$$m^2\,(a_2 - a_1^2) - m\,(a_3 - a_1 a_2) + a_3 a_1 - a_2^2 = 0. \quad \text{..................(xvi)}$$

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* Of course equations of condition hold for the 5th and higher moments in the case of the two binomial components. But they are of small service as the probable errors of these high moments are usually very considerable.

Further $$\lambda_1 = \frac{\mu_1' - m_2}{m_1 - m_2}, \quad \lambda_2 = \frac{\mu_1' - m_1}{m_2 - m_1}, \qquad\text{(xvii)}$$
which determine λ_1 and λ_2.

It is thus quite easy to resolve a series into the sum of two Poisson's binomial limits provided the roots of the above quadratic are real. As illustration I take "Student's" first count of yeast cells on the 400 squares of a haemacytometer. He found:

No. of yeast cells	0	1	2	3	4	5	Total
Frequency	213	128	37	18	3	1	400

giving: mean $= \mu_1' = \cdot 6825$, $\mu_2 = \cdot 8117$, $\mu_3 = 1{\cdot}0876$.

From the above values of "Student" I determined
$$\mu_2' = 1{\cdot}2775, \qquad \mu_3' = 3{\cdot}0675.$$
Whence $$a_1 = \cdot 6825, \qquad a_2 = \cdot 5950, \qquad a_3 = \cdot 6000,$$
and the resulting quadratic is
$$\cdot 129{,}194 m^2 - \cdot 193{,}913 m + \cdot 055{,}475 = 0.$$

The roots are $m_1 = \cdot 3847$ and $m_2 = 1{\cdot}1163$, leading to $\lambda_1 = \cdot 59{,}295$ and $\lambda_2 = \cdot 40{,}705$, or, the series has for its two components
$$\nu_1 = 237{\cdot}18, \qquad m_1 = \cdot 3847,$$
$$\nu_2 = 162{\cdot}82, \qquad m_2 = 1{\cdot}1163.$$

Calculating out the Poisson's series for these components we have:

No. of yeast cells	0	1	2	3	4	5	6	7
1st Compt.	161·44	62·11	11·95	1·53	·15	·01	·00	·00
2nd Compt.	53·32	59·52	32·22	12·36	3·45	·77	·14	·02
Round totals	215	122	44	14	4	1		
Observed	213	128	37	18	3	1		

The test for "goodness of fit" for these six groups gives $\chi^2 = 2{\cdot}82$ and $P = \cdot 73$, or the fit is very good. The negative binomial gave $P = \cdot 52$ and a single Poisson's series only $P = \cdot 04$. But the double Poisson's series places of course one constant more at our disposal than the binomial, and we can do still better with a double binomial, as we have four constants and only six frequencies, while the double Poisson has three constants to six frequencies. It is clear that neither of the above components forming 41 % and 59 % of the total number of cells, and having their means at ·3847 and 1·1163 instead of ·6825, gives any idea of a dominant constitution in the solution sampled. If in this case heterogeneity accounts for the negative binomial, then the difference of the components is not slight, and the heterogeneity being gross would indicate some considerable failure in technique.

If we assume that the counts with a haemacytometer ought to follow the Poisson distribution,—and this seems to be theoretically probable,—then the criterion of the binomial might well be adopted to ascertain the possibility of some failure in technique. The actual binomial in "Student's" first case should be $(\frac{1}{400} + \frac{399}{400})^{273}$ and any binomial with p very small and $np = \cdot 6825$ would effectively represent the series; we could not anticipate getting $n = 273$ and $p = \frac{1}{400}$ closely from the data, but we might certainly anticipate a *positive* binomial, if the theory of a Poisson distribution be correct. If on the other hand we say in this and many similar cases that the negative binomial arises from heterogeneity, then it appears to me that we have saved our theory at the expense of our technique. I propose now to test this point further by considering the component binomials. If the theory of heterogeneity be correct, unless it be very manifold, we might anticipate two binomial components, $\nu_1(p_1+q_1)^n+\nu_2(p_2+q_2)^n$, with n positive and large, both q_1 and q_2 being small, while n_1q_1 and n_2q_2 would be approximately ·3847 and 1·1163, the frequencies ν_1 and ν_2 being roughly in the ratio of 3 to 2.

Returning to "Student's" data we find $\mu_4' = 8{\cdot}9275$, whence

$$a_1 = {\cdot}6825, \quad a_2 = {\cdot}5950, \quad a_3 = {\cdot}6000, \quad a_4 = {\cdot}4800.$$

Substituting in (xii)$^{\text{bis}}$ we obtain for the cubic:

$$-{\cdot}021{,}3269n^3 + {\cdot}028{,}2321n^2 + {\cdot}363{,}3020n + {\cdot}051{,}0825 = 0.$$

This has three real roots, approximately.

$$n' = 4{\cdot}89997, \quad n'' = -{\cdot}14234, \quad \text{and} \quad n''' = -3{\cdot}43390.$$

We will consider in succession these cases: (i) $n' = 4{\cdot}89997$. The first two equations of (xi) provide

$$\begin{aligned} {\cdot}5950 - 2{\cdot}66173P_1 + 19{\cdot}10974P_2 &= 0, \\ {\cdot}6000 - 1{\cdot}72548P_1 + 7{\cdot}71894P_2 &= 0, \end{aligned}$$

leading to $P_1 = {\cdot}55304$, $P_2 = {\cdot}04590$, and the quadratic

$$q^2 - {\cdot}55304q + {\cdot}04590 = 0,$$

whence we deduce the binomial factors

$$q_1 = {\cdot}4514, \quad p_1 = {\cdot}5486, \quad \text{and} \quad q_2 = {\cdot}1017, \quad p_2 = {\cdot}8983.$$

The first two equations of (viii) give

$$1 = \lambda_1 + \lambda_2, \qquad {\cdot}6825/4{\cdot}89997 = {\cdot}451{,}355\lambda_1 + {\cdot}101{,}685\lambda_2,$$

leading to

$$\lambda_1 = {\cdot}107{,}535, \qquad \lambda_2 = {\cdot}892{,}465,$$

or, in a population of 400,

$$\nu_1 = 43{\cdot}014, \qquad \nu_2 = 356{\cdot}986.$$

Accordingly the compound series is given by

$$43{\cdot}014\,({\cdot}5486 + {\cdot}4514)^{4\cdot89997} + 356{\cdot}986\,({\cdot}8983 + {\cdot}1017)^{4\cdot89997},$$

with means of the components at

$$m_1 = 2{\cdot}2118 \quad \text{and} \quad m_2 = {\cdot}4983.$$

We see that neither the sizes of the component populations nor their means have any relation to the previously discussed component Poisson series; further the present series* diverge widely from Poisson series, n is not large nor q_1 or q_2 very small. Calculated to the nearest whole numbers we obtain:

No. of yeast cells	0	1	2	3	4	5
1st Compt.	2	9	15	12	4	1
2nd Compt.	211	117	26	3	0	0
Combination	213	126	41	15	4	1
Observed	213	128	37	18	3	1

which leads to $\chi^2 = 1{\cdot}27$ and $P = {\cdot}93$.

Thus the fit is excellent, but it does not correspond to the heterogeneity of a double Poisson series.

(ii) $n'' = -{\cdot}14234$. Here the first two equations of (xi) provide

$$\begin{aligned} {\cdot}5950 + {\cdot}77965P_1 + {\cdot}16260P_2 &= 0, \\ {\cdot}6000 + 1{\cdot}27469P_1 + 1{\cdot}75727P_2 &= 0, \end{aligned}$$

and give

$$P_1 = -{\cdot}81529, \qquad P_2 = {\cdot}24996,$$

with

$$q^2 + {\cdot}81529q + {\cdot}24996 = 0.$$

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* It should be noted that such fractional binomial series tend ultimately to become negative, although with negligibly small frequencies.

This gives imaginary values of q_1 and q_2 and thus the solution can for the present purpose be discarded.

(iii) $n''' = -3{\cdot}43390$. We deduce

$$\cdot 5950 + 3{\cdot}02614P_1 + 15{\cdot}22557P_2 = 0,$$

$$\cdot 6000 + 3{\cdot}23317P_1 + 16{\cdot}44372P_2 = 0,$$

giving $$P_1 = -1{\cdot}21441, \quad P_2 = \cdot 20229$$

and $$q^2 + 1{\cdot}21441q + \cdot 20229 = 0.$$

Hence $q_1 = -1{\cdot}0151$, $p_1 = 2{\cdot}0151$, $q_2 = -\cdot 1993$, $p_2 = 1{\cdot}1993$.

The λ equations are

$$1 = \lambda_1 + \lambda_2, \quad -\cdot 6845/3{\cdot}4339 = -1{\cdot}0151\lambda_1 - \cdot 1993\lambda_2,$$

leading to $$\lambda_1 = \cdot 000043, \quad \lambda_2 = \cdot 999957,$$

or, $$\nu_1 = \cdot 0172, \quad \nu_2 = 399{\cdot}9828.$$

Thus the component series is

$$\cdot 0172\,(2{\cdot}0151 - 1{\cdot}0151)^{-3\cdot 4339} + 399{\cdot}9828\,(1{\cdot}1993 - \cdot 1993)^{3\cdot 4339},$$

with means at $$m_1 = 3{\cdot}4858 \quad \text{and} \quad m_2 = \cdot 6847.$$

The first of these components is negligible, it contains roughly only ·02 individuals in 400, and the second is sensibly identical with the negative binomial obtained by "Student," i.e.

$$400\,(1{\cdot}1893 - \cdot 1893)^{-3\cdot 6054},$$

with slightly modified constants. It provides:

No. of yeast cells	0	1	2	3	4	5
Calculated	214	122	45	14	4	1
Observed	213	128	37	18	3	1

leading to $\chi^2 = 3{\cdot}12$ and $P = \cdot 68$, which for all practical purposes is as good as the double Poisson.

Conclusions. It having been suggested that the appearance of *negative* binomials as better "fits" than Poisson's series for material that is supposed to follow the law of small numbers is due to heterogeneity, formulae have been provided for testing whether this heterogeneity is due to a second component. If so this component should be small and the first component should substantially agree with the primary Poisson's series. The smallness of the second component would measure the goodness of the technique in haemacytometer or opsonic index counts. Applied to "Student's" first series of counts of yeast cells we obtain (*a*) two Poisson's series neither of which dominates the data or approximates to the primary Poisson's series; (*b*) two positive binomials, neither of which has any approach to a Poisson series or any agreement with the components of (*a*); and lastly (*c*) two *negative* binomials, one of which dominates the series, and agrees with the primary negative binomial. This investigation as far as it goes suggests either that "Student's" first count is really described *homogeneously* by a negative binomial, or, if it be heterogeneous, then the heterogeneity is manifold, and no weight can be given to the results of fitting by the primary Poisson's series.

The general numerical discussion by the formulae of this paper of a variety of data assumed to follow the "law of small numbers" is in hand and will shortly be published.

NOTE.

With this double Part of *Biometrika* is issued an Appendix containing the Records of the late W. F. R. Weldon's Mice Breeding Experiments. It was intended originally to issue these Records as an Appendix to the memoir in which the reduced observations are discussed. While the reductions have made considerable progress there has been unfortunately great delay in their completion owing to Assistants to whom they were entrusted being called away for other work, and owing to the whole Laboratory Staff for the past twelve months having other duties. As the present Records were set up more than two years ago, it is convenient to issue them at once, in anticipation of the explanatory memoir, which, it is hoped, will not be too long delayed. The determinations of colour and extent of pigmentation from the scales are due to F. J. W. upon whom also the more laborious part of the preparation for press of these Records has fallen. K. P.

APPENDIX TO BIOMETRIKA.

W. F. R. WELDON'S MICE BREEDING EXPERIMENTS.

RECORDS OF MATINGS.

For an Explanation of the Symbols used and Notes on the Individual Mice, see the conclusion of the Records.

MATING	OFFSPRING				PARENTS	
					Father	Mother
0*H*1	a^1 ♂ 45 *q*49.46. 2(2*d*)	a^2 ♂ 55 *q*49. 1. 1(3*c*)	a^3 ♂ 45 *q*48. 1. 1(2*c*)	a^4 ♀ – *q* – – –(1*c*)	*W* no. 1 ♂ *p* – – –(*Waβ*)	*Lb* no. 1 ♀ 00*p* – – –(*A* × 1 or 2)
	a^5 ♀ – *q* – – –(1*c*)	a^6 ♀ – *q* – – –(3*c*)	... ...	... ...	... ...	... ...
0*H*6	a^1 ♀ – *q* – – –(2*c*)	... ...	... ...	... ...	*W* no. 6 ♂ *p* – – –(*Wβ*)	*Lb* no. 6 ♀ 00*p* – – –(*A* × 1 or 2)
0*H*7	a^1 ♂ 100*zq*49.46. 7(5*c*)	a^2 ♂ 45 *q*48. 1. 1(2*c*)	a^3 ♂ 55 *q*49.46. 2(2–3*c*)	a^4 ♂ 55 *q*49.21. 2(3*d*)	*W* no. 7 ♂ *p* – – –(*Wa*)	*Bt* no. 7 ♀ 00*p* – – –(*A*1)
	a^5 ♀ 50 *q*48.36. 2(3*c*)	a^6 ♀ 100*zq*48.46.12(5*c*)	a^7 ♀ – *q* – – –(2*d*)	... ...	... ...	... ...
0*H*8	a^1 ♂ 100*zq*48.36. 7(5*c*)	a^2 ♂ 100*zq*49.46. 7(5*c*)	a^3 ♀ – *q* – – –(5*c*)	a^4 ♀ – *q* – – –(5*c*)	*W* no. 7 ♂ *p* – – –(*Wa*)	*Bt* no. 8 ♀ 00*p* – – –(*A*1)
(or 7)					*W* no. 8 ♂ *p* – – –(*Wβ*)	(or 7)
	a^5 ♀ 100*zq* 49.46.12(5*c*)	a^6 – *q* – – –(5*c*)	... ...	... ...	... ...	... ...
0*H*9	a^1 ♂ – *q* – – –(2*c*)	a^2 ♀ 30 *q*46. 1. 1(2*c*)	a^3 ♀ 45 *q*48.46. 1(2*c*)	a^4 ♀ 50 *q*49. 1. 1(2*c*)	*W* no. 1 ♂ *p* – – –(*Waβ*)	*Lb* no. 9 ♀ 00*p* – – –(*A* × 1 or 2)
	a^5 ♀ 25 *q*46. 1. 1(2*c*)	a^6 ♀ 40 *q*49. 1. 1(2*d*)	... ...	... ...	... ...	... ...
0*H*12	a^1 ♂ – *q* – – –(5*c*)	a^2 ♂ – *q* – – –(5*c*)	a^3 ♀ – *q* – – –(5*a*)	a^4 ♀ – *q* – – –(5*c*)	*W* – ♂ *p* – – –(*Wa*)	*Bt* no. 11 ♀ 00*p* – – –(*A*1 ?)
	a^5 ♀ – *q* – – –(5*a*)	... ...	... ...	... ...	... ...	... ...
0*H*13	a^1 ♂ 50 *q*46.46. 1(3*c*)	a^2 ♂ 100*zq*48.46. 7(5*c*)	a^3 ♂ 100*zq*49.46. 7(5*d*)	a^4 ♂ 45 *q*52. 1. 1(3*c*)	*W* no. 12 ♂ *p* – – –(*Wa*)	*Bt* no. 12 ♀ 00*p* – –(*A*1 ?)
0*H*16	a^1 ♂ 50 *q*47. 1. 1(3*c*)	a^2 ♀ 50 *q*48.46. 1(3*c*)	a^3 ♀ 50 *q*48. 1. 1(3*c*)	a^4 ♀ 55 *q*49.46. 1(4*c*)	*W* no. 8 ♂ *p* – – –(*Wβ*)	*Lb* no. 13 ♀ 00*p* – – –(*A* × 1 or 2)
	a^5 ♀ – *q* – – –(4*c*)	... ...	... ...	... ...	... ...	... ...
0*H*20	a^1 ♂ 55 *q*47.46. 1(3*d*)	a^2 ♂ 40 *q*48.46. 1(2*c*)	a^3 ♂ 100*zq*48.46. 7(5*d*)	a^4 ♀ 45 *q*49. 1. 1(2*d*)	*W* no. 1 ♂ *p* – – –(*Waβ*)	*Bt* no. 15 ♀ 00*p* – – –(*A*1 ?)
	a^5 ♀ 25 *q*49. 1. 1(2*d*)	a^6 – *q* – – –(5*d*)	... ...	... ...	... ...	... ...
0*H*27	a^1 ♀ 40 *q*58. 1. 2(2*e*)	... ...	... ...	... ...	*Bt* – ♂ 00*p* – – –(*A*1 ?)	*W* no. 24 ♀ *p* – – –(*Waβ*)
0*H*33	a^1 – *q* – – –(5*c*)	... ...	... ...	... ...	*Bt* – ♂ 00*p* – – –(*A*1 ?)	*W* no. 43 ♀ *p* – – –(*Wa*)
0*H*35	a^1 ♂ – *q* – – –(2*c*)	a^2 ♀ – *q* – – –(3*d*)	a^3 ♀ – *q* – – –(3*d*)	a^4 ♀ – *q* – – –(2*d*)	*W* – ♂ *p* – – –(*Wa* ?)	*Lb* – ♀ *p* – – –(*A* × 1 or 2)
	a^5 ♂ – *q* – – –(4*d*)	a^6 ♂ – *q* – – –(4*d*)	... ...	... ...	... ...	... ...
0*H*45	a^1 ♀ – *q* – – –(2*d*)	a^2 ♂ – *q* – – –(2*d*)	a^3 ♂ – *q* – – –(2*d*)	a^4 ♀ – *q* – – –(2*d*)	*W* no. 6 ♂ *p* – – –(*Wβ*)	*Bt* – ♀ 00*p* – – –(*A*1 ?)
0*H*53	a^1 – *q* – – –(4*c*)	... ...	... ...	... ...	*Bt* – ♂ 00*p* – – –(*A*1 ?)	*W* no. 22 ♀ *p* – – –(*Wβ*)
0*H*59	a^1 ♀ 50 *q*48. 1. 4(3*d*)	a^2 ♂ 100*zq*49.46. 7(6*d*)	... ...	... ...	*Bt* – ♂ 00*p* – – –(*A*1)	*W* no. 34 ♀ *p* – – –(*Wa*)
0*H*61	a^1 ♂ – *q* – – –(2*c*)	a^2 ♂ – *q* – – –(5*c*)	a^3 ♂ – *q* – – –(5*c*)	a^4 ♂ – *q* – – –(5*c*)	*W* no. 39 ♂ *p* – – –(*Waβ*)	*Bt* no. 12 ♀ 00*p* – – –(*A*1 ?)
	a^5 ♀ 80 *q*46. 1. 1(4*c*)	a^6 ♀ 65 *q*49.30. 7(3*c*)	a^7 ♀ – *q* – – –(5*c*)	... ...	... ...	... ...
0*H*62	a^1 ♂ – *q* – – –(2*c*)	a^2 ♂ 40 *q*49. 1. 1(3*c*)	a^3 ♀ 45 *q*48.34. 3(3*c*)	a^4 ♀ 100*zq*49.46. 7(5*c*)	*W* no. 54 ♂ *p* – – –(*Wβ*)	*Bt* – ♀ 00*p* – (*A*1)
	a^5 ♀ 100*zq*49.48.12(5*c*)	a^6 ♀ 100*zq*49.46.12(5*c*)	... ...	... ...	... ...	... ...
0*H*66	a^1 – *q* – – –(5*c*)	... ...	... ...	... ...	*Bt* – ♂ 00*p* – – –(*A*1 ?)	*W* no. 38 ♀ *p* – – –(*Wβ*)
0*H*72	a^1 – *q* – – –(5*c*)	... ...	... ...	... ...	*Bt* – ♂ 00*p* – – –(*A*1 ?)	*W* – ♀ *p* – – –(*Waβ*)
0*H*73	a^1 ♀ – *q* – – –(5*c*)	a^2 ♀ – *q* – – –(5*c*)	a^3 ♀ – *q* – – –(5*c*)	a^4 ♂ – *q* – – –(3*c*)	*W* – ♂ *p* – – –(*Waβ*)	*Bt* – ♀ 00*p* – – –(*A*1)
	a^5 ♀ – *q* – – –(3*c*)	a^6 ♂ – *q* – – –(3*c*)	... ...	... ...	... ...	... ...
0*H*74	a^1 ♂ – *q* – – –(4*c*)	a^2 ♂ – *q* – – –(3*c*)	a^3 ♂ – *q* – – –(4*c*)	a^4 ♂ – *q* – – –(4*c*)	*W* – ♂ *p* – – –(*Waβ*)	*Bt* – ♀ 00*p* – – –(*A*1 ?)
	a^5 ♀ 80 *q*49.46. 2(4*c*)	a^6 ♀ 80 *q*49.48. 1(4*c*)	a^7 ♀ 80 *q*49.46. 1(4*c*)	a^8 ♀ – *q* – – –(4*c*)	... ...	... ...
0*H*75	a^1 ♀ 60 *q*49.48. 1(4*c*)	a^2 ♂ 35 *q*49. 1. 1(2*c*)	a^3 ♀ 45 *q*49. 1. 1(2*c*)	a^4 ♀ 45 *q*49.46. 1(3*c*)	*W* – ♂ *p* – – –(*Waβ*)	*Bt* – ♀ 00*p* – – –(*A*1 ?)
	a^5 ♂ 15 *q*49. 1. 1(1*c*)	... ...	... ...	... ...	... ...	... ...
0*H*77	a^1 ♂ 100*zq*48.46. 7(5*c*)	a^2 ♂ 60 *q*48.46. 4(4*c*)	a^3 ♂ 55 *q*48.46. 1(4*c*)	a^4 ♀ – *q* – – –(5*c*)	*W* no. 16 ♂ *p* – – –(*Waβ*)	*Bt* – ♀ 00*p* – – –(*A*1 ?)
	a^5 ♀ – *q* – – –(5*c*)	a^6 ♀ 100*xq*57.56.56(5*e*)	a^7 ♀ 70 *q*61.61. 1(4*e*)	a^8 ♀ – *q* – – –(5*c*)	... ...	... ...
0*H*78	a^1 ♀ – *q* – – –(5*e*)	a^2 – *q* – – –(5*d*)	a^3 ♀ – *q* – – –(2*e*)	a^4 ♀ – *q* – – –(3*e*)	*W* no. 8 ♂ *p* – – –(*Wβ*)	*Bt* no. 58 ♀ 00*p* – – –(*A*1 ?)
0*H*82	a^1 ♀ 55 *q*48.46.12(3*c*)	a^2 ♀ – *q* – – –(5*c*)	... ...	... ...	*A*1a^1 ♂ 00*p* – – –(*A*2½)	*W* – ♀ *p* – – –(*Wβ*)
0*H*84	a^1 ♀ – *q* – – –(5*c*)	a^2 ♀ 100*zq*49.46. 4(5*c*)	a^3 ♀ 100*zq*35.33.30(5*a*)	a^4 ♀ 100*zq*35.33.30(5*a*)	*W* no. 54 ♂ *p* – – –(*Wβ*)	*Bt* no. 11 ♀ 00*p* – – –(*A*1 ?)
	a^5 ♀ 100*zq*33.30.28(5*a*)	a^6 ♀ 100*zq*39.33.30(5*a*)	a^7 ♂ 100*zq*35.33.28(5*a*)	a^8 ♂ – *q* – – –(5*c*)	... ...	... ...
	a^9 ♂ 100*zq*48.46.28(5*c*)	... ...	... ...	... ...	... ...	... ...
0*H*85	a^1 ♀ – *q* – – –(4*d*)	a^2 ♂ – *q* – – –(3*e*)	a^3 ♀ – *q* – – –(5*d*)	a^4 ♀ – *q* – – –(5–6*d*)	*W* – ♂ *p* – – –(*Wa*)	*Lb* no. 13 ♀ 00*p* – – –(*A* × 1 or 2)
	a^5 ♂ – *q* – – –(5*d*)	a^6 ♂ – *q* – – –(5*e*)	a^7 – *q* – – –(3*e*)	... ...	... ...	... ...

0H 86	a^1 ♂ 100zq49.46. 9(5c)	a^2 ♂ 100zq49.46.28(5c)	a^3 ♀ – q – – –(3c)	a^4 ♀ – q – – –(3c)	W – ♂ p – – –(Waβ)	Bt no. 59 ♀ 00p – – –(A1 ?)
	a^5 ♀ – q – – –(5c)	a^6 ♀ – q – – –(5c)	a^7 ♀ – q – – –(5c)	... ...	... ...	... ...
0H 90	a^1 ♀ – q – – –(5c)	a^2 ♂ – q – – –(5c)	... ...	... ...	W no. 35 ♂ p – – –(Waβ)	Bt – ♀ 00p – – –(A1 ?)
0H 91	a^1 – q – – –(6c)	... ...	... ...	... ...	A1a^2 ♂ 00p – – –($A2\frac{1}{2}$)	W no. 46 ♀ p – – –(Wa)
0H 94	a^1 ♀ 60 q49.46. 1(3c)	a^2 – q – – –(3c)	... ...	... ...	A4a^2 ♂ 00p – – –($A2\frac{1}{2}$)	W no. 33 ♀ p – – –(Wa)
0H 100	a^1 ♂ 40 q49. 1. 1(3c)	a^2 ♂ – q – – –(5c)	a^3 ♂ – q – – –(5c)	a^4 ♂ 100zq49.46. 7(5c)	W no. 54 ♂ p – – –(Wβ)	A1a^5 ♀ 00p – – –($A2\frac{1}{2}$)
	a^5 ♀ 25 q46. 1. 1(2c)	a^6 ♀ 55 q49.46. 9(3c)	a^7 ♀ – q – – –(5c)	a^8 ♀ – q – – –(5c)	... ...	... ...
	a^9 – q – – –(3c)	... ...	... ...	... ...	... ...	... ...
0H 101	a^1 ♂ 45 q49. 1. 1(2c)	a^2 ♂ 50 q49.46. 4(3c)	a^3 ♂ – q – – –(5c)	a^4 ♂ – q – – –(5c)	W no. 54 ♂ p – – –(Wβ)	A1a^6 ♀ 00p – – –($A2\frac{1}{2}$)
	a^5 ♂ 100zq48.46.12(5c)	a^6 ♀ 45 q49.46. 1(3c)	a^7 ♀ – q – – –(4c)	a^8 ♀ – q – – –(5c)	... ...	... ...
	a^9 – q – – –(5c)	... ...	... ...	... ...	... ...	... ...
0H 102	a^1 ♂ 70 q48.46. 2(4c)	a^2 ♂ 55 q48. 1. 2(3c)	a^3 ♂ 95 q48.46. 7(4c)	a^4 ♂ – q – – –(5c)	W no. 54 ♂ p – – –(Wβ)	A1a^7 ♀ 00p – – –($A2\frac{1}{2}$)
	a^5 ♀ – q – – –(5c)	a^6 ♀ – q – – –(5c)	a^7 ♀ 100zq49.46.14(5c)	a^8 – q – – –(4c)	... ...	... ...
	a^9 – q – – –(4c)	... ...	... ...	... ...	... ...	... ...
0H 103	a^1 ♂ – q – – –(5c)	a^2 ♂ – q – – –(5c)	a^3 ♂ – q – – –(5c)	a^4 ♀ 55 q49. 1. 3(3c)	W – ♂ p – – –(Waβ)	A1a^8 ♀ 00p – – –($A2\frac{1}{2}$)
	a^5 ♀ 55 q49.46. 4(3c)	a^6 ♀ 50 q49.46. 3(3c)	a^7 – q – – –(5c)	a^8 – q – – –(2c)	... ...	... ...
0H 104	a^1 ♀ – q – – –(2c)	a^2 ♀ – q – – –(1c)	a^3 ♂ 40 q46. 1. 1(3c)	a^4 ♂ 40 q48.46. 2(3c)	W – ♂ p – – –(Waβ)	A1a^9 ♀ 00p – – –($A2\frac{1}{2}$)
	a^5 – q – – –(4c)	a^6 – q – – –(2c)	a^7 – q – – –(5c)	... ...	... ...	... ...
0H 107	a^1 ♂ 100zq49.46.20(5c)	a^2 ♂ 55 q61.57. 1(3e)	... ...	... ...	A13a^1 ♂ 00p – – –(A × 3)	W no. 24 ♀ p – – –(Waβ)
0H 108	a^1 ♂ – q – – –(5c)	... ...	... ...	... ...	A5a^1 ♂ 00p – – –(A × 3)	W no. 47 ♀ p – – –(Waβ)
0H 110	a^1 ♂ 60 q48.46. 1(3c)	a^2 ♀ 60 q52.46. 2(3c)	... ...	... ...	A14a^2 ♂ 00p – – –(A1)	W no. 44 ♀ p – – –(Waβ)
0H 111	a^1 ♂ – q – – –(5d)	a^2 ♂ – q – – –(5a)	a^3 ♀ – q – – –(5d)	a^4 ♀ – q – – –(5d)	W no. 39 ♂ p – – –(Waβ)	Bt no. 11 ♀ 00p – – –(A1 ?)
	a^5 ♀ – q – – –(5d)	a^6 ♀ – q – – –(5d)	a^7 ♀ – q – – –(5a)	a^8 ♀ 100zq35.35.30(5a)	... ...	... ...
	a^9 ♀ 100zq35.34.29(5a)	a^{10} ♀ – q – – –(5a)	... ...	... ...	... ...	... ...
0H 112	a^1 ♂ 30 q49. 1. 1(2c)	a^2 ♂ – q – – –(5c)	a^3 ♀ 50 q48.46. 1(3c)	a^4 ♀ 35 q46. 1. 1(2c)	W no. 54 ♂ p – – –(Wβ)	A5a^3 ♀ 00p – – –(A × 3)
	a^5 ♀ 40 q49. 1. 1(2c)	... ...	... ...	... ...	... ...	... ...
0H 113	a^1 ♀ – q – – –(4c)	a^2 ♀ 50 q49.46. 2(3c)	a^3 ♀ – q – – –(3c)	a^4 ♀ 100zq49.46.12(5c)	W – ♂ p – – –(Waβ)	A8a^1 ♀ 00p – – –(A × 2 or 3)
	a^5 ♀ 100zq48.46.43(5c)	a^6 ♀ 100zq49.36.20(5c)	a^7 ♂ – q – – –(5c)	a^8 ♂ – q – – –(5c)	... ...	... ...
0H 114	a^1 ♀ – q – – –(5c)	a^2 ♀ 35 q49.46. 1(3c)	a^3 ♀ 100zq49.46.12(5c)	a^4 ♂ 50 q46.30. 5(3c)	W – ♂ p – – –(Waβ)	A8a^2 ♀ 00p – – –(A × 2 or 3)
	a^5 ♂ 25 q49. 1. 2(2c)	a^6 ♂ – q – – –(5c)	... ...	... ...	... ...	... ...
0H 117	a^1 ♀ 80 q49.46. 1(4c)	a^2 ♀ 95 q49.46. 4(4c)	a^3 ♀ – q – – –(3c)	a^4 ♀ 90 q49.36. 1(4c)	W no. 39 ♂ p – – –(Waβ)	A27a^4 ♀ 00p – – –(A × 3)
	a^5 ♀ 65 q49.36. 4(3c)	a^6 ♂ 65 q48.46. 3(3c)	a^7 ♂ 90 q49.46. 9(4c)	... ...	... ...	... ...
0H 118	a^1 ♂ 85 q48.46.11(4c)	a^2 ♀ – q – – –(5c)	a^3 ♀ – q – – –(5c)	a^4 – q – – –(5c)	W no. 39 ♂ p – – –(Waβ)	A27a^5 ♀ 00p – – –(A × 3)
	a^5 – q – – –(4c)	... ...	... ...	... ...	... ...	... ...
0H 120	a^1 ♂ – q – – –(4c)	a^2 ♂ – q – – –(3c)	a^3 ♀ 45 q49. 1. 1(2c)	a^4 ♂ 60 q49.46. 1(3c)	W no. 35 ♂ p – – –(Waβ)	A6a^1 ♀ 00p – – –(A × 2)
	a^5 ♂ – q – – –(2c)	a^6 ♀ 75 q49.46. 1(4c)	a^7 ♂ 50 q48.46. 3(3c)	a^8 ♀ – q – – –(2c)	... ...	... ...
0H 124	a^1 ♂ 100zq49.21.20(4c)	a^2 ♀ 90 q49.34. 1(4c)	a^3 ♀ – q – – –(4c)	a^4 – q – – –(4c)	1H11a^1 ♂ 00p – – –(A × 2)	W – ♀ p – – –(Waβ)
0H 125	a^3 100z 53.51.14	... ...	... ...	... ...	1H14a^1 ♂ 00p – – –(A × 2)	W no. 38 ♀ p – – –(Wβ)
0H 127	a^1 ♀ 50 q53. 1. 1(3c)	a^2 ♂ 50 q52.46. 1(3c)	a^3 ♂ 50 q52. 1. 2(3c)	a^4 ♂ – q – – –(3c)	W no. 35 ♂ p – – –(Waβ)	1H12a^1 ♀ 00p – – –(A × 2)
	a^5 ♂ 65 q49.46. 1(4?c)	... ...	... ...	... ...	... ...	... ...
0H 128	a^1 ♀ 85 q49.46. 1(4c)	a^2 – q – – –(3cw)	a^3 – q – – –(3cw)	a^4 – q – – –(3c)	W no. 39 ♂ p – – –(Waβ)	1H2a^1 ♀ 00p – – –(A × 2)
0H 129	a^1 ♀ – q – – –(3c)	a^2 ♀ 50 q49. 1. 1(3c)	a^3 ♀ 25 q49. 1. 1(2c)	a^4 ♂ 35 q46.46. 3(2c)	W – ♂ p – – –(Waβ)	1H24a^1 ♀ 00p – – –(A × 2)
	a^5 ♂ 35 q49. 1. 1(3c)	a^6 ♂ 35 q49.36. 1(2c)	... ...	... ...	... ...	... ...
0H 130	a^1 – q – – –(2c)	a^2 – q – – –(2c)	... ...	... ...	W no. 36 ♂ p – – –(Waβ)	1H20a^2 ♀ 00p – – –(A × 1)
0H 131	a^1 ♂ 100zq35.33.29(4–5a)	a^2 ♀ 60 q49.46. 3(3d)	a^3 ♀ 95 q35.35. 2(4a)	a^4 ♀ 95 q35.35.29(6a)	W no. 42 ♂ p – – –(Waβ)	1H40a^1 ♀ 00p – – –(A × 2)
	a^5 – q – – –(6a)	... ...	... ...	... ...	... ...	... ...
0H 132	a^1 ♀ 80 q49.46. 7(5c)	a^2 ♀ – q – – –(5cw)	a^3 ♀ 100zp36.35.28(6a)	a^4 ♀ 100zp36.34. 4(6a)	W – ♂ p – – –(Waβ)	1H40a^2 ♀ 00p – – –(A × 2)
	a^5 ♀ – p – – –(6a)	a^6 ♀ 100zp35.35. 4(6a)	a^7 – q – – –(5c)	... ...	... ...	... ...
0H 133	a^1 ♀ 100zq49.46.12(5c)	a^2 – q – – –(3c)	a^3 – q – – –(5c)	a^4 – q – – –(5c)	W no. 31 ♂ p – – –(Wβ)	1H27a^1 ♀ 00p – – –(A × 2)
0H 134	a^1 ♂ 55 p37. 1. 1(3a–b)	a^2 – q – – –(3c)	... ...	... ...	W no. 31 ♂ p – – –(Wβ)	1H27a^2 ♀ 00p – – –(A × 2)

Mating	Offspring				Parents	
					Father	Mother
0H1	a^1 ♂ 45 q49.46. 2(2d)	a^2 ♂ 55 q49. 1. 1(3c)	a^3 ♂ 45 q48. 1. 1(2c)	a^4 ♀ – q – – –(1c)	W no. 1 ♂ p – – –(Waβ)	Lb no. 1 ♀ 00p – – –(A × 1 or 2)
	a^5 ♀ – q – – –(1c)	a^6 ♀ – q – – –(3c)	...	...	...	...
0H6	a^1 ♀ – q – – –(2c)	...	...	...	W no. 6 ♂ p – – –(Wβ)	Lb no. 6 ♀ 00p – – –(A × 1 or 2)
0H7	a^1 ♂ 100zq49.46. 7(5c)	a^2 ♂ 45 q48. 1. 1(2c)	a^3 ♂ 55 q49.46. 2(2–3c)	a^4 ♂ 55 q49.21. 2(3d)	W no. 7 ♂ p – – –(Wa)	Bt no. 7 ♀ 00p – – –(A1)
	a^5 ♀ 50 q48.36. 2(3c)	a^6 ♀ 100zq48.46.12(5c)	a^7 ♀ – q – – –(2d)	...	...	...
0H8 (or 7)	a^1 ♂ 100zq48.36. 7(5c)	a^2 ♂ 100zq49.46. 7(5c)	a^3 ♀ – q – – –(5c)	a^4 ♀ – q – – –(5c)	W no. 7 ♂ p – – –(Wa)	Bt no. 8 ♀ 00p – – –(A1)
					W no. 8 ♂ p – – –(Wβ)	(or 7)
	a^5 ♀ 100zq49.46.12(5c)	a^6 – q – – –(5c)	...	...	...	...
0H9	a^1 ♂ – q – – –(2c)	a^2 ♀ 50 q46. 1. 1(2c)	a^3 ♀ 45 q48.46. 1(2c)	a^4 ♀ [illegible] q49. 1. 1(2c)	W no. 1 ♂ p – – –(Waβ)	Lb no. 9 ♀ 00p – – –(A × 1 or 2)
	a^5 ♀ 25 q46. 1. 1(2c)	a^6 ♀ 40 q49. 1. 1(2d)	...	...	...	...
0H12	a^1 ♂ – q – – –(5c)	a^2 ♂ – q – – –(5c)	a^3 ♀ – q – – –(5a)	a^4 ♀ – q – – –(5c)	W – ♂ p – – –(Wa)	Bt no. 11 ♀ 00p – – –(A1 ?)
	a^5 ♀ – q – – –(5a)	...	...	...	...	...
0H13	a^1 ♂ 50 q46.46. 1(3c)	a^2 ♂ 100zq48.46. 7(5c)	a^3 ♂ 100zq49.46. 7(5d)	a^4 ♂ 45 q52. 1. 1(3c)	W no. 12 ♂ p – – –(Wa)	Bt no. 12 ♀ 00p – – –(A1 ?)
0H16	a^1 ♂ 50 q47. 1. 1(3c)	a^2 ♀ 50 q48.46. 1(3c)	a^3 ♀ 50 q48. 1. 1(3c)	a^4 ♀ 55 q49.46. 1(4c)	W no. 8 ♂ p – – –(Wβ)	Lb no. 13 ♀ 00p – – –(A × 1 or 2)
	a^5 ♀ – q – – –(4c)	...	...	...	...	...
0H20	a^1 ♂ 55 q47.46. 1(3d)	a^2 ♂ 40 q48.46. 1(2c)	a^3 ♂ 100zq48.46. 7(5d)	a^4 ♀ 45 q49. 1. 1(2d)	W no. 1 ♂ p – – –(Waβ)	Bt no. 15 ♀ 00p – – –(A1 ?)
	a^5 ♀ 25 q49. 1. 1(2d)	a^6 – q – – –(5d)	...	...	...	...
0H27	a^1 ♀ 40 q58. 1. 2(2c)	...	...	...	Bt – ♂ 00p – – –(A1 ?)	W no. 24 ♀ p – – –(Waβ)
0H33	a^1 – q – – –(5c)	...	...	...	Bt – ♂ 00p – – –(A1 ?)	W no. 43 ♀ p – – –(Wa)
0H35	a^1 ♂ – q – – –(2c)	a^2 ♀ – q – – –(3d)	a^3 ♀ – q – – –(3d)	a^4 ♀ – q – – –(2d)	W – ♂ p – – –(Wa ?)	Lb – ♀ p – – –(A × 1 or 2)
	a^5 ♀ – q – – –(4d)	a^6 ♂ – q – – –(4d)	...	...	...	...
0H45	a^1 ♀ – q – – –(2d)	a^2 ♂ – q – – –(2d)	a^3 ♂ – q – – –(2d)	a^4 ♀ – q – – –(2d)	W no. 6 ♂ p – – –(Wβ)	Bt – ♀ 00p – – –(A1 ?)
0H53	a^1 – q – – –(1c)	...	...	...	Bt – ♂ 00p – – –(A1 ?)	W no. 22 ♀ p – – –(Wβ)
0H59	a^1 ♀ 50 q48. 1. 4(3d)	a^2 ♂ 100zq49.46. 7(6d)	...	...	Bt – ♂ 00p – – –(A1)	W no. 34 ♀ p – – –(Wa)
0H61	a^1 ♂ – q – – –(2c)	a^2 ♂ – q – – –(5c)	a^3 ♂ – q – – –(5c)	a^4 ♂ – q – – –(5c)	W no. 39 ♂ p – – –(Waβ)	Bt no. 12 ♀ 00p – – –(A1 ?)
	a^5 ♀ 80 q46. 1. 1(4c)	a^6 ♀ 65 q49.30. 7(3c)	a^7 ♀ – q – – –(5c)	...	...	...
0H62	a^1 ♂ – q – – –(2c)	a^2 ♂ 40 q49. 1. 1(3c)	a^3 ♀ 45 q48.34. 3(3c)	a^4 ♀ 100zq49.46. 7(5c)	W no. 54 ♂ p – – –(Wβ)	Bt – ♀ 00p – – –(A1)
	a^5 ♀ 100zq49.48.12(5c)	a^6 ♀ 100zq49.46.12(5c)	...	...	...	...
0H66	a^1 – q – – –(5c)	...	...	...	Bt – ♂ 00p – – –(A1 ?)	W no. 38 ♀ p – – –(Wβ)
0H72	a^1 – q – – –(5c)	...	...	...	Bt – ♂ 00p – – –(A1 ?)	W – ♀ p – – –(Waβ)
0H73	a^1 ♀ – q – – –(5c)	a^2 ♀ – q – – –(5c)	a^3 ♀ – q – – –(5c)	a^4 ♂ – q – – –(3c)	W – ♂ p – – –(Waβ)	Bt – ♀ 00p – – –(A1)
	a^5 ♀ – q – – –(3c)	a^6 ♂ – q – – –(3c)	...	...	...	...
0H74	a^1 ♂ – q – – –(4c)	a^2 ♂ – q – – –(3c)	a^3 ♂ – q – – –(4c)	a^4 ♂ – q – – –(4c)	W – ♂ p – – –(Waβ)	Bt – ♀ 00p – – –(A1 ?)
	a^5 ♀ 80 q49.46. 2(4c)	a^6 ♀ 80 q49.48. 1(4c)	a^7 ♀ 80 q49.46. 1(4c)	a^8 ♀ – q – – –(4c)	...	...
0H75	a^1 ♀ 60 q49.48. 1(4c)	a^2 ♂ 35 q49. 1. 1(2c)	a^3 ♀ 45 q49. 1. 1(2c)	a^4 ♀ 45 q49.46. 1(3c)	W – ♂ p – – –(Waβ)	Bt – ♀ 00p – – –(A1 ?)
	a^5 ♂ 15 q49. 1. 1(1c)	...	...	...	...	...
0H77	a^1 ♂ 100zq48.46. 7(5c)	a^2 ♂ 60 q48.46. 4(4c)	a^3 ♂ 55 q48.46. 1(4c)	a^4 ♀ – q – – –(5c)	W no. 16 ♂ p – – –(Waβ)	Bt – ♀ 00p – – –(A1 ?)
	a^5 ♀ – q – – –(5c)	a^6 ♀ 100zq57.56.56(5c)	a^7 ♀ 70 q61.61. 1(4c)	a^8 ♀ – q – – –(5c)	...	...
0H78	a^1 ♀ – q – – –(5c)	a^2 – q – – –(5d)	a^3 ♀ – q – – –(2c)	a^4 ♀ – q – – –(3c)	W no. 8 ♂ p – – –(Wβ)	Bt no. 58 ♀ 00p – – –(A1 ?)
0H82	a^1 ♀ 55 q48.46.12(3c)	a^2 ♀ – q – – –(5c)	...	...	A1a^1 ♂ 00p – – –(A2½)	W – ♀ p – – –(Wβ)
0H84	a^1 ♀ – q – – –(5c)	a^2 ♀ 100zq49.46. 4(5c)	a^3 ♀ 100zq35.33.30(5c)	a^4 ♀ 100zq35.33.30(5c)	W no. 54 ♂ p – – –(Wβ)	Bt no. 11 ♀ 00p – – –(A1 ?)
	a^5 ♀ 100zq33.30.28(5a)	a^6 ♀ 100zq39.33.30(5a)	a^7 ♂ 100zq35.33.28(5a)	a^8 ♂ – q – – –(5c)	...	...
	a^9 ♂ 100zq48.46.28(5c)	...	...	...	...	...
0H85	a^1 ♀ – q – – –(4d)	a^2 ♂ – q – – –(3c)	a^3 ♀ – q – – –(5d)	a^4 ♀ – q – – –(5–6d)	W – ♂ p – – –(Wa)	Lb no. 13 ♀ 00p – – –(A × 1 or 2)
	a^5 ♂ – q – – –(5d)	a^6 ♂ – q – – –(5c)	a^7 – q – – –(3e)	...	...	...
0H86	a^1 ♂ 100zq49.46. 9(5c)	a^2 ♂ 100zq49.46.28(5c)	a^3 ♀ – q – – –(3c)	a^4 ♀ – q – – –(3c)	W – ♂ p – – –(Waβ)	Bt no. 59 ♀ 00p – – –(A1 ?)
	a^5 ♀ – q – – –(5c)	a^6 ♀ – q – – –(5c)	a^7 ♀ – q – – –(5c)	...	...	...
0H90	a^1 ♀ – q – – –(5c)	a^2 ♂ – q – – –(5c)	...	...	W no. 35 ♂ p – – –(Waβ)	Bt – ♀ 00p – – –(A1 ?)
0H91	a^1 – q – – –(6c)	...	...	...	A1a^2 ♂ 00p – – –(A2½)	W no. 46 ♀ p – – –(Wa)
0H94	a^1 ♀ 60 q49.46. 1(3c)	a^2 – q – – –(3c)	...	...	A4a^2 ♂ 00p – – –(A2½)	W no. 33 ♀ p – – –(Wa)
0H100	a^1 ♂ 40 q49. 1. 1(3c)	a^2 ♂ – q – – –(5c)	a^3 ♂ – q – – –(5c)	a^4 ♂ 100zq49.46. 7(5c)	W no. 54 ♂ p – – –(Wβ)	A1a^5 ♀ 00p – – –(A2½)
	a^5 ♀ 25 q46. 1. 1(2c)	a^6 ♀ 55 q49.46. 9(3c)	a^7 ♀ – q – – –(5c)	a^8 ♀ – q – – –(5c)	...	...
	a^9 – q – – –(3c)	...	...	...	...	...
0H101	a^1 ♂ 45 q49. 1. 1(2c)	a^2 ♂ 50 q49.46. 4(3c)	a^3 ♂ – q – – –(5c)	a^4 ♂ – q – – –(5c)	W no. 54 ♂ p – – –(Wβ)	A1a^6 ♀ 00p – – –(A2½)
	a^5 ♂ 100zq48.46.12(5c)	a^6 ♀ 45 q49.46. 1(3c)	a^7 ♀ – q – – –(4c)	a^8 ♀ – q – – –(5c)	...	...
	a^9 – q – – –(3c)	...	...	...	...	...
0H102	a^1 ♂ 70 q48.46. 2(4c)	a^2 ♂ 55 q48. 1. 2(3c)	a^3 ♂ 95 q48.46. 7(4c)	a^4 ♂ – q – – –(5c)	W no. 54 ♂ p – – –(Wβ)	A1a^7 ♀ 00p – – –(A2½)
	a^5 ♀ – q – – –(5c)	a^6 ♀ – q – – –(5c)	a^7 ♀ 100zq49.46.14(5c)	a^8 – q – – –(4c)	...	...
	a^9 – q – – –(4c)	...	...	...	...	...
0H103	a^1 ♂ – q – – –(5c)	a^2 ♂ – q – – –(5c)	a^3 ♂ – q – – –(5c)	a^4 ♀ 55 q49. 1. 3(3c)	W – ♂ p – – –(Waβ)	A1a^8 ♀ 00p – – –(A2½)
	a^5 ♀ 55 q49.46. 4(3c)	a^6 ♀ 50 q49.46. 3(3c)	a^7 – q – – –(5c)	a^8 – q – – –(2c)	...	...
0H104	a^1 ♀ – q – – –(2c)	a^2 ♀ – q – – –(1c)	a^3 ♂ 40 q46. 1. 1(3c)	a^4 ♂ 40 q48.46. 2(3c)	W – ♂ p – – –(Waβ)	A1a^9 ♀ 00p – – –(A2½)
	a^5 – q – – –(4c)	a^6 – q – – –(2c)	a^7 – q – – –(5c)	...	...	...
0H107	a^1 ♂ 100zq49.46.20(5c)	a^2 ♂ 55 q61.57. 1(3c)	...	...	A13a^1 ♂ 00p – – –(A × 3)	W no. 24 ♀ p – – –(Waβ)
0H108	a^1 ♂ – q – – –(5c)	...	...	...	A5a^1 ♂ 00p – – –(A × 3)	W no. 47 ♀ p – – –(Waβ)
0H110	a^1 ♂ 60 q48.46. 1(3c)	a^2 ♀ 60 q52.46. 2(3c)	...	...	A14a^2 ♂ 00p – – –(A1)	W no. 44 ♀ p – – –(Waβ)
0H111	a^1 ♂ – q – – –(5d)	a^2 ♂ – q – – –(5d)	a^3 ♀ – q – – –(5d)	a^4 ♀ – q – – –(5d)	W no. 39 ♂ p – – –(Waβ)	Bt no. 11 ♀ 00p – – –(A1 ?)
	a^5 ♀ – q – – –(5d)	a^6 ♀ – q – – –(5d)	a^7 ♀ – q – – –(5d)	a^8 ♀ 100zq35.35.30(5d)	...	...
	a^9 ♀ 100zq35.34.20(5a)	a^{10} ♀ – q – – –(5a)	...	...	...	...
0H112	a^1 ♂ 30 q49. 1. 1(2c)	a^2 ♂ – q – – –(5c)	a^3 ♀ 50 q48.46. 1(3c)	a^4 ♀ 35 q46. 1. 1(2c)	W no. 54 ♂ p – – –(Wβ)	A5a^3 ♀ 00p – – –(A × 3)
	a^5 ♀ 40 q49. 1. 1(2c)	...	...	...	...	...
0H113	a^1 ♀ – q – – –(4c)	a^2 ♀ 50 q49.46. 2(3c)	a^3 ♀ – q – – –(3c)	a^4 ♀ 100zq49.46.12(5c)	W – ♂ p – – –(Waβ)	A8a^1 ♀ 00p – – –(A × 2 or 3)
	a^5 ♀ 100zq48.46.43(5c)	a^6 ♀ 100zq49.36.20(5c)	a^7 ♂ – q – – –(5c)	a^8 ♂ – q – – –(5c)	...	...
0H114	a^1 ♀ – q – – –(5c)	a^2 ♀ 35 q49.46. 1(3c)	a^3 ♀ 100zq49.46.12(5c)	a^4 ♂ 50 q46.30. 5(3c)	W – ♂ p – – –(Waβ)	A8a^2 ♀ 00p – – –(A × 2 or 3)
	a^5 ♀ 25 q49. 1. 2(2c)	[illegible]	...	...	...	...
0H117	a^1 ♀ 50 q49.46. 1(4c)	a^2 ♀ 55 q49.46. 1(4c)	a^3 ♀ – q – – –(3c)	a^4 ♀ 50 q49.36. 1(4c)	W no. 39 ♂ p – – –(Waβ)	A27a^1 ♀ 00p – – –(A × 3)
	a^5 ♀ 65 q49.36. 1(3c)	a^6 ♂ 65 q48.46. 3(3c)	a^7 ♂ 90 q49.46. 9(4c)	...	...	...
0H118	a^1 ♂ 85 q48.46.11(4c)	a^2 ♀ – q – – –(5c)	a^3 ♀ – q – – –(5c)	a^4 – q – – –(5c)	W no. 39 ♂ p – – –(Waβ)	A27a^3 ♀ 00p – – –(A × 3)
	a^5 – q – – –(4c)	...	...	...	...	...
0H120	a^1 ♂ – q – – –(4c)	a^2 ♂ – q – – –(3c)	a^3 ♀ 45 q49. 1. 1(2c)	a^4 ♂ 60 q49.46. 1(3c)	W no. 35 ♂ p – – –(Waβ)	A6a^1 ♀ 00p – – –(A × 2)
	a^5 ♂ – q – – –(2c)	a^6 ♀ 75 q49.46. 1(4c)	a^7 ♂ 50 q48.46. 3(3c)	a^8 ♀ – q – – –(2c)	...	...
0H124	a^1 ♂ 100zq49.21.20(4c)	a^2 ♀ 90 q49.34. 1(4c)	a^3 ♀ – q – – –(4c)	a^4 – q – – –(4c)	1H11a^1 ♂ 00p – – –(A × 2)	W – ♀ p – – –(Waβ)
0H125	a^1 100[illegible]				1H14a^1 ♂ 00p – – –(A × 2)	W no. 38 ♀ p – – –(Wβ)
0H127	a^1 ♀ 50 q53. 1. 1(3c)	a^2 ♂ 50 q52.46. 1(3c)	a^3 ♂ 50 q52. 1. 2(3c)	a^4 ♂ – q – – –(3c)	W no. 35 ♂ p – – –(Waβ)	1H12a^1 ♀ 00p – – –(A × 2)
	a^5 ♂ 65 q49.46. 1(4c)	...	...	...	...	...
0H128	a^1 ♀ 85 q49.46. 1(4c)	a^2 – q – – –(3cw)	a^3 – q – – –(3cw)	a^4 – q – – –(3c)	W no. 39 ♂ p – – –(Waβ)	1H2a^1 ♀ 00p – – –(A × 2)
0H129	a^1 ♀ – q – – –(3c)	a^2 ♀ 50 q49. 1. 1(3c)	a^3 ♀ 25 q49. 1. 1(2c)	a^4 ♂ 25 q46.46. 3(2c)	W – ♂ p – – –(Waβ)	1H24a^1 ♀ 00p – – –(A × 2)
	a^5 ♂ 55 q49. 1. 1(3c)	a^6 ♂ 35 q49.36. 1(2c)	...	...	...	...
0H130	a^1 – q – – –(2c)	a^2 – q – – –(2c)	...	...	W no. 36 ♂ p – – –(Waβ)	1H20a^2 ♀ 00p – – –(A × 1)
0H131	a^1 ♀ 100zp35.33.20. 4(5a)	a^2 ♀ 60 q49.46. 3(3d)	a^3 ♀ 35 q35.35. 2(4a)	a^4 ♀ 35 q35.35.20(6a)	W no. 42 ♂ p – – –(Waβ)	1H40a^1 ♀ 00p – – –(A × 2)
	a^5 – q – – –(6a)	...	...	...	...	...
0H132	a^1 ♀ 80 q49.46. 7(5c)	a^2 ♀ – q – – –(5cw)	a^3 ♀ 100zp36.35.28(6a)	a^4 ♀ 100zp36.34. 4(6a)	W – ♂ p – – –(Waβ)	1H40a^2 ♀ 00p – – –(A × 2)
	a^5 ♀ – p – – –(6a)	a^6 ♀ 100zp35.35. 4(6a)	a^7 – q – – –(5c)	...	...	...
0H133	a^1 ♀ 100zq49.46.12(5c)	a^2 – q – – –(3c)	a^3 – q – – –(5c)	a^4 – q – – –(5c)	W no. 31 ♂ p – – –(Wβ)	1H27a^1 ♀ 00p – – –(A × 2)
0H134	a^1 ♂ 55 p37. 1. 1(3a b)	a^2 – q – – –(3c)	...	...	W no. 31 ♂ p – – –(Wβ)	1H27a^2 ♀ 00p – – –(A × 2)

MATING	OFFSPRING				PARENTS	
					Father	Mother
0H 135	a^1 ♂ 50 q49.46. 1(3c)	a^2 ♀ 45 q52.46. 1(2cw)	a^3 ♀ 45 q49.46. 4(2c)	a^4 ♀ 100zq49.48.14(5c)	W no. 35 ♂ p – – –(Waβ)	1H47a^1 ♀ 00p – – –(A × 2)
	a^5 ♀ 100zq49.46.12(5cw)	a^6 ♂ 30 q48. 1. 1(2cw)	...	...	...	...
0H 136	a^1 ♂ 40 q47.28. 2(3c)	a^2 ♂ 25 q52. 1. 1(2c)	a^3 ♂ 25 q48. 1. 1(2c)	a^4 – q – – –(5c)	W no. 39 ♂ p – – –(Waβ)	1H47a^2 ♀ 00p – – –(A × 2)
	a^5 – q – – –(5c)	a^6 – q – – –(5c)	a^7 – q – – –(5c)	...	...	...
0H 138	a^1 ♀ – q – – –(5c)	a^2 ♀ – q – – –(5–6c)	a^3 ♂ – q – – –(5c)	a^4 ♀ 50 q49. 1. 1(3c)	W – ♂ p – – –(Waβ)	1H49a^1 ♀ 00p – – –(A × 2)
0H 139	a^1 ♂ 20 q49. 1. 2(2c)	a^2 ♂ – q – – –(5c)	a^3 ♀ 30 q49. 1. 2(2c)	a^4 ♀ – q – – –(5c)	W no. 40 ♂ p – – –(Waβ)	1H32a^1 ♀ 00p – – –(A × 2)
	a^5 ♀ – q – – –(5c)	a^6 – q – – –(2c)	...	...	...	...
0H 140	a^1 ♀ 35 q53.53. 1(2c)	a^2 ♂ 85 q52.53.12(4c)	a^3 ♀ 35 q48. 1. 1(2c)	a^4 ♀ 25 q49.46. 4(2c)	W no. 40 ♂ p – – –(Waβ)	1H32a^2 ♀ 00p – – –(A × 2)
	a^5 ♂ 40 q49.46. 1(3c)	a^6 ♀ 40 q46. 1. 1(3c)	a^7 – q – – –(3c)	...	...	...
0H 142	a^1 ♂ 45 q49. 1. 1(3c)	a^2 ♂ 40 q49. 1. 1(2c)	a^3 ♂ 100zq49.36. 7(5c)	a^4 ♀ 30 q49. 1. 4(2c)	W no. 42 ♂ p – – –(Waβ)	A51a^1 ♀ 00p – – –(A × 2)
	a^5 ♀ – q – – –(2c)	a^6 ♀ 80 q49.36. 4(5c)	...	...	...	...
0H 143	a^1 ♂ – q – – –(5c)	a^2 ♀ – q – – –(5c)	a^3 ♂ – q – – –(5c)	a^4 ♀ – q – – –(5c)	W no. 35 ♂ p – – –(Waβ)	Bt – ♀ 00p – – –(A × 1)
	a^5 ♂ – q – – –(5c)	a^6 ♀ – q – – –(5c)	a^7 – q – – –(5c)	...	...	...
0H 144	a^1 – q – – –(5c)	a^2 – q – – –(5c)	a^3 – q – – –(5c)	a^4 – q – – –(5c)	W no. 36 ♂ p – – –(Waβ)	A71a^1 ♀ 00p – – –(A2 ?)
	a^5 – q – – –(5c)	a^6 – q – – –(5c)	...	...	...	...
0H 145	a^1 ♂ 100zq49.48. 9(5c)	a^2 ♂ 100zq35.35.30(5a–b)	a^3 ♂ 100zq35.35.30(5a–b)	a^4 ♂ 100zq36.30.28(5a–b)	1H42b^1 ♂ p – – –(3a–bw)	A30a^1 ♀ 00p – – –(A2 ?)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♀ 100zq35.33.30(5–6a-b)	a^7 ♀ 100zq39.35.30(5a–b)	a^8 ♀ 100zq39.35.30(5–6a–b)	...	...
0H 146	a^1 ♂ 90 q49.34. 2(4c)	a^2 ♂ 100zq35.33. 4(5a)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ – q – – (5c)	1H42b^1 ♂ p – – –(3a–bw)	A30a^2 ♀ 00p – – –(A2 ?)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♀ 00 p – – –(alb.)	a^7 ♀ 00 p – – –(alb.)	a^8 – q – – –(5c)	...	...
0H 147	a^1 – q – – –(5c)	a^2 100zq53.51.14(5c)	a^3 – q – – –(5c)	a^4 – q – – –(5c)	W no. 37 ♂ p – – –(Wβ)	A67a^2 ♀ 00p – – –(A × 1)
	a^5 – q – – –(5c)	a^6 – q – – –(5c)	...	...	...	...
0H 148	a^1 ♀ 55 q53. 1. 1(3c)	a^2 ♀ 55 q49.36. 3(3c)	a^3 ♀ 45 q49.36. 1(3c)	a^4 – q – – –(3c)	W no. 36 ♂ p – – –(Waβ)	1H18a^3 ♀ 00p – – –(A × 1)
	a^5 – q – – –(3c)	a^6 – q – – –(3c)	...	...	...	...
0H 150	a^1 ♂ 15 q47. 1. 1(2c)	a^2 55 q47.47. 1(3c)	a^3 35 q47. 1. 1(2c)	a^4 – q – – –(2c)	W no. 31 ♂ p – – –(Wβ)	1H19a^6 ♀ 00p – – –(A × 1)
	a^5 – q – – –(2c)	...	...	...	...	...
0H 151	a^1 ♂ 85 q53.46. 2(5c)	a^2 ♂ 55 q46. 1. 1(3c)	a^3 ♂ 30 q53. 1. 1(2c)	a^4 ♂ 75 q53.46. 1(4c)	W no. 35 ♂ p – – –(Waβ)	1H28b^3 ♀ 00p – – –(A × 2)
	a^5 80 q53.51. 2(5c)	a^6 – q – – –(4c)	...	...	...	...
0H 152	a^1 ♂ 80 q44.47. 2(5d)	a^2 ♂ – q – – –(5c)	a^3 ♀ – q – – –(5c)	a^4 ♀ – q – – –(5c)	W no. 35 ♂ p – – –(Waβ)	1H28b^4 ♀ 00p – – –(A × 2)
	a^5 – q – – –(5c)	a^6 – q – – –(6c)	a^7 – q – – –(5c)	...	...	...
0H 153	a^1 ♀ 60 q44.47. 1(3c)	a^2 ♂ 90 q44.46.11(4c)	a^3 ♂ 95 q 44.53. 2(4d)	a^4 – q – – –(4c)	W no. 35 ♂ p – – –(Waβ)	1H28b^5 ♀ 00p – – –(A × 2)
	a^5 – q – – –(4c)	a^6 – q – – –(3c)	...	...	...	...
0H 154	a^1 ♀ 35 p37. 1. 2(2a–b)	a^2 ♀ 35 q49. 1. 1(2–3c)	a^3 ♂ 10 p35. 1. 4(1–2a–b)	a^4 ♀ – p – – –(2b)	W no. 54 ♂ p – – –(Wβ)	1H58a^2 ♀ 00p – – –(A × 2)
	a^5 – q – – –(3c)	...	...	...	...	...
0H 155	a^1 ♂ 100xq35.35.30(5a)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	...	1H42b^1 ♂ p – – –(3a–bw)	A58a^4 ♀ 00p – – –(A × 3)
0H 156	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ 100zq48.46.20(5b–c)	1H42b^1 ♂ p – – –(3a–bw)	A58a^5 ♀ 00p – – –(A × 3)
0H 159	a^1 ♂ – q – – –(5c)	a^2 ♂ – q – – –(5c)	a^3 ♂ – q – – –(5c)	a^4 ♀ – q – – –(5c)	W no. 31 ♂ p – – –(Wβ)	A141a^3 ♀ 00p – – –(A1½)
	a^5 ♀ 100zq52.46. 7(5c)	a^6 ♀ 75 q52.46. 4(5c)	...	...	...	...
0H 160	a^1 ♂ – q – – –(5c)	a^2 ♂ – q – – –(5c)	a^3 ♀ 100xq49.46. 2(5c)	a^4 ♀ 100zq49.46. 7(5c)	W no. 31 ♂ p – – –(Wβ)	A141a^4 ♀ 00p – – –(A1½)
	a^5 ♀ 100zq49.46. 7(5c)	...	...	...	...	...
0H 161	a^1 ♂ – q – – –(5c)	a^2 ♂ 80 q48.46. 7(5c)	a^3 ♂ – q – – –(5c)	a^4 ♀ – q – – –(5c)	W no. 56 ♂ p – – –(Wβ)	A182a^3 ♀ 00p – – –(A1½)
	a^5 ♀ 100zq49.46.12(5c)	a^6 ♀ – q – – –(5c)	...	...	...	...
0H 162	a^1 ♀ 100zq49.46.11(5c)	a^2 ♂ 80 q48.46. 7(5c)	a^3 – q – – –(5c)	a^4 – q – – –(5c)	W no. 56 ♂ p – – –(Wβ)	A182a^4 ♀ 00p – – –(A1½)
	a^5 – q – – –(5c)	a^6 – q – – –(5c)	...	...	...	...
0H 163	a^1 ♂ 75 q44.51.11(5c)	a^2 ♂ 80 q44.51.11(5c)	a^3 ♀ 80 q44.51.11(5c)	a^4 ♀ 95 q44.51.14(5c)	W no. 56 ♂ p – – –(Wβ)	A182a^5 ♀ 00p – – –(A1½)
	a^5 – q – – –(5c)	...	...	...	...	...

0*H*165	a^1 70 q48.46. 2(4c)	a^2 – q – – –(5c)	a^3 – q – – –(5c)	a^4 – q – – –(5c)	*W* no. 37 ♂ p – – –(*W*β)	*A*143a^2 ♀ 00 p – – –(*A*2)
	a^5 – q – – –(5c)	a^6 – q – – –(5c)	a^7 – q – – –(5c)	... ...	... ...	... ...
0*H*166	a^1 ♂ – q – – –(5c)	a^2 ♂ – q – – –(4–5c)	a^3 ♂ 55 q47.21.11(3–4c)	a^4 ♂ 95 q48.36.28(5c)	*W* no. 37 ♂ p – – –(*W*β)	*A*143a^3 ♀ 00 p – – –(*A*2)
	a^5 ♀ 100zq49.48. 7(5c)	... ...	... ...	... ...	... ...	... ...
0*H*167	a^1 80 q49.46. 1(5c)	a^2 85 q49.46.11(5c)	a^3 – q – – –(5c)	a^4 – q – – –(4c)	*W* no. 37 ♂ p – – –(*W*β)	*A*143a^4 ♀ 00 p – – –(*A*2)
	a^5 – q – – –(4c)	... ...	... ...	... ...	... ...	... ...
0*H*169	a^1 ♀ – q – – –(6c d)	a^2 – q – – –(4c)	—	... ...	*A*155a^1 or a^2 ♂ 00p – – –(*A*2)	*W* no. 24 ♀ p – – –(*W*αβ)
0*H*170	a^1 ♂ 45 q48.46. 2(3c)	a^2 ♂ – q – – –(3c)	a^3 ♀ 100zq49.46.12(5c)	a^4 ♀ – q – – –(5c)	*W* no. 42 ♂ p – – –(*W*αβ)	*A*184a^5 ♀ 00 p – – –(*A* × 1)
	a^5 ♀ 90 q49.35. 2(4c)	... ...	... ...	... ...	... ...	... ...
0*H*171	a^1 40 q49.46. 1(3c)	a^2 50 q48.46. 1(3c)	a^3 45 q49.46. 4(3c)	a^4 45 q49.36. 1(3c)	*W* no. 42 ♂ p – – –(*W*αβ)	*A*184a^6 ♀ 00 p – – –(*A* × 1)
	a^5 – q – – –(5c)	... ...	... ...	... ...	... ...	... ...
0*H*172	a^1 – q – – –(4c)	a^2 – q – – –(4c)	a^3 – q – – –(4c)	a^4 – q – – –(5c)	*W* no. 42 ♂ p – – –(*W*αβ)	*A*184a^7 ♀ 00 p – – –(*A* × 1)
	a^5 – q – – –(5c)	a^6 – q – – –(5c)	a^7 – q – – –(5c)	a^8 – q – – –(5c)	... ...	... ...
0*H*175	a^1 – q – – –(5c)	a^2 – q – – –(5c)	a^3 – q – – –(5c)	a^4 – q – – –(5c)	*W* no. 37 ♂ p – – –(*W*β)	*A*167a^5 ♀ 00 p – – –(*A* × 2)
	a^5 – q – – –(5c)	a^6 – q – – –(5c)	a^7 – q – – –(5c)	... ...	... ...	... ...
0*H*176	a^1 – q – – –(5c)	a^2 – q – – –(5c)	a^3 – q – – –(5c)	a^4 – q – – –(5c)	*W* no. 37 ♂ p – – –(*W*β)	*A*167a^6 ♀ 00 p – – –(*A* × 2)
	a^5 – q – – –(5c)	a^6 – q – – –(5c)	a^7 – q – – –(5c)	... ...	... ...	... ...
0*H*179	a^1 55 q49.36. 4(3c)	a^2 30 q47. 1. 4(3c)	a^3 55 q49.46. 2(3c)	a^4 – q – – –(3c)	*W* no. 42 ♂ p – – –(*W*αβ)	*A*163a^4 ♀ 00 p – – –(*A* × 2)
	a^5 – q – – –(5c)	a^6 – q – – –(5c)	a^7 – q – – –(5c)	a^8 – q – – –(5c)	... ...	... ...
0*H*184	a^1 100zq49.46. 7(5c)	a^2 100zq49.46.12(5c)	a^3 – q – – –(5c)	a^4 – q – – –(5c)	*W* no. 37 ♂ p – – –(*W*β)	*A*154a^2 ♀ 00 p – – –(*A* × 2)
	a^5 – q – – –(5c)	a^6 – q – – –(5c)	a^7 – q – – –(5c)	a^8 – q – – –(5c)	... ...	... ...
	a^9 – q – – –(5c)	a^{10} – q – – –(5c)	... ...	... ...	... ...	... ...
0*H*185	a^1 – q – – –(6c)	a^2 – q – – –(6c)	... ...	... ...	*W* no. 37 ♂ p – – –(*W*β)	*A*157a^1 ♀ 00 p – – –(*A*1)
1*H*1	a^1 – q – – –(5c)	a^2 – q – – –(5c)	a^3 – q – – –(5c)	a^4 – q – – –(5c)	0*H*1a^1 ♂ 45 q49.46. 2(2d)	*Bt* – ♀ 00 p – – –(*A*1 ?)
1*H*2	a^1 ♀ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0*H*1a^2 ♂ 55 q49. 1. 1(3c)	*Bt* – ♀ 00 p – – –(*A*1 ?)
	a^5 ♂ 100zq48.46.11(4d)	a^6 ♂ 90 q57.55. 1(4e)	a^7 ♂ 100zq48.46.11(4d)	a^8 ♂ 100xq57.55.55(5d)	... ...	... ...
	a^9 ♂ 00 p – – –(alb.)	... ...	... ...	... ...	... ...	... ...
1*H*3	a^1 00 p – – –(alb.)	a^2 ♀ – q – – –(6d)	a^3 ♀ 100zq49.46. 7(6c)	a^4 ♀ 100zq46.36.20(5c)	0*H*1a^3 ♂ 45 q48. 1. 1(2c)	*Bt* – ♀ 00 p – – –(*A*1 ?)
	a^5 ♂ 80 q46.36. 5(5c)	a^6 ♂ 00 p – – –(alb.)	a^7 ♂ 00 p – – –(alb.)	a^8 ♂ 00 p – – –(alb.)	... ...	... ...
	a^9 – q – – –(5c)	a^{10} – q – – –(5c)	... ...	... ...	... ...	... ...
1*H*4	a^1 ♂ – q – – –(5d)	a^2 ♀ 00 p – – –(alb.)	a^3 – q – – –(6e)	a^4 – q – – –(5c)	0*H*8 (or 7) a^1 ♂ 100zq48.36. 7(5c)	*Bt* – ♀ 00 p – – –(*A*1 ?)
	a^5 00 p – – –(alb.)	... ...	... ...	... ...	... ...	... ...
1*H*5	a^1 ♀ 100zq49.46.47(6d)	a^2 ♀ 100zq48.46.47(6d)	a^3 ♂ 100zq49.46.47(6d)	a^4 ♂ – q – – –(6c)	0*H*8 (or 7) a^2 ♂ 100zq49.46. 7(5c)	*Bt* – ♀ 00 p – – –(*A*1 ?)
	a^5 ♂ 00 p – – –(alb.)	... ...	... ...	... ...	... ...	... ...
1*H*6	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♂ – q – – –(6d)	a^4 ♂ 100zq49.46.47(6d)	0*H*7a^1 ♂ 100zq49.46. 7(5c)	*Bt* – ♀ 00 p – – –(*A*1 ?)
1*H*7	a^1 ♂ 30 p24. 1. 1(2fw)	a^2 ♀ 40 p37. 1. 1(2a)	a^3 ♀ 35 q49. 1. 2(2b)	b^1 ♂ 50 q49.46. 2(3c)	0*H*9a^1 ♂ q – – –(2c)	0*H*1a^4 ♀ – q – – –(1c)
	b^2 ♂ 40 p37.37. 4(2b)	b^3 ♀ 25 q49. 1. 2(2d)	b^4 ♀ 30 q49. 1. 2(2d)	b^5 – q – – –(2c)	... ...	... ...
1*H*8	a^1 ♀ – p – – –(3b)	a^2 ♀ – p – – –(4a)	a^3 ♀ – p – – –(4a)	a^4 ♀ – q – – –(3c)	0*H*7a^2 ♂ 45 q48. 1. 1(2c)	0*H*1a^5 ♀ – q – – –(1c)
1*H*9	a^1 ♀ 40 p37.36. 3(3a)	a^2 ♀ 40 q58. 1. 1(3c : ew)	a^3 ♀ 70 p42.50. 1(3f)	a^4 ♂ 55 q40. 1. 1(3b)	0*H*7a^3 ♂ 55 q49.46. 2(2–3c)	0*H*1a^6 ♀ – q – –(3c)
	a^5 ♂ 40 q49.46. 1(3b–c)	... ...	... ...	... ...	... ...	... ...
1*H*11	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0*H*20a^1 ♂ 55 q47.46. 1(3d)	*Bt* – ♀ 00 p – – –(*A*1 ?)
	a^5 ♂ 100xq58.58.55(6e)	a^6 00 p – – –(alb.)	... ...	... ...	... ...	... ...
1*H*12	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ – q – – –(6e)	a^3 00 p – – –(alb.)	a^4 – q – – –(6e)	*Bt* – ♂ 00 p – – –(*A*1)	0*H*9a^2 ♀ 30 q46. 1. 1(2c)
	a^5 – q – – –(3c)	... ...	... ...	... ...	... ...	... ...
1*H*13	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 80 q48.46.11(5d)	a^3 ♂ 100xq58.58.55(6e)	... ...	*Bt* – ♂ 00 p – – –(*A*1)	0*H*8 (or 7) a^4 ♀ – q – – –(5c)
1*H*14	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	*Bt* – ♂ 00 p – – –(*A*1)	0*H*8 (or 7) a^5 ♀ 100zq49.46.12(5c)
	a^5 ♀ – q – – –(2e)	a^6 ♀ – q – – –(3e)	... ...	... ...	... ...	... ...

MATING	OFFSPRING				PARENTS	
					Father	Mother
0H135	a^1 ♂ 50 q49.46. 1(3c) a^5 ♀ 100zq49.46.12(5cw)	a^2 ♀ 45 q52.46. 1(2cw) a^6 ♂ 30 q48. 1. 1(2cw)	a^3 ♀ 45 q49.46. 4(2c) … … …	a^4 ♀ 100zq49.48.14(5c) … … …	W no. 35 ♂ p – – –(Waβ)	1H47a^1 ♀ 00p – – –(A × 2)
0H136	a^1 ♂ 40 q47.28. 2(3c) a^5 – q – – –(5c)	a^2 ♂ 25 q52. 1. 1(2c) a^6 – q – – –(5c)	a^3 ♂ 25 q48. 1. 1(2c) a^7 – q – – –(5c)	a^4 – q – – –(5c) … … …	W no. 39 ♂ p – – –(Waβ)	1H47a^2 ♀ 00p – – –(A × 2)
0H138	a^1 ♀ – q – – –(5c)	a^2 ♀ – q – – –(5–6c)	a^3 ♂ – q – – –(5c)	a^4 ♀ 50 q49. 1. 1(3c)	W [illegible] ♂ p – – –(Waβ)	1H49a^1 ♀ 00p – – –(A × 2)
0H139	a^1 ♂ 20 q49. 1. 2(2c) a^5 ♀ – q – – –(5c)	a^2 ♂ – q – – –(5c) a^6 – q – – –(2c)	a^3 ♀ 30 q49. 1. 2(2c) … … …	a^4 ♀ – q – – –(5c) … … …	W no. 40 ♂ p – – –(Waβ)	1H32a^1 ♀ 00p – – –(A × 2)
0H140	a^1 ♀ 35 q53.53. 1(2c) a^5 ♂ 40 q49.46. 1(3c)	a^2 ♂ 85 q52.53.12(4c) a^6 ♀ 40 q46. 1. 1(3c)	a^3 ♀ 35 q48. 1. 1(2c) a^7 – q – – –(3c)	a^4 ♀ 27 q49.46. 4(2c) … … …	W no. 40 ♂ p – – –(Waβ)	1H32a^2 ♀ 00p – – –(A × 2)
0H142	a^1 ♂ 45 q49. 1. 1(3c) a^5 ♀ – q – – –(2c)	a^2 ♂ 40 q49. 1. 1(2c) a^6 ♀ 80 q49.36. 4(5c)	a^3 ♂ 100zq49.36. 7(5c) … … …	a^4 ♀ 30 q49. 1. 1(2c) … … …	W no. 42 ♂ p – – –(Waβ)	A51a^1 ♀ 00p – – –(A × 2)
0H143	a^1 ♂ – q – – –(5c) a^5 ♂ – q – – –(5c)	a^2 ♀ – q – – –(5c) a^6 ♀ – q – – –(5c)	a^3 ♂ – q – – –(5c) a^7 – q – – –(5c)	a^4 ♀ – q – – –(5c) … … …	W no. 35 ♂ p – – –(Waβ)	Bt – ♀ 00p – – –(A × 1)
0H144	a^1 – q – – –(5c) a^5 – q – – –(5c)	a^2 – q – – –(5c) a^6 – q – – –(5c)	a^3 – q – – –(5c) … … …	a^4 – q – – –(5c) … … …	W no. 36 ♂ p – – –(Waβ)	A71a^1 ♀ 00p – – –(A2 ?)
0H145	a^1 ♂ 100zq49.48. 9(5c) a^5 ♂ 00 p – – –(alb.)	a^2 ♂ 100zq35.35.30(5a–b) a^6 ♀ 100zq35.33.30(5–6a–b)	a^3 ♂ 100zq35.35.30(5a–b) a^7 ♀ 100zq39.35.30(5a–b)	a^4 ♂ 100zq36.34.28(5a–b) a^8 ♀ 100zq34.35.30(5–6a–b)	1H42b^1 ♂ p – – –(3a–bw)	A30a^1 ♀ 00p – – –(A2 ?)
0H146	a^1 ♂ 90 q49.34. 2(4c) a^5 ♀ 00 p – – –(alb.)	a^2 ♂ 100zq35.33. 4(5a) a^6 ♀ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.) a^7 ♀ 00 p – – –(alb.)	a^4 ♀ – q – – –(5c) a^8 – q – – –(5c)	1H42b^1 ♂ p – – –(3a–bw)	A30a^2 ♀ 00p – – –(A2 ?)
0H147	a^1 – q – – –(5c) a^5 – q – – –(5c)	a^2 100zq53.51.14(5c) a^6 – q – – –(5c)	a^3 – q – – –(5c) … … …	a^4 – q – – –(5c) … … …	W no. 37 ♂ p – – –(Wβ)	A67a^2 ♀ 00p – – –(A × 1)
0H148	a^1 ♀ 55 q53. 1. 1(3c) a^5 – q – – –(3c)	a^2 ♀ 55 q49.36. 3(3c) a^6 – q – – –(3c)	a^3 ♀ 45 q49.36. 1(3c) … … …	a^4 – q – – –(3c) … … …	W no. 36 ♂ p – – –(Waβ)	1H18a^3 ♀ 00p – – –(A × 1)
0H150	a^1 ♂ 15 q47. 1. 1(2c) a^5 – q – – –(2c)	a^2 55 q47.47. 1(3c) … … …	a^3 35 q47. 1. 1(2c) … … …	a^4 – q – – –(2c) … … …	W no. 31 ♂ p – – –(Wβ)	1H19a^6 ♀ 00p – – –(A × 1)
0H151	a^1 ♂ 85 q53.46. 2(5c) a^5 80 q53.51. 2(5c)	a^2 ♂ 55 q46. 1. 1(3c) a^6 – q – – –(4c)	a^3 ♂ 30 q53. 1. 1(2c) … … …	a^4 ♂ 75 q53.46. 1(4c) … … …	W no. 35 ♂ p – – –(Waβ)	1H28b^3 ♀ 00p – – –(A × 2)
0H152	a^1 ♂ 80 q44.47. 2(5d) a^5 – q – – –(5c)	a^2 ♂ – q – – –(5c) a^6 – q – – –(6c)	a^3 ♀ – q – – –(5c) a^7 – q – – –(5c)	a^4 ♀ – q – – –(5c) … … …	W no. 35 ♂ p – – –(Waβ)	1H28b^4 ♀ 00p – – –(A × 2)
0H153	a^1 ♀ 60 q44.47. 1(3c) a^5 – q – – –(4c)	a^2 ♂ 90 q44.46.11(4c) a^6 – q – – –(3c)	a^3 ♂ 95 q 44.53. 2(4d) … … …	a^4 – q – – –(4c) … … …	W no. 35 ♂ p – – –(Waβ)	1H28b^5 ♀ 00p – – –(A × 2)
0H154	a^1 ♀ 35 p37. 1. 2(2a–b) a^5 – q – – –(3c)	a^2 ♀ 35 q49. 1. 1(2–3c) … … …	a^3 ♂ 10 p35. 1. 4(1–2a–b) … … …	a^4 ♀ – p – – –(2b) … … …	W no. 54 ♂ p – – –(Wβ)	1H58a^2 ♀ 00p – – –(A × 2)
0H155	a^1 ♂ 100zq35.35.30(5a)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	… … …	1H42b^1 ♂ p – – –(3a–bw)	A58a^3 ♀ 00p – – –(A × 3)
0H156	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ 100zq48.46.20(5b–c)	1H42b^1 ♂ p – – –(3a–bw)	A58a^5 ♀ 00p – – –(A × 3)
0H159	a^1 ♂ – q – – –(5c) a^5 ♀ 100zq52.46. 7(5c)	a^2 ♂ – q – – –(5c) a^6 ♀ 75 q52.46. 4(5c)	a^3 ♂ – q – – –(5c) … … …	a^4 ♀ – q – – –(5c) … … …	W no. 31 ♂ p – – –(Wβ)	A141a^3 ♀ 00p – – –(A1½)
0H160	a^1 ♂ – q – – –(5c) a^5 ♀ 100zq49.46. 7(5c)	a^2 ♂ – q – – –(5c) … … …	a^3 ♀ 100xq49.46. 2(5c) … … …	a^4 ♀ 100zq49.46. 7(5c) … … …	W no. 31 ♂ p – – –(Wβ)	A141a^4 ♀ 00p – – –(A1½)
0H161	a^1 ♂ – q – – –(5c) a^5 ♀ 100zq49.46.12(5c)	a^2 ♂ 80 q48.46. 7(5c) a^6 ♀ – q – – –(5c)	a^3 ♂ – q – – –(5c) … … …	a^4 ♀ – q – – –(5c) … … …	W no. 56 ♂ p – – –(Wβ)	A182a^3 ♀ 00p – – –(A1½)
0H162	a^1 ♀ 100zq49.46.11(5c) a^5 – q – – –(5c)	a^2 ♂ 80 q48.46. 7(5c) a^6 – q – – –(5c)	a^3 – q – – –(5c) … … …	a^4 – q – – –(5c) … … …	W no. 56 ♂ p – – –(Wβ)	A182a^4 ♀ 00p – – –(A1½)
0H163	a^1 ♂ 75 q44.51.11(5c) a^5 – q – – –(5c)	a^2 ♂ 80 q44.51.11(5c) … … …	a^3 ♀ 80 q44.51.11(5c) … … …	a^4 ♀ 95 q44.51.14(5c) … … …	W no. 56 ♂ p – – –(Wβ) … … … …	A182a^5 ♀ 00p – – –(A1½) … … … …

0H165	a^1 70 q48.46. 2(4c) a^5 – q – – –(5c)	a^2 – q – – –(5c) a^6 – q – – –(5c)	a^3 – q – – –(5c) a^7 – q – – –(5c)	a^4 – q – – –(5c) … … …	W no. 37 ♂ p – – –(Wβ)	A143a^2 ♀ 00 p – – –(A2)
0H166	a^1 ♂ – q – – –(5c) a^5 ♀ 100zq49.48. 7(5c)	a^2 ♂ – q – – –(4–5c) … … …	a^3 ♂ 55 q47.21.11(3–4c) … … …	a^4 ♂ 95 q48.36.28(5c) … … …	W no. 37 ♂ p – – –(Wβ)	A143a^3 ♀ 00 p – – –(A2
0H167	a^1 80 q49.46. 1(5c) a^5 – q – – –(4c)	a^2 85 q49.46.11(5c) … … …	a^3 – q – – –(5c) … … …	a^4 – q – – –(4c) … … …	W no. 37 ♂ p – – –(Wβ)	A143a^4 ♀ 00 p – – –(A2)
0H169	a^1 ♀ – q – – –(6c–d)	a^2 – q – – –(4c)	— … …	… … …	A155a^1 or a^2 ♂ 00p – – –(A2)	W no. 24 ♀ p – – –(Waβ)
0H170	a^1 ♂ 45 q48.46. 2(3c) a^5 ♀ 90 q49.35. 2(4c)	a^2 ♂ – q – – –(3c) … … …	a^3 ♀ 100zq49.46.12(5c) … … …	a^4 ♀ – q – – –(5c) … … …	W no. 42 ♂ p – – –(Waβ)	A184a^5 ♀ 00 p – – –(A × 1)
0H171	a^1 40 q49.46. 1(3c) a^5 – q – – –(5c)	a^2 50 q48.46. 1(3c) … … …	a^3 45 q49.46. 4(3c) … … …	a^4 45 q49.36. 1(3c) … … …	W no. 42 ♂ p – – –(Waβ)	A184a^6 ♀ 00 p – – –(A × 1)
0H172	a^1 – q – – –(4c) a^5 – q – – –(5c)	a^2 – q – – –(4c) a^6 – q – – –(5c)	a^3 – q – – –(4c) a^7 – q – – –(5c)	a^4 – q – – –(5c) a^8 – q – – –(5c)	W no. 42 ♂ p – – –(Waβ)	A184a^7 ♀ 00 p – – –(A × 1)
0H175	a^1 – q – – –(5c) a^5 – q – – –(5c)	a^2 – q – – –(5c) a^6 – q – – –(5c)	a^3 – q – – –(5c) a^7 – q – – –(5c)	a^4 – q – – –(5c) … … …	W no. 37 ♂ p – – –(Wβ)	A167a^5 ♀ 00 p – – –(A × 2)
0H176	a^1 – q – – –(5c) a^5 – q – – –(5c)	a^2 – q – – –(5c) a^6 – q – – –(5c)	a^3 – q – – –(5c) a^7 – q – – –(5c)	a^4 – q – – –(5c) … … …	W no. 37 ♂ p – – –(Wβ)	A167a^6 ♀ 00 p – – –(A × 2)
0H179	a^1 55 q49.36. 4(3c) a^5 – q – – –(5c)	a^2 30 q47. 1. 4(3c) a^6 – q – – –(5c)	a^3 55 q49.46. 2(3c) a^7 – q – – –(5c)	a^4 – q – – –(3c) a^8 – q – – –(5c)	W no. 42 ♂ p – – –(Waβ)	A163a^4 ♀ 00 p – – –(A × 2)
0H184	a^1 100zq49.46. 7(5c) a^5 – q – – –(5c) a^9 – q – – –(5c)	a^2 100zq49.46.12(5c) a^6 – q – – –(5c) a^{10} – q – – –(5c)	a^3 – q – – –(5c) a^7 – q – – –(5c) … … …	a^4 – q – – –(5c) a^8 – q – – –(5c) … … …	W no. 37 ♂ p – – –(Wβ) … … … …	A154a^2 ♀ 00 p – – –(A × 2) … … … …
0H185	a^1 – q – – –(6c)	a^2 – q – – –(6c)	… … …	… … …	W no. 37 ♂ p – – –(Wβ)	A157a^1 ♀ 00 p – – –(A1)
1H1	a^1 – q – – –(5c)	a^2 – q – – –(5c)	a^3 – q – – –(5c)	a^4 – q – – –(5c)	0H1a^1 ♂ 45 q49.46. 2(2d)	Bt – ♀ 00 p – – –(A1 ?)
1H2	a^1 ♀ 00 p – – –(alb.) a^5 ♂ 100zq48.46.11(4d) a^9 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.) a^6 ♂ 90 q57.55. 1(4e) … … …	a^3 ♂ 00 p – – –(alb.) a^7 ♂ 100zq48.46.11(4d) … … …	a^4 ♂ 00 p – – –(alb.) a^8 ♂ 100xq57.55.55(5d) … … …	0H1a^2 ♂ 45 q49. 1. 1(3c) … … … …	Bt – ♀ 00 p – – –(A1 ?) … … … …
1H3	a^1 00 p – – –(alb.) a^5 ♂ 80 q46.36. 5(5c) a^9 – q – – –(5c)	a^2 ♀ – q – – –(6d) a^6 ♂ 00 p – – –(alb.) a^{10} – q – – –(5c)	a^3 ♀ 100zq49.46. 7(6c) a^7 ♂ 00 p – – –(alb.) … … …	a^4 ♀ 100zq46.36.20(5c) a^8 ♂ 00 p – – –(alb.) … … …	0H1a^3 ♂ 45 q48. 1. 1(2c) … … … …	Bt – ♀ 00 p – – –(A1 ?) … … … …
1H4	a^1 ♂ – q – – –(5d) a^5 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.) … … …	a^3 – q – – –(6e) … … …	a^4 – q – – –(5c) … … …	0H8 (or 7) a^1 ♂ 100zq48.36. 7(5c)	Bt – ♀ 00 p – – –(A1 ?)
1H5	a^1 ♀ 100zq49.46.47(6d) a^5 ♂ 00 p – – –(alb.)	a^2 ♀ 100zq48.46.47(6d) … … …	a^3 ♂ 100zq49.46.47(6d) … … …	a^4 ♂ – q – – –(6c) … … …	0H8 (or 7) a^2 ♂ 100zq49.46. 7(5c)	Bt – ♀ 00 p – – –(A1 ?)
1H6	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♂ – q – – –(6d)	a^4 ♂ 100zq49.46.47(6d)	0H7a^1 ♂ 100zq49.46. 7(5c)	Bt – ♀ 00 p – – –(A1 ?)
1H7	a^1 ♂ 30 p24. 1. 1(2fw) b^2 ♂ 40 p37.37. 4(2b)	a^2 ♀ 40 p37. 1. 1(2a) b^3 ♀ 25 q49. 1. 2(2d)	a^3 ♀ 35 q49. 1. 2(2b) b^4 ♀ 30 q49. 1. 2(2d)	b^1 ♂ 50 q49.46. 2(3c) b^5 – q – – –(2c)	0H9a^1 ♂ – q – – –(2c)	0H1a^4 ♀ – q – – –(1c)
1H8	a^1 ♀ – p – – –(3b)	a^2 ♀ – p – – –(4a)	a^3 ♀ – p – – –(4a)	a^4 ♀ – q – – –(3c)	0H7a^2 ♂ 45 q48. 1. 1(2c)	0H1a^5 ♀ – q – – –(1c
1H9	a^1 ♀ 40 p37.36. 3(3a) a^5 ♂ 40 q49.46. 1(3b–c)	a^2 ♀ 40 q58. 1. 1(3c–cw)	a^3 ♀ 70 p42.50. 1(3f)	a^4 ♂ 55 q49. 1. 1(3b)	0H7a^3 ♂ 55 q49.46. 2(2–3c)	0H1a^6 ♀ – q – – –(3c)
1H11	a^1 ♂ 00 p – – –(alb.) a^5 ♂ 100zq58.58.55(6e)	a^2 ♀ 00 p – – –(alb.) a^6 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0H20a^1 ♂ 55 q47.46. 1(3d)	Bt ♀ 00 p – – –(A1 ?)
1H12	a^1 ♀ 00 p – – –(alb.) a^5 – q – – –(3c)	a^2 ♀ – q – – –(6e) … … …	a^3 00 p – – –(alb.) … … …	a^4 – q – – –(6e) … … …	Bt ♂ 00 p – – –(A1)	0H9a^2 ♀ 30 q46. 1. 1(2c)
1H13	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 80 q48.46.11(5d)	a^3 ♂ 100xq58.58.55(6e)	… … …	Bt – ♂ 00 p – – –(A1)	0H8 (or 7) a^4 ♀ – q – – –(5c)
1H14	a^1 ♂ 00 p – – –(alb.) a^5 ♀ – q – – –(2e)	a^2 ♂ 00 p – – –(alb.) a^6 ♀ – q – – –(3e)	a^3 ♂ 00 p – – –(alb.) … … …	a^4 ♂ 00 p – – –(alb.) … … …	Bt – ♂ 00 p – – –(A1) … … … …	0H8 (or 7) a^5 ♀ 100zq49.46.12(5c) … … … …

MATING	OFFSPRING				PARENTS			
					Father		Mother	
1H15	a^1 ♂ 00 p - - - (alb.)	a^2 ♂ 00 p - - - (alb.)	a^3 ♂ 65 q33.33. 3(3a)	a^4 ♀ 100zq35.35.30(6a)	0H8(or 7)a^2	♂ 100zq49.46.7(5c)	Bt -	♀ 00 p - - - (A1 ?)
	a^5 ♀ 45 q39. 1. 1(3a)	a^6 ♀ 70 q61.58. 1(3e)	a^7 ♀ 00 p - - - (alb.)	a^8 ♀ 00 p - - - (alb.)	...	... ...	...	... ...
1H16	a^1 ♂ 95 q41.50. 4(4g)	a^2 ♂ 55 q58.55. 1(3e)	a^3 ♂ 00 p - - (alb.)	a^4 ♂ 00 p - - - (alb.)	0H7a^1	♂ 100zq49.46.7(5c)	Bt -	♀ 00 p - - - (A1 ?)
	a^5 ♂ 00 p - - - (alb.)	a^6 ♀ 00 p - - - (alb.)	a^7 ♀ 00 p - - - (alb.)	... ...	...	... ...	...	... ...
1H17	a^1 ♂ 85 q49.46.11(5–6c–d)	a^2 ♂ 00 p - - - (alb.)	a^3 ♂ 00 p - - - (alb.)	a^4 ♂ 45 q61. 1. 1(3d)	0H8(or 7)a^1	♂ 100zq48.36.7(5c)	Bt -	♀ 00 p - - - (A1 ?)
	a^5 ♂ 80 q58.58.55(3e)	... ...	... ...	... ...	...	... ...	...	... ...
1H18	a^1 - q - - - (3e)	a^2 ♀ 55 p37. 1. 1(3bw)	a^3 ♀ 00 p - - - (alb.)	a^4 ♂ 35 p40. 1. 1(3b)	0H7a^2	♂ 45 q48. 1.1(2c)	0H9a^3	♀ 45 q48.46. 1(2c)
	a^5 00 p - - - (alb.)	a^6 00 p - - - (alb.)	a^7 00 p - - - (alb.)	... ...	...	... ...	...	... ...
1H19	a^1 - q - - - (2cw)	a^2 - q - - - (4ew)	a^3 ♀ - q - - - (2c)	a^4 ♀ 30 q49. 1. 1(2c)	0H7a^3	♂ 55 q49.46.2(2–3c)	0H9a^4	♀ 50 q49. 1. 1(2c)
	a^5 ♀ 00 p - - - (alb.)	a^6 ♀ 00 p - - - (alb.)	b^1 ♀ 40 q58. 1. 1(2e)	b^2 40 p35. 1. 1(2b)	...	... ...	...	... ...
1H20	a^1 ♀ 00 p - - - (alb. w)	a^2 ♀ 00 p - - - (alb.)	a^3 ♂ 45 q49. 1. 1(3cw)	a^4 ♂ - q - - - (2e)	0H9a^1	♂ - q - - - (2c)	0H6a^1	♀ - q - - - (2c)
	a^5 ♂ - p - - - (2f)	a^6 - q - - - (2cw)	... ...	... ...	...	... ...	...	... ...
1H24	a^1 ♀ 00 p - - - (alb.)	a^2 ♀ 100zq49.46.47(6d)	a^3 ♂ 00 p - - - (alb.)	a^4 ♂ 00 p - - - (alb.)	0H13a^2	♂ 100zq48.46.7(5c)	Bt -	♀ 00 p - - - (A1 ?)
	a^5 ♂ 95 q49.46. 2(4d)	a^6 ♂ 90 q49.46. 4(4d)	... ...	... ...	...	... ...	...	... ...
1H26	a^1 ♀ 100zq49.46.11(5d)	a^2 ♀ 00 p - - - (alb.)	a^3 ♀ - q - - - (6e)	a^4 ♂ 100zq46.46.11(5d)	0H13a^4	♂ 45 q52. 1.1(3c)	Bt -	♀ 00 p - - - (A × 1 or 2)
1H27	a^1 ♀ 00 p - - - (alb.)	a^2 ♀ 00 p - - - (alb.)	a^3 ♂ 00 p - - - (alb.)	a^4 ♂ 00 p - - - (alb.)	Bt -	♂ 00 p - - - (A1)	0H7a^5	♀ 50 q48.36. 2(3c)
	a^5 ♂ 00 p - - - (alb.)	a^6 ♂ 100xq58.58.57(6e)	... ...	... ...	...	... ...	...	... ...
1H28	a^1 ♀ 100zq49.46.47(5–6c–d)	a^2 ♀ 100zq49.48.43(5c)	b^1 ♂ - q - - - (6d)	b^2 ♂ 100zq48.34. 9(5d)	Bt -	♂ 00 p - - - (A1)	0H7a^6	♀ 100zq48.46.12(5c)
	b^3 ♀ 00 p - - - (alb.)	b^4 ♀ 00 p - - - (alb.)	b^5 ♀ 00 p - - - (alb.)	... ...	...	... ...	...	... ...
1H29	a^1 ♂ 00 p - - - (alb.)	a^2 ♂ 60 q49.46. 1(4c)	a^3 ♀ - q - - - (2c)	a^4 ♀ 55 q49 46. 2(3d)	0H16a^1	♂ 50 q47. 1.1(3c)	0H9a^5	♀ 25 q46. 1. 1(2c)
	a^5 - q - - - (2e)	b^1 ♀ 50 q49. 1. 2(2cw)	b^2 ♂ 50 p23. 1. 2(3f)	b^3 ♂ 00 p - - - (alb.)	...	... ...	...	... ...
1H30	a^1 ♂ 35 p23. 1. 1(2f)	a^2 ♀ 100zq48.21.43(5cw)	a^3 ♀ 45 p36. 1. 1(3b)	a^4 ♀ 45 p23. 1. 2(2f)	0H12a^1	♂ - q - - - (5c)	0H9a^6	♀ 40 q49. 1. 1(2d)
	a^5 ♀ 60 q49.46. 3(3d)	a^6 - q - - - (2dw)	... ...	... ...	...	... ...	...	... ...
1H31	a^1 00 p - - - (alb.)	... ...	... ...	... ...	0H12a^2	♂ - q - - - (5c)	Bt -	♀ 00 p - - - (A1)
1H32	a^1 ♀ 00 p - - - (alb.)	a^2 ♀ 00 p - - - (alb.)	a^3 ♀ 100zq49.46. 2(4c)	a^4 ♀ 100xq58.57.55(6e)	A1a^3	♂ 00 p - - - (A2½)	0H7a^7	♀ - q - - - (2d)
	a^5 ♀ 100zq49.46. 4(5–6c)	a^6 ♂ 00 p - - - (alb.)	a^7 - q - - - (6c)	... ...	...	... ...	...	... ...
1H33	a^1 ♀ 00 p - - - (alb.)	a^2 ♀ 00 p - - - (alb.)	a^3 ♀ 00 p - - - (alb.)	a^4 ♀ 100yq58.58.55(6e)	0H8(or 7)a^1	♂ 100zq48.36.7(5c)	Bt -	♀ 00 p - - - (A1 ?)
	a^5 ♂ 00 p - - - (alb.)	a^6 ♂ - q - - - (6e)	a^7 - q - - - (6e)	a^8 - q - - - (6e)	...	... ...	...	... ...
1H34	b^1 ♂ 00 p - - - (alb.)	b^2 ♂ 95 q48.46. 1(5c)	b^3 ♂ 100yq49.46.47(5c)	b^4 ♀ - q - - - (6d)	Bt -	♂ 00 p - - - (A1 ?)	0H16a^2	♀ 50 q48.46. 1(3c)
1H35	b^1 ♂ 100zq49.46.14(4d)	b^2 ♀ 100zq49.46.43(5d)	b^3 ♂ 100zq48.46.43(5d)	b^4 ♂ 100zq48.46.12(5d)	Bt -	♂ 00 p - - - (A1 ?)	0H16a^3	♀ 50 q48. 1. 1(3c)
	b^5 ♀ 100zq46.46.14(5d)	b^6 ♂ 00 p - - - (alb.)	... ...	... ...	...	... ...	...	... ...
1H36	a^1 ♂ 00 p - - - (alb.)	a^2 ♀ 00 p - - - (alb.)	a^3 ♀ 75 q48.46. 1(3c–d)	a^4 ♀ 30 r37. 1. 1(2c)	0H35a^1	♂ - q - - - (2c)	0H16a^4	♀ 55 q49.46. 1(4c)
	a^5 ♀ 15 q?37. 1. 1(1aw)	... ...	... ...	... ...	...	... ...	...	... ...
1H37	a^1 ♀ 65 q49.48. 1(2a)	a^2 ♀ - r - - - (2a)	a^3 ♀ - q - - - (3c–dw)	a^4 ♂ 25 p40. 1. 1(2aw)	0H35a^1	♂ - q - - - (2c)	0H16a^5	♀ - q - - - (4c)
	a^5 ♂ 25 q47. 1. 1(2c)	... ...	... ...	... ...	...	... ...	...	... ...
1H38	a^1 ♂ 00 p - - - (alb.)	a^2 ♀ 00 p - - - (alb.)	a^3 ♀ 100zq48.46.43(5–6d)	a^4 ♀ 100zq49.46. 9(5–6d)	Bt -	♂ 00 p - - - (A1)	0H20a^4	♀ 45 q49. 1. 1(2d)
1H39	a^1 ♀ 00 p - - - (alb.)	a^2 ♀ 00 p - - - (alb.)	a^3 ♂ 00 p - - - (alb.)	a^4 ♂ 00 p - - - (alb.)	Bt -	♂ 00 p - - - (A1)	0H20a^5	♀ 25 q49. 1. 1(2d)
1H40	a^1 ♀ 00 p - - - (alb.)	a^2 ♀ 00 p - - - (alb.)	a^3 ♀ 100zq35.33.29(6a)	a^4 ♂ 00 p - - - (alb.)	Bt -	♂ 00 p - - - (A1 ?)	0H12a^3	♀ - q - - - (5a)
	a^5 ♂ 00 p - - - (alb.)	a^6 ♂ 00 p - - - (alb.)	a^7 - q - - - (6a)	... ...	...	... ...	...	... ...
1H41	a^1 ♀ 00 p - - - (alb.)	a^2 ♂ 00 p - - - (alb.)	a^3 ♂ 00 p - - - (alb.)	a^4 ♂ 100xq61.61.57(6e)	Bt -	♂ 00 p - - - (A1 ?)	0H12a^4	♀ - q - - - (5c)
	a^5 ♂ 100zq49.46.14(5c)	... ...	... ...	... ...	...	... ...	...	... ...
1H42	a^1 ♂ 00 p - - - (alb. w)	a^2 ♂ 90 p39.35. 4(4a)	a^3 ♀ 45 q49. 1. 1(3c)	a^4 ♀ 35 q46. 1. 2(3c)	0H1a^2	♂ 55 q49. 1.1(3c)	0H12a^5	♀ - q - - - (5a)
	b^1 ♂ 55 p35.33. 1(3a–bw)	b^2 ♂ 65 q39. 1. 3(4a)	b^3 ♂ - q - - - (1cw)	b^4 ♀ 100yq35.33.30(6a)	...	... ...	...	... ...
	b^5 ♀ 00 p - - - (alb.)	... ...	... ...	... ...	...	... ...	...	... ...
1H43	a^1 ♀ 00 p - - - (alb.)	a^2 - q - - - (6e)	a^3 00 p - - - (alb.)	a^4 ♂ - q - - - (6e)	0H9a^1	♂ - q - - - (2c)	Bt -	♀ 00 p - - - (A1 ?)

	a^5 ♂ 100zq49.46.12(5c–d)	...	...	...	...	...	...	...
1H44	a^1 ♀ 00 p– – –(alb.)	a^2 ♀ 00 p– – –(alb.)	a^3 ♀ 100zq49.46. 9(5c–d)	a^4 ♀ 85 q48.46. 1(4c)	0H1a^1	♂ 45 q49.46.2(2d)	Bt –	♀ 00 p– – –(A1?)
	a^5 45 q49.48. 1(3c)	...	...	...	...	...	...	...
1H45	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 00 p– – –(alb.)	a^3 ♂ 00 p– – –(alb.)	a^4 00 p– – –(alb.)	0H8(or7)a^2	♂ 100zq49.46.7(5c)	Bt –	♀ 00 p– – –(A×1or2)
1H46	a^1 – q– – –(3c)	a^2 – q– – –(3c)	a^3 – q– – –(3c)	a^4 00 p– – –(alb.)	0H7a^2	♂ 45 q48. 1.1(2c)	Bt –	♀ 00 p– – –(A1)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	a^8 00 p– – –(alb.)	...	...	...	...
1H47	a^1 ♀ 00 p– – –(alb.)	a^2 ♀ 00 p– – –(alb.)	a^3 ♀ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	0H7a^3	♂ 55 q49.46.2(2–3c)	Bt –	♀ 00 p– – –(A×1or2)
	a^5 ♂ 100xq58.57.55(6e)	a^6 ♂ 100xq58.57.55(6e)	...	...	...	...	...	...
1H48	a^1 ♀ 00 p– – –(alb.)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♂ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	0H13a^2	♂ 100zq48.46.7(5c)	Bt –	♀ 00 p– – –(A1)
1H49	a^1 ♀ 00 p– – –(alb.)	a^2 ♀ 100zq48.36. 7(6*g)	a^3 ♂ 100xq58.57.55(6e)	a^4 ♂ 00 p– – –(alb.)	0H13a^3	♂ 100zq49.46.7(5d)	Bt –	♀ 00 p– – –(A1)
	a^5 ♂ 00 p– – –(alb.)	b^1 ♂ 55 q46. 1. 1(3c)	b^2 ♀ 100zq47.46.14(5d)	b^3 ♂ 00 p– – –(alb.)	...	...	...	...
	b^4 ♂ 00 p– – –(alb.)	b^5 ♀ – q– – –(6e)	...	...	...	...	...	...
1H51	a^1 ♂ 30 q46. 1. 1(4c)	a^2 ♀ 55 q48. 1. 3(4c)	a^3 ♀ 75 q58. 1. 2(4e)	a^4 ♀ 00 p– – –(alb.)	0H12a^2	♂ – q– – –(5c)	Lb –	♀ 00 p– – –(A×1or2)
1H52	a^1 ♂ 100xq57.38.19(6e)	a^2 ♂ 100xq57.57.55(6e)	a^3 ♂ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	0H20a^2	♂ 40 q48.46.1(2c)	Bt –	♀ 00 p– – –(A1?)
	a^5 ♂ 00 p– – –(alb.)	...	...	...	...	...	...	...
1H55	a^1 45 q37.37. 1(3*a)	a^2 ♀ 00 p– – –(alb.)	a^3 ♂ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	A1a^1	♂ 00 p– – –(A2??)	0H1a^5	♀ – q– – –(1c)
	a^5 ♂ 00 p– – –(alb.)	a^6 ♂ 100xq41.41.50(6g)	...	...	...	...	...	...
1H56	a^1 ♂ – q– – –(4e)	a^2 ♀ 90 q48.46.12(4c)	a^3 ♀ 100zq49.48.12(5–6d)	a^4 ♀ 00 p– – –(alb.)	A14a^1	♂ 00 p– – –(A1)	0H1a^6	♀ – q– – –(3c)
1H57	a^1 ♀ 90 q48.46. 1(4c)	a^2 ♀ 100zq49.36.12(5–6d)	a^3 ♀ 00 p– – –(alb.)	a^4 ♀ 00 p– – –(alb.)	0H8(or7)a^1	♂ 100zq48.36.7(5c)	Lb –	♀ 00 p– – –(A×1or 2)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	a^7 ♂ 85 q48.46. 1(4c)	a^8 ♂ 60 q49.46. 1(4c)	...	...	...	...
1H58	a^1 ♀ 30 q39. 1. 1(2a)	a^2 ♀ 00 p– – –(alb.)	a^3 ♀ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	A8a^3	♂ 00 p– – –(A×2or3)	0H6a^1	♀ – q– – –(2c)
1H59	a^1 00 p– – –(alb.)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♂ 00 p– – –(alb.)	a^4 ♀ 100zq48.46.12(5c)	A4a^4	♂ 00 p– – –(A2??)	0H7a^7	♀ – q– – –(2d)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	...	...	...	...	...
1H60	a^1 ♂ 100xq57.57.55(6e)	a^2 ♂ 00 p– – –(alb.)	a^3 ♂ 75 q46.46. 4(4c)	a^4 ♂ 55 q49.46. 2(4c)	0H13a^4	♂ 45 q52. 1.1(3c)	Bt –	♀ 00 p– – –(A1?)
	a^5 – q– – –(6e)	a^6 00 p– – –(alb.)	...	...	...	...	...	...
1H61	a^1 ♂ 00 p– – –(alb.)	a^2 ♀ 100zq46.46.11(5c)	...	...	0H12a^2	♂ – q– – –(5c)	0H8(or7)a^5	♀ 100zq49.46.12(5c)
1H63	a^1 ♂ 45 q41. 1. 1(2g)	a^2 ♀ 100xq58.58.55(6e)	a^3 ♂ 00 p– – –(alb.)	a^4 ♀ 100zq48.48.12(5d)	A4a^1	♂ 00 p– – –(A2??)	0H9a^3	♀ 45 q48.46. 1(2c)
	a^5 ♂ 00 p– – –(alb.)	...	...	...	...	...	...	...
1H64	a^1 ♂ – q– – –(2e)	a^2 ♀ 35 q49. 1. 1(2d)	a^3 ♀ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	A5a^1	♂ 00 p– – –(A×3)	0H9a^2	♀ 30 q46. 1. 1(2c)
1H65	a^1 ♀ 100xq58.58.55(6e)	a^2 ♀ 00 p– – –(alb.)	a^3 ♂ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	A14a^3	♂ 00 p– – –(A1)	0H27a^1	♀ 40 q58. 1. 2(2e)
	b^1 ♀ – q– – –(6e)	b^2 ♀ 100xq58.58.56(6e)	b^3 ♀ 00 p– – –(alb.)	...	...	...	...	...
1H66	a^1 ♂ – q– – –(3e)	a^2 ♂ 85 p23.23. 1(4f)	a^3 ♀ 35 q46. 1. 1(3c)	a^4 ♂ – q– – –(2aw)	0H1a^3	♂ 45 q48. 1.1(2c)	0H35a^2	♀ – q– – –(3d)
	a^5 ♂ 20 q47. 1. 1(1c)	a^6 ♂ – p– – –(2f)	...	...	...	...	...	...
1H67	a^1 00 p– – –(alb.)	a^2 – q– – –(3c)	...	...	0H9a^1	♂ – q– – –(2c)	0H35a^3	♀ – q– – –(3d)
1H68	a^1 ♀ – q– – –(1e)	a^2 ♀ 10 q47. 1. 1(1cw)	a^3 – q– – –(1cw)	a^4 00 p– – –(alb. w)	0H35a^5	♂ – q– – –(4d)	0H35a^4	♀ – q– – –(2d)
	a^5 – q– – –(3c)	...	...	...	...	...	...	...
1H70	a^1 ♀ 45 q58. 1. 2(3e)	a^2 ♂ 55 q37.33.30(3a)	a^3 ♀ – q– – –(2c)	...	0H45a^2	♂ – q– – –(2d)	0H59a^1	♀ 50 q48. 1. 4(3d)
1H71	a^1 ♀ – q– – –(3c)	a^2 00 p– – –(alb.)	...	...	0H45a^3	♂ – q– – –(2d)	0H75a^1	♀ 60 q49.48. 1(4c)
1H72	a^1 ♂ – q– – –(3c)	...	...	...	0H75a^2	♂ 35 q49. 1.1(2c)	0H45a^4	♀ – q– – –(2d)
1H73	a^1 ♀ 70 q48.46. 1(3c)	a^2 ♀ 100zq49.46.43(5d)	a^3 ♀ 100zq48.46.12(5c)	...	0H59a^2	♂ 100zq49.46.7(6d)	0H73a^1	♀ – q– – –(5c)
1H74	a^1 ♂ – p– – –(3a)	b^1 ♂ 35 p23. 1. 1(2a)	b^2 ♀ 50 p36.36. 2(3a–b)	...	0H77a^1	♂ 100zq48.46.7(5c)	0H73a^2	♀ – q– – –(5c)
1H75	a^1 00 p– – –(alb.)	...	...	...	1H4a^1	♂ – q– – –(5d)	0H73a^3	♀ – q– – –(5c)
1H76	a^1 ♀ 60 q58.58. 1(3e)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	...	0H77a^2	♂ 60 q48.46.4(4c)	0H75a^3	♀ 45 q49. 1. 1(2c)
1H77	a^1 ♂ – q– – –(4e)	...	...	...	0H77a^3	♂ 55 q48.46.1(4c)	0H75a^4	♀ 45 q49.46. 1(3c)
1H78	a^1 ♂ – q– – –(3c)	a^2 ♂ 00 p– – –(alb.)	...	...	0H75a^5	♂ 15 q49. 1.1(1c)	1H4a^2	♀ 00 p– – –(alb.)
1H80	a^1 ♂ 00 p– – –(alb.)	a^2 ♀ 00 p– – –(alb.)	a^3 ♂ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	0H1a^1	♂ 45 q49.46.2(2d)	Bt –	♀ 00 p– – –(A1)
	a^5 ♀ 00 p– – –(alb.)	a^6 ♀ 100zq37.36. 4(5g)	a^7 ♂ 100xq57.57.55(6e)	...	...	...	...	...
1H81	a^1 ♀ – q– – –(6e)	a^2 ♀ – q– – –(3e)	a^3 ♂ – q– – –(4e)	a^4 ♂ 00 p– – –(alb.)	A12a^6	♂ 00 p– – –(A×2)	{0H77a^4	♀ – q– – –(5c)
	a^5 ♀ 00 p– – –(alb.)	a^6 ♀ 00 p– – –(alb.)	a^7 ♂ 00 p– – –(alb.)	a^8 ♀ – q– – –(6e)	...	...	0H77a^5	♀ – q– – –(5c)
	a^9 ♀ – q– – –(6e)	...	...	...	...	...	0H77a^6	♀ 100xq57.56.56(5e)

MATING	OFFSPRING				PARENTS			
					Father		Mother	
1H15	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 65 q33.33. 3(3a)	a^4 ♀ 100zq35.35.30(6a)	0H8(or 7)a^2	♂ 100zq49.46.7(5c)	Bt –	♀ 00 p – – –(A1?)
	a^5 ♀ 45 q39. 1. 1(3a)	a^6 ♀ 70 q61.58. 1(3e)	a^7 ♀ 00 p – – –(alb.)	a^8 ♀ 00 p – – –(alb.)	...	...	...	...
1H16	a^1 ♂ 95 q41.50. 4(4g)	a^2 ♂ 55 q58.55. 1(3e)	a^3 ♂ 00 p – – –(alb.)	a^1 ♂ 00 p – – –(alb.)	0H7a^1	♂ 100zq49.46.7(5c)	Bt –	♀ 00 p – – –(A1?)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♀ 00 p – – –(alb.)	a^7 ♀ 00 p – – –(alb.)	...	...	...	...	...
1H17	a^1 ♂ 85 q49.46.11(5–6c–d)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 45 q61. 1. 1(3d)	0H8(or 7)a^1	♂ 100zq48.36.7(5c)	Bt	♀ 00 p – – –(A1?)
	a^5 ♂ 80 q58.58.55(3a)	...	...	...	...	...	...	...
1H18	a^1 – q – – –(3e)	a^2 ♀ 55 p37. 1. 1(3bw)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 35 p40. 1. 1(3b)	0H7a^2	♂ 45 q48. 1.1(2c)	0H9a^3	♀ 45 q48.46. 1(2c)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	a^7 00 p – – –(alb.)	...	...	...	...	...
1H19	a^1 – q – – –(2cw)	a^2 – q – – –(4cw)	a^3 ♀ – q – – –(2c)	a^4 ♀ 30 q49. 1. 1(2c)	0H7a^3	♂ 55 q49.46.2(2–3c)	0H9a^4	♀ 50 q49. 1. 1(2c)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♀ 00 p – – –(alb.)	b^1 ♀ 40 q58. 1. 1(2e)	b^2 40 p35. 1. 1(2b)	...	...	...	...
1H20	a^1 ♀ 00 p – – –(alb. w)	a^2 ♀ 00 p – – –(alb.)	a^3 ♂ 45 q49. 1. 1(3cw)	a^4 ♂ – q – – –(2e)	0H9a^1	♂ – q – – –(2c)	0H6a^1	♀ – q – – –(2c)
	a^5 ♂ – p – – –(2f)	a^6 – q – – –(2cw)	...	...	...	...	...	...
1H24	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 100zq49.46.17(6d)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0H13a^2	♂ 100zq48.46.7(5c)	Bt	♀ 00 p – – –(A1?)
	a^5 ♂ 95 q49.46. 2(4d)	a^6 ♂ 90 q49.46. 4(4d)	...	...	...	...	...	...
1H26	a^1 ♀ 100zq49.46.11(5d)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ – q – – –(6e)	a^4 ♂ 100zq46.46.11(5d)	0H13a^4	♂ 45 q52. 1.1(3c)	Bt –	♀ 00 p – – –(A × 1 or 2)
1H27	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	Bt –	♂ 00 p – – –(A1)	0H7a^5	♀ 50 q48.36. 2(3c)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♂ 100xq58.58.57(6e)	...	...	...	...	...	...
1H28	a^1 ♀ 100zq49.46.17(5–6c–d)	a^2 ♀ 100zq49.48.43(5c)	b^1 ♂ – q – – –(6d)	b^2 ♂ 100zq48.34. 9(5d)	Bt	♂ 00 p – – –(A1)	0H7a^6	♀ 100zq48.46.12(5c)
	b^3 ♀ 00 p – – –(alb.)	b^4 ♀ 00 p – – –(alb.)	b^5 ♀ 00 p – – –(alb.)	...	...	...	...	...
1H29	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 60 q49.46. 1(4e)	a^3 ♀ – q – – –(2c)	a^4 ♀ 55 q49.46. 2(3d)	0H16a^1	♂ 50 q47. 1.1(3e)	0H9a^5	♀ 25 q46. 1. 1(2c)
	a^5 – q – – –(2e)	b^1 ♀ 50 q49. 1. 2(2cw)	b^2 ♂ 50 p23. 1. 2(3f)	b^3 ♂ 00 p – – –(alb.)	...	...	...	...
1H30	a^1 ♂ 35 p23. 1. 1(2f)	a^2 ♀ 100zq48.21.43(5ew)	a^3 ♀ 45 p36. 1. 1(3b)	a^4 ♀ 45 p23. 1. 2(2f)	0H12a^1	♂ – q – – –(5c)	0H9a^6	♀ 40 q49. 1. 1(2d)
	a^5 ♀ 60 q49.46. 3(3d)	a^6 – q – – –(2dw)	...	...	...	...	...	...
1H31	a^1 00 p – – –(alb.)	...	...	...	0H12a^2	♂ – q – – –(5c)	Bt –	♀ 00 p – – –(A1)
1H32	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 100zq49.46. 2(4c)	a^4 ♀ 100xq58.57.55(6e)	A1a^3	♂ 00 p – – –(A2½)	0H7a^7	♀ – q – – –(2d)
	a^5 ♀ 100zq49.46. 4(5–6c)	a^6 ♂ 00 p – – –(alb.)	a^7 – q – – –(6e)	...	...	...	...	...
1H33	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ 100yq58.58.55(6e)	0H8(or 7)a^1	♂ 100zq48.36.7(5c)	Bt	♀ 00 p – – –(A1?)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♂ – q – – –(6e)	a^7 – q – – –(6e)	a^8 – q – – –(6e)	...	...	...	...
1H34	b^1 ♂ 00 p – – –(alb.)	b^2 ♂ 95 q48.46. 1(5c)	b^3 ♂ 100yq49.46.47(5e)	b^4 ♀ – q – – –(6d)	Bt –	♂ 00 p – – –(A1?)	0H16a^2	♀ [illegible] q48.46. 1(3c)
1H35	b^1 ♂ 100zq49.46.14(4d)	b^2 ♀ 100zq49.46.43(5d)	b^3 ♂ 100zq48.46.13(5d)	b^4 ♂ 100zq48.46.12(5d)	Bt	♂ 00 p – – –(A1?)	0H16a^3	♀ 50 q48. 1. 1(3c)
	b^5 ♀ 100zq46.46.14(5d)	b^6 ♂ 00 p – – –(alb.)	...	...	...	...	...	...
1H36	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 75 q48.46. 1(3c–d)	a^4 ♀ 30 r37. 1. 1(2c)	0H35a^1	♂ – q – – –(2c)	0H16a^4	♀ 55 q49.46. 1(4c)
	a^5 ♀ 15 q?37. 1. 1(1aw)	...	...	...	...	...	...	...
1H37	a^1 ♀ 65 q49.48. 1(2a)	a^2 ♀ – r – – –(2a)	a^3 ♀ – q – – –(3c–dw)	a^4 ♂ 25 p40. 1. 1(2aw)	0H35a^1	♂ – q – – –(2c)	0H16a^5	♀ – q – – –(4c)
	a^5 ♂ 25 q47. 1. 1(2c)	...	...	...	...	...	...	...
1H38	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 100zq48.46.43(5–6d)	a^4 ♀ 100zq49.46. 9(5–6d)	Bt –	♂ 00 p – – –(A1)	0H20a^4	♀ 45 q49. 1. 1(2d)
1H39	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	Bt –	♂ 00 p – – –(A1)	0H20a^5	♀ 25 q49. 1. 1(2d)
1H40	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 100zq35.33.29(6a)	a^4 ♂ 00 p – – –(alb.)	Bt –	♂ 00 p – – –(A1?)	0H12a^3	♀ – q – – –(5a)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♂ 00 p – – –(alb.)	a^7 – q – – –(6a)	...	...	...	...	...
1H41	a^1 ♀ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 100xq61.61.57(6e)	Bt –	♂ 00 p – – –(A1?)	0H12a^4	♀ – q – – –(5c)
	a^5 ♂ 100zq49.46.14(5c)	...	...	...	...	...	...	...
1H42	a^1 ♂ 00 p – – –(alb. w)	a^2 ♂ 90 p39.35. 4(4a)	a^3 ♀ 45 q49. 1. 1(3c)	a^4 ♀ 35 q46. 1. 2(3c)	0H1a^2	♂ 55 q49. 1.1(3c)	0H12a^5	♀ – q – – –(5a)
	b^1 ♂ 55 p35.33. 1(3a–bw)	b^2 ♂ 65 q39. 1. 3(4a)	b^3 ♂ – q – – –(1cw)	b^4 ♀ 100yq35.33.30(6a)	...	...	...	...
	b^5 ♀ 00 p – – –(alb.)	...	...	...	...	...	...	...
1H43	a^1 ♀ 00 p – – –(alb.)	a^2 – q – – –(6e)	a^3 00 p – – –(alb.)	a^4 ♂ – q – – –(6e)	0H9a^1	♂ – q – – –(2c)	Bt –	♀ 00 p – – –(A1?)

	a^5 ♂ 100zq49.46.12(5c–d)	...	...	...	...	...	...	...
1H44	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 100zq49.46. 9(5c–d)	a^4 ♀ 85 q48.46. 1(4c)	0H1a^1	♂ 45 q49.46.2(2d)	Bt –	♀ 00 p – – –(A1?)
	a^5 45 q49.48. 1(3c)	...	...	...	...	...	...	...
1H45	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 00 p – – –(alb.)	0H8(or 7)a^2	♂ 100zq49.46.7(5c)	Bt –	♀ 00 p – – –(A × 1 or 2)
1H46	a^1 – q – – –(3c)	a^2 – q – – –(3c)	a^3 – q – – –(3c)	a^4 00 p – – –(alb.)	0H7a^2	♂ 45 q48. 1.1(2c)	Bt –	♀ 00 p – – –(A1)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	a^7 00 p – – –(alb.)	a^8 00 p – – –(alb.)	...	...	...	...
1H47	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0H7a^3	♂ 55 q49.46.2(2–3c)	Bt –	♀ 00 p – – –(A × 1 or 2)
	a^5 ♂ 100xq58.57.55(6e)	a^6 ♂ 100xq58.57.55(6e)	...	...	...	...	...	...
1H48	a^1 ♀ 00 p – – –(alb.)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0H13a^2	♂ 100zq48.46.7(5c)	Bt –	♀ 00 p – – –(A1)
1H49	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 100zq48.36. 7(6*g)	a^3 ♂ 100xq58.57.55(6e)	a^4 ♂ 00 p – – –(alb.)	0H13a^3	♂ 100zq49.46.7(5d)	Bt	♀ 00 p – – –(A1)
	a^5 ♂ 00 p – – –(alb.)	b^1 ♂ 55 q46. 1. 1(3c)	b^2 ♀ 100zq47.46.14(5d)	b^3 ♂ 00 p – – –(alb.)	...	...	...	...
	b^4 ♂ 00 p – – –(alb.)	b^5 ♀ – q – – –(6e)	...	...	...	...	...	...
1H51	a^1 ♂ 30 q46. 1. 1(4c)	a^2 ♀ 55 q48. 1. 3(4c)	a^3 ♀ 75 q58. 1. 2(4e)	a^4 ♀ 00 p – – –(alb.)	0H12a^2	♂ – q – – –(5c)	Lb –	♀ 00 p – – –(A × 1 or 2)
1H52	a^1 ♂ 100zq57.38.19(6e)	a^2 ♂ 100xq57.57.55(6e)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0H20a^2	♂ 40 q48.46.1(2c)	Bt	♀ 00 p – – –(A1?)
	a^5 ♂ 00 p – – –(alb.)	...	...	...	...	...	...	...
1H55	a^1 45 q37.37. 1(3*a)	a^2 ♀ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	A1a^1	♂ 00 p – – –(A2??)	0H1a^5	♀ – q – – –(1c)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♂ 100xq41.41.50(6g)	...	...	...	...	...	...
1H56	a^1 ♂ – q – – –(4e)	a^2 ♀ 90 q48.46.12(4c)	a^3 ♀ 100zq49.48.12(5–6d)	a^4 ♀ 00 p – – –(alb.)	A14a^1	♂ 00 p – – –(A1)	0H1a^6	♀ – q – – –(3c)
1H57	a^1 ♀ 90 q48.46. 1(4c)	a^2 ♀ 100zq49.36.12(5–6d)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ 00 p – – –(alb.)	0H8(or 7)a^1	♂ 100zq48.36.7(5c)	Lb –	♀ 00 p – – –(A × 1 or 2)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	a^7 ♂ 85 q48.46. 1(4c)	a^8 ♂ 60 q49.46. 1(4c)	...	...	...	...
1H58	a^1 ♀ 30 q39. 1. 1(2a)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	A8a^1	♂ 00 p – – –(A × 2 or 3)	0H6a^1	♀ – q – – –(2c)
1H59	a^1 00 p – – –(alb.)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ 100zq48.46.12(5c)	A4a^4	♂ 00 p – – –(A2??)	0H7a^7	♀ – q – – –(2d)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	a^7 00 p – – –(alb.)	...	...	...	...	...
1H60	a^1 ♂ 100xq57.57.55(6e)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 75 q46.46. 4(4c)	a^4 ♂ 55 q49.46. 2(4e)	0H13a^4	♂ 45 q52. 1.1(3c)	Bt	♀ 00 p – – –(A1?)
	a^5 – q – – –(6e)	a^6 00 p – – –(alb.)	...	...	...	...	...	...
1H61	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 100zq46.46.11(5c)	...	...	0H12a^2	♂ – q – – –(5c)	0H8(or 7)a^1	♀ 100zq49.46.12(5c)
1H63	a^1 ♀ 45 q41. 1. 1(2g)	a^2 ♀ 100xq58.58.55(6e)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ 100zq48.48.12(5d)	A4a^1	♂ 00 p – – –(A2?)	0H9a^1	♀ 45 q48.46. 1(2c)
	a^5 ♀ 00 p – – –(alb.)	...	...	...	...	...	...	...
1H64	a^1 ♂ – q – – –(2e)	a^2 ♀ 35 q49. 1. 1(2d)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	A5a^1	♂ 00 p – – –(A × 3)	0H9a^2	♀ 30 q46. 1. 1(2c)
1H65	a^1 ♀ 100xq58.58.55(6e)	a^2 ♀ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	A14a^1	♂ 00 p – – –(A1)	0H27a^1	♀ 40 q58. 1. 2(2c)
	b^1 ♀ – q – – –(6e)	b^2 ♀ 100xq58.58.56(6e)	b^3 ♀ 00 p – – –(alb.)	...	...	...	...	...
1H66	a^1 ♂ – q – – –(3e)	a^2 ♂ 85 p23.23. 1(4f)	a^3 ♀ 35 q46. 1. 1(3c)	a^4 ♂ – q – – –(2aw)	0H1a^3	♂ 45 q48. 1.1(2c)	0H35a^2	♀ – q – – –(3d)
	a^5 ♂ 20 q47. 1. 1(1c)	a^6 ♂ – p – – –(2f)	...	...	...	...	...	...
1H67	a^1 00 p – – –(alb.)	a^2 – q – – –(3c)	...	...	0H9a^1	♂ – q – – –(2c)	0H35a^3	♀ – q – – –(3d)
1H68	a^1 ♀ – q – – –(1e)	a^2 ♀ 10 q47. 1. 1(1cw)	a^3 – q – – –(1cw)	a^4 00 p – – –(alb. w)	0H35a^5	♂ – q – – –(4d)	0H35a^4	♀ – q – – –(2d)
	a^5 – q – – –(3c)	...	...	...	...	...	...	...
1H70	a^1 ♀ 45 q58. 1. 2(3a)	a^2 ♂ 55 q37.33.30(3a)	a^3 ♀ – q – – –(2c)	...	0H45a^2	♂ – q – – –(2d)	0H59a^1	♀ 50 q48. 1. 4(3d)
1H71	a^1 ♀ – q – – –(3c)	a^2 00 p – – –(alb.)	...	...	0H45a^3	♂ – q – – –(2d)	0H75a^1	♀ 60 q49.48. 1(4c)
1H72	a^1 ♂ – q – – –(3e)	...	...	...	0H75a^2	♂ 35 q49. 1.1(2c)	0H45a^4	♀ – q – – –(2d)
1H73	a^1 ♀ 70 q48.46. 1(3c)	a^2 ♀ 100zq49.46.43(5d)	a^3 ♀ 100zq48.46.12(5c)	...	0H59a^2	♂ 100zq49.46.7(6d)	0H73a^1	♀ – q – – –(5c)
1H74	a^1 ♂ – p – – –(3a)	b^1 ♂ 35 p23. 1. 1(2a)	b^2 ♀ 50 p36.36. 2(3a–b)	...	0H77a^1	♂ 100zq48.46.7(5c)	0H73a^2	♀ – q – – –(5c)
1H75	a^1 00 p – – –(alb.)	...	...	...	1H4a^1	♂ – q – – –(5d)	0H73a^3	♀ – q – – –(5c)
1H76	a^1 ♀ 60 q58.58. 1(3e)	a^2 00 p – – –(alb.)	a^3 00 p – – –(alb.)	...	0H77a^2	♂ 60 q48.46.4(4c)	0H75a^3	♀ 45 q49. 1. 1(2c)
1H77	a^1 ♂ – q – – –(4e)	...	...	...	0H77a^3	♂ 55 q48.46.1(4c)	0H75a^4	♀ 45 q49.46. 1(3c)
1H78	a^1 ♂ – q – – –(3c)	a^2 ♂ 00 p – – –(alb.)	...	...	0H75a^5	♂ 15 q49. 1.1(1c)	1H4a^2	♀ 00 p – – –(alb.)
1H80	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0H1a^1	♂ 45 q49.46.2(2d)	Bt –	♀ 00 p – – –(A1)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♀ 100zq37.36. 4(5g)	a^7 ♂ 100xq57.57.55(6e)	...	...	...	...	...
1H81	a^1 ♀ – q – – –(6e)	a^2 ♀ – q – – –(3c)	a^3 ♂ – q – – –(4c)	a^4 ♂ 00 p – – –(alb.)	A12a^6	♂ 00 p – – –(A × 2)	{0H77a^4	♀ – q – – –(5c)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♀ 00 p – – –(alb.)	a^7 ♂ 00 p – – –(alb.)	a^8 ♀ – q – – –(6e)	...	...	{0H77a^5	♀ – q – – –(5c)
	a^9 ♀ – q – – –(6e)	...	...	...	...	...	{0H77a^6	♀ 100xq57.56.56(5e)

Mating	Offspring				Parents	
					Father	Mother
1H83	b^1 ♂ 00 p – – – (alb.)	b^2 ♀ – q – – – (5e)	b^3 ♀ – q – – – (5e)	b^4 ♂ – q – – – (6e)	A12a^6 ♂ 00 p – – – (A × 2)	0H77a^6 ♀ 100xq57.56.56(5e)
	b^5 ♀ 60 q 61.60. 2(4e)	... ...	... ...	... ...	... ...	... ...
1H84	a^1 – q – – – (4c : g)	a^2 – q – – – (3c : g)	a^3 – q – – – (3e)	... ...	A14a^2 ♂ 00 p – – – (A1)	0H77a^7 ♀ 70 q61.61. 1(4e)
						0H77a^8 ♀ – q – – – (5c)
1H85	b^1 ♀ 00 p – – – (alb.)	b^2 ♀ 00 p – – – (alb.)	b^3 ♀ 00 p – – – (alb.)	b^4 ♂ 00 p – – – (alb.)	A14a^2 ♂ 00 p – – – (A1)	0H77a^8 ♀ – q – – – (5c)
	b^5 ♂ 00 p – – – (alb.)	b^6 ♂ 00 p – – – (alb.)	... ...	... ...	... ...	... ...
1H86	a^1 ♂ – q – – – (5c)	a^2 ♂ – q – – – (5c)	a^3 ♂ 00 p – – – (alb.)	a^4 ♀ 00 p – – – (alb.)	A12a^7 ♂ 00 p – – – (A × 2)	0H84a^1 ♀ – q – – – (5c)
1H87	a^1 ♀ 100xq 58.58.57(6e)	a^2 ♂ 00 p – – – (alb.)	a^3 ♀ 00 p – – – (alb.)	a^4 ♀ 00 p – – (alb.)	A12a^7 ♂ 00 p – – – (A × 2)	0H84a^2 ♀ 100zq 49.46. 4(5c)
	a^5 ♀ – q – – – (6c)	... ...	... ...	... ...	... ...	... ...
1H88	a^1 00 p – – – (alb.)	a^2 00 p – – – (alb.)	a^3 00 p – – – (alb.)	a^4 – q – – – (6e)	0H13a^1 ♂ 50 q46.46. 1(3c)	A15a^1 ♀ 00 p – – – (A × 3)
						A15a^2 ♀ 00 p – – – (A × 3)
	a^5 – q – – – (5c)	a^6 – q – – – (5c)	b^1 ♂ 00 p – – – (alb.)	b^2 ♀ 00 p – – – (alb.)	... ...	... ...
	b^3 ♀ 00 p – – – (alb.)	b^4 ♀ 00 p – – – (alb.)	b^5 00 p – – – (alb.)	b^6 ♀ – q – – – (6e)	... ...	... ...
1H90	a^1 ♀ 00 p – – – (alb.)	a^2 ♀ 00 p – – – (alb.)	a^3 ♂ – q – – – (6e)	... ...	0H7a^1 ♂ 100zq49.46. 7(5c)	1H6a^1 ♀ 00 p – – – (alb.)
						1H6a^2 ♀ 00 p – – – (alb.)
1H92	a^1 ♀ 50 q 49.46. 1(3c)	a^2 ♂ 60 q 58. 1. 1(4e)	a^3 ♂ 100xq57.57.55(6e)	a^4 ♂ 00 p – – – (alb.)	0H73a^6 ♂ – q – – – (3c)	A1a^9 ♀ 00 p – – – (A2 ??)
	a^5 ♂ 00 p – – – (alb.)	a^6 ♀ 00 p – – – (alb.)	... ...	... ...	... ...	... ...
1H94	a^1 ♀ – q – – – (5dw)	a^2 ♂ – p – – – (6aw)	a^3 ♀ 00 p – – – (alb. w)	a^4 ♂ 00 p – – – (alb.)	0H84a ? ♂ – q – – – (5 ?)	0H84a ? ♀ – q – – – (5 ?)
1H95	a^1 ♀ – q – – – (3e)	a^2 ♂ – q – – – (2e)	a^3 ♂ 100zp37.35. 4(6*b–c)	a^4 ♂ 100x r : p 41.41.50(6* b–c)	0H62a^1 or a^2 ♂ – q – – – (2 or 3c)	0H62a^{3-6} ♀ – q48 or 49 (3 or 5c)
	a^5 ♂ 00 p – – – (alb. w)	a^6 00 p – – – (alb.)	... ...	... ...	... ...	... ...
1H96	a^1 ♂ 100zq 48.46. 4(5c)	a^2 ♂ 100zq 35.33.28(5–6b)	a^3 ♀ 00 p – – – (alb.)	a^4 ♀ 00 p – – – (alb.)	0H84a^7 ♂ 100zq35.33.28(5a)	0H12a^3 ♀ – q – – – (5a)
	b^1 ♀ 00 p – – – (alb.)	b^2 ♀ – p – – – (5aw)	b^3 ♀ 100zp 35.33.28(5a–b)	b^4 – q – – – (5a)	... ...	... ...
1H97	a^1 ♀ 100zq 33.30.28(5–6b?)	b^1 ♀ 00 p – – – (alb.)	b^2 100zp 37.35. 6(5b)	b^3 – q – – – (5c)	0H111a^2 ♂ – q – – – (5a)	0H12a^5 ♀ – q – – – (5a)
	b^4 – p – – – (3b)	... ...	... ...	... ...	... ...	... ...
1H98	a^1 ♂ – q – – – (4e)	a^2 ♂ 80 q61.57. 1(4e)	a^3 ♂ 100zq 49.46. 4(6c)	a^4 ♀ 100zq 49.46.30(5c)	0H111a^1 ♂ – q – – – (5d)	0H7a^6 ♀ 100zq 48.46.12(5c)
	a^5 ♀ 100zq 48.46. 4(5–6c)	a^6 85 p26. 9. 3(5–6b)	a^7 00 p – – – (alb.)	b^1 80 p37.37. 2(5b)	... ...	... ...
	b^2 – p – – – (6f)	b^3 – q – – – (5c)	b^4 – q – – – (5c)	b^5 00 p – – – (alb.)	... ...	... ...
	b^6 00 p – – – (alb.)	... ...	... ...	... ...	... ...	... ...
1H99	a^1 ♂ 100zp 39.35.28(6b)	a^2 – q – – – (5c)	a^3 ♂ 80 q48.46. 7(5d)	a^4 00 p – – – (alb.)	0H84a^8 ♂ – q – – – (5c)	0H82a^2 ♀ – q – – – (5c)
1H100	a^1 ♀ 00 p – – – (alb. w)	a^2 ♂ 30 q46. 1. 1(2c)	a^3 ♂ – q – – – (2d)	a^4 ♀ 65 q49.46. 1(3c–d)	0H13a^4 ♂ 45 q52. 1. 1(3c)	0H82a^1 ♀ 55 q48.46.12(3c)
	a^5 ♀ 30 p 37. 1. 1(2b)	a^6 ♂ 85 p36.36. 1(4b)	b^1 ♂ 55 q49.46. 1(3c)	b^2 ♂ 60 q55.46. 1(4c)	... ...	... ...
	b^3 ♂ 00 p – – – (alb.)	b^4 ♀ 65 q58.58. 4(4e)	b^5 ♀ 00 p – – – (alb.)	b^6 ♀ 00 p – – – (alb.)	... ...	... ...
1H101	a^1 100xp 24.24.17(6f)	a^2 ♂ 00 p – – – (alb. w)	a^3 ♂ 65 q61.60. 3(3e)	a^4 – q – – – (5e)	0H84a^9 ♂ 100zq 48.46.28(5c)	0H85a^4 ♀ – q – – – (5–6d)
	a^5 ♀ – q – – – (6e)	a^7 75 49.46. 9	b^1 ♂ 00 p – – – (alb.)	b^2 ♀ 100xq49.48.46(6c)	... ...	... ...
	b^3 ♀ 100xq 58.58.55(6ew)	b^4 ♂ 00 p – – – (alb.)	b^5 ♀ 00 p – – – (alb.)	b^6 ♀ – p – – – (6*b–g)	... ...	... ...
1H102	a^1 ♂ 100zq 52.51. 9(5dw)	a^2 – p – – – (5bw)	a^3 ♂ 100zq 37.37. 4(5b–c)	a^4 ♀ 100xq58.57.57(6e)	0H20a^3 ♂ 100zq 48.46. 7(5d)	0H85a^3 ♀ – q – – – (5d)
	a^5 – q – – – (1cw)	a^6 00 p – – – (alb. w)	b^1 ♂ 25 p37. 1. 1(1–2b)	b^2 ♂ – p – – – (6f)	... ...	... ...
	b^3 ♂ – p – – – (6fw)	b^4 ♀ 100xp24.24.15(6fw)	b^5 – q – – – (3c)	... ...	... ...	... ...
1H103	a^1 ♀ 55 q 34.34. 1(3bw)	a^2 – p – – – (3bw)	a^3 ♀ – q – – – (4b–c)	b^1 ♀ 60 dr? 40.39.1(4b–c)	0H120a^4 ♂ 60 q49.46. 1(3c)	0H85a^1 ♀ – q – – – (4d)
	b^2 ♀ 55 p or dr 40.35. 1(4b)	b^3 ♀ 80 p36.34. 3(4b)	b^4 ♀ 65 q61.60. 1(3e)	b^5 – q – – – (3b)	... ...	... ...
	b^6 – q – – – (3b–c)	... ...	... ...	... ...	... ...	... ...
1H104	a^1 ♀ 00 p – – – (alb.)	a^2 ♂ – q – – – (2e)	a^3 ♀ 50 p37.36. 3(3b)	a^4 ♀ 00 p – – – (alb.)	0H120a^7 ♂ 50 q48.46. 3(3c)	0H120a^3 ♀ 45 q49. 1. 1 (2c)
	a^5 – q – – – (2cw)	b^1 35 q46. 1. 1(2cw)	b^2 ♂ – q – – – (3b–c)	b^3 20 q47. 1. 1(1cw)	... ...	... ...

	b^4 00 p – – –(alb.)	…	…	…	…	…
1*H*105	a^1♂ 40 p 37. 1. 1(3b)	a^2 30 q44. 1. 1(2c)	a^3♂ 50 p23.23. 1(3f)	a^4♂ 40 p37. 1. 1(2b)	0*H*120a^2♂ – q – – –(3c)	0*H*120a^6 ♀ 75 q49.46. 1(4c)
	b^1 60 p 37.37.33(3b)	b^2 00 p – – –(alb. w)	b^3 75 p37.33. 1(4b)	b^4 – p – – –(4b)	…	…
	b^5 – q – – –(3c)	b^6 – q – – –(4e)	…	…	…	…
1*H*106	a^1 60 q 48.46. 7(3cw)	b^1 60 q44. 1. 1(3c)	b^2 35 p26. 1. 2(2f)	b^3 55 q44.47.29(3dw)	0*H*120a^1♂ – q – – –(4c)	0*H*1a^6 ♀ – q – – –(3c)
	b^4♂ 00 p – – –(alb.)	…	…	…	…	…
1*H*107	a^1♂ 95dr 37.37.28(4b–c)	a^2♀ 30 p37. 1. 1(1–2b)	…	…	0*H*110a^1♂ 60 q48.46. 1(3c)	0*H*16a^5 ♀ – q – – –(4c)
1*H*108	a^1 – p – – –(3b)	a^2 25 q60. 1. 1(2e)	a^3 00 p – – –(alb.)	a^4 00 p – – –(alb.)	0*H*101a^1♂ 45 q49. 1. 1(2c)	0*H*9a^5 ♀ 25 q46. 1. 1(2c)
	a^5 00 p – – –(alb.)	…	…	…	…	…
1*H*109	a^1 – p – – –(2bw)	a^2 70 p37.37. 4(3bw)	a^3 – q – – –(2c)	a^4 00 p – – –(alb.)	0*H*101a^2♂ 50 q49.46. 4(3c)	0*H*101a^6 ♀ 45 q49.46. 1(3c)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb. w)	b^1♂ 25 q58. 1. 2(2e)	b^2♂ – q – – –(3c)	…	…
	b^3♂ 00 p – – –(alb.)	b^4♂ 00 p – – –(alb.)	b^5 00 p – – –(alb.)	…	…	…
1*H*110	a^1 85 p 39.33. 3(5–6a–bw)	a^2 100zp37.39.34(6b)	…	…	0*H*101a^4♂ – q – – –(5c)	0*H*101a^8 ♀ – q – – –(5c)
1*H*111	a^1♂ 100xp 19.19. 9(6a)	a^2 80 p26.37. 2(5bw)	a^3♂ 100zq 48.46. 7(5c)	a^4 – q – – –(6c)	0*H*101a^3♂ – q – – –(5c)	0*H*61a^7 ♀ – q – – –(5c)
	a^5♂ 95 p 35. 1. 3(4bw)	a^6 – p – – –(4b)	b^1♂ 50 p37. 1. 2(3b)	b^2♂ 60 q58.55. 1(3e)	…	…
	b^3♂ 100xr 41.41.50(5–6g)	b^4♂ 00 p – – –(alb.)	b^5♂ 00 p – – –(alb.)	b^6♀ 100xq58.57.55(6e)	…	…
	b^7♀ – q – – –(5–6c:ew)	…	…	…	…	…
1*H*112	a^1 10 q 47. 1. 1(1cw)	a^2♀ 35 q44. 3. 1(2cw)	a^3 – p – – –(2f)	a^4♂ 00 p – – –(alb.)	0*H*75a^5 ♂ 15 q49. 1. 1(1c)	0*H*61a^6 ♀ 65 q49.30. 7(3c)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	b^1♀ 30 p26. 1. 1(2b–f)	b^2♂ 35 q48. 1. 3(2c)	…	…
	b^3♂ 00 p – – –(alb.)	…	…	…	…	…
1*H*113	a^1♂ 45 p 37. 2. 2(2–3b–cw)	a^2♂ 65 q58. 1. 1(3e)	a^3♀ 00 p – – –(alb.)	a^4♀ 60 q48.36. 4(3cw)	0*H*120a^2♂ – q – – –(3c)	0*H*101a^7 ♀ – q – – –(4c)
	a^5♀ 95 q 49.48.12(4c)	b^1♂ 95 q48.46. 4(3–4c)	b^2♂ 75 q58.55. 2(3e)	b^3♂ 00 p – – –(alb.)	…	…
	b^4♀ 40 q 49. 1. 1(2c)	b^5♀ 25 q49. 1. 1(1c)	b^6♀ 00 p – – –(alb.)	…	…	…
1*H*114	a^1♀ 05 p 36. 1. 1(1b)	a^2♂ 10 q58. 1. 1(1e)	a^3 – p or r –(2b)	a^4 – p – – –(2b)	0*H*62a^1 ♂ – q – – –(2c)	0*H*6a^1 ♀ – q – – –(2c)
	a^5♂ 35 q 48. 1. 1(2c)	a^6♂ 00 p – – –(alb. w)	…	…	…	…
1*H*115	a^1♂ 30 q 47.46. 1(1c)	a^2 – q – – –(3c)	a^3♀ 85 q58.55. 1(4e)	a^4 – q – – –(2ew)	0*H*100a^1♂ 40 q49. 1. 1(3c)	0*H*74a^5 ♀ 80 q49.46. 2(4c)
	a^5♂ 00 p – – –(alb.)	b^1♀ 45 q58.57. 2(3e)	b^2♀ – q – – –(4e)	b^3♂ 65 q49.46. 3(4c)	…	…
	b^4 00 p – – –(alb.)	…	…	…	…	…
1*H*116	a^1♀ 40 p 36. 5. 5(3b)	a^2♂ 95 q58.57. 2(4e)	a^3♀ – q – – –(4e)	a^4♀ – q – – –(4c)	0*H*102a^3♂ 95 q48.46. 7(4c)	0*H*74a^6 ♀ 80 q49.48. 1(4c)
	a^5♂ – q – – –(4e)	a^6♂ 00 p – – –(alb.)	b^1♂ 40 q49.46. 3(3c)	b^2♂ 65 q48.46. 3(4c)	…	…
	b^3♂ 00 p – – –(alb.)	…	…	…	…	…
1*H*117	a^1♂ 60 q 49.46. 1(5c)	a^2♂ – q – – –(6e)	a^3♂ 00 p – – –(alb.)	b^1♀ 100xq41.41.50(5g)	0*H*102a^4♂ – q – – –(5c)	0*H*62a^5 ♀ 100zq 49.48.12(5c)
	b^2♀ – p – – –(3f)	b^3♀ 45 q58. 1. 1(3e)	b^4♂ 00 p – – –(alb.)	…	…	…
1*H*118	a^1♀ – p – – –(2b)	a^2♂ 55 q49. 1. 2(3c)	a^3♂ 40 q61.60. 1(3e)	a^4♀ 20 q48. 1. 1(2cw)	0*H*45a^3 ♂ – q – – –(2d)	0*H*62a^3 ♀ 45 q48.34. 3(3c)
	a^5♀ 45 q 49.46. 1(3cw)	b^1♂ – p – – –(3a–b)	b^2♂ – q – – –(3c)	b^3♂ 40 q48. 1. 1(3c-dw)	…	…
	b^4♀ 00 p – – –(alb.)	b^5♀ 00 p – – –(alb.)	b^6♂ 00 p – – –(alb.)	b^7♀ – q – – –(3–4e)	…	…
1*H*119	a^1♀ 100xq 60.60.45(6e)	a^2♀ 100zq 52.46.43(5c)	a^3♂ 55 q49. 1. 1(3c)	a^4 – q – – –(2c)	0*H*100a^4♂ 100zq 49.46. 7(5c)	0*H*62a^6 ♀ 100zq 49.46.12(5c)
	a^5♂ 00 p – – –(alb.)	a^6♂ 00 p – – –(alb. w)	b^1 00 p – – –(alb.)	b^2 00 p – – –(alb.)	…	…
	b^3 00 p – – –(alb.)	b^4 00 p – – –(alb.)	b^5 00 p – – –(alb.)	…	…	…
1*H*120	a^1♀ 100zp 37.34. 3(5b-cw)	a^2 00 p – – –(alb. w)	b^1♂ 95 q58.55. 2(4e)	b^2♂ 80 q48.46. 2(5c)	0*H*59a^2 ♂ 100zq 49.46. 7(6d)	0*H*62a^4 ♀ 100zq 49.46. 7(5c)
	b^3♂ 00 p – – –(alb.)	b^4♀ 100zq 49.46.12(5c)	b^5♀ 100zq 49.48.43(5c)	b^6 – q – – –(4c)	…	…
1*H*121	a^1 – q – – –(4c)	a^2 90 q61.60. 1(4e)	a^3♀ – q – – –(4d)	a^4 40 q48. 1. 1(2d)	0*H*102a^1♂ 70 q48.46. 2(4c)	0*H*110a^2 ♀ 60 q52.46. 2(3c)
	a^5♂ 55 q 48.46. 2(3c)	a^6♂ 45 p37. 1. 3(2aw)	a^7♀ 00 p – – –(alb.)	b^1 – q – – –(3c(w?))	…	…
	b^2♀ 75 p 39.34. 1(4a–b)	b^3♂ – p – – –(4b)	b^4♀ – q – – –(3c–d)	b^5 – p – – –(4a–b)	…	…
	b^6♂ – p – – –(3b)	…	…	…	…	…
1*H*122	a^1 00 p – – –(alb.)	a^2♀ – q – – –(3e)	a^3♂ – q – – –(4e)	a^4♂ 55 p23. 1. 1(3f)	0*H*102a^2♂ 55 q48. 1. 2(3c)	0*H*73a^5 ♀ – q – – –(3c)
	a^5♂ 25 q 46. 1. 1(2c)	a^6♂ – q – – –(2c)	a^7♀ – q – – –(3c)	b^1♀ 25 q49. 1. 1(1c)	…	…

MATING	OFFSPRING				PARENTS	
					Father	Mother
1H83	b^1 ♂ 00 p – – –(alb.)	b^2 ♀ – q – – –(5e)	b^3 ♀ – q – – –(5e)	b^4 ♂ – q – – (6e)	A12a^6 ♂ 00 p – – –(A×2)	0H77a^6 ♀ 100xq57.56.56 5
	b^5 ♀ 60 q 61.60. 2(4e)	… … …	… … …	… … …	… … …	… … …
1H84	a^1 – q – – –(4c:g)	a^2 – q – – –(3c:g)	a^3 – q – – –(3e)	… … …	A14a^2 ♂ 00 p – – –(A1)	0H77a^7 ♀ 70 q61.61. 1(4e)
						0H77a^8 ♀ q 5e
1H85	b^1 ♀ 00 p – – –(alb.)	b^2 ♀ 00 p – – –(alb.)	b^3 ♀ 00 p – – –(alb.)	b^4 ♂ 00 p – – –(alb.)	A14a^3 ♂ 00 p – – –(A1)	0H77a^8 ♀ q 5e
	b^5 ♂ 00 p – – –(alb.)	b^6 ♂ 00 p – – –(alb.)	… … …	… … …	… … …	… … …
1H86	a^1 ♂ – q – – –(5c)	a^2 ♂ – q – – –(5c)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ 00 p – – –(alb.)	A12a^7 ♂ 00 p – – –(A×2)	0H84a^1 ♀ – q – – –(5c)
1H87	a^1 ♀ 100xq 58.58.57 6	a^2 ♂ 00 p (alb.)	a^3 ♀ 00 p – (alb.)	a^4 ♀ 00 p – (alb.)	A12a^7 ♂ 00 p (A×2)	0H84a^2 ♀ 100zq 49.46. 4(5
	a^6 ♀ – q – – –(6c)	… … …	… … …	… … …	… … …	… … …
1H88	a^1 00 p – – –(alb.)	a^2 00 p – – –(alb.)	a^3 00 p – – –(alb.)	a^4 – q – – –(6e)	0H13a^1 ♂ 50 q46.46. 1(3c)	A15a^1 ♀ 00 p – – –(A×3)
						A15a^2 ♀ 00 p – – –(A×3)
	a^5 – q – – –(5c)	a^6 – q – – –(5c)	b^1 ♂ 00 p – – –(alb.)	b^2 ♀ 00 p – – –(alb.)	… … …	… … …
	b^3 ♀ 00 p – – –(alb.)	b^4 ♀ 00 p – – –(alb.)	b^5 00 p – – –(alb.)	b^6 ♀ – q – – –(6e)	… … …	… … …
1H90	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♂ – q – – –(6e)	… … …	0H7a^1 ♂ 100zq49.46. 7(5c)	1H6a^1 ♀ 00 p – – –(alb.)
						1H6a^2 ♀ 00 p – – –(alb.)
1H92	a^1 ♀ 50 q 49.46. 1(3c)	a^2 ♂ 60 q 58. 1. 1(4e)	a^3 ♂ 100xq57.57.55(6e)	a^4 ♂ 00 p – – –(alb.)	0H73a^6 ♂ – q – – –(3c)	A1a^9 ♀ 00 p – – –(A2 ??)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♀ 00 p – – –(alb.)	… … …	… … …	… … …	… … …
1H94	a^1 ♀ – q – – –(5dw)	a^2 ♂ – p – – –(6aw)	a^3 ♀ 00 p – – –(alb. w)	a^4 ♂ 00 p – – –(alb.)	0H84a ? ♂ – q – – –(5 ?)	0H84a ? ♀ – q – – –(5 ?)
1H95	a^1 ♀ q 3	a^2 ♂ – q 2	a^3 ♂ 100 p37.35 4(6*b	a^4 ♂ 100x q 41.41.50(6*	0H62a^9 ♂ q (2 or 3	0H62a^{1-9} ♀ q 48 or 49 (3 or 5
				r : p} b–c)}	or a^4}	
	a^5 ♂ 00 p – – –(alb. w)	a^0 00 p – – –(alb.)	… … …	… … …	… … …	… … …
1H96	a^1 ♂ 100zq 48.46. 4(5c)	a^2 ♂ 100zq 35.33.28(5–6b)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ 00 p – – –(alb.)	0H84a^7 ♂ 100zq35.33.28(5a)	0H12a^3 ♀ – q – – –(5a)
	b^1 ♀ 00 p – – –(alb.)	b^2 ♀ – p – – –(5aw)	b^3 ♀ 100zp 35.33.28(5a–b)	b^4 – q – – –(5a)	… … …	… … …
1H97	a^1 ♀ 100zq 33.30.28(5–6b)	b^1 ♀ 00 p – – –(alb.)	b^2 100zp 37.35. 6(5b)	b^3 – q – – –(5c)	0H111a^2 ♂ – q – – –(5a)	0H12a^5 ♀ – q – 5a)
	b^4 p 3b	… … …	… … …	… … …	… … …	… … …
1H98	a^1 ♂ q 4	a^2 ♂ 80 q61.57. 1(4e)	a^3 ♂ 100zq 49.46. 4 6	a^4 ♀ 100zq 49.46.30(5e)	0H111a^1 ♂ q 5d	0H7a^6 ♀ 100zq 48.46.12(5e)
	a^5 ♀ 100zq 48.46. 4(5–6c)	a^0 85 p26. 9. 3(5–6b)	a^7 00 p – – –(alb.)	b^1 80 p37.37. 2(5b)	… … …	…
	b^2 – p – – –(6f)	b^3 – q – – –(5c)	b^4 – q – – –(5c)	b^5 00 p – – –(alb.)		
	b^6 00 p – – –(alb.)	… … …	… … …	… … …	… … …	… … …
1H99	a^1 ♂ 100zp 39.35.28(6b)	a^2 – q – – –(5c)	a^3 ♂ 80 q48.46. 7(5d)	a^4 00 p – – –(alb.)	0H84a^8 ♂ – q – – –(5c)	0H82a^2 ♀ – q – – –(5c)
1H100	a^1 ♀ 00 p (alb. w)	a^2 ♂ 30 q46. 1. 1(2c)	a^3 ♂ q (2d)	a^4 ♀ 65 q49.46. 1(3d)	0H13a^4 ♂ 45 q52. 1. 1(3c)	0H82a^1 ♀ 75 q48.46.12(3c)
	a^5 ♀ 80 p 37. 1. 1(2b)	a^6 ♂ 85 p36.36. 1(4b)	b^1 ♂ 55 q49.46. 1(3c)	b^2 ♂ 60 q55.46. 1(4c)	… … …	…
	b^3 ♂ 00 p – – –(alb.)	b^4 ♀ 65 q58.58. 4(4e)	b^5 ♀ 00 p – – –(alb.)	b^6 ♀ 00 p – – –(alb.)	… … …	…
1H101	a^1 100xp 24.24.17(6f)	a^2 ♂ 00 p (alb. w)	a^3 ♂ 65 q61.60. 3(3e)	a^4 q (5e)	0H84a^7 ♂ 100zq 48.46.28(5c)	0H85a^4 ♀ q 5–6d
	a^5 ♀ – q – – –(6e)	a^7 75 49.46. 9	b^1 ♂ 00 p – – –(alb.)	b^2 ♀ 100xq49.48.46(6c)	… … …	… … …
	b^3 ♀ 100xq 58.58.55(6ew)	b^4 ♂ 00 p – – –(alb.)	b^5 ♀ 00 p – – –(alb.)	b^6 ♀ – p – – –(6*b–g)	… … …	… … …
1H102	a^1 ♂ 100zq 52.51. 9(5dw)	a^2 p 5bw	a^3 ♂ 100zq 37.37. 4(5b	a^4 ♀ 100zq 58.57.57(6	0H20a^3 ♂ 100zq 48.46. 7(5d)	0H85a^3 ♀ q 5d
	a^5 q – – –(1cw)	a^0 00 p – – –(alb. w)	b^1 ♂ 25 p37. 1. 1(1–2b)	b^2 ♂ – p – – –(6f)	… … …	… … …
	b^3 ♂ – p – – –(6fw)	b^4 ♀ 100xp24.24.15(6fw)	b^5 – q – – –(3c)	… … …	… … …	… … …
1H103	a^1 ♀ 55 q 34.34. 1(3bw)	a^2 – p – – –(3bw)	a^3 ♀ – q – – (4b–c)	b^1 ♀ 60 dr? 40.39.1(4b–c)	0H120a^4 ♂ 60 q49.46. 1(3c)	0H85a^1 ♀ q (4d)
	b^2 ♀ 55 p or dr} 40.35. 1(4b)	b^3 ♀ 80 p36.34. 3(4b)	b^4 ♀ 65 q61.60. 1(3e)	b^5 – q – – –(3b)	… … …	… … …
	b^6 – q – – –(3b–c)	… … …	… … …			
1H104	a^1 ♀ 00 p – – –(alb.)	a^2 ♂ – q – – –(2e)	a^3 ♀ 50 p37.36. 3(3b)	a^4 ♀ 00 p – – –(alb.)	0H120a^7 ♂ 50 q48.46. 3(3c)	0H120a^3 ♀ 45 q49. 1. 1(2c)
	a^5 – q – – –(2cw)	b^1 35 q46. 1. 1(2cw)	b^2 ♂ – q – – –(3b–c)	b^3 20 q47. 1. 1(1cw)	… … …	… … …
	b^4 00 p (alb.)	…	…	…	… … …	… … …
1H105	a^1 ♂ 40 p 37. 1. 1(3b)	a^2 30 q44. 1. 1(2c)	a^3 ♂ 50 p23.23. 1(3f)	a^4 ♂ 40 p37. 1. 1(2b)	0H120a^2 ♂ – q – – –(3c)	0H120a^6 ♀ 75 q49.46. 1(4c)
	b^1 60 p 37.37.33(3b)	b^2 00 p – – –(alb. w)	b^3 75 p37.33. 1(4b)	b^4 – p – – –(4b)		… … …
	b^5 q (3	b^6 – q – – –(4e)	… … …	… … …	… … …	… … …
1H106	a^1 60 q 48.46. 7(3cw)	b^1 60 q44. 1. 1(3c)	b^2 35 p26. 1. 2(2f)	b^3 55 q44.47.29(3dw)	0H120a^1 ♂ – q – – –(4c)	0H1a^0 ♀ q – –(3c)
	b^4 ♂ 00 p (alb.	…	… … …	… …	… … …	… … …
1H107	a^1 ♂ 95dr 37.37.28(4b	a^2 ♀ 30 p37. 1. 1(1–2b)	… … …	… … …	0H110a^1 ♂ 60 q48.46. 1(3c)	0H16a^5 ♀ – q – – –(4c)
1H108	a^1 p 3b	a^2 25 q60. 1. 1(2e)	a^3 00 p – – –(alb.)	a^4 00 p – – –(alb.)	0H101a^1 ♂ 45 q49. 1. 1(2c)	0H9a^5 ♀ 25 q46. 1. 1(2c)
	a^5 00 p – – –(alb.)	…	… … …	… … …	… … …	… … …
1H109	a^1 p – – –(2bw)	a^2 70 p37.37. 4(3bw)	a^3 – q – – –(2c)	a^4 00 p – – –(alb.)	0H101a^2 ♂ 50 q49.46. 4(3c)	0H101a^6 ♀ 45 q49.46. 1(3c)
	a^5 00 p – – –(alb.)	a^0 00 p – – –(alb. w)	b^1 ♂ 25 q58. 1. 2(2e)	b^2 ♂ q – – (3c)	… … …	… … …
	b^3 ♂ 00 p – – –(alb.)	b^4 ♂ 00 p – – –(alb.)	b^5 00 p – – –(alb.)	… …	… … …	… … …
1H110	a^1 85 p 39.33. 3(5–6a	a^2 100zp37.39.34(6b)	… … …	… … …	0H101a^4 ♂ – q – – –(5c)	0H101a^8 ♀ q – –(5c)
	bw)					
1H111	a^1 ♂ 100xp 19.19. 9(6a)	a^2 80 p26.37. 2(5bw)	a^3 ♂ 100zq 48.46. 7(5c)	a^4 – q – – –(6c)	0H101a^3 ♂ – q – – –(5c)	0H61a^7 ♀ – q – – –(5c)
	a^5 ♂ 95 p 35. 1. 3(4bw)	a^0 – p – – –(4b)	b^1 ♂ 50 p37. 1. 2(3b)	b^2 ♂ 60 q58.55. 1(3e)	… … …	… … …
	b^3 ♂ 100xr 41.41.50(5–6g)	b^4 ♂ 00 p – – –(alb.)	b^5 ♂ 00 p – – –(alb.)	b^6 ♀ 100xq58.57.55(6e)	… … …	… … …
	b^7 ♀ q – – –(5–6c	… … …	… … …	… … …	… … …	… … …
	cw)					
1H112	a^1 10 q 47. 1. 1(1cw)	a^2 ♀ 35 q44. 3. 1(2cw)	a^3 – p – – –(2f)	a^4 ♂ 00 p – – –(alb.)	0H75a^5 ♂ 15 q49. 1. 1(1c)	0H61a^6 ♀ 65 q49.30. 7(3c)
	a^5 00 p – –(alb.)	a^6 00 p – – –(alb.)	b^1 ♀ 30 p26. 1. 1(2b–f)	b^2 ♂ 35 q48. 1. 3(2c)	… … …	… … …
	b^3 ♂ 00 p – –(alb.)					
1H113	a^1 ♂ 55 p 37. 2. 2(2–3b	a^2 ♂ 65 q58. 1. 1(3e)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ 60 q48.36. 4(3cw)	0H120a^2 ♂ – q – – –(3c)	0H101a^7 ♀ – q – – –(4c)
	cw)					
	a^5 ♀ 90 q 49.48.12(4c)	b^1 ♂ 95 q48.46. 4(3–4c)	b^2 ♂ 75 q58.55. 2(3e)	b^3 ♂ 00 p – – –(alb.)	… … …	… … …
	b^4 ♀ 40 q 49. 1. 1(2c)	b^5 ♀ 25 q49. 1. 1(1c)	b^6 ♀ 00 p – – –(alb.)			
1H114	a^1 ♀ 05 p 36. 1. 1(1b)	a^2 ♂ 10 q58. 1. 1(1e)	a^3 – p or r –(2b)	a^4 – p – – –(2b)	0H62a^1 ♂ – q – – –(2c)	0H6a^1 ♀ – q – – –(2c)
	a^5 ♂ 55 q 48. 1. 1(2c)	a^6 ♂ 00 p – – –(alb. w)	… … …	… …	… … …	… … …
1H115	a^1 ♂ 30 q 47.46. 1(1c)	a^2 – q – – –(3c)	a^3 ♀ 85 q58.55. 1(4e)	a^4 q – – –(2ew)	0H100a^1 ♂ 40 q49. 1. 1(3c)	0H74a^5 ♀ 80 q49.46. 2(4c)
	a^5 ♂ 00 p – – –(alb.)	b^1 ♀ 45 q58.57. 2(3e)	b^2 ♀ – q – – –(4e)	b^3 ♂ 65 q49.46. 3(4c)	… … …	… … …
	b^3 00 p – – –(alb.)					
1H116	a^1 ♀ 40 p 36. 5. 5(3b)	a^2 ♂ 95 q58.57. 2(4e)	a^3 ♀ – q – – –(4e)	a^4 ♀ q – – –(4c)	0H102a^3 ♂ 95 q48.46. 7(4c)	0H74a^0 ♀ 80 q49.48. 1(4c)
	a^5 ♂ q 4e	a^6 ♂ 00 p – – –(alb.)	b^1 ♂ 40 q49.46. 3(3c)	b^2 ♂ 65 q48.46. 3(4c)	… … …	… … …
	b^2 ♂ 00 p alb					
1H117	a^1 ♂ 60 q 49.46. 1(5e	a^2 ♂ – q – – –(6e)	a^3 ♂ 00 p – – –(alb.)	b^1 ♀ 100xq41.41.50(5g)	0H102a^4 ♂ – q – – –(5c)	0H62a^5 ♀ 100zq 49.48.12(5c)
	b^2 ♀ p (3f)	b^3 ♀ 45 q58. 1. 1(3e)	b^4 ♂ 00 p – – –(alb.)	… … …	… … …	… … …
1H118	a^1 ♀ p (2b)	a^2 ♀ 55 q49. 1. 2(3c)	a^3 ♂ 40 q61.60. 1(3e)	a^4 ♀ 20 q48. 1. 1(2cw)	0H45a^3 ♂ – q – – –(2d)	0H62a^3 ♀ 45 q48.34. 3(3c)
	a^5 ♀ 45 q 49.46. 1(3cw)	b^1 ♂ – p – – –(3a–b)	b^2 ♀ q 3	b^3 ♂ 40 q48. 1. 1(3c–dw)		… … …
	b^4 ♀ 00 p alb.)	b^5 ♀ 00 p – – –(alb.)	b^6 ♂ 00 p – – –(alb.)	b^7 ♀ – q – 3(4c)	… … …	… … …
1H119	a^1 ♀ 100xq 60.60.45(6e	a^2 ♀ 100zq 52.46.43(5	a^3 ♂ 55 q49. 1. 1(3c)	a^4 – q – – –(2c)	0H100a^4 ♂ 100zq 49.46. 7(5c)	0H62a^6 ♀ 100zq49.46.12(5c)
	a^5 ♂ 00 p – – –(alb.)	a^0 ♂ 00 p – – –(alb. w)	b^1 00 p – – –(alb.)	b^2 00 p – – –(alb.)	… … …	… … …
	b^3 00 p – – –(alb.)	b^4 00 p – – –(alb.)	b^5 00 p – – –(alb.)		… … …	… … …
1H120	a^1 ♀ 100 p 37.34. 3(5b–cw)	a^2 00 p – – –(alb. w)	b^1 ♂ 95 q58.55. 2(4c)	b^2 ♂ 80 q48.46. 2(5c)	0H59a^2 ♂ 100zq49.46. 7(6d)	0H62a^4 ♀ 100zq49.46. 7(5c)
	b^3 ♂ 00 p – – –(alb.)	b^1 ♀ 100zq 49.46.12(5c)	b^5 ♀ 100zq 49.48.43(5c)	b^6 – q – – –(4c)	… … …	… … …
1H121	a^1 – q – 1	a^2 90 q61.60. 1(4e)	a^3 ♀ – q – – –(4d)	a^4 40 q48. 1. 1(2d)	0H102a^1 ♂ 70 q48.46. 2(4c)	0H110a^2 ♀ 60 q52.46. 2(3c)
	a^5 ♂ 55 q 48.46. 2(3c)	a^6 ♂ 45 p37. 1. 3(2aw)	a^7 ♀ 00 p – – –(alb.)	b^1 – q – – –(3c(w?))	… … …	… … …
	b^2 ♀ 75 p 39.34. 1(4a–b)	b^3 ♂ – p – – –(4b)	b^4 ♀ – q – – –(3c–d)	b^5 – p – – –(4a–b)	… … …	… … …
	b^6 ♂ p – – –(3b)					
1H122	a^1 00 p – – –(alb.)	a^2 ♀ q 3	a^3 ♂ – q – – –(4e)	a^4 ♂ 55 p23. 1. 1(3f)	0H102a^2 ♂ 55 q48. 1. 2(3c)	0H73a^5 ♀ – q – – –(3c)
	a^5 ♂ 25 q 46. 1. 1(2c)	a^6 ♂ q 2	a^7 ♀ – q – – –(3c)	b^1 ♀ 25 q49. 1. 1(1c)	… … …	… … …

MATING	OFFSPRING				PARENTS	
					Father	Mother
1*H*122	b^2 ♀ 70 q49.46. 4(4c)	b^3 ♂ – p – – –(3a–b)	b^4 ♂ – p – – –(3a–b)	... ...	... ...	... ...
1*H*123	a^1 ♂ 80 q48.51. 4(5c)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 – q – – –(5c)	0*H*100a^3 ♂ – q – – –(5c)	0*H*100a^8 ♀ – q – – –(5c)
	b^1 ♂ 60 q58.55. 1(3e)	b^2 ♂ – q – – –(6e)	b^3 ♂ 100xq57.56.55(6e)	b^4 ♂ 80 q46.46. 4(5c–d)	... ...	... ...
	b^5 ♂ 100zp35.33.28(5–6b)	b^6 40 q44. 1. 1(3c)	... ...	... ...	... ...	... ...
1*H*124	a^1 ♂ – q – – –(3e)	a^2 ♀ 90 p37.37. 2(5b–c)	a^3 ♂ – q – – –(6e)	a^4 ♀ – q – – –(4c)	0*H*100a^2 ♂ – q – – –(5c)	0*H*111a^3 ♀ – q – – –(5d)
	a^5 – q – – –(5cw)	b^1 ♀ 65 q58. 1. 1(3–4e)	b^2 ♀ 00 p – – –(alb. w)	b^3 ♂ 65 q58.54. 2(4e)	... ...	... ...
	b^4 ♂ 85 p37.37. 2(5b)	b^5 – p – – –(5b)	... ...	... ...	... ...	... ...
1*H*125	a^1 ♀ 85 q53.51. 4(5c)	a^2 – q – – –(5c)	a^3 ♀ 80 q39.39. 4(5b–c)	a^4 – q – – –(6c)	0*H*85a^5 ♂ – q – – –(5d)	0*H*100a^7 ♀ – q – – –(5c)
	a^5 – q – – –(5c)	a^6 – p – – –(5bw)	a^7 – q – – –(3c)	... ...	... ...	... ...
1*H*126	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ – p – – –(2b)	a^3 ♂ – q – – –(1cw)	a^4 ♀ – q – – –(2c)	0*H*112a^1 ♂ 30 q49. 1. 1(2c)	0*H*45a^4 ♀ – q – – –(2d)
	a^5 ♂ – q – – –(3c)	a^6 – p – – –(2b)	a^7 – q – – –(1ew)	b^1 ♀ 25 p37.37. 5(2–3a–b)	... ...	... ...
	b^2 ♂ – q – – –(1–2ew)	b^3 ♀ 20 q49. 1. 1(2c)	b^4 ♂ 05 p37. 1. 1(1a)	b^5 ♂ 00 p – – –(alb.)	... ...	... ...
1*H*127	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 100zq47.51.20(5c)	a^3 ♂ 100zq48.46. 9(5c)	a^4 ♀ – q – – –(5e)	0*H*112a^2 ♂ – q – – –(5c)	0*H*102a^6 ♀ – q – – –(5c)
	a^5 – q – – –(5c)	b^1 ♀ – q – – –(3c)	b^2 ♀ 85 p37.36. 4(5-6bw)	b^3 ♀ 100xq41.41.50(6g)	... ...	... ...
	b^4 ♂ – p – – –(5–6b)	b^5 ♂ – p – – –(5–6b)	... ...	... ...	... ...	... ...
1*H*128	a^1 00 p – – –(alb. w)	a^2 ♂ 00 p – – –(alb. w)	a^3 ♂ 100xp26.26.17(6fw)	a^4 ♂ 100zp36.35.30(5a–f)	0*H*86a^2 ♂ 100zq49.46.28(5c)	0*H*102a^5 ♀ – q – – –(5c)
	a^5 ♂ 100zp37.35.30(4a–f)	a^6 ♀ – q – – –(4e)	a^7 85 q44.47. 2(5cw)	b^1 ♂ 100zp35.35. 4(5–6b)	... ...	... ...
	b^2 ♀ 00 p – – –(alb.)	... ...	... ...	... ...	... ...	... ...
1*H*129	a^1 – q – – –(5c)	a^2 ♂ – q – – –(6e)	a^3 ♀ 100xp24.23.15(6f)	a^4 ♀ 100zp39.35.30(5b)	0*H*86a^1 ♂ 100zq49.46. 9(5c)	0*H*102a^7 ♀ 100zq49.46.14(5c)
	b^1 20 q46. 1. 1(1–2c)	b^2 100zp37.37. 4(5bw)	b^3 95 q52.46. 7(5cw)	b^4 100xq60.60.54(5e)	... ...	... ...
	b^5 100xq60.60.54(6ew)	b^6 65 q61. 1. 1(4e)	b^7 00 p – – –(alb.)	... ...	... ...	... ...
1*H*130	a^1 ♀ 50 q61.60. 1(4ew)	a^2 ♂ 35 p19.34. 3(3bw)	a^3 ♀ 05 q47. 1. 2(1cw)	a^4 ♂ 00 p – – –(alb.)	0*H*104a^4 ♂ 40 q48.46. 2(3c)	0*H*94a^1 ♀ 60 q49.46. 1(3c)
	a^5 ♀ 00 p – – –(alb.)	b^1 ♂ 30 p37. 1. 1(2a–b)	b^2 ♀ – p – – –(2a–b)	b^3 ♂ 35 p39.35. 1(2a–b)	... ...	... ...
	b^4 ♂ 15 p37. 1. 3(2b)	b^5 ♂ – q – – –(3–4e)	b^6 ♀ 00 p – – –(alb.)	... ...	... ...	... ...
1*H*131	a^1 ♂ 45 q49.46. 1(3c)	a^2 ♀ 00 p – – –(alb.)	a^3 ♂ – q – – –(3c)	a^4 ♂ – q – – –(2e)	0*H*104a^3 ♂ 40 q46. 1. 1(3c)	*A*43a^1 ♀ 00 p – – –(*A*2 ?)
1*H*133	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 45 q61.60. 1(2e)	b^1 00 p – – –(alb.)	b^2 00 p – – –(alb.)	0*H*114a^6 ♂ – q – – –(5c)	0*H*114a^1 ♀ – q – – –(5c)
	b^3 100zq40.40.34(5c)	b^4 – q – – –(5c)	b^5 – q – – –(5c)	... ...	... ...	... ...
1*H*134	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb. w)	b^1 ♂ – q – – –(2c)	b^2 ♂ – q – – –(2c)	0*H*117a^6 ♂ 65 q48.46. 3(3c)	0*H*114a^2 ♀ 35 q49.46. 1(3c)
	b^3 ♂ – q – – –(2c)	b^4 ♂ 00 p – – –(alb.)	... ...	... ...	... ...	... ...
1*H*135	a^1 ♀ 100zp37.37. 2(5b?–c)	a^2 ♂ 30 q60. 1. 1(2e)	a^3 00 p – – –(alb. w)	a^4 – q – – –(6e)	0*H*113a^7 ♂ – q – – –(5c)	0*H*113a^4 ♀ 100zq49.46.12(5c)
	b^1 80 q53.51.14(5dw)	b^2 00 p – – –(alb.)	... ...	... ...	... ...	... ...
1*H*136	a^1 ♀ 100zq48.46. 9(5cw)	a^2 ♀ 100zp33.33.28(5a)	a^3 ♀ – q – – –(5a)	a^4 ♀ 35 p39. 1. 1(2b)	0*H*113a^8 ♂ – q – – –(5c)	0*H*84a^3 ♀ 100zq35.33.30(5a)
	a^5 ♂ 35 q49. 1. 3(2c)	b^1 ♀ 90 p39.35. 2(4a)	b^2 ♀ 100zp33.33.30(5–6a)	b^3 ♀ 100zp37.35.28(5*b)	... ...	... ...
1*H*137	a^1 ♀ 60 r48.48. 2(3g*)	b^1 ♀ – q – – –(3c–d)	b^2 00 p – – –(alb.)	b^3 00 p – – –(alb.)	0*H*117a^7 ♂ 90 q49.46. 9(4c)	0*H*113a^2 ♀ 50 q49.46. 2(3c)
1*H*138	a^1 ♂ 100zq48.46.11(5cw)	a^2 – q – – –(5c)	a^3 – q – – –(3c : g)	a^4 00 p – – –(alb. w)	0*H*107a^1 ♂ 100zq49.46.20(5c)	0*H*113a^6 ♀ 100zq49.36.20(5c)
	b^1 ♀ 85 p39.35. 2(5bw)	b^2 ♀ 100xq42.42.50(6gw)	b^3 ♂ 100zq49.47. 9(5c)	b^4 ♀ 00 p – – –(alb.)	... ...	... ...
	b^5 – p – – –(5b)	... ...	... ...	... ...	... ...	... ...
1*H*139	a^1 ♀ 60 p26.23. 2(3fw)	a^2 ♂ – p – – –(1–2f)	a^3 ♀ 00 p – – –(alb.)	b^1 ♂ 100xq58.57.55(6e)	0*H*107a^2 ♂ 55 q61.57. 1(3e)	0*H*78a^1 ♀ – q – – –(5e)
	b^2 ♂ 00 p – – –(alb. w)	... ...	... ...	... ...	... ...	... ...
1*H*140	a^1 55 q52.51. 1(3c)	a^2 100zq52.51. 9(6c)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ 00 p – – –(alb.)	*A*26a^1 ♂ 00 p – – –(*A*2 ?)	0*H*113a^5 ♀ 100zq48.46.43(5c)
	a^5 ♂ 00 p – – –(alb.)	a^6 00 p – – –(alb.)	... ...	... ...	... ...	... ...
1*H*142	a^1 ♀ 100xq58.57.55(6e)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	b^1 ♂ 80 q48.46. 2(5c)	0*H*103a^1 ♂ – q – – –(5c)	0*H*111a^5 ♀ – q – – –(5d)
	b^2 ♀ 80 q48.46. 3(5c)	b^3 ♀ 100xq58.57.55(6e)	b^4 – p – – –(6f)	b^5 – q – – –(4c)	... ...	... ...
1*H*143	a^1 ♂ 100zq48.46. 2(5c)	a^2 ♀ 100zq47.51.12(6c)	a^3 ♀ 100zq51.46.14(6c)	a^4 ♂ 00 p – – –(alb.)	0*H*103a^2 ♂ – q – – –(5c)	*A*43a^2 ♀ 00 p – – –(*A*2 ?)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♀ 00 p – – –(alb.)	b^1 ♀ 95 q46.33. 1(4c)	b^2 ♂ 00 p – – –(alb.)	... ...	... ...

	b^3 ♂ 100zq 49.46.14(5c)	b^4 00 p – – – (alb.)	...	...	...	...
1H144	a^1 ♂ – q – – – (2e)	a^2 ♂ 00 p – – – (alb.)	b^1 ♀ 00 p – – – (alb.)	b^2 ♂ – q – – – (5c)	0H114a^4 ♂ 50 q46.30. 5(3c)	A43a^3 ♀ 00 p – – – (A2?)
	b^3 ♂ 00 p – – – (alb.)	b^4 ♂ 00 p – – – (alb.)	...	...	...	...
1H145	a^1 ♂ 00 p – – – (alb. w)	a^2 ♀ 00 p – – – (alb.)	a^3 ♀ 00 p – – – (alb.)	a^4 ♀ 00 p – – – (alb.)	0H114a^5 ♂ 25 q49. 1. 2(2c)	0H112a^5 ♀ 40 q49. 1. 1(2c)
	a^5 ♂ 35 q51. 1. 2(2c)	b^1 ♀ – q – – – (1c)	b^2 ♂ – p – – – (1fw)	b^3 ♀ 20 q46. 1. 3(2c)	...	...
	b^4 ♂ 30 p37. 1. 1(2b)	...	...	...	...	...
1H146	a^1 ♀ 100zq 48.47. 9(5c)	a^2 ♀ 100xq 48.51.47(6c)	a^3 ♀ 00 p – – – (alb.)	a^4 ♀ 00 p – – – (alb.)	A26a^3 ♂ 00 p – – – (A2?)	0H112a^4 ♀ 35 q46. 1. 1(2c)
	a^5 ♀ 00 p – – – (alb.)	a^6 – q – – – (6e)	b^1 ♀ 100zq 52.46.20(5c–d)	b^2 ♀ 100zq 49.46.20(5c)	...	...
	b^3 ♀ 00 p – – – (alb.)	b^4 ♂ 00 p – – – (alb.)	b^5 – q – – – (5c)	...	...	...
1H147	a^1 ♀ 65 q44. 3. 3(3c)	a^2 ♀ 00 p – – – (alb.)	a^3 ♀ 00 p – – – (alb.)	a^4 ♀ 00 p – – – (alb.)	A33a^1 ♂ 00 p – – – (A2?)	0H112a^3 ♀ 50 q48.46. 1(3c)
	a^5 ♂ 00 p – – – (alb.)	...	...	...	...	...
1H148	a^1 ♂ 100xq 58.58.55(6e)	a^2 ♂ 80 q49.46. 4(5c)	a^3 ♀ 100zq 48.46. 4(5c)	a^4 ♀ 00 p – – – (alb.)	0H103a^3 ♂ – q – – – (5c)	0H111a^6 ♀ – q – – – (5d)
	a^5 80 q49.48. 3(5c)	a^6 ♀ 00 p – – – (alb.)	b^1 ♂ – p – – – (4f)	b^2 ♂ 00 p – – – (alb.)	...	...
	b^3 ♀ – q – – – (5c)	b^4 ♀ 00 p – – – (alb.)	b^5 ♂ 00 p – – – (alb. w)	...	...	...
1H149	a^1 ♂ – q – – – (5–6c)	a^2 ♀ 00 p – – – (alb.)	a^3 ♀ 00 p – – – (alb.)	a^4 ♀ 00 p – – – (alb.)	A46a^2 ♂ 00 p – – – (A2?)	0H111a^4 ♀ – q – – – (5d)
	a^5 ♂ 00 p – – – (alb.)	a^6 – q – – – (5c)	b^1 ♂ 00 p – – – (alb.)	b^2 ♂ 00 p – – – (alb.)	...	...
	b^3 ♂ 100zq 49.46.20(5d)	b^4 ♂ – q – – – (6d)	b^5 ♀ 00 p – – – (alb.)	b^6 ♀ 100zq 49.48.12(5d)	...	...
1H150	a^1 ♂ – q – – – (4cw)	a^2 ♂ 100zq 48.46. 7(4c)	a^3 – q – – – (5c)	b^1 ♂ – q – – – (4c)	0H90a^2 ♂ – q – – – (5c)	0H117a^4 ♀ 90 q49.36. 1(4c)
	b^2 ♂ – q – – – (w)	b^3 ♀ – q – – – (5c)	...	...	...	...
1H151	a^1 ♀ 100zq 49.48.34(5c)	a^2 ♀ – q – – – (6c)	a^3 ♀ – q – – – (6c)	a^4 ♀ 80 q49.48.34(5c)	A46a^3 ♂ 00 p – – – (A2?)	0H117a^5 ♀ 65 q49.36. 4(3c)
	a^5 ♂ 00 p – – – (alb.)	b^1 ♀ 100zq 49.48.47(6c)	b^2 ♀ – q – – – (5c)	b^3 ♀ 00 p – – – (alb.)	...	...
	b^4 ♂ 00 p – – – (alb.)	b^5 ♂ 00 p – – – (alb.)	...	...	...	...
1H152	a^1 ♀ 100zq 49.48.47(6c)	a^2 ♀ 00 p – – – (alb.)	a^3 ♀ 00 p – – – (alb.)	a^4 ♂ 00 p – – – (alb.)	A46a^1 ♂ 00 p – – – (A2?)	0H103a^4 ♀ 55 q49. 1. 3(3c)
	a^5 ♀ 00 p – – – (alb.)	a^6 ♂ 00 p – – – (alb.)	c^1 ♂ 00 p – – – (alb.)	c^2 ♀ 00 p – – – (alb.)	...	...
	c^3 ♀ 00 p – – – (alb.)	c^4 ♀ 00 p – – – (alb.)	c^5 ♂ 80 q48.46. 4(5c)	c^6 ♂ – q – – – (5c)	...	...
1H153	a^1 ♂ 75 q40.34. 1(4b–c)	a^2 ♀ 75 q61.60. 1(3e)	a^3 ♂ 00 p – – – (alb.)	a^4 ♂ 00 p – – – (alb.)	A25a^3 ♂ 00 p – – – (A2??)	0H103a^5 ♀ 55 q49.46. 4(3c)
	a^5 00 p – – – (alb.)	...	...	...	...	...
1H154	a^1 ♀ 100xq 58.58.55(6e)	a^2 ♂ – q – – – (6e)	a^3 ♂ – q – – – (5b–c)	a^4 ♂ 00 p – – – (alb.)	A25a^4 ♂ 00 p – – – (A2??)	0H103a^6 ♀ 50 q49.46. 3(3c)
	a^5 ♀ 00 p – – – (alb.)	...	...	...	...	...
1H155	a^1 ♂ 90 q 52.46. 1(4c)	a^2 ♂ 85 q49.46. 4(5c)	a^3 ♂ 00 p – – – (alb.)	a^4 ♂ 00 p – – – (alb.)	A32a^7 ♂ 00 p – – – (A2?)	0H100a^5 ♀ 25 q46. 1. 1(2c)
	a^5 – q – – – (6e)	a^6 – q – – – (4c)	...	...	...	...
1H156	a^1 ♀ 100xq 58.57.55(6e)	a^2 ♀ 00 p – – – (alb.)	a^3 ♀ 00 p – – – (alb.)	a^4 ♂ 00 p – – – (alb.)	A32a^8 ♂ 00 p – – – (A2?)	0H100a^6 ♀ 55 q49.46. 9(3c)
1H157	a^1 ♀ – q – – – (5a)	a^2 ♀ – q – – – (5a)	a^3 ♀ 100zq 49.48.14(5d)	a^4 ♀ 00 p – – – (alb.)	A48a^4 ♂ 00 p – – – (A2!?)	0H84a^4 ♀ 100zq 35.33.30(5a)
	a^5 ♂ 00 p – – – (alb.)	a^6 ♀ 00 p – – – (alb.)	...	...	...	...
1H158	a^1 ♂ – q – – – (5c)	a^2 ♀ 100zq 49.48. 9(5c–d)	a^3 ♀ 100zq 49.48.34(5c)	a^4 ♀ 00 p – – – (alb.)	A81a^2 ♂ 00 p – – – (A2??)	0H84a^5 ♀ 100zq 33.30.28(5a)
	a^5 00 p – – – (alb.)	a^6 00 p – – – (alb.)	b^1 ♂ 100zq 46.46. 9(5c)	b^2 ♂ 00 p – – – (alb.)	...	...
	b^3 ♂ 100zq 33.33. 4(5a)	b^4 ♂ 100zq 35.30.28(5a)	b^5 ♀ 100zq 49.48.14(5c)	b^6 ♀ – q – – – (5c)	...	...
1H159	a^1 ♀ – q – – – (5c)	a^2 ♀ 00 p – – – (alb.)	a^3 ♂ 00 p – – – (alb.)	...	A81a^3 ♂ 00 p – – – (A2??)	0H84a^6 ♀ 100zq 39.33.30(5a)
1H160	a^1 00 p – – – (alb.)	a^2 00 p – – – (alb.)	a^3 – q – – – (5c)	a^4 – q – – – (5c)	A93a^2 ♂ 00 p – – – (A2×)	0H111a^7 ♀ – q – – – (5a)
	a^5 – q – – – (5c)	a^6 – q – – – (5c)	...	...	...	...
1H161	a^1 ♂ 100zq 35.33.28(5a–b)	a^2 ♂ 100zq 48.46. 7(5c)	a^3 ♀ 100zq 49.48. 9(5c)	a^4 ♂ 00 p – – – (alb.)	A28a^1 ♂ 00 p – – – (A2?)	0H111a^8 ♀ 100zq 35.35.30(5a)
	a^5 ♂ 00 p – – – (alb.)	a^6 ♂	...	...	...	...
1H162	a^1 ♀ – q – – – (5c)	a^2 ♂ – q – – – (5a–b)	a^3 ♀ 00 p – – – (alb.)	...	A34a^1 ♂ 00 p – – – (A×3)	0H111a^9 ♀ 100zq 35.34.29(5a)
1H163	a^1 ♂ – q – – – (5a–b)	a^2 ♂ – q – – – (5a)	a^3 ♀ – q – – – (4c)	a^4 ♂ – q – – – (4c)	A81a^4 ♂ 00 p – – – (A2??)	0H111a^{10} ♀ – q – – – (5a)
1H164	a^1 ♂ 100xq 58.58.55(6e)	a^2 ♂ 100zq 36.36. 9(5c)	a^3 ♂ 00 p – – – (alb.)	b^1 ♀ – q – – – (6e)	0H120a^4 ♂ 60 q49.46. 1(3c)	A133a^1 ♀ 00 p – – – (A×2)
	b^2 ♂ – q – – – (6e)	b^3 ♂ 00 p – – – (alb.)	b^4 ♀ 00 p – – – (alb.)	b^5 ♀ 00 p – – – (alb.)	...	...
	b^6 ♀ 00 p – – – (alb.)	b^7 ♂ 00 p – – (alb.)	...	...	...	...
1H165	a^1 ♂ 80 q49.46.11(5c)	a^2 ♂ 00 p – – – (alb.)	...	...	0H101a^2 ♂ 50 q49.46. 4(3c)	A133a^2 ♀ 00 p – – – (A×2)
1H166	a^1 ♂ 00 p – – – (alb.)	a^2 ♂ 00 p – – – (alb.)	a^3 ♂ 00 p – – – (alb.)	a^4 ♂ – q – – – (5c)	0H120a^1 ♂ – q – – – (4c)	A133a^3 ♀ 00 p – – – (A×2)
	a^5 ♂ – q – – – (5c)	a^6 ♂ 00 p – – – (alb.)	a^7 ♀ 00 p – – – (alb.)	a^8 – q – – – (5d)	...	...

Mating	Offspring				Parents	
					Father	Mother
1H122	b^2♀ 70 q49.46. 4(4c)	b^3♂ – p – – –(3a–b)	b^4♂ – p – – –(3a–b)	... ...	... ...	... ...
1H123	a^1♂ 80 q48.51. 4(5c)	a^2♂ 00 p – – –(alb.)	a^3♀ 00 p – – –(alb.)	a^4 – q – – –(5c)	0H100a^3♂ – q – – –(5c)	0H100a^8♀ – q – – –(5c)
	b^1♂ 60 q58.55. 1(3e)	b^2♂ – q – (6e)	b^3♂ 100xq57.56.55(6e)	b^4♂ 80 q46.46. 4(5c–d)	... ...	... ...
	b^5♂ 100zp35.33.28(5–6b)	b^6 40 q44. 1. 1(3c)	... ...	... ...	... ...	... ...
1H124	a^1♂ – q (3e)	a^2♀ 90 p37.37. 2(5b–c)	a^3♂ – q – (6e)	a^4♀ q – – (4c)	0H100a^2♂ – q – – –(5c)	0H111a^9♀ – q – – (5d)
	a^5 – q – – –(5cw)	b^1♀ 65 q58. 1. 1(3–4e)	b^2♀ 00 p – – –(alb. w)	b^1♂ 65 q58.54. 2(4e)	... ...	... ...
	b^4♂ 85 p37.37. 2(5b)	b^5 – p – – –(5b)	... ...	... ...	... ...	... ...
1H125	a^1♀ 85 q53.51. 4(5c)	a^2 – q – –(5c)	a^3♀ 80 q39.39. 4(5b–c)	a^4 q – – –(6c)	0H85a^5 ♂ – q – – –(5d)	0H100a^7♀ – q – (5c)
	a^5 – q – – –(5c)	a^6 – p – (5bw)	a^7 – q – –(3c)	... ...	... ...	... ...
1H126	a^1♂ 00 p – – –(alb.)	a^2♀ – p – – –(2b)	a^3♂ – q – – –(1cw)	a^4♀ – q – – –(2c)	0H112a^1♂ 30 q49. 1. 1(2c)	0H45a^4 ♀ – q – – (2d)
	a^5♂ – q – – (3c)	a^6 p – – (2b)	a^7 – q – – –(1ew)	b^1♀ 25 p37.37. 5(2–3a–b)	... ...	... ...
	b^2♂ – q – – –(1–2ew)	b^3♀ 20 q49. 1. 1(2c)	b^4♂ 05 p37. 1. 1(1a)	b^6♂ 00 p – – –(alb.)	... ...	... ...
1H127	a^1♀ 00 p – – (alb.)	a^2♀ 100zq47.51.20(5c)	a^3♂ 100zq48.46. 9(5c)	a^4♀ q – (5c)	0H112a^2♂ q – – –(5c)	0H102a^6♀ q – – (5c)
	a^5 – q – – –(5c)	b^1♀ – q – – –(3c)	b^2♀ 85 p37.36. 4(5-6bw)	b^3♀ 100xq41.41.50(6g)	... ...	... ...
	b^4♂ – p – – –(5–6b)	b^5♂ – p – – –(5–6b)	... ...	... ...	... ...	... ...
1H128	a^1 00 p – (alb. w)	a^2♂ 00 p – – (alb. w)	a^3♂ 100vp26.26.17(6fw)	a^4♂ 100zp36.35.30(5a–f)	0H86a^2 ♂ 100zq49.46.28(5c)	0H102a^5♀ – q – – (5c)
	a^5♂ 100zp37.35.30(4a–f)	a^6♀ – q – – –(4e)	a^7 85 q44.47. 2(5cw)	b^1♂ 100zp35.35. 4(5–6b)	... ...	... ...
	b^2♀ 00 p – – –(alb.)	... ...	... ...	... ...	... ...	... ...
1H129	a^1 – q – – –(5c)	a^2♂ – q – – –(6e)	a^3♀ 100xp24.23.15(6f)	a^4♀ 100zp39.35.30(5b)	0H86a^1 ♂ 100zq49.46. 9(5c)	0H102a^7♀ 100zq49.46.14(5c)
	b^1 20 q46. 1. 1(1–2c)	b^2 100zp37.37. 4(5bw)	b^3 95 q52.46. 7(5cw)	b^4 100xq60.60.54(5e)	... ...	... ...
	b^5 100xq60.60 54(6ew)	b^6 65 q61. 1. 1(4e)	b^7 00 p – – –(alb.)	... ...	... ...	... ...
1H130	a^1♀ 50 q61.60. 1(4ew)	a^2♂ 35 p19.34. 3(3bw)	a^3♀ 05 q47. 1. 2(1cw)	a^4♂ 00 p – – –(alb.)	0H104a^4♂ 40 q48.46. 2(3c)	0H94a^1 ♀ 60 q49.46. 1(3c)
	a^5♀ 00 p – – –(alb.)	b^1♂ 30 p37. 1. 1(2a–b)	b^2♀ – p – – –(2a–b)	b^3♂ 35 p39.35. 1(2a–b)	... ...	... ...
	b^4♂ 15 p37. 1. 3(2b)	b^5♂ – q – – –(3 4e)	b^6♀ 00 p – – –(alb.)	... ...	... ...	... ...
1H131	a^1♂ 45 q49.46. 1(3c)	a^2♀ 00 p – – –(alb.)	a^3♂ – q – – –(3c)	a^4♂ – q – – –(2e)	0H104a^3♂ 40 q46. 1. 1(3c)	A43a^1 ♀ 00 p – – –(A2 ?)
1H133	a^1♂ 00 p – – –(alb.)	a^2♂ 45 q61.60. 1(2e)	b^1 00 p – – –(alb.)	b^2 00 p – – –(alb.)	0H114a^9♂ – q – – –(5c)	0H114a^1♀ – q – – –(5c)
	b^3 100zq40.40.34(5c)	b^4 – q – – –(5c)	b^5 – q – – –(5c)	... ...	... ...	... ...
1H134	a^1♂ 00 p – –(alb.)	a^2♂ 00 p (alb. w)	b^1♂ – q – – –(2c)	b^2♂ q – –(2c)	0H117a^6♂ 65 q48.46. 3(3c)	0H114a^2♀ 35 q49.46. 1(3c)
	b^3♂ – q (2c)	b^4♂ 00 p – – –(alb.)	... ...	... ...	... ...	... ...
1H135	a^1♀ 100zp37.37. 2(5b?–c)	a^2♂ 30 q60. 1. 1(2e)	a^3 00 p (alb. w)	a^4 – q – (6e)	0H113a^7♂ – q – – –(5c)	0H113a^4♀ 100zq49.46.12(5c)
	b^1 80 q53.51.14(5dw)	b^2 00 p – – –(alb.)	... ...	... ...	... ...	... ...
1H136	a^1♀ 100zq48.46. 9(5cw)	a^2♀ 100zp33.33.28(5a)	a^3♀ – q – – –(5a)	a^4♀ 35 p39. 1. 1(2b)	0H113a^8♂ – q – – –(5c)	0H84a^3 ♀ 100zq35.33.30(5a)
	a^5♂ 35 q49. 1. 3(2c)	b^1♀ 90 p39.35. 2(4a)	b^2♀ 100zp33.33.30(5–6a)	b^3♀ 100zp37.35.28(5*b)	... ...	... ...
1H137	a^1♀ 60 r48.48. 2(3g*)	b^1♀ – q – – –(3c–d)	b^2 00 p (alb.)	b^3 00 p – –(alb.)	0H117a^7♂ 90 q49.46. 9(4c)	0H113a^2♀ 50 q49.46. 2(3c)
1H138	a^1♂ 100zq48.46.11(5cw)	a^2 – q – – –(5c)	a^3 – q – – –(3c : g)	a^4 00 p – – –(alb. w)	0H107a^1♂ 100zq49.46.20(5c)	0H113a^6♀ 100zq49.36.20(5c)
	b^1♀ 85 p39.35. 2(5bw)	b^2♀ 100xq42.42.50(6gw)	b^3♂ 100zq49.47. 9(5c)	b^4♀ 00 p – – –(alb.)	... ...	... ...
	b^5 – p – – –(5b)	... ...	... ...	... ...	... ...	... ...
1H139	a^1♀ 60 p26.23. 2(3fw)	a^2♂ – p – – –(1–2f)	a^3♀ 00 p – – –(alb.)	b^1♂ 100xq58.57.55(6e)	0H107a^2♂ 55 q61.57. 1(3e)	0H78a^1 ♀ – q – – –(5e)
	b^2♂ 00 p (alb. w)	... ...	... ...	... ...	... ...	... ...
1H140	a^1 55 q52.51. 1(3c)	a^2 100zq52.51. 9(6c)	a^3♀ 00 p (alb.)	a^4♀ 00 p – (alb.)	A26a^1 ♂ 00 p – – –(A2 ?)	0H113a^5♀ 100zq48.46.43(5c)
	a^5♂ 00 p – – –(alb.)	a^6 00 p – – –(alb.)	... ...	... ...	... ...	... ...
1H142	a^1♀ 100xq58.57.55(6e)	a^2♀ 00 p – – –(alb.)	a^3♀ 00 p – – –(alb.)	b^1♂ 80 q48.46. 2(5c)	0H103a^1♂ – q – – –(5c)	0H111a^5♀ – q – – –(5d)
	b^2♀ 80 q48.46. 3(5c)	b^3♀ 100xq58.57.55(6e)	b^4 p – – (6f)	b^5 q – (4c)	... ...	... ...
1H143	a^1♂ 100zq48.46. 2(5c)	a^2♀ 100zq47.51.12(6c)	a^3♀ 100zq51.46.14(6c)	a^4♂ 00 p – – –(alb.)	0H103a^2♂ – q – – –(5c)	A43a^2 ♀ 00 p – – –(A2 ?)
	a^5♀ 00 p – – –(alb.)	a^6♀ 00 p – – –(alb.)	b^1♀ 95 q46.33. 1(4c)	b^2♂ 00 p – – –(alb.)	... ...	... ...
	b^3♂ 100zq49.46.14(5c)	b^4 00 p – – –(alb.)				
1H144	a^1♂ – q – – –(2e)	a^2♂ 00 p – – –(alb.)	b^1♀ 00 p – – –(alb.)	b^2♂ – q – – –(5c)	0H114a^4♂ 50 q46.30. 5(3c)	A43a^3 ♀ 00 p – – –(A2 ?)
	b^3♂ 00 p – – –(alb.)	b^4♂ 00 p – – –(alb.)				
1H145	a^1♂ 00 p – – –(alb. w)	a^2♀ 00 p – – –(alb.)	a^3♀ 00 p – – –(alb.)	a^4♀ 00 p – – –(alb.)	0H114a^5♂ 25 q49. 1. 2(2c)	0H112a^5♀ 40 q49. 1. 1(2c)
	a^5♂ 35 q51. 1. 2(2c)	b^1♀ – q – (1c)	b^2♂ p – – (1fw)	b^3♀ 20 q46. 1. 3(2c)	... ...	... ...
	b^4♂ 30 p37. 1. 1(2b)	... ...	... ...	... ...	... ...	... ...
1H146	a^1♀ 100zq48.47. 9(5c)	a^2♀ 100xq48.51.47(6c)	a^3♀ 00 p – – –(alb.)	a^4♀ 00 p – – –(alb.)	A26a^3 ♂ 00 p – – –(A2 ?)	0H112a^4♀ 35 q46. 1. 1(2c)
	a^5♀ 00 p – – –(alb.)	a^6 – q – – –(6e)	b^1♀ 100zq52.46.20(5c–d)	b^2♀ 100zq49.46.20(5c)	... ...	... ...
	b^3♀ 00 p – –(alb.)	b^4♂ 00 p – – –(alb.)	b^5 – q – – –(5c)		... ...	... ...
1H147	a^1♀ 65 q44. 3. 3(3c)	a^2♀ 00 p – – –(alb.)	a^3♀ 00 p – (alb.)	a^4♀ 00 p – – (alb.)	A33a^1 ♂ 00 p – – (A2 ?)	0H112a^3♀ 50 q48.46. 1(3c)
	a^5♂ 00 p – (alb.)	... ...	... ...	... ...	... ...	... ...
1H148	a^1♂ 100xq58.58.55(6e)	a^2♂ 80 q49.46. 4(5c)	a^3♀ 100zq48.46. 4(5c)	a^4♀ 00 p – – –(alb.)	0H103a^3♂ – q – – –(5c)	0H111a^6♀ – q – – –(5d)
	a^5 80 q49.48. 3(5c)	a^6♀ 00 p – – –(alb.)	b^1♂ – p – – –(4f)	b^2♂ 00 p – – –(alb.)	... ...	... ...
	b^3♀ – q – – –(5c)	b^4♀ 00 p – – –(alb.)	b^5♂ 00 p – (alb. w)		... ...	... ...
1H149	a^1♂ – q – – –(5–6c)	a^2♀ 00 p – – –(alb.)	a^3♀ 00 p – – –(alb.)	a^4♀ 00 p – – –(alb.)	A46a^2 ♂ 00 p – – –(A2 ?)	0H111a^4♀ – q – – (5d)
	a^5♂ 00 p – – –(alb.)	a^6 – q – – (5c)	b^1♂ 00 p – – (alb.)	b^2♂ 00 p – – (alb.)	... ...	... ...
	b^3♂ 100zq49.46.20(5d)	b^4♂ – q – – (6d)	b^5♀ 00 p – – (alb.)	b^6♀ 100zq49.48.12(5d)	... ...	... ...
1H150	a^1♂ – q – (4cw)	a^2♂ 100zq48.46. 7(4c)	a^3 q – – –(5c)	b^1♂ q – (4c)	0H90a^2 ♂ – q – – (5c)	0H117a^4♀ 90 q49.36. 1(4c)
	b^2♂ – q – – [illegible]	b^3♀ – q – – (5c)	... ...	... ...	... ...	... ...
1H151	a^1♀ 100zq49.48.34(5c)	a^2♀ – q – – –(6c)	a^3♀ – q – – –(6c)	a^4♀ 80 q49.48.34(5c)	A46a^3 ♂ 00 p – – (A2 ?)	0H117a^5♀ 65 q49.36. 4(3c)
	a^5♂ 00 p – – (alb.)	b^1♀ 100zq49.48.47(6c)	b^2♀ – q – – (5c)	b^3♀ 00 p – – –(alb.)	... ...	... ...
	b^4♂ 00 p – – (alb.)	b^5♂ 00 p – (alb.)	... ...	... ...	... ...	... ...
1H152	a^1♀ 100zq49.48.47(6c)	a^2♀ 00 p – – (alb.)	a^3♀ 00 p – – –(alb.)	a^4♂ 00 p – – (alb.)	A46a^1 ♂ 00 p – – (A2 ?)	0H103a^4♀ 55 q49. 1. 3(3c)
	a^5♀ 00 p – – (alb.)	a^6♂ 00 p – – (alb.)	c^1♂ 00 p – (alb.)	c^2♀ 00 p – – (alb.)	... ...	... ...
	c^3♀ 00 p – – –(alb.)	c^4♀ 00 p – – –(alb.)	c^5♂ 80 q48.46. 4(5c)	c^6♂ – q – – –(5c)	... ...	... ...
1H153	a^1♂ 75 q40.34 1(4b–c)	a^2♀ 75 q61.60. 1(3e)	a^3♂ 00 p – (alb.)	a^4♂ 00 p – – (alb.)	A25a^3 ♂ 00 p – (A2 ??)	0H103a^5♀ 55 q49.46. 4(3c)
	a^5 00 p – – (alb.)	... ...	... ...	... ...	... ...	... ...
1H154	a^1♀ 100xq58.58.55(6e)	a^2♂ – q – – –(6e)	a^3♂ – q – – –(5b–c)	a^4♂ 00 p – – –(alb.)	A25a^4 ♂ 00 p – – –(A2 ??)	0H103a^6♀ 50 q49.46. 3(3c)
	a^5♀ 00 p – – –(alb.)	... ...	... ...	... ...	... ...	... ...
1H155	a^1♂ 90 q52.46. 1(4c)	a^2♂ 85 q49.46. 4(5c)	a^3♂ 00 p – – –(alb.)	a^4♂ 00 p – – –(alb.)	A32a^7 ♂ 00 p – – –(A2 ?)	0H100a^5♀ 25 q46. 1. 1(2c)
	a^5 q – – –(6e)	a^6 – q – – –(4c)	... ...	... ...	... ...	... ...
1H156	a^1♀ 100xq58.57.55(6e)	a^2♀ 00 p – – –(alb.)	a^3♀ 00 p – – –(alb.)	a^4♂ 00 p – – –(alb.)	A32a^9 ♂ 00 p – – (A2 ?)	0H100a^6♀ 55 q49.46. 9(3c)
1H157	a^1♀ – q – (5a)	a^2♀ – q – (5a	a^3♀ 100zq49.48.14(5d)	a^4♀ 00 p – (alb.)	A48a^4 ♂ 00 p – – (A2 ??)	0H84a^4 ♀ 100zq35.33.30(5a)
	a^5♂ 00 p – alb.)	a^6♀ 00 p – (alb.)	... ...	... ...	... ...	... ...
1H158	a^1♂ – q – – –(5c)	a^2♀ 100zq49.48. 9(5c–d)	a^3♀ 100zq49.48.34(5c)	a^4♀ 00 p – – –(alb.)	A81a^2 ♂ 00 p – – (A2 ??)	0H84a^5 ♀ 100zq33.30.28(5a)
	a^5 00 p – – –(alb.)	a^6 00 p – (alb.)	b^1♂ 100zq46.46. 9(5c)	b^2♂ 00 p – – –(alb.)	... ...	... ...
	b^3♂ 100zq33.33. 4(5a)	b^4♂ 100zq35.30.28(5a)	b^5♀ 100zq49.48.14(5c)	b^6♀ – q – – –(5c)	... ...	... ...
1H159	a^1♀ – q – – –(5c)	a^2♀ 00 p – – –(alb.)	a^3♂ 00 p – – –(alb.)		A81a^3 ♂ 00 p – – (A2 ??)	0H84a^6 ♀ 100zq39.33.30(5a)
1H160	a^1 00 p – – (alb.)	a^2 00 p – (alb.)	a^3 q – (5c)	a^4 – q – – (5c)	A93a^2 ♂ 00 p – – (A2 ×)	0H111a^7♀ q – – –(5a)
	a^5 – q – – (5c)	a^6 – q – 5c)	... ...	... ...	... ...	... ...
1H161	a^1♂ 100zq35.33.28(5a–b)	a^2♂ 100zq48.46. 7(5c)	a^3♀ 100zq49.48. 9(5c)	a^4♂ 00 p – – –(alb.)	A28a^1 ♂ 00 p – – (A2 ?)	0H111a^8♀ 100zq35.35.30(5a)
	a^5♂ 00 p – – (alb.)	a^6♂	... ...	... ...	... ...	... ...
1H162	a^1♀ – q – – –(5c)	a^2♂ – q – – –(5a–b)	a^3♀ 00 p – – –(alb.)	... ...	A34a^1 ♂ 00 p – – –(A × 3)	0H111a^9♀ 100zq35.34.29(5a)
1H163	a^1♂ – q – – –(5a–b)	a^2♂ – q – – –(5a)	a^3♀ – q – – –(4c)	a^4♂ – q – – –(4c)	A81a^4 ♂ 00 p – (A2 ??)	0H111a^{10}♀ – q – – (5a)
1H164	a^1♂ 100xq58.58.55(6e)	a^2♂ 100zq36.36. 9(5c)	a^3♂ 00 p – – –(alb.)	b^1♀ – q – – –(6e)	0H120a^4♂ 60 q49.46. 1(3c)	A133a^1 ♀ 00 p – – (A × 2)
	b^2♂ – q – – –(6e)	b^3♂ 00 p – (alb.)	b^4♀ 00 p – – (alb.)	b^5♀ 00 p – – –(alb.)	... ...	... ...
	b^6♀ 00 p – – –(alb.)	b^7♂ 00 p – – (alb.)	... ...	... ...	... ...	... ...
1H165	a^1♂ 80 q49.46.11(5c)	a^2♂ 00 p – – –(alb.)			0H101a^2♂ 50 q49.46. 4(3c)	A133a^2 ♀ 00 p – – (A × 2)
1H166	a^1♂ 00 p – – –(alb.)	a^2♂ 00 p – – –(alb.)	a^3♂ 00 p – – –(alb.)	a^4♂ – q – – –(5c)	0H120a^1♂ – q – – (4c)	A133a^3 ♀ 00 p – – –(A × 2)
	a^5♂ – q – – (5c)	a^6♂ 00 p – – (alb.)	a^7♀ 00 p – (alb.)	a^8 – q – (5d)	... ...	... ...

MATING	OFFSPRING				PARENTS	
					Father	Mother
1*H*167	a^1 ♀ – *q* – – – (5*c*–*d*)	a^2 ♀ 100*xq*58.58.55(6*e*)	a^3 ♀ 100*zq*49.46.21(6*c*)	a^4 ♂ 100*zq*48.46.47(6*c*)	0*H*13a^4 ♂ 45 *q*52. 1. 1(3*c*)	*A*133a^4 ♀ 00 *p* – – – (*A* × 2)
	a^5 ♂ 100*zq*48.46.12(6*d*)	a^6 ♂ 00 *p* – – – (alb.)	a^7 ♀ 00 *p* – – – (alb.)	a^8 ♂ 00 *p* – – – (alb.)	... ...	... ...
1*H*168	a^1 ♂ 00 *p* – – – (alb.)	a^2 ♂ 00 *p* – – – (alb.)	a^3 ♂ 100*zq*39.33.28(4*a*)	a^4 ♀ 00 *p* – – – (alb.)	*A*133a^6 ♂ 00 *p* – – – (*A* × 2)	0*H*84a^4 ♀ 100*zq*35.33.30(5*a*)
	a^5 ♂ 100*zq*46.46. 9(5–6*c*)	a^6 ♀ 100*zq*33.33.28(5*a*)	a^7 ♀ 100*zq*35.30.28(5*a*)	... ...	... ...	... ...
1*H*169	a^1 ♀ 00 *p* – – – (alb.)	a^2 ♀ 00 *p* – – – (alb.)	a^3 ♂ 100*xq*58.58.55(6*e*)	a^4 ♀ 100*zq*49.48.47(5*c*–*d*)	*A*133a^7 ♂ 00 *p* – – – (*A* × 2)	0*H*112a^3 ♀ 50 *q*48.46. 1(3*c*)
	a^5 ♂ 80 *q*46.21. 7(5*c*)	a^6 ♀ 100*zq*48.46. 4(6*c*)	a^7 ♂ 00 *p* – – – (alb.)	a^8 ♀ – *q* – – – (3–4*c*)	... ...	... ...
1*H*170	a^1 ♀ 00 *p* – – – (alb.)	a^2 ♂ 85 *q*48.51. 2(5*c*–*d*)	a^3 ♂ 00 *p* – – – (alb.)	a^4 ♀ 100*xq*26.26.26(6*g*)	0*H*101a^4 ♂ – *q* – – – (5*c*)	*A*136a^1 ♀ 00 *p* – – – (*A* × 1)
	a^5 ♀ 85 *q*40.40. 4(5*c*)	... ...	... ...	... ...	... ...	... ...
1*H*171	a^1 ♂ 100*zq*52.48.47(6*c*–*d*)	a^2 ♀ 00 *p* – – – (alb.)	a^3 ♀ – *q* – – – (6*c*–*d*)	a^4 ♀ – *q* – – – (6*c*–*d*)	*A*136a^4 ♂ 00 *p* – – – (*A* × 1)	0*H*118a^2 ♀ – *q* – – – (5*c*)
	a^5 ♀ 00 *p* – – – (alb.)	... ...	... ...	... ...	... ...	... ...
1*H*172	a^1 ♀ 00 *p* – – – (alb.)	a^2 ♀ 100*xq*60.60.45(6*e*)	a^3 ♂ 00 *p* – – – (alb.)	a^4 – *q* – – – (3-4*c*:*e*)	*A*133a^6 ♂ 00 *p* – – – (*A* × 2)	0*H*1a^6 ♀ – *q* – – – (3*c*)
	a^5 ♂ – *q* – – – (4*c*)	... ...	... ...	... ...	... ...	... ...
1*H*173	a^1 ♂ 00 *p* – – – (alb.)	a^2 ♂ 00 *p* – – – (alb.)	a^3 ♀ 00 *p* – – – (alb.)	a^4 ♀ – *q* – – – (6*e*)	*A*179a^1 ♂ 00 *p* – – – (*A*2 ×)	0*H*62a^4 ♀ 100*zq*49.46. 7(5*c*)
	a^5 ♂ 100*xq*57.57.55(6*e*)	a^6 ♀ 100*zq*47.51. 9(5*c*)	... ...	... ...	... ...	... ...
1*H*175	a^1 ♀ 00 *p* – – – (alb.)	a^2 ♀ 00 *p* – – – (alb.)	a^3 ♀ 100*zq*47.51.25(5*d*)	a^4 ♀ 100*zq*51.51. 7(5*c*)	*A*179a^3 ♂ 00 *p* – – – (*A*2 ×)	0*H*102a^5 ♀ – *q* – – – (5*c*)
	a^5 ♂ 100*zq*40.36. 4(5*b*–*c*)	a^6 ♂ 100*zq*40.36. 9(5*b*–*c*)	a^7 ♂ – *q* – – – (4*e*)	... ...	... ...	... ...
1*H*178	a^1 ♀ 00 *p* – – – (alb.)	a^2 ♀ 00 *p* – – – (alb.)	a^3 ♂ 00 *p* – – – (alb).	a^4 ♂ 00 *p* – – – (alb.)	*A*150a^2 ♂ 00 *p* – – – (*A*1 or 2)	0*H*113a^4 ♀ 100*zq*49.46.12(5*c*)
	a^5 ♂ 00 *p* – – – (alb.)	a^6 ♀ 100*xq*51.51.47(5–6*c*)	a^7 ♀ 80 *q* 51.51. 2(5*c*)	a^8 ♀ – *q* – – – (2–3*c*)	... ...	... ...
1*H*179	a^1 ♀ 00 *p* – – – (alb.)	a^2 ♀ 100*xq*51.51.47(6*d*)	a^3 ♂ 100*zq*48.46. 7(5*c*)	a^4 ♂ 100*zq*52.47. 9(5*d*)	*A*150a^3 ♂ 00 *p* – – – (*A*1 or 2)	0*H*7a^6 ♀ 100*zq*48.46.12(5*c*)
	a^5 ♀ – *q* – – – (5*c*)	a^6 ♂ – *q* – – – (5*c*)	... ...	... ...	... ...	... ...
1*H*180	a^1 ♂ 00 *p* – – – (alb.)	a^2 ♂ 00 *p* – – – (alb.)	a^3 ♀ 100*zq*51.51. 4(5*c*)	a^4 ♀ 100*zq*51.46. 9(5*c*)	*A*150a^4 ♂ 00 *p* – – – (*A*1 or 2)	0*H*111a^3 ♀ – *q* – – – (5*d*)
	a^5 ♀ 100*zq*51.51. 9(5*c*)	a^6 ♀ 100*zq*52.51. 9(5*c*)	a^7 ♀ 100*xq*51.47.47(6*c*)	... ...	... ...	... ...
1*H*181	a^1 ♂ 00 *p* – – – (alb.)	a^2 ♂ 00 *p* – – – (alb.)	a^3 ♂ 00 *p* – – – (alb.)	a^4 ♂ 95 *q*58.57. 1(5*e*)	*A*150a^5 ♂ 00 *p* – – – (*A*1 or 2)	0*H*113a^6 ♀ 100*zq*49.36.20(5*c*)
	a^5 ♂ – *q* – – – (6*e*)	a^6 ♂ 80 *q*52.51. 4(5*c*)	... ...	... ...	... ...	... ...
1*H*182	a^1 ♀ – *q* – – – (6*e*)	a^2 ♂ 00 *p* – – – (alb.)	a^3 ♀ 00 *p* – – – (alb.)	a^4 ♂ 00 *p* – – – (alb.)	*A*153a^1 ♂ 00 *p* – – – (*A*1 or 2)	0*H*111a^5 ♀ – *q* – – – (5*d*)
	a^5 ♂ 85 *q*52.51. 4(5*c*)	a^6 ♂ 100*zq*52.51.34(5*c*)	... ...	... ...	... ...	... ...
1*H*183	a^1 ♀ 75 *q*51.25. 2(5*c*)	a^2 ♂ 80 *q*52.51. 7(5*c*)	a^3 ♂ 00 *p* – – – (alb.)	a^4 ♀ 00 *p* – – – (alb.)	*A*153a^2 ♂ 00 *p* – – – (*A*1 or 2)	0*H*100a^8 ♀ – *q* – – – (5*c*)
	a^5 ♀ 00 *p* – – – (alb.)	a^6 ♂ 00 *p* – – – (alb.)	... ...	... ...	... ...	... ...
1*H*184	a^1 ♂ 00 *p* – – – (alb.)	a^2 ♀ 00 *p* – – – (alb.)	a^3 ♀ 00 *p* – – – (alb.)	a^4 ♀ 00 *p* – – – (alb.)	*A*153a^3 ♂ 00 *p* – – – (*A*1 or 2)	0*H*102a^6 ♀ – *q* – – – (5*c*)
	a^5 ♀ 00 *p* – – – (alb.)	a^6 ♀ 00 *p* – – – (alb.)	a^7 ♀ 00 *p* – – – (alb.)	a^8 00 *p* – – – (alb.)	... ...	... ...
	a^9 – *q* – – – (6*e*)	... ...	... ...	... ...	... ...	... ...
1*H*185	a^1 ♀ – *q* – – – (4*e*)	a^2 ♀ – *q* – – – (6*e*)	a^3 ♀ – *q* – – – (6*e*)	a^4 ♂ – *q* – – – (6*e*)	*A*153a^4 ♂ 00 *p* – – – (*A*1 or 2)	0*H*61a^7 ♀ – *q* – – – (5*c*)
	a^5 ♂ – *q* – – – (6*e*)	a^6 ♂ 00 *p* – – – (alb.)	a^7 ♂ 00 *p* – – – (alb.)	a^8 – *q* – – – (6*e*)	... ...	... ...
	a^9 – *q* – – – (5*c*)	... ...	... ...	... ...	... ...	... ...
1*H*186	a^1 ♀ 00 *p* – – – (alb.)	a^2 ♀ 00 *p* – – – (alb.)	a^3 ♀ 00 *p* – – – (alb.)	a^4 ♂ 00 *p* – – – (alb.)	0*H*100a^4 ♂ 100*zq*49.46. 7(5*c*)	*A*147a^4 ♀ 00 *p* – – – (*A*2 ×)
	a^5 ♀ 00 *p* – – – (alb.)	a^6 ♂ 00 *p* – – – (alb.)	a^7 ♂ 100*zq*52.47.12(5*d*)	a^8 ♀ 95 *q*52.46.12(5*c*)	... ...	... ...
1*H*187	b^1 ♀ 55 *q*58.57. 1(3*e*)	b^2 60 *q*48.46. 1(3*c*)	b^3 00 *p* – – – (alb.)	... ...	0*H*59a^2 ♂ 100*zq*49.46. 7(6*d*)	*A*147a^5 ♀ 00 *p* – – – (*A*2 ×)
1*H*188	a^1 ♂ – *q* – – – (5*e*)	a^2 ♂ 100*xq*41.50.26(5–6*g*)	... ...	... ...	0*H*107a^1 ♂ 100*zq*49.46.20(5*c*)	*A*147a^6 ♀ 00 *p* – – – (*A*2 ×)
1*H*189	a^1 ♂ 00 *p* – – – (alb.)	a^2 ♀ 00 *p* – – – (alb.)	a^3 ♀ 00 *p* – – – (alb.)	a^4 ♂ – *q* – – – (6*e*)	0*H*103a^3 ♂ – *q* – – – (5*c*)	*A*147a^7 ♀ 00 *p* – – – (*A*2 ×)
	a^5 ♂ 100*zq*52.51.43(5–6*c*)	a^6 ♀ 30 *q*47.47. 1(2*c*–*d*)	a^7 ♀ – *q* – – – (3*e*)	... ...	... ...	... ...
1*H*190	a^1 ♀ 85 *q*61.60. 1(3–4*e*)	a^2 ♂ 100*xq*58.57.57(6*e*)	a^3 ♂ – *q* – – – (6*e*)	a^4 ♂ – *q* – – – (6*e*)	0*H*100a^2 ♂ – *q* – – – (5*c*)	*A*152a^4 ♀ 00 *p* – – – (*A*1 or 2)
	a^5 ♀ 90 *q*52.46.14(5*c*)	... ...	... ...	... ...	... ...	... ...
1*H*191	a^1 ♀ 00 *p* – – – (alb.)	a^2 ♀ 00 *p* – – – (alb.)	a^3 ♀ – *q* – – – (6*e*)	a^4 ♀ – *q* – – – (6*e*)	0*H*101a^3 ♂ – *q* – – – (5*c*)	*A*152a^5 ♀ 00 *p* – – – (*A*1 or 2)
	a^5 ♂ 70 *q*49.51. 1(3*c*)	... ...	... ...	... ...	... ...	... ...
1*H*192	a^1 00 *p* – – – (alb.)	a^2 00 *p* – – – (alb.)	a^3 90 *q*48.46.14(5*c*)	a^4 100*zq*52.46.12(5*d*)	0*H*103a^1 ♂ – *q* – – – (5*c*)	*A*152a^6 ♀ 00 *p* – – – (*A*1 or 2)
	a^5 100*zq*55.46.43(6*e*)	a^6 ♂ – *q* – – – (4*e*)	a^7 – *q* – – – (5*c*)	... ...	... ...	... ...

1H193	a^{1} ♂ 100xq58.58.55(6e)	a^{2} ♀ 00 p– – –(alb.)	… … …	… … …	0H112a^{2} ♂ – q– – –(5c)	A185a^{3} ♀ 00 p– – –(A1 or 2 or A2×)
1H194	a^{1} ♀ 00 p– – –(alb.)	a^{2} ♂ 00 p– – –(alb.)	a^{3} ♀ 00 p– – –(alb.)	a^{4} ♂ 00 p– – –(alb.)	0H111a^{1} ♂ – q– – –(5d)	A142a^{4} ♀ 00 p– – –(A1 or 2)
	a^{5} ♂ 00 p– – –(alb.)	a^{6} ♂ 100xq48.48.47(6c : e)	a^{7} ♂ 100zq48.46.43(5c)	a^{8} ♀ 95 q52.46.12(5d)	… … … …	… … … …
	a^{9} ♂ – q– – –(3e)	… … …	… … …	… … …	… … … …	… … … …
1H195	a^{1} ♂ 00 p– – –(alb.)	a^{2} ♀ 00 p– – –(alb.)	a^{3} ♂ 00 p– – –(alb.)	a^{4} ♂ – q– – –(6e)	0H84a^{9} ♂ 100zq48.46.28(5c)	A142a^{5} ♀ 00 p– – –(A1 or 2)
	a^{5} ♀ 90 q49.46.12(5c)	a^{6} ♀ 100zq49.46.12(5c)	a^{7} ♀ 100zq52.46.43(5c)	a^{8} ♂ 100zq52.47.12(5d)	… … … …	… … … …
1H196	a^{1} ♂ 00 p– – –(alb.)	a^{2} ♀ 00 p– – –(alb.)	a^{3} ♀ 00 p– – –(alb.)	a^{4} ♂ 00 p– – –(alb.)	0H90a^{2} ♂ – q– – –(5c)	A142a^{6} ♀ 00 p– – –(A1 or 2)
	a^{5} ♂ 00 p– – –(alb.)	a^{6} ♂ 100yq55.53.12(6d)	a^{7} ♂ 100xq51.51.47(6d)	a^{8} ♂ 100xq49.46.47(6d)	… … … …	… … … …
	a^{9} ♂ 100xq48.46.47(6d)	… … …	… … …	… … …	… … … …	… … … …
1H197	a^{1} ♀ 00 p– – –(alb.)	a^{2} ♀ 00 p– – –(alb.)	a^{3} ♀ 00 p– – –(alb.)	a^{4} ♀ 00 p– – –(alb.)	0H85a^{5} ♂ – q– – –(5d)	A150a^{6} ♀ 00 p– – –(A1 or 2)
	a^{5} ♀ 95 q49.51.14(5c)	a^{6} ♀ 90 q48.51.14(5c)	… … …	… … …	… … … …	… … … …
1H198	a^{1} ♀ 00 p– – –(alb.)	a^{2} ♀ 00 p– – –(alb.)	a^{3} ♀ – q– – –(6e)	a^{4} ♂ – q– – –(6e)	0H100a^{3} ♂ – q– – –(5c)	A153a^{5} ♀ 00 p– – –(A1 or 2)
	a^{5} ♀ – q– – –(6e)	a^{6} ♀ 90 q52.46.11(5c–d)	a^{7} ♀ 85 q49.46.11(5c–d)	a^{8} ♂ – q– – –(4e)	… … … …	… … … …
	a^{9} – q– – –(4c)	… … …	… … …	… … …	… … … …	… … … …
1H199	a^{1} ♂ 100zq48.46.43(5c)	a^{2} ♀ 00 p– – –(alb.)	a^{3} ♀ 00 p– – –(alb.)	a^{4} ♂ 00 p– – –(alb.)	0H102a^{4} ♂ – q– – –(5c)	A153a^{6} ♀ 00 p– – –(A1 or 2)
	a^{5} ♂ 00 p– – –(alb.)	a^{6} ♀ 00 p– – –(alb.)	a^{7} ♂ – q– – –(4e)	… … …	… … … …	… … … …
1H200	a^{1} ♂ 50 q40. 1. 1(3b–c)	a^{2} ♀ 00 p– – –(alb.)	a^{3} ♀ 00 p– – –(alb.)	a^{4} ♂ 00 p– – –(alb.)	0H114a^{6} ♂ – q– – –(5c)	A146a^{3} ♀ 00 p– – –(A2×)
	a^{5} ♀ 00 p– – –(alb.)	… … …	… … …	… … …	… … … …	… … … …
1H201	a^{1} ♂ – q– – –(5a)	a^{2} ♂ 00 p– – –(alb.)	a^{3} ♂ 00 p– – –(alb.w)	a^{4} ♀ – q– – –(5a)	0H84a^{7} ♂ 100zq35.33.28(5a)	0H12a^{5} ♀ – q– – –(5a)
	a^{5} ♀ – q– – –(5a)	a^{6} ♀ 00 p– – –(alb.)	a^{7} ♀ – q– – –(2–3a)	… … …	… … … …	… … … …
1H202	a^{1} 85 p39.35. 3(4a)	a^{2} ♂ – p– – –(5a)	a^{3} ♂ 00 p– – –(alb.)	a^{4} ♀ 00 p– – –(alb.)	0H111a^{2} ♂ – q– – –(5a)	0H111a^{7} ♀ – q– – –(5a)
1H203	a^{1} ♀ 85 q53.51. 1(5c)	a^{2} ♀ 85 q51.51. 2(5c)	a^{3} ♂ 55 q33. 6. 6(5a)	a^{4} ♂ 100yq33.30.30(5a)	A182a^{1} ♂ 00 p– – –(A1 or 2)	0H12a^{3} ♀ – q– – –(5a)
	a^{5} ♀ 00 p– – –(alb.)	a^{6} ♂ 00 p– – –(alb.)	… … …	… … …	… … … …	… … … …
1H204	a^{1} ♂ 80 q52.51. 4(5c)	a^{2} ♂ – q– – –(5c)	a^{3} ♂ – q– – –(5a)	a^{4} ♀ – q– – –(5a)	A182a^{2} ♂ 00 p– – –(A1 or 2)	0H111a^{9} ♀ 100zq35.34.29(5a)
	a^{5} ♀ 00 p– – –(alb.)	a^{6} ♂ 00 p– – –(alb.)	a^{7} ♂ 00 p– – –(alb.)	… … …	… … … …	… … … …
1H205	a^{1} ♀ 100zq35.33.30(5a)	a^{2} ♀ – q– – –(5a)	a^{3} ♂ 90 q53.51. 9(5c)	a^{4} ♂ 90 q51.51. 7(5c)	A183a^{2} ♂ 00 p– – –(A1 or 2)	0H111a^{8} ♀ 100zq35.35.30(5a)
	a^{5} ♂ 80 q53.51. 7(5c)	a^{6} ♂ – q– – –(5c)	… … …	… … …	… … … …	… … … …
1H206	a^{1} ♂ 00 p– – –(alb.)	a^{2} ♂ 00 p– – –(alb.)	a^{3} ♀ 00 p– – –(alb.)	a^{4} ♀ – q– – –(6e)	0H113a^{7} ♂ – q– – –(5c)	A160a^{1} ♀ 00 p– – –(A2×)
	a^{5} ♂ 80 q52.51. 7(5c)	a^{6} ♀ 85 q53.51. 4(4c)	a^{7} ♂ – q– – –(3–4c)	a^{8} ♀ 60 q44.47. 4(3–4c)	… … … …	… … … …
	a^{9} ♀ 85 q52.51. 4(5c)	… … …	… … …	… … …	… … … …	… … … …
1H207	a^{1} ♀ 00 p– – –(alb.)	a^{2} ♂ 00 p– – –(alb.)	a^{3} ♂ 00 p– – –(alb.)	a^{4} ♂ – q– – –(6e)	0H113a^{8} ♂ – q– – –(5c)	A160a^{2} ♀ 00 p– – –(A2×)
	a^{5} ♂ – q– – –(6e)	a^{6} ♀ – q– – –(6e)	a^{7} ♂ 80 q44.47. 2(4d)	a^{8} – q– – –(4c)	… … … …	… … … …
	a^{9} – q– – –(5c)	… … …	… … …	… … …	… … … …	… … … …
1H208	a^{1} ♂ 95 q53.51. 1(4c)	a^{2} ♀ 100xq41.41.50(6g)	a^{3} ♂ 00 p– – –(alb.)	a^{4} ♀ 00 p– – –(alb.)	A158a^{1} ♂ 00 p– – –(A1)	0H62a^{5} ♀ 100zq49.48.12(5c)
	a^{5} ♀ 00 p– – –(alb.)	a^{6} ♂ 00 p– – –(alb.)	a^{7} 00 p– – –(alb.)	… … …	… … … …	… … … …
1H209	a^{1} ♂ 00 p– – –(alb.)	a^{2} ♀ 100zq40.37. 2(5b–c)	… … …	… … …	A158a^{2} ♂ 00 p– – –(A1)	0H84a^{3} ♀ 100zq35.33.30(5a)
1H210	a^{1} ♂ 00 p– – –(alb.)	a^{2} ♂ 00 p– – –(alb.)	a^{3} ♀ 95 p37.35.34(4b)	… … …	0H102a^{3} ♂ 95 q48.46. 7(4c)	0H84a^{6} ♀ 100zq39.33.30(5a)
1H211	a^{1} ♀ 40 q60.60. 1(3e)	a^{2} ♀ 35 q53. 1. 1(2c)	a^{3} ♂ 100zq41.26. 8(5c)	a^{4} ♂ 100xq58.58.60(6e)	0H114a^{6} ♂ 25 q49. 1. 2(2c)	A146a^{4} ♀ 00 p– – –(A2×)
	a^{5} ♀ 00 p– – –(alb.)	a^{6} ♀ 00 p– – –(alb.)	a^{7} ♀ 00 p– – –(alb.)	… … …	… … … …	… … … …
1H212	a^{1} ♂ 60 q58.57. 1(3e)	a^{2} ♂ – q– – –(6e)	a^{3} ♀ – q– – –(6e)	a^{4} ♂ 00 p– – –(alb.)	0H112a^{1} ♂ 30 q49. 1. 1(2c)	A146a^{5} ♀ 00 p– – –(A2×)
	a^{5} ♂ 00 p– – –(alb.)	a^{6} ♀ 00 p– – –(alb.)	a^{7} ♂ 00 p– – –(alb.)	… … …	… … … …	… … … …
1H213	a^{1} ♂ 00 p– – –(alb.)	a^{2} ♀ 00 p– – –(alb.)	a^{3} ♀ 85 q49.51. 1(4c)	a^{4} ♀ 85 q51.51. 1(4c)	0H102a^{2} ♂ 55 q48. 1. 2(3c)	A146a^{6} ♀ 00 p– – –(A2×)
	a^{5} ♂ 40 r40. 1. 3(3b–c)	a^{6} ♂ 50 q47.51. 3(3c)	a^{7} ♂ 75 q51.51. 4(4c)	… … …	… … … …	… … … …
1H214	a^{1} ♂ 00 p– – –(alb.)	a^{2} ♂ 00 p– – –(alb.)	a^{3} ♂ 00 p– – –(alb.)	a^{4} ♂ 00 p– – –(alb.)	0H120a^{2} ♂ – q– – –(3c)	A146a^{7} ♀ 00 p– – –(A2×)
	a^{5} ♀ – q– – –(6e)	a^{6} ♀ 100zq52.46.43(5–6, c–d)	a^{7} ♀ – q– – –(4e)	a^{8} ♂ – q– – –(3–4e)	… … … …	… … … …
1H216	a^{1} ♀ 55 q41.50. 5(3g)	a^{2} ♀ – q– – –(4e)	… … …	… … …	0H104a^{4} ♂ 40 q48.46. 2(3c)	A158a^{4} ♀ 00 p– – –(A1)
1H217	a^{1} ♂ 00 p– – –(alb.)	a^{2} ♂ 00 p– – –(alb.)	a^{3} ♀ 00 p– – –(alb.)	a^{4} ♂ 55 q40.37. 3(3b–c)	A147a^{1} ♂ 00 p– – –(A2×)	0H85a^{1} ♀ – q– – –(4d)
	a^{5} ♀ 75 q41.50. 2(4g)	a^{6} ♀ 35 q47. 1. 1(3c)	… … …	… … …	… … … …	… … … …

Mating	Offspring				Parents	
					Father	Mother
1H167	a^1 ♀ – q – – –(5c–d)	a^2 ♀ 100xq58.58.55(6e)	a^3 ♀ 100zq49.46.21(6c)	a^4 ♂ 100zq48.46.47(6c)	0H13a^4 ♂ 45 q52. 1. 1(3c)	A133a^4 ♀ 00 p – – –(A×2)
	a^5 ♂ 100zq48.46.12(6d)	a^6 ♂ 00 p – – –(alb.)	a^7 ♀ 00 p – – –(alb.)	a^8 ♂ 00 p – – –(alb.)	...	...
1H168	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 100zq39.33.28(4a)	a^4 ♀ 00 p – – –(alb.)	A133a^0 ♂ 00 p – – –(A×2)	0H84a^4 ♀ 100zq35.33.30(5a)
	a^5 ♂ 100zq46.46. 9(5–6c)	a^6 ♀ 100zq33.33.28(5a)	a^7 ♀ 100zq35.30.28(5a)	...	...	...
1H169	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♂ 100xq58.58.55(6e)	a^4 ♀ 100zq49.48.47(5c–d)	A133a^7 ♂ 00 p – – –(A×2)	0H112a^3 ♀ 50 q48.46. 1(3c)
	a^5 ♂ 80 q46.21. 7(5c)	a^6 ♀ 100zq48.46. 4(6e)	a^7 ♂ 00 p – – –(alb.)	a^8 ♀ – q – – –(3–4c)	...	...
1H170	a^1 ♀ 00 p – – –(alb.)	a^2 ♂ 85 q48.51. 2(5c–d)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ 100xq26.26.26(6g)	0H101a^4 ♂ – q – – –(5c)	A136a^1 ♀ 00 p – – –(A×1)
	a^5 ♀ 85 q40.40. 4(5c)	...	...	...	...	...
1H171	a^1 ♂ 100zq52.48.47(6c–d)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ – q – – –(6c–d)	a^4 ♀ – q – – –(6c–d)	A136a^4 ♂ 00 p – – –(A×1)	0H118a^2 ♀ – q – – –(5c)
	a^5 ♀ 00 p – – –(alb.)	...	...	...	...	...
1H172	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 100xq60.60.45(6e)	a^3 ♂ 00 p – – –(alb.)	a^4 – q – – –(3-4c:e)	A133a^6 ♂ 00 p – – –(A×2)	0H1a^0 ♀ – q – – –(3c)
	a^5 ♂ – q – – –(4c)	...	...	...	...	...
1H173	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ – q – – –(6e)	A179a^1 ♂ 00 p – – –(A2×)	0H62a^4 ♀ 100zq49.46. 7(5c)
	a^5 ♂ 100xq57.57.55(6e)	a^6 ♀ 100zq47.51. 9(5c)	...	...	...	...
1H175	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 100zq47.51.25(5d)	a^4 ♀ 100zq51.51. 7(5c)	A179a^3 ♂ 00 p – – –(A2×)	0H102a^6 ♀ – q – – –(5c)
	a^5 ♂ 100zq40.36. 4(5b–c)	a^6 ♂ 100zq40.36. 9(5b–c)	a^7 ♂ – q – – –(4e)	...	...	...
1H178	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.).	a^4 ♂ 00 p – – –(alb.)	A150a^2 ♂ 00 p – – –(A1 or 2)	0H113a^4 ♀ 100zq49.46.12(5c)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♀ 100xq51.51.47(5–6c)	a^7 ♀ 80 q51.51. 2(5c)	a^8 ♀ – q – – –(2–3c)	...	...
1H179	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 100xq51.51.47(6d)	a^3 ♂ 100zq48.46. 7(5c)	a^4 ♂ 100zq52.47. 9(5d)	A150a^3 ♂ 00 p – – –(A1 or 2)	0H7a^0 ♀ 100zq48.46.12(5c)
	a^5 ♀ – q – – –(5c)	a^6 ♂ – q – – –(5c)	...	...	...	...
1H180	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 100zq51.51. 4(5c)	a^4 ♀ 100zq51.46. 9(5c)	A150a^4 ♂ 00 p – – –(A1 or 2)	0H111a^3 ♀ – q – – –(5d)
	a^5 ♀ 100zq51.51. 9(5c)	a^6 ♀ 100zq52.51. 9(5c)	a^7 ♀ 100xq51.47.47(6c)	...	...	...
1H181	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 95 q58.57. 1(5e)	A150a^5 ♂ 00 p – – –(A1 or 2)	0H113a^6 ♀ 100zq49.36.20(5c)
	a^5 ♂ – q – – –(6e)	a^6 ♂ 80 q52.51. 4(5c)	...	...	...	...
1H182	a^1 ♀ – q – – –(6e)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	A153a^1 ♂ 00 p – – –(A1 or 2)	0H111a^5 ♀ – q – – –(5d)
	a^5 ♂ 85 q52.51. 4(5c)	a^6 ♂ 100zq52.51.34(5c)	...	...	...	...
1H183	a^1 ♀ 75 q51.25. 2(5c)	a^2 ♂ 80 q52.51. 7(5c)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ 00 p – – –(alb.)	A153a^2 ♂ 00 p – – –(A1 or 2)	0H100a^8 ♀ – q – – –(5c)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♂ 00 p – – –(alb.)	...	...	...	...
1H184	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ 00 p – – –(alb.)	A153a^3 ♂ 00 p – – –(A1 or 2)	0H102a^0 ♀ – q – – –(5c)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♀ 00 p – – –(alb.)	a^7 ♀ 00 p – – –(alb.)	a^8 00 p – – –(alb.)	...	...
	a^9 – q – – –(6e)	...	...	...	...	...
1H185	a^1 ♀ – q – – –(4e)	a^2 ♀ – q – – –(6e)	a^3 ♀ – q – – –(6e)	a^4 ♂ – q – – –(6e)	A153a^4 ♂ 00 p – – –(A1 or 2)	0H61a^7 ♀ – q – – –(5c)
	a^5 ♂ – q – – –(6e)	a^6 ♂ 00 p – – –(alb.)	a^7 ♂ 00 p – – –(alb.)	a^8 – q – – –(6e)	...	...
	a^9 – q – – –(5e)	...	...	...	...	...
1H186	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0H100a^4 ♂ 100zq49.46. 7(5c)	A147a^4 ♀ 00 p – – –(A2×)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♂ 00 p – – –(alb.)	a^7 ♂ 100zq52.47.12(5d)	a^8 ♀ 95 q52.46.12(5c)	...	...
1H187	b^1 ♀ 55 q58.57. 1(3e)	b^2 60 q48.46. 1(3c)	b^3 00 p – – –(alb.)	...	0H59a^2 ♂ 100zq49.46. 7(6d)	A147a^5 ♀ 00 p – – –(A2×)
1H188	a^1 ♂ – q – – –(5e)	a^2 ♂ 100xq41.50.26(5–6g)	...	...	0H107a^1 ♂ 100zq49.46.20(5c)	A147a^6 ♀ 00 p – – –(A2×)
1H189	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ – q – – –(6e)	0H103a^3 ♂ – q – – –(5c)	A147a^7 ♀ 00 p – – –(A2×)
	a^5 ♂ 100zq52.51.43(5–6c)	a^6 ♀ 30 q47.47. 1(2c–d)	a^7 ♀ – q – – –(3e)	...	...	...
1H190	a^1 ♀ 85 q61.60. 1(3–4e)	a^2 ♂ 100xq58.57.57(6e)	a^3 ♂ – q – – –(6e)	a^4 ♂ – q – – –(6e)	0H100a^2 ♂ – q – – –(5c)	A152a^4 ♀ 00 p – – –(A1 or 2)
	a^5 ♀ 90 q52.46.14(5c)	...	...	...	...	...
1H191	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ – q – – –(6e)	a^4 ♀ – q – – –(6e)	0H101a^3 ♂ – q – – –(5c)	A152a^5 ♀ 00 p – – –(A1 or 2)
	a^5 ♂ 70 q49.51. 1(3c)	...	...	...	...	...
1H192	a^1 00 p – – –(alb.)	a^2 00 p – – –(alb.)	a^3 90 q48.46.14(5c)	a^4 100zq52.46.12(5d)	0H103a^1 ♂ – q – – –(5c)	A152a^6 ♀ 00 p – – –(A1 or 2)
	a^5 100zq55.46.43(6e)	a^6 ♂ – q – – –(4e)	a^7 – q – – –(5c)	...	...	...
1H193	a^1 ♂ 100xq58.58.55(6e)	a^2 ♀ 00 p – – –(alb.)	...	...	0H112a^2 ♂ – q – – –(5c)	A185a^3 ♀ 00 p – – –(A1 or 2 or A2×)
1H194	a^1 ♀ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0H111a^1 ♂ – q – – –(5d)	A142a^4 ♀ 00 p – – –(A1 or 2)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♂ 100xq48.48.47(6c:e)	a^7 ♂ 100zq48.46.43(5c)	a^8 ♀ 95 q52.46.12(5d)	...	...
	a^9 ♂ – q – – –(3e)	...	...	...	...	...
1H195	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ – q – – –(6e)	0H84a^9 ♂ 100zq48.46.28(5c)	A142a^5 ♀ 00 p – – –(A1 or 2)
	a^5 ♀ 90 q49.46.12(5c)	a^6 ♀ 100zq49.46.12(5c)	a^7 ♀ 100zq52.46.43(5e)	a^8 ♂ 100zq52.47.12(5d)		
1H196	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0H90a^2 ♂ – q – – –(5c)	A142a^6 ♀ 00 p – – –(A1 or 2)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♂ 100yq55.53.12(6d)	a^7 ♂ 100xq51.51.47(6d)	a^8 ♂ 100xq49.46.47(6d)	...	...
	a^9 ♂ 100xq48.46.47(6d)	...	...	...	...	...
1H197	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ 00 p – – –(alb.)	0H85a^5 ♂ – q – – –(5d)	A150a^6 ♀ 00 p – – –(A1 or 2)
	a^5 ♀ 95 q49.51.14(5c)	a^6 ♀ 90 q48.51.14(5c)	...	...	...	...
1H198	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ – q – – –(6e)	a^4 ♂ – q – – –(6e)	0H100a^3 ♂ – q – – –(5c)	A153a^5 ♀ 00 p – – –(A1 or 2)
	a^5 ♀ – q – – –(6e)	a^6 ♀ 90 q52.46.11(5c–d)	a^7 ♀ 85 q49.46.11(5c–d)	a^8 ♂ – q – – –(4e)	...	...
	a^9 – q – – –(4c)	...	...	...	...	...
1H199	a^1 ♂ 100zq48.46.43(5c)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0H102a^4 ♂ – q – – –(5c)	A153a^0 ♀ 00 p – – –(A1 or 2)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♀ 00 p – – –(alb.)	a^7 ♂ – q – – –(4e)	...	...	...
1H200	a^1 ♂ 50 q40. 1. 1(3b–c)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0H114a^6 ♂ – q – – –(5c)	A146a^3 ♀ 00 p – – –(A2×)
	a^5 ♀ 00 p – – –(alb.)	...	...	...	...	...
1H201	a^1 ♂ – q – – –(5a)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.w)	a^4 ♀ – q – – –(5a)	0H84a^7 ♂ 100zq35.33.28(5a)	0H12a^5 ♀ – q – – –(5a)
	a^5 ♀ – q – – –(5a)	a^6 ♀ 00 p – – –(alb.)	a^7 ♀ – q – – –(2–3a)	...	...	...
1H202	a^1 85 p39.35. 3(4a)	a^2 ♂ – p – – –(5a)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ 00 p – – –(alb.)	0H111a^2 ♂ – q – – –(5a)	0H111a^7 ♀ – q – – –(5a)
1H203	a^1 ♀ 85 q53.51. 1(5c)	a^2 ♀ 85 q51.51. 2(5e)	a^3 ♂ 55 q33. 6. 6(5a)	a^4 ♂ 100yq33.30.30(5a)	A182a^1 ♂ 00 p – – –(A1 or 2)	0H12a^3 ♀ – q – – –(5a)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♂ 00 p – – –(alb.)	...	...	...	...
1H204	a^1 ♂ 80 q52.51. 4(5c)	a^2 ♂ – q – – –(5c)	a^3 ♂ – q – – –(5a)	a^4 ♀ – q – – –(5a)	A182a^2 ♂ 00 p – – –(A1 or 2)	0H111a^9 ♀ 100zq35.34.29(5a)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♂ 00 p – – –(alb.)	a^7 ♂ 00 p – – –(alb.)	...	...	...
1H205	a^1 ♀ 100zq35.33.30(5a)	a^2 ♀ – q – – –(5a)	a^3 ♂ 90 q53.51. 9(5c)	a^4 ♂ 90 q51.51. 7(5c)	A183a^2 ♂ 00 p – – –(A1 or 2)	0H111a^8 ♀ 100zq35.35.30(5a)
	a^5 ♂ 80 q53.51. 7(5e)	a^6 ♂ – q – – –(5c)	...	...	...	...
1H206	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ – q – – –(6e)	0H113a^7 ♂ – q – – –(5c)	A160a^1 ♀ 00 p – – –(A2×)
	a^5 ♂ 80 q52.51. 7(5e)	a^6 ♀ 85 q53.51. 4(4c)	a^7 ♂ – q – – –(3–4c)	a^8 ♀ 60 q44.47. 4(3–4c)	...	...
	a^9 ♀ 85 q52.51. 4(5e)	...	...	...	...	...
1H207	a^1 ♀ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ – q – – –(6e)	0H113a^8 ♂ – q – – –(5c)	A160a^2 ♀ 00 p – – –(A2×)
	a^5 ♂ – q – – –(6e)	a^6 ♀ – q – – –(6e)	a^7 ♂ 80 q44.47. 2(4d)	a^8 – q – – –(4c)	...	...
	a^9 – q – – –(5c)	...	...	...	...	...
1H208	a^1 ♂ 95 q53.51. 1(4c)	a^2 ♀ 100xq41.41.50(6g)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ 00 p – – –(alb.)	A158a^1 ♂ 00 p – – –(A1)	0H62a^5 ♀ 100zq49.48.12(5c)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♂ 00 p – – –(alb.)	a^7 00 p – – –(alb.)	...	...	...
1H209	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 100zq40.37. 2(5b–c)	...	...	A158a^2 ♂ 00 p – – –(A1)	0H84a^3 ♀ 100zq35.33.30(5a)
1H210	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 95 p37.35.34(4b)	...	0H102a^3 ♂ 95 q48.46. 7(4c)	0H84a^6 ♀ 100zq39.33.30(5a)
1H211	a^1 ♀ 40 q60.60. 1(3e)	a^2 ♀ 35 q53. 1. 1(2c)	a^3 ♂ 100zq41.26. 8(5c)	a^4 ♂ 100xq58.58.60(6e)	0H114a^5 ♂ 25 q49. 1. 2(2c)	A146a^4 ♀ 00 p – – –(A2×)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♀ 00 p – – –(alb.)	a^7 ♀ 00 p – – –(alb.)	...	...	...
1H212	a^1 ♂ 60 q58.57. 1(3e)	a^2 ♂ – q – – –(6e)	a^3 ♀ – q – – –(6e)	a^4 ♂ 00 p – – –(alb.)	0H112a^1 ♂ 30 q49. 1. 1(2c)	A146a^5 ♀ 00 p – – –(A2×)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♀ 00 p – – –(alb.)	a^7 ♂ 00 p – – –(alb.)	...	...	...
1H213	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 85 q49.51. 1(4c)	a^4 ♀ 85 q51.51. 1(4e)	0H102a^2 ♂ 55 q48. 1. 2(3c)	A146a^6 ♀ 00 p – – –(A2×)
	a^5 ♂ 40 r40. 1. 3(3b–c)	a^6 ♂ 50 q47.51. 3(3c)	a^7 ♂ 75 q51.51. 4(4c)	...	...	...
1H214	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0H120a^2 ♂ – q – – –(3c)	A146a^7 ♀ 00 p – – –(A2×)
	a^5 ♀ – q – – –(6e)	a^6 ♀ 100zq52.46.43(5–6, c–d)	a^7 ♀ – q – – –(4e)	a^8 ♂ – q – – –(3–4e)	...	...
1H216	a^1 ♀ 55 q41.50. 5(3g)	a^2 ♀ – q – – –(4e)	...	...	0H104a^4 ♂ 40 q48.46. 2(3c)	A158a^4 ♀ 00 p – – –(A1)
1H217	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 55 q40.37. 3(3b–c)	A147a^1 ♂ 00 p – – –(A2×)	0H85a^1 ♀ – q – – –(4d)
	a^6 ♀ 75 q41.50. 2(4g)	a^6 ♀ 35 q47. 1. 1(3c)	...	...	...	...

MATING	OFFSPRING				PARENTS	
					Father	Mother
1H218	a^1 ♂ 00 p – – – (alb.)	a^2 ♀ 00 p – – – (alb.)	a^3 ♀ 00 p – – – (alb.)	a^4 ♂ 40 q40.37. 3(3b–c)	A147a^2 ♂ 00 p – – – (A2 ×)	0H112a^5 ♀ 40 q49. 1. 1(2c)
	a^5 ♂ 30 q51. 1. 2(3c)	a^6 ♀ 60 q58.57. 1(3–4e)	a^7 ♂ 50 q58. 1. 1(3–4e)	... ...	... ...	... ...
1H219	a^1 ♀ 00 p – – – (alb.)	a^2 ♂ 00 p – – – (alb.	a^3 ♂ 00 p – – – (alb.)	a^4 ♀ 100zq51.51. 4(5c)	A152a^1 ♂ 00 p – – – (A1 or 2)	0H111a^6 ♀ – q – – – (5d)
	a^5 ♀ – q – – – (6e)	a^6 ♀ 100xq58.58.55(6e)	... ...	... ...	... ...	... ...
1H220	a^1 ♀ 85 q48.46.11(5c)	a^2 ♂ 100zq49.46. 7(5c)	a^3 – q – – – (5c)	a^4 00 p – – – (alb.)	A152a^2 ♂ 00 p – – – (A1 or 2)	0H120a^3 ♀ 45 q49. 1. 1(2c)
	a^5 00 p – – – (alb.)	... ...	... ...	... ...	... ...	... ...
1H221	a^1 ♀ 100xq58.58.52(6e)	a^2 ♀ 100xq58.58.57(6e)	a^3 ♂ 85 q47.51. 4(5c)	a^4 ♂ 00 p – – – (alb.)	A152a^3 ♂ 00 p – – – (A1 or 2)	0H101a^6 ♀ 45 q49.46. 1(3c)
1H222	a^1 ♀ 00 p – – – (alb.)	a^2 ♂ 00 p – – – (alb.)	a^3 ♂ 00 p – – – (alb.)	a^4 ♀ 00 p – – – (alb.)	A159a^1 ♂ 00 p – – – (A1)	0H117a^4 ♀ 90 q49.36. 1(4c)
	a^5 ♂ 100xq40.37.36(5b–c)	a^6 ♀ 100xq41.21.19(5b–c)	a^7 ♂ 80 q52.51. 3(3c)	a^8 – q – – – (3–4c)	... ...	... ...
	a^9 – q – – – (5c)	... ...	... ...	... ...	... ...	... ...
1H223	a^1 ♀ 00 p – – – (alb.)	a^2 ♂ 100zq48.46. 7(5c)	a^3 ♀ 90 q41.23. 1(4g)	... ...	A159a^2 ♂ 00 p – – – (A1)	0H6a^1 ♀ – q – – – (2c)
1H224	a^1 ♂ 30 q36. 1. 1(2a)	a^2 ♀ 20 q61. 1. 1(2e)	a^3 25 p?23. 1. 1(2–3f)	a^4 – p? – – – (3f)	0H13a^4 ♂ 45 q52. 1. 1(3c)	0H113a^2 ♀ 50 q49.46. 2(3c)
	a^5 – q – – – (1cw)	a^6 00 p – – – (alb.)	... ...	... ...	... ...	... ...
1H225	a^1 ♂ 35 q49.48. 1(3c)	a^2 ♂ 85 q57.55. 1(4e)	a^3 ♂ – p – – – (2a)	a^4 ♂ 00 p – – – (alb.)	0H101a^2 ♂ 50 q49.46. 4(3c)	0H120a^6 ♀ 75 q49.46. 1(4c)
	a^5 – q – – – (3c)	a^6 00 p – – – (alb. w)	a^7 – p – – – (3f)	... ...	... ...	... ...
1H226	a^1 ♂ 00 p – – – (alb.)	a^2 ♂ 00 p – – – (alb.)	a^3 ♀ 45 p39. 1. 1(3b)	a^4 ♀ 55 q61.60. 1(4ew)	0H120a^4 ♂ 60 q49.46. 1(3c)	0H61a^6 ♀ 65 q49.30. 7(3c)
	a^5 ♀ 00 p – – – (alb.)	a^6 ♀ 55 p37.39. 1(3bw)	... ...	... ...	... ...	... ...
1H227	a^1 ♂ 35 q48. 1. 1(2cw)	a^2 ♂ 00 p – – – (alb.)	a^3 ♀ 60 p22. 1. 1(4f)	a^4 ♀ 60 q57.55. 1(3ew)	0H45a^3 ♂ – q – – – (2d)	0H74a^5 ♀ 80 q49.46. 2(4c)
	a^5 ♀ 65 p37.35. 1(3–4bw)	a^6 ♀ 25 q58. 1. 1(2e)	... ...	... ...	... ...	... ...
1H228	a^1 ♂ 45 q48. 1. 3(3c)	a^2 ♀ 55 q52. 1. 1(3d)	a^3 ♂ – q – – – (2c)	a^4 ♂ 00 p – – – (alb.)	0H114a^4 ♂ 50 q46.30. 5(3c)	0H117a^5 ♀ 65 q49.36. 4(3c)
	a^5 ♂ 00 p – – – (alb.)	a^6 ♂ 00 p – – – (alb. w)	... ...	... ...	... ...	... ...
1H229	a^1 ♂ 20 p22. 1. 1(3fw)	a^2 ♀ 60 q49. 1. 1(4c)	a^3 ♀ 35 q48. 1. 5(3c)	a^4 25 q47. 1. 1(2c)	0H100a^1 ♂ 40 q49. 1. 1(3c)	0H112a^4 ♀ 35 q46. 1. 1(2c)
	a^5 ♂ 50 p?18. 1. 1(2f)	a^6 – p – – – (2fw)	... ...	... ...	... ...	... ...
1H230	a^1 ♀ 85 p35.35. 3(4b)	... ...	... ...	... ...	0H110a^1 ♂ 60 q48.46. 1(3c)	0H82a^1 ♀ 55 q48.46.12(3c)
1H231	a^1 ♂ 65r?40.36. 4(3b–c)	a^2 ♂ – q – – – (3c)	a^3 ♂ 00 p – – – (alb.)	a^4 ♀ 50 q40. 1. 1(3c)	0H120a^7 ♂ 50 q48.46. 3(3c)	0H103a^5 ♀ 55 q49.46. 4(3c)
	a^5 ♀ 50 q49.46. 1(3c–d)	a^6 ♀ 50 q58.56. 1(3e)	a^7 ♀ 85 q57.55. 1(4–5e)	a^8 00 p – – – (alb.)	... ...	... ...
1H232	a^1 ♂ 65 q49.33. 1(3c)	a^2 ♂ 65 q58.57. 1(4e)	a^3 ♀ 55 p39.39. 2(3b)	a^4 ♀ 35 p37. 1. 1(2b)	0H102a^1 ♂ 70 q48.46. 2(4c)	0H59a^1 ♀ 50 q48. 1. 4(3d)
	a^5 ♀ 00 p – – – (alb.)	a^6 – p – – – (1bw)	... ...	... ...	... ...	... ...
1H233	a^1 ♂ 30 q46. 1. 3(1–2c)	a^2 ♂ 35 q48. 1. 3(2c)	a^3 ♂ – q – – – (2c)	a^4 ♂ – q – – – (2c)	0H75a^5 ♂ 15 q49. 1. 1(1c)	0H100a^6 ♀ 55 q49.46. 9(3c)
	a^5 ♂ 00 p – – – (alb.)	a^6 – q – – – (2c)	a^7 – q – – – (2c)	a^8 – q – – – (2ew)	... ...	... ...
	a^9 00 p – – – (alb.)	... ...	... ...	... ...	... ...	... ...
1H234	a^1 ♂ 100zq49.48.20(5cw)	a^2 ♂ 100xp37.19. 9(5–6f)	a^3 ♀ 100xp22.15.15(5–6f)	a^4 ♀ 40 q49. 1. 1(3c)	0H103a^2 ♂ – q – – – (5c)	0H27a^1 ♀ 40 q58. 1. 2(2e)
	a^5 – p – – – (2fw)	... ...	... ...	... ...	... ...	... ...
1H235	a^1 80 q53.51. 4(5c)	a^2 – q – – – (4dw)	a^3 – q – – – (3e)	a^4 – q – – – (2c)	0H101a^4 ♂ – q – – – (5c)	0H100a^5 ♀ 25 q46. 1. 1(2c)
	a^5 00 p – – – (alb.)	a^6 00 p – – – (alb.)	... ...	... ...	... ...	... ...
1H236	a^1 – p – – – (4a)	a^2 – q – – – (4c)	a^3 – q – – – (3c)	b^1 – q – – – (3d)	0H118a^1 ♂ 85 q48.46.11(4c)	0H35a^2 ♀ – q – – – (3d)
	b^2 – q – – – (3c)	b^3 – p – – – (3b)	b^4 00 p – – – (alb.)	... ...	... ...	... ...
1H237	a^1 ♂ 60 p37. 3. 3(3bw)	a^2 ♀ 30 p37. 1. 1(2aw)	a^3 ♀ 95 p37.34. 3(5bw)	a^4 ♂ 95 p23.36. 3(5bw)	0H138a^3 ♂ – q – – – (5c)	0H128a^1 ♀ 85 q49.46. 1(4c)
	a^5 ♀ 80 p37.33. 4(5b)	a^6 ♀ 100zq53.51. 9(5c)	a^7 ♀ 100zq48.36. 9(5c)	... ...	... ...	... ...
1H238	a^1 ♀ 00 p – – – (alb.)	a^2 ♂ 00 p – – – (alb.)	a^3 ♀ 100zq52.46.20(5d)	a^4 ♀ – q – – – (5d)	0H140a^5 ♂ 40 q49.46. 1(3c)	0H133a^1 ♀ 100zq49.46.12(5c)
	a^5 ♂ 35 p34. 1. 1(2b)	a^6 ♂ 40 p37. 1. 4(3b)	a^7 ♀ 15 p37. 1. 4(1bw)	... ...	... ...	... ...
1H239	a^1 ♂ 90 p18.18. 3(4fw)	a^2 ♀ 85 p37.36. 3(4bw)	a^3 ♀ 90 p37.36. 3(5b)	a^4 ♀ 35 q41. 1. 1(3g)	0H152a^2 ♂ – q – – – (5c)	0H140a^6 ♀ 40 q46. 1. 1(3c)
	a^5 ♀ 60 q49.46. 4(4c)	a^6 – q – – – (3c)	a^7 – p – – – (3b)	... ...	... ...	... ...
1H240	a^1 ♂ 30 p35. 1. 1(3aw)	a^2 ♂ 90 p35.33.28(4a)	a^3 ♀ 35 p37. 3. 3(3b)	a^4 ♂ 55 q35.35. 1(3a)	0H129a^4 ♂ 35 q46.46. 3(2c)	0H131a^3 ♀ 95 q35.35. 2(4a)
	a^5 ♀ 85 q35.33. 3(4a)	a^6 ♂ 65 q48.34. 3(3c)	a^7 ♀ 00 p – – – (alb.)	... ...	... ...	... ...

1H241	a^1 40 p37. 1. 1(3b)	a^2 – p– – –(2aw)	a^3 – q– – –(2cw)	a^4 – q– – –(3c)	0H129a^6 ♂ 35 q49.36. 1(2c)	0H131a^2 ♀ 60 q49.46. 3(3d)
	a^5 – q– – –(3dw)	... ...	... ...	... ...	... ...	... ...
1H242	a^1 ♂ 35 q48.46. 1(2–3c)	a^2 ♀ 00 p– – –(alb.)	a^3 ♀ – p– – –(5a)	a^4 ♂ – q– – –(3c)	0H136a^1 ♂ 40 q47.28. 2(3c)	0H131a^4 ♀ 95 q35.35.29(6a)
	a^5 ♂ – q– – –(3c)	... ...	... ...	... ...	... ...	... ...
1H243	a^1 ♂ 100zq35.35.28(5aw)	a^2 ♂ 90 q36.33. 1(5a)	a^3 ♂ 00 p– – –(alb.)	a^4 ♂ 60 q48.46. 3(3c)	0H131a^1 ♂ 100zq35.33.29(4–5a)	0H129a^1 ♀ – q– – –(3c)
	a^5 ♀ 55 p37.29. 3(3b)	... ...	... ...	... ...	... ...	... ...
1H244	a^1 ♂ 40 q51. 2. 2(3cw)	a^2 ♂ 60 q49.46. 4(3c)	a^3 ♀ – q– – –(3c)	a^4 ♀ 35 p37.35. 1(2a–b)	0H129a^5 ♂ 35 q49. 1. 1(3c)	0H129a^3 ♀ 25 q49. 1. 1(2c)
	a^5 ♂ 35 q46. 1. 1(2–3c)	a^6 ♀ 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H245	a^1 ♀ – q– – –(3–4c)	a^2 ♀ 65 q53. 1. 3(3c)	a^3 ♂ 20 p37. 1. 1(2bw)	a^4 ♂ 35 p37.37. 1(3b)	0H127a^5 ♂ 65 q49.46. 1(4?c)	0H129a^2 ♀ 50 q49. 1. 1(3c)
	a^5 ♂ 70 p37.37.30(3b)	a^6 ♀ 55 p37.19. 4(3b)	... ...	... ...	... ...	... ...
1H246	a^1 ♀ 90 q47.51. 3(4c)	a^2 ♀ 100xq48.51.47(6d)	a^3 ♂ 100xq53.51.47(6d)	a^4 ♂ – q– – –(6d)	0H127a^3 ♂ 50 q52. 1. 2(3c)	0H143a^2 ♀ – q– – –(5c)
	a^5 ♂ 100xp37.37. 9(5b)	a^6 ♂ 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H248	a^1 ♀ 00 p– – –(alb. w)	a^2 ♂ 100xq53.51.47(6dw)	a^3 ♂ 100zq53.51. 9(5cw)	a^4 ♂ 100zp37.37. 9(5bw)	0H135a^1 ♂ 50 q49.46. 1(3c)	0H143a^6 ♀ – q– – –(5c)
	a^5 ♀ – q– – –(5c–d)	a^6 ♂ 00 p– – –(alb.)	a^7 ♀ – q– – –(4c)	... ...	... ...	... ...
1H249	a^1 ♀ 90 q53.46.11(5cw)	a^2 ♂ 00 p– – –(alb. w)	a^3 ♂ 00 p– – –(alb. w)	a^4 ♀ 100xq53.51.47(6d)	0H143a^1 ♂ – q– – –(5c)	0H138a^1 ♀ – q– – –(5c)
	a^5 ♂ 100xq52.46.47(6d)	a^6 ♀ – q– – –(4c)	a^7 ♀ 70 q46.47. 2(3c)	... ...	... ...	... ...
1H250	a^1 ♂ 45 q44. 3. 3(3c–d)	a^2 ♂ – q– – –(5c)	a^3 ♂ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	0H143a^3 ♂ – q– – –(5c)	0H138a^4 ♀ 50 q49. 1. 1(3c)
	a^5 ♂ 00 p– – –(alb.)	... ...	... ...	... ...	... ...	... ...
1H252	a^1 ♂ – q– – –(4e)	a^2 ♂ 00 p– – –(alb. w)	... ...	... ...	0H124a^1 ♂ 100zq49.21.20(4c)	0H154a^2 ♀ 35 q49. 1. 1(2–3c)
1H253	a^1 90 q39.36. 1(4a)	a^2 100xq39.35.36(5a)	... ...	... ...	0H145a^2 ♂ 100zq35.35.30(5a–b)	0H145a^6 ♀ 100zq35.33.30(5–6a–b)
1H254	a^1 ♀ – p– – –(5a–bw)	a^2 ♀ 00 p– – –(alb.)	... ...	... ...	0H155a^1 ♂ 100xq35.35.30(5a)	0H145a^7 ♀ 100zq39.35.30(5a-b)
1H255	a^1 ♂ 100zq35.33.28(5aw)	a^2 ♀ 00 p– – –(alb. w)	a^3 ♂ 00 p– – –(alb.)	a^4 ♀ 100xq30.30.28(5a)	0H146a^2 ♂ 100zq35.33. 4(5a)	0H145a^8 ♀ 100zq39.35.30(5–6a–b)
	a^5 – p– – –(5a)	... ...	... ...	... ...	... ...	... ...
1H256	a^1 ♀ 100zq49.46.12(5cw)	a^2 ♂ 85 p37.37. 3(5b)	a^3 ♂ 100zq48.46.12(5c)	a^4 ♀ 70 p37.37. 2(4b)	0H145a^1 ♂ 100zq49.48. 9(5c)	0H146a^4 ♀ – q– – –(5c)
	a^5 – p– – –(4bw)	... ...	... ...	... ...	... ...	... ...
1H257	a^1 ♂ 00 p– – –(alb. w)	a^2 ♀ 00 p– – –(alb. w)	a^3 ♂ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	0H145a^5 ♂ 00 p– – –(alb.)	0H146a^5 ♀ 00 p– – –(alb.)
	a^5 ♂ 00 p– – –(alb.)	a^6 ♂ 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H258	a^1 ♂ 00 p– – –(alb.)	a^2 ♂ 00 p– – –(alb.)	... ...	... ...	0H146a^3 ♂ 00 p– – –(alb.)	0H146a^6 ♀ 00 p– – –(alb.)
1H260	a^1 ♀ 00 p– – –(alb.)	b^1 – q– – –(5c)	b^2 00 p– – –(alb.)	b^3 00 p– – –(alb.)	A167a^1 ♂ 00 p– – –(A × 2)	0H117a^2 ♀ 95 q49.46. 4(4c)
1H261	a^1 ♀ 75 q48.48. 1(4c)	a^2 ♀ 80 q48.46. 4(5c)	a^3 ♀ 00 p– – –(alb.)	a^4 ♀ 00 p– – –(alb.)	A163a^1 ♂ 00 p– – –(A × 2)	0H117a^1 ♀ 80 q49.46. 1(4c)
	a^5 ♂ 00 p– – –(alb.)	a^6 ♀ 00 p– – –(alb.)	a^7 ♀ 00 p– – –(alb.)	... ...	... ...	... ...
1H262	a^1 ♂ 100xp19. 6. 6(5b)	a^2 ♀ 45 q58. 1. 1(3e)	a^3 ♀ 20 p37. 1. 1(2bw)	a^4 ♀ 40 p37. 1. 1(3b)	0H104a^3 ♂ 40 q46. 1. 1(3c)	0H111a^4 ♀ – q– – –(5d)
	a^5 00 p– – –(alb.)	a^6 – q– – –(5c)	... ...	... ...	... ...	... ...
1H263	a^1 ♀ 35 q53. 4. 4(3cw)	a^2 ♀ 00 p– – –(alb. w)	a^3 ♀ 100zq48.46.20(5c)	a^4 ♀ 00 p– – –(alb.)	0H139a^2 ♂ – q– – –(5c)	0H124a^2 ♀ 90 q49.34. 1(4c)
	a^5 ♂ 40 q49. 1. 1(2–3c)	... ...	... ...	... ...	... ...	... ...
1H264	a^1 100xq53.51.47(6c)	a^2 100xq52.51.47(6c)	a^3 80 q52.51.11(5c)	a^4 100xq61.61.54(6e)	A150a^3 ♂ 00 p– – –(A1 or 2)	0H85a^3 ♀ – q– – –(5d)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	... ...	... ...	... ...
1H265	a^1 35 q48. 1. 1(2c)	a^2 – p– – –(4b)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	0H102a^2 ♂ 55 q48. 1. 2(3c)	0H112a^3 ♀ 50 q48.46. 1(3c)
	a^5 00 p– – –(alb.)	... ...	... ...	... ...	... ...	... ...
1H266	a^1 85 q39.35. 3(5a)	a^2 80 p30.28. 4(4aw)	a^3 100zq39.39.30(5a)	a^4 100zq35.33.28(5a)	0H84a^7 ♂ 100zq35.33.28(5a)	0H84a^4 ♀ 100zq35.33.30(5a)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H267	a^1 00 p– – –(alb.)	a^2 – p– – –(5b)	a^3 – p– – –(5b)	a^4 100zq48.46.43(5cw)	0H103a^3 ♂ – q– – –(5c)	0H118a^2 ♀ – q– – –(5c)
	a^5 – p– – –(5b)	a^6 – q– – –(4c)	a^7 – p– – –(3b)	... ...	... ...	... ...
1H268	a^1 100zq49.46. 9(5c)	a^2 – q– – –(3c)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	A150a^4 ♂ 00 p– – –(A1 or 2)	0H73a^5 ♀ – q– – –(3c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H269	a^1 ♀ 65 p33.28. 2(5a)	a^2 ♀ 100zp33.30.30(5a–b)	a^3 ♀ 70 p35.33. 2(5a–b)	a^4 100zq48.46. 4(5c)	0H111a^2 ♂ – q– – –(5a)	0H84a^5 ♀ 100zq33.30.28(5a)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb. w)	a^7 00 p– – –(alb.)	... ...	... ...	... ...
1H270	a^1 ♀ 100xq58.58.55(6e)	a^2 ♀ 90 q58.55. 2(4–5e)	a^3 – q– – –(6e)	a^4 – q– – –(5c)	A81a^2 ♂ 00 p– – –(A2 ??)	0H85a^4 ♀ – q– – –(5-6d)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	... ...	... ...	... ...

MATING	OFFSPRING				PARENTS	
					Father	Mother
1H218	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 00 p – – –(alb.)	a^4 ♂ 40 q40.37. 3(3b–c)	A147a^2 ♂ 00 p – – –(A2×)	0H112a^5 ♀ 40 q49. 1. 1(2c)
	a^5 ♂ 30 q51. 1 2(3c)	a^6 ♀ 60 q58.57 1(3–4c)	a^7 ♂ 50 q58 1. 1(3-4c)	...	...	...
1H219	a^1 ♀ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ 100zq51.51. 4(5c)	A152a^1 ♂ 00 p – – –(A1 or 2)	0H111a^6 ♀ – q – – (5d)
	a^5 ♀ – q – – –(6e)	a^6 ♀ 100xq58.58.55(6e)	...	...	...	...
1H220	a^1 ♀ 85 q48.46.11(5c)	a^2 ♂ 100zq49.46. 7(5c)	a^3 – q – (5c)	a^4 00 p – (alb.)	A152a^2 ♂ 00 p – –(A1 or 2)	0H120a^3 ♀ 45 q49. 1. 1(2c)
	a^5 00 p – – –(alb.)	...	...	...	...	...
1H221	a^1 ♀ 100xq58.58.52(6e)	a^2 ♀ 100xq58.58.57(6e)	a^3 ♂ 85 q47.51. 4(5c)	a^4 ♂ 00 p –(alb.)	A152a^3 ♂ 00 p – – –(A1 or 2)	0H101a^6 ♀ 45 q49.46. 1(3c)
1H222	a^1 ♀ 00 p – (alb.)	a^2 ♂ 00 p – (alb.)	a^3 ♂ 00 p – alb.)	a^4 ♀ 00 p –(alb.)	A159a^1 ♂ 00 p – – –(A1)	0H117a^4 ♀ 90 q49.36. 1(4c)
	a^5 ♂ 100xq40.37.36(5b–c)	a^6 ♀ 100xq41.21.19(5b–c)	a^7 ♂ 80 q52.51. 3(3c)	a^8 – q – – –(3–4c)	...	...
	a^9 – q – – –(5c)	...	...	...	...	...
1H223	a^1 ♀ 00 p (alb.)	a^2 ♂ 100zq48.46. 7(5c)	a^3 ♀ 90 q41.23. 1(4g)	...	A159a^2 ♂ 00 p – –(A1)	0H6a^1 ♀ – q – (2c)
1H224	a^1 ♂ 30 q36. 1. 1(2a)	a^2 ♀ 20 q61. 1. 1(2e)	a^3 25 p?23. 1. 1(2–3f)	a^4 – p? – (3f)	0H13a^4 ♂ 45 q52. 1. 1(3c)	0H113a^2 ♀ 50 q49.46. 2(3c)
	a^5 – q – – –(1cw)	a^6 00 p – – –(alb.)	...	...	...	...
1H225	a^1 ♂ 35 q49.48. 1(3c)	a^2 ♂ 85 q57.55. 1(4e)	a^3 ♂ p – –(2a)	a^4 ♂ 00 p – – –(alb.)	0H101a^2 ♂ 50 q49.46. 4(3c)	0H120a^6 ♀ 75 q49.46 1(4c)
	a^5 – q – – –(3c)	a^6 00 p – – –(alb. w)	a^7 – p – – –(3f)	...	...	...
1H226	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 45 p39. 1. 1(3b)	a^4 ♀ 55 q61.60. 1(4ew)	0H120a^4 ♂ 60 q49.46. 1(3c)	0H61a^6 ♀ 65 q49.30. 7(3c)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♀ 55 p37.39. 1(3bw)	...	...	...	...
1H227	a^1 ♂ 35 q48. 1. 1(2cw)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 60 p22. 1. 1(4f)	a^4 ♀ 60 q57.55. 1(3ew)	0H45a^3 ♂ – q – – –(2d)	0H74a^5 ♀ 80 q49.46. 2(4c)
	a^5 ♀ 65 p37.35. 1(3–4bw)	a^6 ♀ 25 q58. 1. 1(2e)	...	...	...	...
1H228	a^1 ♂ 45 q48 1 3(3c)	a^2 ♀ 75 q52 1 1(3d)	a^3 ♂ – q –(2c)	a^4 ♂ 00 p – – –(alb.)	0H114a^4 ♂ 50 q46.30 5(3c)	0H117a^5 ♀ 65 q49.36. 4(3c)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♂ 00 p – – –(alb. w)	...	...	...	...
1H229	a^1 ♂ 20 p22. 1. 1(3fw)	a^2 ♀ 60 q49. 1. 1(4c)	a^3 ♀ 35 q48. 1. 5(3c)	a^4 25 q47. 1. 1(2c)	0H100a^1 ♂ 40 q49. 1. 1(3c)	0H112a^4 ♀ 35 q46. 1. 1(2c)
	a^5 ♂ 50 p?18. 1. 1(2f)	a^6 – p – – –(2fw)	...	...	...	...
1H230	a^1 ♀ 85 p35.35. 3(4b)	...	...	...	0H110a^1 ♂ 60 q48.46. 1(3c)	0H82a^1 ♀ 55 q48.46.12(3c)
1H231	a^1 ♂ 65 p?40.36. 4(3b–c)	a^2 ♂ – q – (3c)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ 50 q40. 1. 1(3c)	0H120a^7 ♂ 50 q48.46. 3(3c)	0H103a^5 ♀ 55 q49.46. 4(3c)
	a^5 ♀ 70 q49.46 1(3–d)	a^6 ♀ 50 q58.56 1(3e)	a^7 ♀ 85 q57.55. 1(4–5e)	a^8 00 p – – –(alb.)	...	...
1H232	a^1 ♂ 65 q49.33 1(3c)	a^2 ♂ 65 q58.57. 1(4e)	a^3 ♀ 55 p39.39. 2(3b)	a^4 ♀ 35 p37. 1. 1(2b)	0H102a^1 ♂ 70 q48.46. 2(4c)	0H59a^1 ♀ 50 q48. 1. 4(3d)
	a^5 ♀ 00 p – – –(alb.)	a^6 – p – – (1bw)	...	...	...	...
1H233	a^1 ♂ 30 q46. 1. 3(1–2c)	a^2 ♂ 35 q48. 1. 3(2c)	a^3 ♂ – q – – –(2c)	a^4 ♂ – q – – –(2c)	0H75a^5 ♂ 15 q49. 1. 1(1e)	0H100a^6 ♀ 55 q49.46. 9(3c)
	a^5 ♂ 00 p – (alb.)	a^6 q –(2c)	a^7 – q – – –(2c)	a^8 – q – – –(2ew)	...	...
	a^9 00 p – – alb.)	...	...	...	...	...
1H234	a^1 ♂ 100zq49.48.20(5cw)	a^2 ♂ 100xp37.19. 9(5–6f)	a^3 ♀ 100zp22.15.15(5–6f)	a^4 ♀ 40 q49. 1 1(3c)	0H103a^2 ♂ q (5c)	0H27a^1 ♀ 40 q58 1. 2(2c)
	a^5 – p – – –(2fw)	...	...	...	...	...
1H235	a^1 80 q53.51. 4(5c)	a^2 – q – – –(4dw)	a^3 – q – – –(3e)	a^4 – q – – –(2c)	0H101a^4 ♂ – q – – –(5c)	0H100a^5 ♀ 25 q46. 1. 1(2c)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	...	...	...	...
1H236	a^1 – p – – –(4a)	a^2 – q – – –(4c)	a^3 – q – – –(3c)	b^1 – q – – –(3d)	0H118a^1 ♂ 85 q48.46.11(4c)	0H35a^2 ♀ – q – – –(3d)
	b^2 – q – – –(3c)	b^3 – p – – –(3b)	b^4 00 p – – –(alb.)	...	...	...
1H237	a^1 ♂ 60 p37. 3. 3(3bw)	a^2 ♀ 30 p37. 1. 1(2aw)	a^3 ♀ 95 p37.34. 3(5bw)	a^4 ♂ 95 p23.36. 3(5bw)	0H138a^3 ♂ – q – – –(5c)	0H128a^1 ♀ 85 q49.46. 1(4c)
	a^5 ♀ 80 p37.33. 4(5b)	a^6 ♀ 100zq53.51. 9(5c)	a^7 ♀ 100zq48.36. 9(5c)	...	...	...
1H238	a^1 ♀ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 100zq52.46.20(5d)	a^4 ♀ – q – – –(5d)	0H140a^5 ♂ 40 q49.46. 1(3c)	0H133a^1 ♀ 100zq49.46.12(5c)
	a^5 ♂ 35 p34 1. 1(2b)	a^6 ♂ 40 p37 1. 4(3b)	a^7 ♀ 15 p37. 1. 4(1bw)	...	...	...
1H239	a^1 ♂ 90 p18.18. 3(4fw)	a^2 ♀ 85 p37.36. 3(4bw)	a^3 ♀ 90 p37.36. 3(5b)	a^4 ♀ 35 q41. 1. 1(3g)	0H152a^2 ♂ – q – – –(5c)	0H140a^6 ♀ 40 q46. 1. 1(3c)
	a^5 ♀ 60 q49.46 4(4c)	a^6 q (3c)	a^7 – p – – –(3b)	...	...	...
1H240	a^1 ♂ 30 p35. 1. 1(3aw)	a^2 ♂ 90 p35.33.28(4a)	a^3 ♀ 35 p37. 3. 3(3b)	a^4 ♂ 55 q35.35. 1(3a)	0H129a^4 ♂ 35 q46.46. 3(2c)	0H131a^3 ♀ 95 q35.35. 2(4a)
	a^5 ♀ 85 q35.33. 3(4a)	a^6 ♂ 65 q48.34. 3(3c)	a^7 ♀ 00 p – – –(alb.)	...	...	...
1H241	a^1 40 p37. 1. 1(3b)	a^2 – p – – –(2aw)	a^3 – q –(2cw)	a^4 q (3c)	0H129a^6 ♂ 55 q49.36 1(2c)	0H131a^2 ♀ 60 q49.46 3(3d)
	a^5 q – –(3dw)	...	...	...	...	...
1H242	a^1 ♂ 35 q48.46. 1(2–3c)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ – p – – –(5a)	a^4 ♂ – q – – –(3c)	0H136a^1 ♂ 40 q47.28. 2(3c)	0H131a^4 ♀ 95 q35.35.29(6a)
	a^5 ♂ – q – – –(3c)	...	...	...	...	...
1H243	a^1 ♂ 100zq35.35.28(5aw)	a^2 ♂ 90 q36.33. 1(5a)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 60 q48.46. 3(3c)	0H131a^1 ♂ 100zq35.33.29(4–5a)	0H129a^1 ♀ – q – – – 3c
	a^5 ♀ 75 p37.29. 3(3b)	...	...	...	...	...
1H244	a^1 ♂ 40 q51. 2. 2(3cw)	a^2 ♂ 60 q49.46. 4(3c)	a^3 ♀ – q – – –(3c)	a^4 ♀ 35 p37.35. 1(2a–b)	0H129a^5 ♂ 55 q49. 1 1(3c)	0H129a^1 ♀ 25 q49. 1. 1(2c)
	a^5 ♂ 35 q46. 1. 1(2–3c)	a^6 ♀ 00 p (alb.)	...	...	...	...
1H245	a^1 ♀ – q – – –(3–4c)	a^2 ♀ 65 q53. 1. 3(3c)	a^3 ♂ 20 p37. 1. 1(2bw)	a^4 ♂ 35 p37.37 1(3b)	0H127a^5 ♂ 65 q49.46. 1(4?c)	0H129a^2 ♀ 50 q49. 1. 1(3c)
	a^5 ♂ 70 p37.37.30(3b)	a^6 ♀ 55 p37.19. 4(3b)	...	...	...	...
1H246	a^1 ♀ 90 q47.51. 3(4c)	a^2 ♀ 100xq48.51.47(6d)	a^3 ♂ 100xq53.51.47(6d)	a^4 ♂ – q – – –(6d)	0H127a^3 ♂ 50 q52. 1. 2(3c)	0H143a^2 ♀ q – – –(5c)
	a^5 ♂ 100xp37.37. 9(5b)	a^6 ♂ 00 p –(alb.)	...	...	...	...
1H248	a^1 ♀ 00 p – – –(alb. w)	a^2 ♂ 100xq53.51.47(6dw)	a^3 ♂ 100zq53.51. 9(5cw)	a^4 ♂ 100zp37.37. 9(5bw)	0H135a^1 ♂ 50 q49.46. 1(3c)	0H143a^6 ♀ – q – (5c)
	a^5 ♀ – q – – –(5c–d)	a^6 ♂ 00 p – – –(alb.)	a^7 ♀ – q – – –(4c)	...	...	...
1H249	a^1 ♀ 90 q53.46.11(5cw)	a^2 ♂ 00 p – – –(alb. w)	a^3 ♂ 00 p – – –(alb. w)	a^4 ♀ 100xq53.51.47(6d)	0H143a^1 ♂ – q – – –(5c)	0H138a^1 ♀ – q – – –(5c)
	a^5 ♂ 100xq52.46.47(6d)	a^6 ♀ q (4c)	a^7 ♀ 70 q46.47. 2(3c)	...	...	...
1H250	a^1 ♂ 45 q44. 3. 3(3c–d)	a^2 ♂ – q – – –(5c)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	0H143a^3 ♂ – q – – –(5c)	0H138a^4 ♀ 50 q49. 1. 1(3c)
	a^5 ♂ 00 p – – –(alb.)	...	...	...	...	...
1H252	a^1 ♂ – q – – –(4e)	a^2 ♂ 00 p – – –(alb. w)	...	...	0H124a^1 ♂ 100zq49.21.20(4c)	0H154a^2 ♀ 35 q49. 1. 1(2–3c)
1H253	a^1 90 q39.36 1(4a)	a^2 100xq39.35.36(5a)	...	...	0H145a^2 ♂ 100zq35.35.30(5a–b)	0H145a^6 ♀ 100zq35.33.30(5–6 a–b)
1H254	a^1 ♀ – p – – –(5a–bw)	a^2 ♀ 00 p – – –(alb.)	...	...	0H155a^1 ♂ 100xq35.35.30(5a)	0H145a^7 ♀ 100zq39.35.30(5a-b
1H255	a^1 ♂ 100zq35.33.28(5aw)	a^2 ♀ 00 p – –(alb. w)	a^3 ♂ 00 p – (alb.)	a^4 ♀ 100xq30.30.28(5a)	0H146a^2 ♂ 100zq35.33. 4(5a)	0H145a^8 ♀ 100zq39.35.30(5–6 a–b)
	a^5 – p – – –(5a)					
1H256	a^1 ♀ 100zq49.46.12(5cw)	a^2 ♂ 85 p37.37. 3(5b)	a^3 ♂ 100zq48.46.12(5c)	a^4 ♀ 70 p37.37. 2(4b)	0H145a^1 ♂ 100zq49.48. 9(5c)	0H146a^4 ♀ – q –(5c)
	a^5 – p – –(4bw)	...	...	...	...	...
1H257	a^1 ♂ 00 p – –(alb. w)	a^2 ♀ 00 p (alb. w)	a^3 ♂ 00 p – –(alb.)	a^4 ♂ 00 p – (alb.)	0H145a^5 ♂ 00 p – (alb.)	0H146a^5 ♀ 00 p – – (alb.)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♂ 00 p – – –(alb.)	...	...	...	...
1H258	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	...	...	0H146a^3 ♂ 00 p – –(alb.)	0H146a^6 ♀ 00 p – – (alb.)
1H260	a^1 ♀ 00 p –(alb.)	b^1 – q –(5c)	b^2 00 p (alb.)	b^3 00 p –(alb.)	A167a^1 ♂ 00 p –(A×2)	0H117a^2 ♀ 95 q49.46 4(4c)
1H261	a^1 ♀ 75 q48.48. 1(4c)	a^2 ♀ 80 q48.46. 4(5c)	a^3 ♀ 00 p (alb.)	a^4 ♀ 00 p – –(alb.)	A163a^1 ♂ 00 p – –(A×2)	0H117a^1 ♀ 80 q49.46. 1(4c)
	a^5 ♂ 00 p –(alb.)	a^6 ♀ 00 p –(alb.)	a^7 ♀ 00 p (alb.)			
1H262	a^1 ♂ 100zp19. 6. 6(5b)	a^2 ♀ 45 q58 1 1(3c)	a^3 ♀ 20 p37. 1 1(2bw)	a^4 ♀ 40 p37. 1 1(3b)	0H104a^3 ♂ 40 q46. 1. 1(3c)	0H111a^4 ♀ q – (5d)
	a^5 00 p (alb.)	a^6 – q – –(5c)	...	...	...	...
1H263	a^1 ♀ 35 q53. 4 4(3cw)	a^2 ♀ 00 p – –(alb. w)	a^3 ♀ 100zq48.46.20(5c)	a^4 ♀ 00 p – – (alb.)	0H139a^2 ♂ q – – (5c)	0H124a^2 ♀ 90 q49.34. 1(4c)
	a^5 ♂ 40 q49. 1. 1(2–3c)	...	...	...	...	...
1H264	a^1 100xq53.51.47(6c)	a^2 100xq52.51.47(6c)	a^3 80 q52.51.11(5c)	a^4 100xq61.61.54(6e)	A150a^1 ♂ 00 p – (A1 or 2)	0H85a^1 ♀ q – –(5d)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	a^7 00 p – – –(alb.)	...	...	...
1H265	a^1 35 q48. 1. 1(2c)	a^2 – p – – –(4b)	a^3 00 p – – –(alb.)	a^4 00 p – (alb.)	0H102a^2 ♂ 55 q48. 1. 2(3c)	0H112a^3 ♀ 50 q48.46. 1(3c)
	a^5 00 p – –(alb.)	...	...	...	...	...
1H266	a^1 85 q39.35. 3(5a)	a^2 80 p30.28. 4(4aw)	a^3 100zq39.39.30(5a)	a^4 100zq35.33.28(5a)	0H84a^7 ♂ 100zq35.33.28(5a)	0H84a^4 ♀ 100zq35.33.30(5a)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	...	...	...	...
1H267	a^1 00 p – – –(alb.)	a^2 – p – – –(5b)	a^3 – p – – –(5b)	a^4 100zq48.46.43(5cw)	0H103a^3 ♂ q – – (5c)	0H118a^2 ♀ q – – –(5c)
	a^5 – p – –(5b)	a^6 q (4c)	a^7 – p (3b)	...	...	...
1H268	a^1 100zq49.46. 9(5c)	a^2 – q – – –(3c)	a^3 00 p – – –(alb.)	a^4 00 p – –(alb.)	A150a^4 ♂ 00 p – (A1 or 2)	0H73a^5 ♀ – q – – –(3c)
	a^5 00 p –(alb.)	a^6 00 p – – –(alb.)	...	...	...	...
1H269	a^1 ♀ 65 p33.28. 2(5a)	a^2 ♀ 100zp33.30.30(5a–b)	a^3 ♀ 70 p35.33. 2(5a–b)	a^4 100zq48.46. 4(5c)	0H111a^2 ♂ – q – –(5a)	0H84a^6 ♀ 100zq33.30.28(5a)
	a^5 00 p – –(alb.)	a^6 00 p – – –(alb. w)	a^7 00 p – – –(alb.)	...	...	...
1H270	a^1 ♀ 100xq58.58.55(6e)	a^2 ♀ 90 q58.55. 2(4–5e)	a^3 – q – – –(6e)	a^4 – q – – (5c)	A81a^2 ♂ 00 p – – (A2??)	0H85a^4 ♀ – q – – –(5-6d)
	a^5 00 p – (alb.)	a^6 00 p – (alb.)	a^7 00 p – (alb.)	...	...	...

MATING	OFFSPRING				PARENTS	
					Father	Mother
1H271	a^1 80 q58.58. 2(4e)	a^2 75 q48.34. 4(4c)	a^3 – q– – –(5c)	a^4 00 p– – –(alb.)	A153a^1 ♂ 00 p– – –(A1 or 2)	0H45a^4 ♀ – q– – –(2d)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	...	...	...
1H272	a^1 65 q49.46. 1(4c)	a^2 100zq49.46.20(5c)	a^3 85 q49.46. 1(4c)	a^4 100zq48.46. 9(5c)	A150a^5 ♂ 00 p– – –(A1 or 2)	0H94a ♀ – q– – –(3c)
	a^5 – q– – –(4c)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	...	...	...
1H273	a^1 100zq49.34. 9(5c)	a^2 ♂ 100xq58.58.55(6e)	a^3 – q– – –(5c)	a^4 00 p– – –(alb.)	A153a^2 ♂ 00 p– – –(A1 or 2)	0H114a^2 ♀ 35 q49.46. 1(3c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	...	...	...	...
1H274	a^1 ♀ 100xq57.57.55(6e)	a^2 ♀ 100xq58.57.55(6e)	a^3 ♀ 100zq48.46.14(5-6c:e)	a^4 ♀ 100xq58.58.55(6e)	0H86a^1 ♂ 100zq49.46. 9(5c)	A182a^6 ♀ 00 p– – –(A1 or 2)
	a^5 100xq57.57.55(6e)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	...	...	...
1H275	a^1 – q– – –(3c)	a^2 – p– – –(3a)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	0H139a^1 ♂ 20 q49. 1. 2(2c)	0H148a^3 ♀ 45 q49.36. 1(3c)
	a^5 00 p– – –(alb.w)	...	...	...	...	...
1H276	a^1 ♀ 75 48.46. 1	a^2 ♀ 55 p37.39. 4(4b)	a^3 ♀ 100zp35.35.28(5b)	...	0H156a^1 ♂ 00 p– – –(alb.)	0H155a^2 ♀ 00 p– – –(alb.)
1H277	a^1 00 p– – –(alb.)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	0H156a^2 ♂ 00 p– – –(alb.)	0H155a^3 ♀ 00 p– – –(alb.)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	...	...	...
1H278	a^1 – q– – –(5c)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	0H145a^4 ♂ 100zq36.30.28(5a–b)	0H156a^3 ♀ 00 p– – –(alb.)
	a^5 00 p– – –(alb.)	...	...	...	...	...
1H279	a^1 00 p– – –(alb.)	...	...	...	0H146a^1 ♂ 90 q49.34. 2(4c)	0H156a^4 ♀ 100zq48.46.20(5b–c)
1H280	a^1 85 q49.48. 1(4c)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	...	0H145a^3 ♂ 100zq35.35.30(5a–b)	0H146a^7 ♀ 00 p– – –(alb.)
1H281	a^1 ♀ 100zp39.39. 6(5b)	a^2 – q– – –(4c)	a^3 – q– – –(5cw)	a^4 – p– – –(5b)	0H160a^2 ♂ – q– – –(5c)	0H161a^4 ♀ – q– – –(5c)
1H283	a^1 ♀ 80 p39.35. 2(5b)	...	...	...	0H159a^3 ♂ – q– – –(5c)	0H161a^6 ♀ – q– – –(5c)
1H286	a^1 – q– – –(5c)	a^2 – q– – –(3c)	a^3 – p– – –(5b)	a^4 00 p– – –(alb.w)	0H161a^2 ♂ 80 q48.46. 7(5c)	0H170a^4 ♀ – q– – –(5c)
	a^5 00 p– – –(alb.)	...	...	...	...	...
1H287	a^1 85 q40.37. 9(5b–c)	a^2 85 q51.51. 2(5c)	a^3 75 q61. 1. 1(3ew)	a^4 80 p37.37. 1(4b)	0H161a^3 ♂ – q– – –(5c)	0H170a^5 ♀ 90 q49.35. 2(4c)
1H288	a^1 35 q47. 1. 1(2c)	a^2 30 q47. 1. 1(2c)	a^3 00 p– – –(alb.)	...	0H120a^7 ♂ 50 q48.46. 3(3c)	0H120a^3 ♀ 45 q49. 1. 1(2c)
1H289	a^1 100zq49.48.12(5cw)	a^2 95 q49.46.11(5cw)	a^3 100xp15.15.15(6f)	a^4 00 p– – –(alb.)	0H100a^3 ♂ – q– – –(5c)	0H100a^8 ♀ – q– – –(5c)
	a^5 00 p– – –(alb.)	...	...	...	...	...
1H290	a^1 100xq49.48.47(6c)	a^2 80 q49.46. 1(4c)	a^3 95 q49.34. 9(4c)	a^4 00 p– – –(alb.)	0H90a^2 ♂ – q– – –(5c)	0H117a^4 ♀ 90 q49.36. 1(4c)
	a^5 00 p– – –(alb.)	...	...	...	...	...
1H291	a^1 100xp26.26.22(5fw)	a^2 30 p37. 1. 1(2b)	a^3 85 p37.35. 3(5b)	a^4 – q– – –(3c)	0H111a^1 ♂ – q– – –(5d)	0H7a^6 ♀ 100zq48.46.12(5c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	...	...	...
1H292	a^1 100yp23.23.15(5f)	a^2 100yp19.19. 9(5f)	a^3 85 p37.35. 4(5b)	a^4 90 q61.54. 1(4e)	0H102a^4 ♂ – q– – –(5c)	0H62a^5 ♀ 100zq49.48.12(5c)
	a^5 100zq49.46.11(5c)	a^6 – q– – –(5c)	...	...	...	...
1H293	a^1 100zp39.35.30(5b)	a^2 60 q47.46. 1(3c)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	0H100a^2 ♂ – q– – –(5c)	0H111a^3 ♀ – q– – –(5d)
	a^5 00 p– – –(alb.)	...	...	...	...	...
1H295	a^1 100zp36.34.28(5b-fw)	a^2 45 q50. 1. 1(2c)	a^3 100zp37.37. 4(5b)	a^4 25 p26. 1. 1(2f)	0H112a^2 ♂ – q– – –(5c)	0H102a^6 ♀ – q– – –(5c)
1H296	a^1 100zq48.46.11(5c)	a^2 50 p37. 1. 1(3b)	a^3 100zq46.46. 7(5c)	a^4 100zp36.36.30(5b)	0H103a^1 ♂ – q– – –(5c)	0H111a^5 ♀ – q– – –(5d)
1H297	a^1 100zq48.46.12(5c)	a^2 100zq55.46.12(5c)	a^3 100zq49. 9. 9(5c)	a^4 100zq49.25. 2(5c)	0H59a^2 ♂ 100zq49.46. 7(6d)	0H62a^4 ♀ 100zq49.46. 7(5c)
	a^5 – q– – –(4c)	...	...	...	...	...
1H298	a^1 50 q44. 1. 1(2c)	a^2 100xq58.58.55(5ew)	a^3 100xp24.24.15(5fw)	a^4 – q– – –(5c)	0H103a^3 ♂ – q– – –(5c)	0H111a^6 ♀ – q– – –(5d)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	...	...	...	...
1H300	a^1 85 q49.46. 2(4c)	a^2 65 q61.61. 1(3ew)	a^3 40 p37. 1. 1(3b)	a^4 80 p37.35. 4(4b)	0H120a^4 ♂ 60 q49.46. 1(3c)	0H85a^1 ♀ – q– – –(4d)
	a^5 80 p23.34. 4(4b)	a^6 00 p– – –(alb.)	...	...	...	...
1H301	a^1 50 q49.46. 1(2c)	a^2 30 q49. 1. 1(2c)	a^3 – p– – –(2b–c)	a^4 – q– – –(2ew)	0H114a^5 ♂ 25 q49. 1. 2(2c)	0H112a^5 ♀ 40 q49. 1. 1(2c)
	a^5 30 q49. 1. 1(2c)	...	...	...	...	...
1H302	a^1 100xq58.58.55(6e)	a^2 100zq55.46.14(6d)	a^3 90 p36.36. 9(5b)	a^4 35 p40. 1. 1(2b)	0H86a^2 ♂ 100zq49.46.28(5c)	0H102a^5 ♀ – q– – –(5c)
	a^5 55 q55. 1. 1(3c)	a^6 100zq55.46.12(6c)	a^7 00 p– – –(alb.)	a^8 00 p– – –(alb.)	...	...

1H304	a^1	65 q49.34. 1(3c)	a^2	30 p37. 1. 1(2b)	a^3	35 p37. 1. 1(2b)	a^4	35 q44. 1. 1(2c)	0H140a^5 ♂	40 q49.46. 1(3c)	0H140a^6 ♀	40 q46. 1. 1(3c)
	a^5	00 p– – –(alb.)		… … …		… … …		… … …	…	… … …	…	… … …
1H305	a^1	80 q48.46. 7(5c)	a^2	100zp35.33.30(5aw)	a^3	45 q35. 1. 3(2a)	a^4	00 p– – –(alb.)	0H138a^3 ♂	– q– – –(5c)	0H131a^4 ♀	95 q35.35.29(6a)
1H306	a^1	100xq58.58.57(6ew)	a^2	100zq49.46.47(5c)	a^3	80 q49.46.12(3c)	a^4	80 p39.35. 3(4b)	0H152a^2 ♂	– q– – –(5c)	0H154a^2 ♀	35 q49. 1. 1(2–3c)
	a^5	– q– – –(3c)	a^6	00 p– – –(alb.)		… … …		… … …	…	… … …	…	… … …
1H308	a^1	– q– – –(2c)	a^2	20 p37. 1. 1(2b)	a^3	50 p37. 1. 1(3bw)	a^4	40 p37. 9. 9(2bw)	0H127a^3 ♂	50 q52. 1. 2(3c)	0H129a^1 ♀	– q– – –(3c)
	a^5	50 q55.46. 1(2c)	a^6	00 p– – –(alb.w)		… … …		… … …	…	… … …	…	… … …
1H309	a^1	100xq49.48.47(6cw)	a^2	00 p– – –(alb.)	a^3	00 p– – –(alb.w)	a^4	00 p– – –(alb.)	0H143a^1 ♂	– q– – –(5c)	0H138a^4 ♀	50 q49. 1. 1(3c)
	a^5	00 p– – –(alb.)	a^6	00 p– – –(alb.)		… … …		… … …	…	… … …	…	… … …
1H310	a^1	100zq55.46.12(5d)	a^2	100zq48.46.14(5c)	a^3	100zp37.36. 3(5b)	a^4	85 q48.46. 7(5c)	0H143a^3 ♂	– q– – –(5c)	0H138a^1 ♀	– q– – –(5c)
	a^5	00 p– – –(alb.)	a^6	00 p– – –(alb.)		… … …		… … …	…	… … …	…	… … …
1H311	a^1	70 p37.34. 1(4bw)	a^2	100zp37.35. 9(5b)	a^3	80 p23.35. 4(4b)	a^4	100xq46.47.47(6d)	0H135a^1 ♂	50 q49.46. 1(3c)	0H143a^6 ♀	– q– – –(5c)
	a^5	90 q49.46. 4(4c)	a^6	95 q49.46. 1(4c)	a^7	– p– – –(2b)		… … …	…	… … …	…	… … …
1H312	a^1	50 q46. 1. 1(3c)	a^2	30 q46. 1. 1(2c)	a^3	30 q47. 1. 1(2c)	a^4	55 q48.46. 1(3c)	0H129a^4 ♂	35 q46.46. 3(2c)	0H129a^3 ♀	25 q49. 1. 1(2c)
	a^5	– p– – –(2b)	a^6	– q– – –(4c)	a^7	00 p– – –(alb.w)	a^8	00 p– – –(alb.)	…	… … …	…	… … …
1H313	a^1	45 q58. 1. 1(2ew)	a^2	100x 58.58.55(6e)	a^3	55 p37. 1. 3(3b)	a^4	100zq49.48. 7(5cw)	0H139a^2 ♂	– q– – –(5c)	0H128a^1 ♀	85 q49.46. 1(4c)
	a^5	00 p– – –(alb.w)		… … …		… … …		… … …	…	… … …	…	… … …
1H314	a^1	80 q49.46. 1(3c)	a^2	80 q48.46.11(4c)	a^3	50 q49.46. 3(2c)	a^4	80 q48.46.11(6c)	0H143a^5 ♂	– q– – –(5c)	0H129a^2 ♀	50 q49. 1. 1(3c)
	a^5	80 p37.35. 3(5b)		… … …		… … …		… … …	…	… … …	…	… … …
1H318	a^1	100zp37.35. 1(5b)	a^2	55 q47.47. 1(3c)	a^3	100zq46.46. 9(5c)	a^4	00 p– – –(alb.)	0H85a^5 ♂	– q– – –(5d)	0H6a^1 ♀	– q– – –(2c)
1H319	a^1	100zq33.33.28(5a)	a^2	100zq35.34.28(5a)	a^3	– q– – –(5a)	a^4	– q– – –(5a)	0H84a^7 ♂	100zq35.33.28(5a)	0H111a^9 ♀	100zq35.34.29(5a)
	a^5	00 p– – –(alb.)		… … …		… … …		… … …	…	… … …	…	… … …
1H320	a^1	50 q61.60. 1(2e)	a^2	95 p36.35. 4(4b)	a^3	75 q46.46. 1(4c)	a^4	45 q49.46. 1(3c)	0H13a^4 ♂	45 q52. 1. 1(3c)	0H103a^4 ♀	55 q49. 1. 3(3c)
	a^5	80 q58.55. 1(4e)	a^6	00 p– – –(alb.)	a^7	00 p– – –(alb.)		… … …	…	… … …	…	… … …
1H321	a^1	90 r37.37. 2(4b–c)	a^2	30 p37.37. 1(2b)	a^3	80 q52.47. 4(4c)	a^4	50 q58. 1. 1(2e)	0H101a^2 ♂	50 q49.46. 4(3c)	0H113a^2 ♀	50 q49.46. 2(3c)
	a^5	00 p– – –(alb.)	a^6	00 p– – –(alb.)		… … …		… … …	…	… … …	…	… … …
1H322	a^1	20 q47. 1. 1(1c)	a^2	– p– – –(3bw)	a^3	00 p– – –(alb.w)	a^4	00 p– – –(alb.w)	0H101a^4 ♂	– q– – –(5c)	0H61a^6 ♀	65 q49.30. 7(3c)
	a^5	00 p– – –(alb.)	a^6	00 p– – –(alb.)		… … …		… … …	…	… … …	…	… … …
1H323	a^1	100xq58.58.57(5ew)	a^2	15 p37. 1. 1(1b)	a^3	85 p37.37. 3(5b)	a^4	60 q46.46. 1(3c)	0H101a^3 ♂	– q– – –(5c)	0H82a^1 ♀	55 q48.46.12(3c)
	a^5	00 p– – –(alb.)	a^6	00 p– – –(alb.w)		… … …		… … …	…	… … …	…	… … …
1H324	a^1	45 58. 1. 1(2e)	a^2	no details		… … …		… … …	0H45a^3 ♂	– q– – –(2d)	0H59a^1 ♀	50 q48. 1. 4(3d)
1H325	a^1	30 p37. 1. 1(2bw)	a^2	40 q48. 1. 1(3c)	a^3	– q– – –(2cw)		… … …	0H110a^1 ♂	60 q48.46. 1(3c)	0H120a^6 ♀	75 q49.46. 1(4c)
1H326	a^1	80 p37.35. 3(5b)	a^2	90 q48.21.11(5c)	a^3	90 r40. 1. 1(3-4b-c)	a^4	– q– – –(5c)	0H102a^1 ♂	70 q48.46. 2(4c)	0H111a^4 ♀	– q– – –(5d)
	a^5	– p– – –(5fw)	a^6	00 p– – –(alb.)	a^7	00 p– – –(alb.)		… … …	…	… … …	…	… … …
1H328	a^1	60 p35. 1. 2(3a)	a^2	45 p35. 2. 2(2a)	a^3	– q– – –(4a)	a^4	– q– – –(5cw)	0H111a^2 ♂	– q– – –(5a)	0H110a^2 ♀	60 q52.46. 2(3c)
	a^5	00 p– – –(alb.)	a^6	00 p– – –(alb.)		… … …		… … …	…	… … …	…	… … …
1H329	a^1	100zq48.46.20(5c)	a^2	80 q48.46. 4(5c)	a^3	45 q44. 1. 3(2c)	a^4	100yp35.34.28(5b)	0H86a^1 ♂	100zq49.46. 9(5c)	0H111a^8 ♀	100zq35.35.30(5a)
	a^5	100zp37.37. 4(5b)	a^6	– q– – –(5c)	a^7	– q– – –(2a)	a^8	00 p– – –(alb.)	…	… … …	…	… … …
1H331	a^1	85 p48.48. 4(5c)	a^2	30 q49. 1. 1(2cw)	a^3	80 q49.48. 1(3c)	a^4	100zp37.34. 3(5b)	0H104a^3 ♂	40 q46. 1. 1(3c)	0H114a^1 ♀	– q– – –(5c)
	a^5	00 p– – –(alb.)	a^6	00 p– – –(alb.)		… … …		… … …	…	… … …	…	… … …
1H333	a^1	100xq58.58.57(6e)	a^2	100xq58.58.57(6e)	a^3	100zq48.48.14(5c)	a^4	– q– – –(5c)	A152a^3 ♂	00 p– – –(A1 or 2)	0H100a^6 ♀	55 q49.46. 9(3c)
	a^5	00 p– – –(alb.)	a^6	00 p– – –(alb.)		… … …		… … …	…	… … …	…	… … …
1H334	a^1	– q– – –(5c)	a^2	00 p– – –(alb.)	a^3	00 p– – –(alb.)	a^4	00 p– – –(alb.)	A153a^1 ♂	00 p– – –(A1 or 2)	0H74a^5 ♀	80 q49.46. 2(4c)
	a^5	00 p– – –(alb.)		… … …		… … …		… … …	…	… … …	…	… … …
1H335	a^1	95 q49.46.12(4c)	a^2	00 p– – –(alb.)	a^3	00 p– – –(alb.)	a^4	00 p– – –(alb.)	A159a^2 ♂	00 p– – –(A1)	0H117a^5 ♀	65 q49.36. 4(3c)
	a^5	00 p– – –(alb.)	a^6	00 p– – –(alb.)		… … …		… … …	…	… … …	…	… … …
1H336	a^1	100zq48.46. 7(5c)	a^2	75 q48.46. 1(3c)	a^3	100zq46.46.14(5c)	a^4	90 q48.46.14(4c)	A133a^6 ♂	00 p– – –(A × 2)	0H103a^5 ♀	55 q49.46. 4(3c)
	a^5	00 p– – –(alb.)	a^6	00 p– – –(alb.)	a^7	00 p– – –(alb.)	a^8	00 p– – –(alb.)	…	… … …	…	… … …
1H337	a^1	55 p26.26. 1(3f)	a^2	35 p39. 1. 1(2bw)	a^3	40 r40. 1. 1(2c)	a^4	00 p– – –(alb.w)	0H102a^2 ♂	55 q48. 1. 2(3c)	0H27a^1 ♀	40 q58. 1. 2(2e)
1H341	a^1	100xq49.48.47(6c)	a^2	100zq49.46.20(5c)	a^3	100xq49.46.47(6c)	a^4	100zq48.36. 9(5c)	0H102a^3 ♂	95 q48.46. 7(4c)	A30a^1 ♀	00 p– – –(A2 ?)
	a^5	100zq49.36.11(5c)	a^6	00 p– – –(alb.)	a^7	00 p– – –(alb.)		… … …	…	… … …	…	… … …

MATING	OFFSPRING				PARENTS	
					Father	Mother
1H271	a^1 80 q58.58. 2(4e)	a^2 75 q48.34. 4(4c)	a^3 – q– – –(5c)	a^4 00 p– – –(alb.)	A153a^1 ♂ 00 p– – –(A1 or 2)	0H45a^4 ♀ – q– – –(2d)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	... ...	... ...	... ...
1H272	a^1 65 q49.46. 1(4c)	a^2 100zq49.46.20(5c)	a^3 85 q49.46. 1(4c)	a^4 100zq48.46. 9(5c)	A150a^5 ♂ 00 p– – –(A1 or 2)	0H94a ♀ – q– – –(3c)
	a^5 – q– – –(4c)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	... ...	... ...	... ...
1H273	a^1 100zq49.34. 9(5c)	a^2 ♂ 100xq58.58.55(6e)	a^3 – q– – –(5c)	a^4 00 p– – –(alb.)	A153a^2 ♂ 00 p– – –(A1 or 2)	0H114a^2 ♀ 35 q49.46. 1(3c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H274	a^1 ♀ 100xq57.57.55(6e)	a^2 ♀ 100xq58.57.55(6e)	a^3 ♀ 100zq48.46.14(5-6c:e)	a^4 ♀ 100xq58.58.55(6e)	0H86a^1 ♂ 100zq49.46. 9(5c)	A182a^6 ♀ 00 p– – –(A1 or 2)
	a^5 100xq57.57.55(6e)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	... ...	... ...	... ...
1H275	a^1 – q– – –(3c)	a^2 – p– – –(3a)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	0H139a^1 ♂ 30 q49. 1. 2(2c)	0H148a^3 ♀ 45 q49.36. 1(3c)
	a^5 00 p– – –(alb.w)	... ...	... ...	... ...	... ...	... ...
1H276	a^1 ♀ 75 48.46. 1	a^2 ♀ 55 p37.39. 4(4b)	a^3 ♀ 100zp35.35.28(5b)	... ...	0H156a^1 ♂ 00 p– – –(alb.)	0H155a^2 ♀ 00 p– – –(alb.)
1H277	a^1 00 p– – –(alb.)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	0H156a^2 ♂ 00 p– – –(alb.)	0H155a^1 ♀ 00 p– – –(alb.)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	... ...	... ...	... ...
1H278	a^1 – q– – –(5c)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	0H145a^1 ♂ 100zq36.30.28(5a–b)	0H156a^3 ♀ 00 p– – –(alb.)
	a^5 00 p– – –(alb.)	... ...	... ...	... ...	... ...	... ...
1H279	a^1 00 p– – –(alb.)	... ...	... ...	... ...	0H146a^1 ♂ 90 q49.34. 2(4c)	0H156a^4 ♀ 100zq48.46.20(5b–c)
1H280	a^1 85 q49.48. 1(4c)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	... ...	0H145a^3 ♂ 100zq35.35.30(5a–b)	0H146a^7 ♀ 00 p– – –(alb.)
1H281	a^1 ♀ 100zp39.39. 6(5b)	a^2 – q– – –(4c)	a^3 – q– – –(5cw)	a^4 – p– – –(5b)	0H160a^2 ♂ – q– – –(5c)	0H161a^4 ♀ – q– – –(5c)
1H283	a^1 ♀ 80 p39.35. 2(5b)	... ...	... ...	... ...	0H159a^3 ♂ – q– – –(5c)	0H161a^6 ♀ – q– – –(5c)
1H286	a^1 – q– – –(5c)	a^2 – q– – –(3c)	a^3 – p– – –(5b)	a^4 00 p– – –(alb.w)	0H161a^2 ♂ 80 q48.46. 7(5c)	0H170a^4 ♀ – q– – –(5c)
	a^5 00 p– – –(alb.)	... ...	... ...	... ...	... ...	... ...
1H287	a^1 85 q40.37. 9(5b–c)	a^2 85 q51.51. 2(5c)	a^3 75 q61. 1. 1(3cw)	a^4 80 p37.37. 1(4b)	0H161a^3 ♂ – q– – –(5c)	0H170a^5 ♀ 90 q49.35. 2(4c)
1H288	a^1 35 q47. 1. 1(2c)	a^2 30 q47. 1. 1(2c)	a^3 00 p– – –(alb.)	... ...	0H120a^7 ♂ 50 q48.46. 3(3c)	0H120a^3 ♀ 45 q49. 1. 1(2c)
1H289	a^1 100zq49.48.12(5cw)	a^2 95 q49.46.11(5cw)	a^3 100xp15.15.15(6f)	a^4 00 p– – –(alb.)	0H100a^3 ♂ – q– – –(5c)	0H100a^6 ♀ – q– – –(5c)
	a^5 00 p– – –(alb.)	... ...	... ...	... ...	... ...	... ...
1H290	a^1 100xq49.48.47(6c)	a^2 80 q49.46. 1(4c)	a^3 95 q49.34. 9(4c)	a^4 00 p– – –(alb.)	0H90a^2 ♂ – q– – –(5c)	0H117a^4 ♀ 90 q49.36. 1(4c)
	a^5 00 p– – –(alb.)	... ...	... ...	... ...	... ...	... ...
1H291	a^1 100xp26.26.22(5fw)	a^2 30 p37. 1. 1(2b)	a^3 85 p37.35. 3(5b)	a^4 – q– – –(3c)	0H111a^1 ♂ – q– – –(5d)	0H7a^6 ♀ 100zq48.46.12(5c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	... ...	... ...	... ...
1H292	a^1 100yp23.23.15(5f)	a^2 100yp19.19. 9(5f)	a^3 85 p37.35. 4(5b)	a^4 90 q61.54. 1(4e)	0H102a^4 ♂ – q– – –(5c)	0H62a^5 ♀ 100zq49.48.12(5c)
	a^5 100zq49.46.11(5c)	a^6 – q– – –(5c)	... ...	... ...	... ...	... ...
1H293	a^1 100zp39.35.30(5b)	a^2 60 q47.46. 1(3c)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	0H100a^2 ♂ – q– – –(5c)	0H111a^3 ♀ – q– – –(5d)
	a^5 00 p– – –(alb.)	... ...	... ...	... ...	... ...	... ...
1H295	a^1 100zp36.34.28(5b-fw)	a^2 45 q50. 1. 1(2c)	a^3 100zp37.37. 4(5b)	a^4 25 p26. 1. 1(2f)	0H112a^2 ♂ – q– – –(5c)	0H102a^6 ♀ – q– – –(5c)
1H296	a^1 100zq48.46.11(5c)	a^2 50 p37. 1. 1(3b)	a^3 100zq46.46. 7(5c)	a^4 100zp36.36.30(5b)	0H103a^1 ♂ – q– – –(5c)	0H111a^5 ♀ – q– – –(5d)
1H297	a^1 100zq48.46.12(5c)	a^2 100zq55.46.12(5c)	a^3 100zq49. 9. 9(5c)	a^4 100zq49.25. 2(5c)	0H59a^2 ♂ 100zq49.46. 7(6d)	0H62a^4 ♀ 100zq49.46. 7(5c)
	a^5 – q– – –(4c)	... ...	... ...	... ...	... ...	... ...
1H298	a^1 50 q44. 1. 1(2c)	a^2 100xq58.58.55(5ew)	a^3 100xp24.24.15(5fw)	a^4 – q– – –(5c)	0H103a^3 ♂ – q– – –(5c)	0H111a^6 ♀ – q– – –(5d)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H300	a^1 85 q49.46. 2(4e)	a^2 65 q61.61. 1(3ew)	a^3 40 p37. 1. 1(3b)	a^4 80 p37.35. 4(4b)	0H120a^4 ♂ 60 q49.46. 1(3c)	0H85a^1 ♀ – q– – –(4d)
	a^5 80 p23.34. 4(4b)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H301	a^1 50 q49.46. 1(2c)	a^2 30 q49. 1. 1(2c)	a^3 – p– – –(2b–c)	a^4 – q– – –(2cw)	0H114a^5 ♂ 25 q49. 1. 2(2c)	0H112a^5 ♀ 40 q49. 1. 1(2c)
	a^5 30 q49. 1. 1(2c)	... ...	... ...	... ...	... ...	... ...
1H302	a^1 100xq58.58.55(6e)	a^2 100zq55.46.14(6d)	a^3 90 p36.36. 9(5b)	a^4 75 p40. 1. 1(2b)	0H86a^2 ♂ 100zq49.46.28(5c)	0H102a^5 ♀ – q– – –(5c)
	a^5 55 q55. 1. 1(3c)	a^6 100zq55.46.12(6c)	a^7 00 p– – –(alb.)	a^8 00 p– – –(alb.)	... ...	... ...

MATING	OFFSPRING				Father	Mother
1H304	a^1 65 q49.34. 1(3c)	a^2 30 p37. 1. 1(2b)	a^3 [illegible] p37. 1. 1(2b)	a^4 35 q44. 1. 1(2c)	0H140a^5 ♂ 40 q49.46. 1(3c)	0H140a^6 ♀ 40 q46. 1. 1(3c)
	a^5 00 p– – –(alb.)	... ...	... ...	... ...	... ...	... ...
1H305	a^1 80 q48.46. 7(5c)	a^2 100zp35.33.30(5aw)	a^3 45 q35. 1. 3(2a)	a^4 00 p– – –(alb.)	0H138a^3 ♂ – q– – –(5c)	0H131a^4 ♀ 95 q35.35.29(6a)
1H306	a^1 100xq58.58.57(6ew)	a^2 100zq49.46.47(5c)	a^3 80 q49.46.12(3c)	a^4 80 p39.35. 3(4b)	0H152a^2 ♂ – q– – –(5c)	0H154a^2 ♀ 35 q49. 1. 1(2–3c)
	a^5 – q– – –(3c)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H308	a^1 – q– – –(2c)	a^2 20 p37. 1. 1(2b)	a^3 50 p37. 1. 1(3bw)	a^4 40 p37. 9. 9(2bw)	0H127a^3 ♂ 50 q52. 1. 2(3c)	0H129a^1 ♀ – q– – –(3c)
	a^5 50 q55.46. 1(2c)	a^6 00 p– – –(alb.w)	... ...	... ...	... ...	... ...
1H309	a^1 100xq49.48.47(6cw)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.w)	a^4 00 p– – –(alb.)	0H143a^1 ♂ – q– – –(5c)	0H138a^4 ♀ 50 q49. 1. 1(3c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H310	a^1 100zq55.46.12(5d)	a^2 100zq48.46.14(5c)	a^3 100zp37.36. 3(5b)	a^4 85 q48.46. 7(5c)	0H143a^2 ♂ – q– – –(5c)	0H138a^1 ♀ – q– – –(5c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H311	a^1 70 p37.34. 1(4bw)	a^2 100zp37.35. 9(5b)	a^3 80 p23.35. 4(4b)	a^4 100xq46.47.47(6d)	0H135a^1 ♂ 50 q49.46. 1(3c)	0H143a^6 ♀ – q– – –(5c)
	a^5 90 q49.46. 4(4c)	a^6 95 q49.46. 1(4c)	a^7 – p– – –(2b)	... ...	... ...	... ...
1H312	a^1 50 q46. 1. 1(3c)	a^2 30 q46. 1. 1(2c)	a^3 30 q47. 1. 1(2c)	a^4 55 q48.46. 1(3c)	0H129a^4 ♂ 35 q46.46. 3(2c)	0H129a^3 ♀ 25 q49. 1. 1(2c)
	a^5 – p– – –(2b)	a^6 – q– – –(4c)	a^7 00 p– – –(alb.w)	a^8 00 p– – –(alb.)	... ...	... ...
1H313	a^1 45 q58. 1. 1(2ew)	a^2 100xq58.58.55(6e)	a^3 55 p37. 1. 3(3b)	a^4 100zq49.48. 7(5cw)	0H139a^2 ♂ – q– – –(5c)	0H128a^1 ♀ 85 q49.46. 1(4c)
	a^5 00 p– – –(alb.w)	... ...	... ...	... ...	... ...	... ...
1H314	a^1 80 q49.46. 1(3c)	a^2 80 q48.46.11(4c)	a^3 50 q49.46. 3(2c)	a^4 80 q48.46.11(6c)	0H143a ♂ – q– – –(5c)	0H129a^2 ♀ 50 q49. 1. 1(3c)
	a^5 80 p37.35. 3(5b)		... ...	... ...	... ...	... ...
1H318	a^1 100zp37.35. 1(5b)	a^2 75 q47.47. 1(3c)	a^3 100zq46.46. 9(5c)	a^4 00 p– – –(alb.)	0H85a^5 ♂ – q– – –(5d)	0H6a^1 ♀ – q– – –(2c)
1H319	a^1 100zq33.33.28(5a)	a^2 100zp35.34.28(5a)	a^3 – q– – –(5a)	a^4 – q– – –(5a)	0H84a^7 ♂ 100zq35.33.28(5a)	0H111a^9 ♀ 100zq35.34.29(5a)
	a^5 00 p– – –(alb.)		... ...	... ...	... ...	... ...
1H320	a^1 50 q61.60. 1(2e)	a^2 90 p36.35. 4(4b)	a^3 75 q46.46. 1(4c)	a^4 45 q49.46. 1(3c)	0H13a^4 ♂ 45 q52. 1. 1(3c)	0H103a^4 ♀ 55 q49. 1. 3(3c)
	a^5 80 q58.55. 1(4e)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	... ...	... ...	... ...
1H321	a^1 90 r37.37. 2(4b–c)	a^2 30 p37.37. 1(2b)	a^3 80 q52.47. 4(4c)	a^4 50 q58. 1. 1(2e)	0H101a^2 ♂ 50 q49.46. 4(3c)	0H113a^2 ♀ 50 q49.46. 2(3c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H322	a^1 20 q47. 1. 1(1c)	a^2 – p– – –(3bw)	a^3 00 p– – –(alb.w)	a^4 00 p– – –(alb.w)	0H101a^4 ♂ – q– – –(5c)	0H61a^0 ♀ 65 q49.30. 7(3c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H323	a^1 100xq58.58.57(5ew)	a^2 15 p37. 1. 1(1b)	a^3 85 p37.37. 3(5b)	a^4 60 q46.46. 1(3c)	0H101a^3 ♂ – q– – –(5c)	0H82a^1 ♀ 55 q48.46.12(3c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.w)	... ...	... ...	... ...	... ...
1H324	a^1 45 q58. 1. 1(2e)	a^2 no details	... ...	... ...	0H45a^3 ♂ – q– – –(2d)	0H59a^1 ♀ 50 q48. 1. 4(3d)
1H325	a^1 30 p37. 1. 1(2bw)	a^2 40 q48. 1. 1(3c)	a^3 – q– – –(2cw)	... ...	0H110a^1 ♂ 60 q48.46. 1(3c)	0H120a^0 ♀ 75 q49.46. 1(4c)
1H326	a^1 80 p37.35. 3(5b)	a^2 90 q48.21.11(5c)	a^3 90 r40. 1. 1(3-4b-c)	a^4 – q– – –(5c)	0H102a^1 ♂ 70 q48.46. 2(4c)	0H111a^4 ♀ – q– – –(5d)
	a^5 – p– – –(5fw)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	... ...	... ...	... ...
1H328	a^1 60 p35. 1. 2(3a)	a^2 45 p35. 2. 2(2a)	a^3 – q– – –(4a)	a^4 – q– – –(5cw)	0H111a^2 ♂ – q– – –(5a)	0H110a^2 ♀ 60 q52.46. 2(3c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H329	a^1 100zq48.46.20(5c)	a^2 80 q48.46. 4(5c)	a^3 45 q44. 1. 3(2c)	a^4 100yp35.34.28(5b)	0H86a^1 ♂ 100zq49.46. 9(5c)	0H111a^8 ♀ 100zq35.35.30(5a)
	a^5 100zp37.37. 4(5b)	a^6 – q– – –(5c)	a^7 – q– – –(2a)	a^8 00 p– – –(alb.)	... ...	... ...
1H331	a^1 85 p48.48. 4(5c)	a^2 30 q49. 1. 1(2cw)	a^3 80 q49.48. 1(3c)	a^4 100zp37.34. 3(6b)	0H104a^3 ♂ 40 q46. 1. 1(3c)	0H114a^1 ♀ – q– – –(5c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H333	a^1 100xq58.58.57(6e)	a^2 100xq58.58.57(6e)	a^3 100zq48.48.14(5c)	a^4 – q– – –(5c)	A152a^3 ♂ 00 p– – –(A1 or 2)	0H100a^0 ♀ 57 q49.46. 9(3c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H334	a^1 – q– – –(5c)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	A153a^1 ♂ 00 p– – –(A1 or 2)	0H74a^5 ♀ 80 q49.46. 2(4c)
	a^5 00 p– – –(alb.)	... ...	... ...	... ...	... ...	... ...
1H335	a^1 95 q49.46.12(4c)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	A159a^3 ♂ 00 p– – –(A1)	0H117a^5 ♀ 65 q49.36. 4(3c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
1H336	a^1 100zq48.46. 7(5c)	a^2 75 q48.46. 1(3c)	a^3 100zq46.46.14(5c)	a^4 90 q48.46.14(4c)	A133a^0 ♂ 00 p– – –(A × 2)	0H103a^5 ♀ 55 q49.46. 4(3c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	a^8 00 p– – –(alb.)	... ...	... ...
1H337	a^1 55 p26.26. 1(3f)	a^2 35 p39. 1. 1(2bw)	a^3 40 r40. 1. 1(2c)	a^4 00 p– – –(alb.w)	0H102a^2 ♂ 55 q48. 1. 2(3c)	0H27a^1 ♀ 40 q58. 1. 2(2c)
1H341	a^1 100xq49.48.47(6c)	a^2 100zq49.46.20(5c)	a^3 100xq49.46.47(6c)	a^4 100zq48.36. 9(5c)	0H102a^3 ♂ 95 q48.46. 7(4c)	A130a^1 ♀ 00 p– – –(A2 ?)
	a^5 100zq49.36.11(5c)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	... ...	... ...	... ...

MATING	OFFSPRING				PARENTS	
					Father	Mother
2H11	a^1 ♂ 45 p40.39. 1(2b)	a^2 ♀ – p– – –(2–3b)	... ...	... ...	1H70a^2 ♂ 55 q37.33.30(3a)	1H19b^1 ♀ 40 q58. 1. 1(2e)
2H12	a^1 ♀ 90 p37.30. 4(5a–b)	a^2 ♀ 100zp37.30. 3(5a–b)	a^3 ♀ 100zp39.39.34(5b–c)	a^4 ♀ – p– – –(4a–b)	1H42a^2 ♂ 90 p39.35. 4(4a)	1H129a^4 ♀ 100zp39.35.30(5b)
	a^5 ♂ 40 p39. 1. 1(2b)	... ...	... ...	... ...	... ...	... ...
2H13	a^1 ♂ 100xp26.23.18(6f)	a^2 ♂ 100zp26.23.19(6f)	a^3 ♂ 100xp22.23.17(6f)	a^4 ♂ 100zp23.23.18(6f)	1H122a^4 ♂ 55 p23. 1. 1(3f)	1H129a^3 ♀ 100xp24.23.15(6f)
2H15	a^1 ♀ 80 p36. 1. 3(4b)	a^2 ♂ – p– – –(3b)	a^3 ♂ – p– – –(3b)	a^4 ♀ 40 p39.35. 1(3b)	1H111a^5 ♂ 95 p35. 1. 3(4bw)	1H136a^4 ♀ 35 p39. 1. 1(2b)
	a^5 ♂ 00 p– – –(alb.)	... ...	... ...	... ...	... ...	... ...
2H18	a^1 – p– – –(6b–c)	a^2 – q– – –(6d)	a^3 – q– – –(6d)	a^4 – p– – –(6f)	1H154a^2 ♂ – q– – –(6e)	1H152a^1 ♀ 100zq49.48.47(6c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	... ...	... ...	... ...	... ...
2H19	a^1 ♂ 00 p– – –(alb.)	a^2 – q– – –(5c)	a^3 – q– – –(5c)	a^4 – q– – –(3d)	1H161a^2 ♂ 100zq48.46. 7(5c)	1H154a^1 ♀ 100xq58.58.55(6e)
	a^5 – q– – –(3c)	... ...	... ...	... ...	... ...	... ...
2H20	a^1 ♂ 00 p– – –(alb.)	a^2 ♀ 00 p– – –(alb.)	a^3 ♂ 100zq39.35.28(5a)	a^4 ♂ – q– – –(5d)	1H161a^6 ♂ – – – – –	1H151a^4 ♀ 80 q49.48.34(5c)
	a^5 ♀ – q– – –(6d)	a^6 ♀ – q– – –(4a)	a^7 – q– – –(5a)	... ...	... ...	... ...
2H22	a^1 ♂ 100yq51.47. 2(4c)	a^2 ♀ 100zq49.46.43(5c)	a^3 ♀ – q– – –(5c)	a^4 ♂ 00 p– – –(alb.)	1H16a^2 ♂ 55 q58.55. 1(3e)	1H151a^1 ♀ 100zq49.48.34(5c)
	a^5 ♀ – q– – –(4c)	... ...	... ...	... ...	... ...	... ...
2H23	a^1 ♂ 100zp39.39.30(5b)	a^2 ♂ 100zq49.48.47(5–6c)	a^3 ♀ 100zq48.48.20(5c)	a^4 ♂ 00 p– – –(alb.)	1H148a^2 ♂ 80 q49.46. 4(5c)	1H161a^3 ♀ 100zq49.48. 9(5c)
	a^5 ♂ 00 p– – –(alb.)	... ...	... ...	... ...	... ...	... ...
2H25	a^1 ♀ 00 p– – –(alb.)	a^2 ♀ 00 p– – –(alb.)	a^3 ♂ 00 p– – –(alb.)	a^4 ♀ – q– – –(6e)	1H60a^1 ♂ 100xq57.57.55(6e)	1H158a^3 ♀ 100zq49.48.34(5c)
2H26	a^1 ♀ – q– – –(3c)	a^2 ♀ 00 p– – –(alb.)	a^3 100zq48.46. 4(4c)	a^4 00 p– – –(alb.)	1H60a^3 ♂ 75 q46.46. 4(4c)	1H157a^1 ♀ – q– – –(5a)
	a^5 – q– – –(5a)	a^6 – q– – –(4a)	... ...	... ...	... ...	... ...
2H27	a^1 ♂ – q– – –(6ew)	a^2 ♀ 75 p35.33.28(5aw)	a^3 ♀ 100zq35.33.28(5a)	a^4 ♀ 100zq35.33.28(5a)	1H80a^7 ♂ 100xq57.57.55(6e)	1H157a^2 ♀ – q– – –(5a)
	a^5 ♂ – q– – –(3e)	... ...	... ...	... ...	... ...	... ...
2H28	a^1 ♂ 100zq55.46. 7(5c)	a^2 ♀ 100zp26.26. 9(5b)	a^3 ♂ – q– – –(6e)	a^4 ♀ – q– – –(6e)	1H98a^1 ♂ – q– – –(4e)	1H157a^3 ♀ 100zq49.48.14(5d)
	a^5 ♂ – q– – –(5c)	... ...	... ...	... ...	... ...	... ...
2H29	a^1 ♀ 00 p– – –(alb.)	a^2 ♂ 100zq48.46.12(5c)	a^3 ♀ 100zq48.46.43(5c)	a^4 ♀ – q– – –(6e)	1H98a^3 ♂ 100zq49.46. 4(6c)	1H156a^1 ♀ 100xq58.57.55(6e)
2H32	a^1 ♀ 100zq49.46.12(5c)	a^2 ♂ 90 q46.51.11(5c)	a^3 ♂ – q– – –(5c)	a^4 ♂ – q– – –(5c)	1H96a^1 ♂ 100zq48.46. 4(5c)	1H3a^3 ♀ 100zq49.46. 7(6c)
	a^5 ♀ – q– – –(4c)	... ...	... ...	... ...	... ...	... ...
2H33	a^1 ♀ 100zq39.35.30(5a–b)	a^2 ♂ 100zq39.35.30(5a–b)	a^3 ♀ 100zq40.37.17(5b–c)	a^4 ♂ – q– – –(5b-cw)	1H96a^2 ♂ 100zq35.33.28(5–6b)	1H3a^4 ♀ 100zq46.36.20(5c)
	a^5 ♂ – p– – –(4bw)	a^6 ♂ 100zq49.25. 7(5c)	... ...	... ...	... ...	... ...
2H34	a^1 ♂ 60 q44.21. 2(3c)	a^2 ♀ – q– – –(3c : e)	a^3 – q– – –(6e)	a^4 ♂ 00 p– – –(alb.)	1H135a^2 ♂ 30 q60. 1. 1(2e)	1H3a^2 ♀ – q– – –(6d)
	a^5 – q– – –(4c)	a^6 – q– – –(6e)	a^7 – p– – –(3b)	... ...	... ...	... ...
2H36	a^1 ♀ 100zq49.46.12(5c)	a^2 ♂ 100xq35.33.30(5a)	a^3 ♀ 55 q39.33. 4(3a)	... ...	1H101a^3 ♂ 65 q61.60. 3(3e)	1H97a^1 ♀ 100zq33.30.28(5–6b?)
2H37	a^1 ♂ – q– – –(4e)	a^2 ♀ – q– – –(3–4e)	a^3 ♀ – q– – –(6e)	a^4 ♀ 00 p– – –(alb.)	1H144a^1 ♂ – q– – –(2e)	1H88b^6 ♀ – q– – –(6e)
	a^5 ♀ 00 p– – –(alb.)	a^6 ♂ 00 p– – –(alb.)	... ...	... ...	... ...	... ...
2H38	a^1 ♂ 60 q48. 1. 4(3b–c)	a^2 ♀ 55 p40.39. 1(3bw)	a^3 ♀ 100zq49.48. 7(5d)	a^4 ♀ – p– – –(5b)	1H114a^5 ♂ 35 q48. 1. 1(2c)	1H65b^1 ♀ – q– – –(6e)
	a^5 ♀ – p– – –(5b)	a^6 ♂ 00 p– – –(alb.)	... ...	... ...	... ...	... ...
2H39	a^1 ♀ 95 p37.34. 1(5b)	a^2 ♀ 100zp37.37. 9(5b)	a^3 ♂ 80 q49.48. 7(5c)	a^4 ♂ – q– – –(1c)	1H114a^2 ♂ 10 q58. 1. 1(1e)	1H98a^4 ♀ 100zq49.46.30(5c)
	a^5 ♀ – q– – –(2c)	a^6 ♂ – q– – –(2c)	a^7 ♀ – q– – –(2c)	b^1 – p– – –(5b)	... ...	... ...
	b^2 – q– – –(2c)	b^3 – q– – –(3c)	b^4 – q– – –(4c)	b^5 – q– – –(5c)	... ...	... ...
2H40	a^1 ♂ 100zq49.46. 9(5b–c)	a^2 ♂ 100zq46.36.28(5c)	a^3 ♀ – q– – –(5b–c)	... ...	1H131a^1 ♂ 45 q49.46. 1(3c)	1H98a^5 ♀ 100zq48.46. 4(5–6c)
2H41	a^1 ♂ 00 p– – –(alb.)	a^2 ♂ – q– – –(6e)	a^3 ♀ – q– – –(6e)	a^4 ♀ – q– – –(6e)	1H111a^3 ♂ 100zq48.46. 7(5c)	1H65b^2 ♀ 100xq58.58.56(6e)
	a^5 ♀ 45 q47.47. 1(3c)	... ...	... ...	... ...	... ...	... ...
2H43	a^1 ♂ – q– – –(4e)	a^2 ♂ 65 p26.23. 1(4f)	a^3 ♀ – q– – –(3e)	a^4 ♂ – q– – –(3e)	1H113a^2 ♂ 65 q58. 1. 1(3e)	1H87a^1 ♀ 100xq58.58.57(6e)
	a^5 – p– – –(3f)	a^6 – q– – –(6e)	a^7 – q– – –(6e)	a^8 – p– – –(3f)	... ...	... ...
2H44	a^1 ♂ 100zq48.46.12(5c)	a^2 ♂ 75 q49.46. 2(5b–c)	a^3 ♀ 00 p– – –(alb.)	a^4 ♂ 65 q49.36.11(3c)	1H100a^2 ♂ 30 q46. 1. 1(2c)	1H32a^5 ♀ 100zq49.46. 4(5–6c)
	a^5 ♀ 65 q46.46. 4(3–4c)	a^6 ♀ – q– – –(3c)	a^7 ♀ 40 p37.33. 1(3a)	b^1 100zq48.35.30(5b–c)	... ...	... ...

	b^2 100zq49.46.12(5c)	b^3 30 q48. 1. 1(2c)	b^4 00 p– – –(alb.)	b^5 00 p– – –(alb.)	… … … …	… … … …
2H45	a^1♂ 100zq49.46.11(4c)	a^2♀ – q– – –(5c)	a^3♂ – q– – –(5c)	a^4♀ 00 p– – –(alb.)	1H143a^1♂ 100zq48.46. 2(5c)	1H80a^6 ♀ 100zq37.36. 4(5g)
	a^5♂ 00 p– – –(alb.)	a^6♀ 00 p– – –(alb.)	… … …	… … …	… … … …	… … … …
2H46	a^1 – p– – –(6b)	… … …	… … …	… … …	1H117a^1♂ 60 q49.46. 1(5c)	1H121a^3♀ – q– – –(4d)
2H48	a^1♂ 100xq35.33.28(5a)	a^2♀ – q– – –(5a)	a^3♂ 00 p– – –(alb.)	a^4♀ 00 p– – –(alb.)	1H115a^1♂ 30 q47.46. 1(1c)	1H136a^3♀ – q– – –(5a)
	a^5♂ – p– – –(4a)	… … …	… … …	… … …	… … … …	… … … …
2H49	a^1♂ – q– – –(6e)	a^2♂ 100xq58.57.55(6e)	a^3♀ 100xq58.57.55(6e)	a^4♂ 100zq49.46.11(5c–d)	1H119a^3♂ 55 q49. 1. 1(3c)	1H142a^1♀ 100xq58.57.55(6e)
	a^5♂ – q– – –(6e)	a^6♀ 100zq49.48.14(5c–d)	… … …	… … …	… … … …	… … … …
2H50	a^1♀ 30 q46. 1. 1(3c)	a^2♂ – q– – –(3–4e)	a^3♀ 40 q46.46. 1(3c)	a^4♂ 55 q46. 1. 1(2c)	1H121a^5♂ 55 q48.46. 2(3c)	1H115a^3♀ 85 q58.55. 1(4e)
	a^5♀ 00 p– – –(alb.)	b^1 – q– – –(2c)	b^2 – q– – –(3e)	b^3 – q– – –(4e)	… … … …	… … … …
	b^4 – q– – –(4e)	b^5 00 p– – –(alb.)	… … …	… … …	… … … …	… … … …
2H51	a^1♀ – q– – –(3g)	a^2♀ – q– – –(3c)	a^3♀ – q– – –(3c)	a^4♂ – q– – –(3e)	1H122a^5♂ 25 q46. 1. 1(2c)	1H113a^5♀ 95 q49.48.12(4c)
	a^5 – q– – –(1c)	… … …	… … …	… … …	… … … …	… … … …
2H52	a^1♂ 100zp36.36. 2(5b)	a^2 75 q48.46.11(5c)	a^3 60 q46.46. 7(3c)	a^4 100zq49.46.12(5c)	1H99a^3 ♂ 80 q48.46. 7(5d)	1H100a^4♀ 65 q49.46. 1(3c–d)
	a^5♀ 00 p– – –(alb.)	… … …	… … …	… … …	… … … …	… … … …
2H53	a^1♂ – q– – –(3e)	a^2♂ – q– – –(3e)	a^3♂ – q– – –(6e)	a^4♂ – q– – –(3–4e)	1H118a^2♂ 55 q49. 1. 2(3c)	1H101a^5♀ – q– – –(6e)
	a^5♀ – q– – –(4–5e)	a^6♂ – q– – –(5c–d)	a^7♀ – q– – –(6e)	a^8 00 p– – –(alb.w)	… … … …	… … … …
	b^1 – q– – –(3c)	b^2 – p– – –(3b)	b^3 – q– – –(3c)	b^4 – q– – –(6e)	… … … …	… … … …
	b^5 – q– – –(5cw)	… … …	… … …	… … …	… … … …	… … … …
2H55	a^1♀ 25 p39. 1. 4(1-2a-b)	a^2♀ 35 p39.39. 3(2a–b)	a^3♂ 10 p37. 1. 1(1a–b)	a^4 – p– – –(5a)	1H99a^1 ♂ 100zp39.35.28(6b)	1H118a^1♀ – p– – –(2b)
2H59	a^1♂ 00 p– – –(alb.)	a^2♀ 00 p– – –(alb.)	a^3♂ 00 p– – –(alb.)	a^4♀ 00 p– – –(alb.)	1H157a^5♂ 00 p– – –(alb.)	1H152a^3♀ 00 p– – –(alb.)
	a^5♀ 00 p– – –(alb.)	… … …	… … …	… … …	… … … …	… … … …
2H60	a^1♂ 00 p– – –(alb.)	a^2♂ 00 p– – –(alb.)	a^3♀ 00 p– – –(alb.)	a^4♀ 00 p– – –(alb.)	1H153a^3♂ 00 p– – –(alb.)	1H152a^5♀ 00 p– – –(alb.)
	a^5♀ 00 p– – –(alb.)	… … …	… … …	… … …	… … … …	… … … …
2H61	a^1♀ 00 p– – –(alb.)	… … …	… … …	… … …	1H153a^4♂ 00 p– – –(alb.)	1H16a^6 ♀ 00 p– – –(alb.)
2H62	a^1♀ 00 p– – –(alb.)	a^2♂ 00 p– – –(alb.)	a^3♂ 00 p– – –(alb.)	a^4♂ 00 p– – –(alb.)	1H143a^4♂ 00 p– – –(alb.)	1H16a^7 ♀ 00 p– – –(alb.)
	a^5♀ 00 p– – –(alb.)	… … …	… … …	… … …	… … … …	… … … …
2H63	a^1♂ 00 p– – –(alb.)	a^2♂ 00 p– – –(alb.)	a^3♂ 00 p– – –(alb.)	a^4♀ 00 p– – –(alb.)	1H112a^4♂ 00 p– – –(alb.)	1H85b^3 ♀ 00 p– – –(alb.)
	a^5♂ 00 p– – –(alb.)	a^6♂ 00 p– – –(alb.)	b^1 00 p– – –(alb.)	b^2 00 p– – –(alb.)	… … … …	… … … …
	b^3 00 p– – –(alb.)	b^4 00 p– – –(alb.)	b^5 00 p– – –(alb.)	… … …	… … … …	… … … …
2H64	a^1♀ 00 p– – –(alb.w)	a^2♂ 00 p– – –(alb.)	a^3♀ 00 p– – –(alb.)	a^4♀ 00 p– – –(alb.)	1H115a^5♂ 00 p– – –(alb.)	1H96a^3 ♀ 00 p– – –(alb.)
	a^5♀ 00 p– – –(alb.)	a^6♂ 00 p– – –(alb.)	… … …	… … …	… … … …	… … … …
2H65	a^1♂ 00 p– – –(alb.)	a^2♂ 00 p– – –(alb.)	a^3♂ 00 p– – –(alb.)	a^4♀ 00 p– – –(alb.)	1H88b^1 ♂ 00 p– – –(alb.)	1H96a^4 ♀ 00 p– – –(alb.)
	a^5♂ 00 p– – –(alb.)	… … …	… … …	… … …	… … … …	… … … …
2H66	a^1♂ 00 p– – –(alb.)	a^2♂ 00 p– – –(alb.)	a^3♀ 00 p– – –(alb.)	a^4♀ 00 p– – –(alb.)	1H119a^5♂ 00 p– – –(alb.)	1H146a^5♀ 00 p– – –(alb.)
2H67	a^1♂ 00 p– – –(alb.)	a^2♀ 00 p– – –(alb.)	a^3♂ 00 p– – –(alb.)	a^4♂ 00 p– – –(alb.)	1H123a^2♂ 00 p– – –(alb.)	1H88b^2 ♀ 00 p– – –(alb.)
	a^5♀ 00 p– – –(alb.)	a^6♀ 00 p– – –(alb.)	… … …	… … …	… … … …	… … … …
2H69	a^1 00 p– – –(alb.)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	1H149a^5♂ 00 p– – –(alb.)	1H88b^4 ♀ 00 p– – –(alb.)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	… … …	… … …	… … … …	… … … …
2H70	b^1 00 p– – –(alb.)	b^2 00 p– – –(alb.)	… … …	… … …	1H16a^3 ♂ 00 p– – –(alb.)	1H123a^3♀ 00 p– – –(alb.)
2H71	a^1♂ 00 p– – –(alb.)	a^2♂ 00 p– – –(alb.)	a^3♂ 00 p– – –(alb.)	a^4♀ 00 p– – –(alb.)	1H16a^4 ♂ 00 p– – –(alb.)	1H149a^4♀ 00 p– – –(alb.)
	a^5♂ 00 p– – –(alb.)	… … …	… … …	… … …	… … … …	… … … …
2H73	a^1 00 p– – –(alb.)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	… … …	1H39a^4 ♂ 00 p– – –(alb.)	1H142a^3♀ 00 p– – –(alb.)
2H75	a^1♀ 00 p– – –(alb.)	a^2♂ 00 p– – –(alb.)	a^3♂ 00 p– – –(alb.)	a^4♂ 00 p– – –(alb.)	1H85b^5 ♂ 00 p– – –(alb.)	1H140a^4♀ 00 p– – –(alb.)
	a^5♂ 00 p– – –(alb.)	… … …	… … …	… … …	… … … …	… … … …
2H76	a^1 00 p– – –(alb.)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	1H92a^4 ♂ 00 p– – –(alb.)	1H147a^4♀ 00 p– – –(alb.)
	a^5 00 p– – –(alb.w)	a^6 00 p– – –(alb.)	… … …	… … …	… … … …	… … … …
2H77	a^1♂ 00 p– – –(alb.)	a^2♂ 00 p– – –(alb.)	a^3♀ 00 p– – –(alb.)	a^4♀ 00 p– – –(alb.)	1H92a^5 ♂ 00 p– – –(alb.)	1H143a^5♀ 00 p– – –(alb.)
	a^5♂ 00 p– – –(alb.)	a^6♂ 00 p– – –(alb.)	b^1 00 p– – –(alb.)	b^2 00 p– – –(alb.)	… … … …	… … … …
	b^3 00 p– – –(alb.)	b^4 00 p– – –(alb.)	… … …	… … …	… … … …	… … … …

MATING	OFFSPRING				PARENTS	
					Father	Mother
2H11	a^1 ♂ 45 p40.39. 1(2b)	a^2 ♀ – p– – –(2–3b)	...	...	1H70a^2 ♂ 55 q37.33.30(3a)	1H19b^1 ♀ 40 q58. 1. 1(2e)
2H12	a^1 ♀ 00 p37.30. 4(5a–b)	a^2 ♀ 100zp37.30. 3(5a–b)	a^3 ♀ 100zp39.39.34(5b–c)	a^4 ♀ – p– – –(4a–b)	1H42a^2 ♂ 90 p39.35. 4(4a)	1H129a^4 ♀ 100zp39.35.30(5b)
	a^6 ♂ 40 p39. 1. 1(2b)	...	...	...	...	...
2H13	a^1 ♂ 100xp26.23.18(6f)	a^2 ♂ 100zp26.23.19(6f)	a^3 ♂ 100xp22.23.17(6f)	a^4 ♂ 100zp23.23.18(6f)	1H122a^4 ♂ 55 p23. 1. 1(3f)	1H129a^1 ♀ 100xp24.23.15(6f)
2H15	a^1 ♀ 80 p36. 1. 3(4b)	a^2 ♂ – p– – –(3b)	a^3 ♂ – p– – –(3b)	a^4 ♀ 40 p39.35. 1(3b)	1H111a^5 ♂ 95 p35. 1. 3(4bc)	1H136a^4 ♀ 35 p39. 1. 1(2b)
	a^6 ♂ 00 p– – –(alb.)	...	...	...	...	...
2H18	a^1 – p– – –(6b–c)	a^2 – q– – –(6d)	a^3 – q– – –(6d)	a^4 – p– – –(6f)	1H154a^2 ♂ – q– – –(6e)	1H152a^1 ♀ 100zq49.48.47(6c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	...	...	...	...
2H19	a^1 ♂ 00 p– – –(alb.)	a^2 – q– – –(5c)	a^3 – q– – –(5c)	a^4 – q– – –(3d)	1H161a^2 ♂ 100zq48.46. 7(5c)	1H154a^1 ♀ 100xq58.58.55(6e)
	a^5 – q– – –(3c)	...	...	...	...	...
2H20	a^1 ♂ 00 p– – –(alb.)	a^2 ♀ 00 p– – –(alb.)	a^3 ♂ 100zp39.35.28(5c)	a^4 ♂ – q– – –(5b)	1H161a^6 ♂	1H151a^4 ♀ 80 q49.48.34(5e)
	a^5 ♀ – q– – –(6d)	a^6 ♀ – q– – –(4a)	a^7 – q– – –(5a)	...	...	...
2H22	a^1 ♂ 100yq51.47. 2(4e)	a^2 ♀ 100zq49.46.43(5e)	a^3 ♀ – q– – –(5e)	a^4 ♂ 00 p– – –(alb.)	1H16a^2 ♂ 55 q58.55. 1(3e)	1H151a^3 ♀ 100zq49.48.34(5e)
	a^5 ♀ – q– – –(4c)	...	...	...	...	...
2H23	a^1 ♂ 100zp39.39.30(5b)	a^2 ♂ 100zq49.48.47(5–6e)	a^3 ♀ 100zq48.48.20(5e)	a^4 ♂ 00 p– – –(alb.)	1H148a^2 ♂ 80 q49.46. 4(5e)	1H161a^3 ♀ 100zq49.48. 9(5e)
	a^6 ♂ 00 p– – –(alb.)	...	...	...	...	...
2H25	a^1 ♀ 00 p– – –(alb.)	a^2 ♀ 00 p– – –(alb.)	a^3 ♂ 00 p– – –(alb.)	a^4 ♀ – q– – –(6e)	1H60a^1 ♂ 100xq57.57.55(6e)	1H158a^3 ♀ 100zq49.48.34(5e)
2H26	a^1 ♀ – q– – –(3e)	a^2 ♀ 00 p– – –(alb.)	a^3 100zq48.46. 4(4e)	a^4 00 p– – –(alb.)	1H60a^1 ♂ 75 q46.46. 1(4e)	1H157a^1 ♀ – q– – –(5a)
	a^5 – q– – –(5a)	a^6 – q– – –(4a)	...	...	...	...
2H27	a^1 ♂ – q– – –(6ew)	a^2 ♀ 75 p35.33.28(5aw)	a^3 ♀ 100zq35.33.28(5a)	a^4 ♀ 100zq35.33.28(5a)	1H80a^7 ♂ 100xq57.57.55(6e)	1H157a^2 ♀ – q– – –(5a)
	a^5 ♂ – q– – –(3e)	...	...	...	...	...
2H28	a^1 ♂ 100zq55.46. 7(5c)	a^2 ♀ 100zp26.26. 9(5b)	a^3 ♂ – q– – –(6e)	a^4 ♀ – q– – –(6e)	1H98a^1 ♂ – q– – –(4e)	1H157a^3 ♀ 100zq49.48.14(5d)
	a^5 ♂ – q– – –(5c)	...	...	...	...	...
2H29	a^1 ♀ 00 p– – –(alb.)	a^2 ♂ 100zq48.46.12(5e)	a^3 ♀ 100zq48.46.43(5e)	a^4 ♀ – q– – –(6e)	1H98a^3 ♂ 100zq49.46. 4(6e)	1H156a^1 ♀ 100xq58.57.55(6e)
2H32	a^1 ♀ 100zq49.46.12(5c)	a^2 ♂ 90 q46.51.11(5c)	a^3 ♂ – q– – –(5c)	a^4 ♂ – q– – –(5c)	1H96a^1 ♂ 100zq48.46. 4(5c)	1H3a^3 ♀ 100zq49.46. 7(6c)
	a^5 ♀ – q– – –(4e)	...	...	...	...	...
2H33	a^1 ♀ 100zq39.35.30(5a–b)	a^2 ♂ 100zq39.35.30(5a–b)	a^3 ♀ 100zq40.37.17(5b–c)	a^4 ♂ – q– – –(5b–cw)	1H96a^2 ♂ 100zq35.33.28(5–6b)	1H3a^1 ♀ 100zq46.36.20(5c)
	a^5 ♂ – p– – –(4bw)	a^6 ♂ 100zq49.25. 7(5c)	...	...	...	...
2H34	a^1 ♂ 60 q44.21. 2(3c)	a^2 ♀ – q– – –(3c : e)	a^3 – q– – –(6e)	a^4 ♂ 00 p– – –(alb.)	1H135a^2 ♂ 30 q60. 1. 1(2e)	1H3a^2 ♀ – q– – –(6d)
	a^5 – q– – –(4c)	a^6 – q– – –(6e)	a^7 – p– – –(3b)	...	...	...
2H36	a^1 ♀ 100zq49.46.12(5c)	a^2 ♂ 100xq35.33.30(5a)	a^3 ♀ 55 q39.33. 4(3a)	...	1H101a^3 ♂ 65 q61.60. 3(3e)	1H97a^1 ♀ 100zq33.30.28(5–6b?)
2H37	a^1 ♂ – q– – –(4e)	a^2 ♀ – q– – –(3–4e)	a^3 ♀ – q– – –(6e)	a^4 ♀ 00 p– – –(alb.)	1H144a^1 ♂ – q– – –(2e)	1H88b^6 ♀ – q– – –(6e)
	a^5 ♀ 00 p– – –(alb.)	a^6 ♂ 00 p– – –(alb.)	...	...	...	...
2H38	a^1 ♂ 60 q48. 1. 4(3b–c)	a^2 ♀ 55 p40.39. 1(3bw)	a^3 ♀ 100zq49.48. 7(5d)	a^4 ♀ – p– – –(5b)	1H114a^5 ♂ 35 q48. 1. 1(2c)	1H65b^1 ♀ – q– – –(6e)
	a^5 ♀ – p– – –(5b)	a^6 ♂ 00 p– – –(alb.)	...	...	...	...
2H39	a^1 ♀ 95 p37.34. 1(5b)	a^2 ♀ 100zp37.37. 9(5b)	a^3 ♂ 80 q49.48. 7(5c)	a^4 ♂ – q– – –(1e)	1H114a^2 ♂ 10 q58. 1. 1(1e)	1H98a^4 ♀ 100zq49.46.30(5c)
	a^5 ♀ – q– – –(2c)	a^6 ♂ – q– – –(2e)	a^7 ♀ – q– – –(2c)	b^1 – p– – –(5b)	...	...
	b^2 – q– – –(2c)	b^3 – q– – –(3c)	b^4 – q– – –(4c)	b^5 – q– – –(5c)	...	...
2H40	a^1 ♂ 100zq49.46. 9(5b–c)	a^2 ♂ 100zq46.36.28(5c)	a^3 ♀ – q– – –(5b–c)		1H131a^1 ♂ 45 q49.46. 1(3e)	1H98a^5 ♀ 100zq48.46. 4(5–6e)
2H41	a^1 ♂ 00 p– – –(alb.)	a^2 ♂ – q– – –(6e)	a^3 ♀ – q– – –(6e)	a^4 ♀ – q– – –(6e)	1H111a^3 ♂ 100zq48.46. 7(5c)	1H65b^2 ♀ 100xq58.58.56(6e)
	a^5 ♀ 45 q47.47. 1(3e)	...	...	...	...	...
2H43	a^1 ♂ – q– – –(4e)	a^2 ♂ 65 p26.23. 1(4f)	a^3 ♀ – q– – –(3e)	a^4 ♂ – q– – –(3e)	1H113a^2 ♂ 65 q58. 1. 1(3e)	1H87a^1 ♀ 100xq58.58.57(6e)
	a^5 – p– – –(3f)	a^6 – q– – –(6e)	a^7 – q– – –(6e)	a^8 – p– – –(3f)	...	...
2H44	a^1 ♂ 100zq48.46.12(5e)	a^2 ♂ 75 q49.46. 2(5b–c)	a^3 ♀ 00 p– – –(alb.)	a^4 ♂ 65 q49.36.11(3c	1H100a^2 ♂ 30 q46. 1. 1(2c)	1H32a^5 ♀ 100zq49.46. 4(5–6c)
	a^5 ♀ 65 q46.46. 4(3–4e)	a^6 ♀ – q– – –(3e)	a^7 ♀ 40 p37.33. 1(3a)	b^1 100zq48.35.30(5b–c	...	...
	b^2 100zq49.46.12(5c)	b^3 30 q48. 1. 1(2c)	b^4 00 p– – –(alb.)	b^5 00 p– – –(alb.)	...	...
2H45	a^1 ♂ 100zq49.46.11(4e)	a^2 ♀ – q– – –(5e)	a^3 ♂ – q– – –(5e)	a^4 ♀ 00 p– – –(alb.)	1H143a^1 ♂ 100zq48.46. 2(5e)	1H80a^6 ♀ 100zq37.36. 4(5g)
	a^5 ♂ 00 p– – –(alb.)	a^6 ♀ 00 p– – –(alb.)	...	...	...	...
2H46	a^1 – p– – –(6b)	...	...	...	1H117a^1 ♂ 60 q49.46. 1(5c)	1H121a^3 ♀ – q– – –(4d)
2H48	a^1 ♂ 100xq35.33.28(5a)	a^2 ♀ – q– – –(5a)	a^3 ♂ 00 p– – –(alb.)	a^4 ♀ 00 p– – –(alb.)	1H115a^3 ♂ 30 q47.46. 1(4e)	1H136a^4 ♀ – q– – –(5a)
	a^5 ♂ – p– – –(4a)	...	...	...	...	...
2H49	a^1 ♀ – q– – –(6e)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♀ 100xq58.57.55(6e)	a^4 ♂ 100zq49.46.11(5c–d)	1H119a^1 ♂ 55 q49. 1. 1(3e)	1H142a^1 ♀ 100xq58.57.55(6e)
	a^5 ♂ – q– – –(6e)	a^6 ♀ 100zq49.48.14(5c–d)	...	...	...	...
2H50	a^1 ♀ 30 q46. 1. 1(3c)	a^2 ♂ – q– – –(3–4e)	a^3 ♀ 40 q46.46. 1(3c)	a^4 ♂ 55 q46. 1. 1(2c)	1H121a^5 ♂ 55 q48.46. 2(3e)	1H115a^3 ♀ 85 q58.55. 1(4e)
	a^5 ♀ 00 p– – –(alb.)	b^1 – q– – –(2e)	b^2 – q– – –(3e)	b^3 – q– – –(4e)	...	...
	b^4 – q– – –(4e)	b^6 00 p– – –(alb.)	...	...	...	...
2H51	a^1 ♀ – q– – –(3g)	a^2 ♀ – q– – –(3c)	a^3 ♀ – q– – –(3c)	a^4 ♂ – q– – –(3e)	1H122a^5 ♂ 25 q46. 1. 1(2c)	1H113a^5 ♀ 95 q49.48.12(4c)
	a^5 – q– – –(1c)	...	...	...	...	...
2H52	a^1 ♂ 100zp36.36. 2(5b)	a^2 75 q48.46.11(5c)	a^3 60 q46.46. 7(3c)	a^4 100zq49.46.12(5c)	1H99a^1 ♂ 80 q48.46. 7(5d)	1H100a^4 ♀ 65 q49.46. 1(3c–d)
	a^5 ♀ 00 p– – –(alb.)	...	...	...	...	...
2H53	a^1 ♂ – q– – –(3e)	a^2 ♂ – q– – –(3e)	a^3 ♂ – q– – –(6e)	a^4 ♂ – q– – –(3–4e)	1H118a^2 ♂ 55 q49. 1. 2(3c)	1H101a^3 ♀ – q– – –(6e)
	a^5 ♀ – q– – –(4–5e)	a^6 ♂ – q– – –(5c–d)	a^7 ♀ – q– – –(6e)	a^8 00 p– – –(alb.,e)	...	...
	b^1 – q– – –(3c)	b^2 – p– – –(3b)	b^3 – q– – –(3c)	b^4 – q– – –(6e)	...	...
	b^6 – q– – –(5cw)	...	...	...	...	...
2H55	a^1 ♀ 25 p39. 1. 4(1-2a-b)	a^2 ♀ 35 p39.39. 3(2a–b)	a^3 ♂ 10 p37. 1. 1(1a–b)	a^4 – p– – –(5a)	1H99a^1 ♂ 100zp39.35.28(6b)	1H118a^1 ♀ – p– – –(2b)
2H59	a^1 ♂ 00 p– – –(alb.)	a^2 ♀ 00 p– – –(alb.)	a^3 ♂ 00 p– – –(alb.)	a^4 ♀ 00 p– – –(alb.)	1H157a^5 ♂ 00 p– – –(alb.)	1H152a^3 ♀ 00 p– – –(alb.)
	a^5 ♀ 00 p– – –(alb.)	...	...	...	...	...
2H60	a^1 ♂ 00 p– – –(alb.)	a^2 ♂ 00 p– – –(alb.)	a^3 ♀ 00 p– – –(alb.)	a^4 ♀ 00 p– – –(alb.)	1H153a^3 ♂ 00 p– – –(alb.)	1H152a^5 ♀ 00 p– – –(alb.)
	a^5 ♀ 00 p– – –(alb.)	...	...	...	...	...
2H61	a^1 ♀ 00 p– – –(alb.)	...	...	...	1H153a^4 ♂ 00 p– – –(alb.)	1H16a^6 ♀ 00 p– – –(alb.)
2H62	a^1 ♀ 00 p– – –(alb.)	a^2 ♂ 00 p– – –(alb.)	a^3 ♂ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	1H143a^4 ♂ 00 p– – –(alb.)	1H16a^7 ♀ 00 p– – –(alb.)
	a^5 ♀ 00 p– – –(alb.)	...	...	...	...	...
2H63	a^1 ♂ 00 p– – –(alb.)	a^2 ♂ 00 p– – –(alb.)	a^3 ♂ 00 p– – –(alb.)	a^4 ♀ 00 p– – –(alb.)	1H112a^4 ♂ 00 p– – –(alb.)	1H85b^3 ♀ 00 p– – –(alb.)
	a^5 ♂ 00 p– – –(alb.)	a^6 ♂ 00 p– – –(alb.)	b^1 00 p– – –(alb.)	b^2 00 p– – –(alb.)	...	...
	b^3 00 p– – –(alb.)	b^4 00 p– – –(alb.)	b^5 00 p– – –(alb.)	...	...	...
2H64	a^1 ♀ 00 p– – –(alb.,w)	a^2 ♂ 00 p– – –(alb.)	a^3 ♀ 00 p– – –(alb.)	a^4 ♀ 00 p– – –(alb.)	1H115a^5 ♂ 00 p– – –(alb.)	1H96a^3 ♀ 00 p– – –(alb.)
	a^5 ♀ 00 p– – –(alb.)	a^6 ♂ 00 p– – –(alb.)	...	...	...	...
2H65	a^1 ♂ 00 p– – –(alb.)	a^2 ♂ 00 p– – –(alb.)	a^3 ♂ 00 p– – –(alb.)	a^4 ♀ 00 p– – –(alb.)	1H88b^1 ♂ 00 p– – –(alb.)	1H96a^4 ♀ 00 p– – –(alb.)
	a^5 ♂ 00 p– – –(alb.)	...	...	...	...	...
2H66	a^1 ♂ 00 p– – –(alb.)	a^2 ♂ 00 p– – –(alb.)	a^3 ♀ 00 p– – –(alb.)	a^4 ♀ 00 p– – –(alb.)	1H119a^6 ♂ 00 p– – –(alb.)	1H146a^5 ♀ 00 p– – –(alb.)
2H67	a^1 ♂ 00 p– – –(alb.)	a^2 ♀ 00 p– – –(alb.)	a^3 ♂ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	1H123a^2 ♂ 00 p– – –(alb.)	1H88b^2 ♀ 00 p– – –(alb.)
	a^5 ♀ 00 p– – –(alb.)	a^6 ♀ 00 p– – –(alb.)	...	...	...	...
2H69	a^1 00 p– – –(alb.)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	1H149a^5 ♂ 00 p– – –(alb.)	1H88b^4 ♀ 00 p– – –(alb.)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	...	...	...	...
2H70	b^1 00 p– – –(alb.)	b^2 00 p– – –(alb.)	...	...	1H16a^3 ♂ 00 p– – –(alb.)	1H123a^3 ♀ 00 p– – –(alb.)
2H71	a^1 ♂ 00 p– – –(alb.)	a^2 ♂ 00 p– – –(alb.)	a^3 ♂ 00 p– – –(alb.)	a^4 ♀ 00 p– – –(alb.)	1H16a^4 ♂ 00 p– – –(alb.)	1H149a^4 ♀ 00 p– – –(alb.)
	a^5 ♂ 00 p– – –(alb.)	...	...	...	...	...
2H73	a^1 00 p– – –(alb.)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	...	1H39a^4 ♂ 00 p– – –(alb.)	1H142a^3 ♀ 00 p– – –(alb.)
2H75	a^1 ♀ 00 p– – –(alb.)	a^2 ♂ 00 p– – –(alb.)	a^3 ♂ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	1H85b^5 ♂ 00 p– – –(alb.)	1H140a^4 ♀ 00 p– – –(alb.)
	a^5 ♂ 00 p– – –(alb.)	...	...	...	...	...
2H76	a^1 00 p– – –(alb.)	a^2 00 p– – –(alb.)	a^3 00 p– – –(alb.)	a^4 00 p– – –(alb.)	1H92a^4 ♂ 00 p– – –(alb.)	1H147a^4 ♀ 00 p– – –(alb.)
	a^5 00 p– – –(alb.,w)	a^6 00 p– – –(alb.)	...	...	...	...
2H77	a^1 ♂ 00 p– – –(alb.)	a^2 ♂ 00 p– – –(alb.)	a^3 ♀ 00 p– – –(alb.)	a^4 ♀ 00 p– – –(alb.)	1H92a^5 ♂ 00 p– – –(alb.)	1H143a^5 ♀ 00 p– – –(alb.)
	a^5 ♂ 00 p– – –(alb.)	a^6 ♂ 00 p– – –(alb.)	b^1 00 p– – –(alb.)	b^2 00 p– – –(alb.)	...	...
	b^3 00 p– – –(alb.)	b^4 00 p– – –(alb.)	...	...	...	...

Mating	Offspring				Parents	
					Father	Mother
2H79	a^1♀ 85 q49.51. 4(3c)	a^2♀ – q – – –(4c)	a^3♀ – q – – –(4c)	a^4♂ – q – – –(4c)	1H70a^2 ♂ 55 q37.33.30(3a)	A159a^3 ♀ 00 p – – –(A1)
	a^5♀ – q – – –(3e)	a^6♂ – q – – –(3e)	a^7♀ – q – – –(3c)	a^8♂ – q – – –(6e)	... ...	... ...
	a^9♀ – q – – –(6e)	... ...	... ...	... ...	... ...	... ...
2H80	a^1♀ 100xq58.57.56(6e)	a^2♀ 60 q58. 1. 1(3–4e)	... ...	... ...	1H122a^4 ♂ 55 p23. 1. 1(3f)	A159a^4 ♀ 00 p – – –(A1)
2H82	a^1♀ 95 q41.41. 1(4g)	a^2♀ 85 q41.50. 3(4g)	a^3♀ – q – – –(4g)	a^4♂ 00 p – – –(alb.)	1H16a^1 ♂ 95 q41.50. 4(4g)	A158a^5 ♀ 00 p – – –(A1)
	a^5♂ 00 p – – –(alb.)	a^6♂ 00 p – – –(alb.)	... ...	... ...	... ...	... ...
2H83	a^1♀ – q – – –(4e)	a^2♂ – q – – –(4e)	a^3♀ – q – – –(6e)	a^4♀ – q – – –(6e)	1H60a^1 ♂ 100xq57.57.55(6e)	A91a^2 ♀ 00 p – – –(A1½)
	a^5♂ 00 p – – –(alb.)	a^6♂ 00 p – – –(alb.)	a^7♀ 00 p – – –(alb.)	a^8♂ 00 p – – –(alb.)	... ...	... ...
	a^9 00 p – – –(alb.)	... ...	... ...	... ...	... ...	... ...
2H85	a^1♀ – p – – –(3b)	a^2♀ – p – – –(3b)	a^3♂ 30 p37. 1. 1(2–3b)	a^4♂ 45 p37. 1. 1(2–3b)	1H74b^1 ♂ 35 p23. 1. 1(2a)	1H117b^2 ♀ – p – – –(3f)
	a^5♂ 30 p36. 1. 2(2–3b)	a^6 – p – – –(3b)	... ...	... ...	... ...	... ...
2H87	a^1 95 q46.46. 4(4b–c)	a^2 00 p – – –(alb.)	a^3 00 p – – –(alb.)	a^4 00 p – – –(alb.)	A153a^4 ♂ 00 p – – –(A1½)	1H103b^1 ♀ 60r? 40.39. 1(4b–c)
2H88	a^1 95 q40.37. 9(4b–c)	a^2 90 q48.39.30(4b–c)	a^3 85 q37.36. 3(4b–c)	a^4 00 p – – –(alb.)	A158a^2 ♂ 00 p – – –(A1)	1H103b^2 ♀ 55 p or r40.35. 1(4b)
2H90	a^1 – q – – –(5b)	a^2 – q – – –(5c)	a^3 – q – – –(5c)	a^4 – q – – –(5c)	A147a^1 ♂ 00 p – – –(A2 ×)	1H96b^3 ♀ 100zp 35.33.28(5a–b)
	a^5 – q – – –(5c)	a^6 – q – – –(5c)	... ...	... ...	... ...	... ...
2H92	a^1 60 q48.46. 1(3c)	a^2 – q – – –(3c)	a^3 – q – – –(4c)	... ...	1H150a^2 ♂ 100zq48.46. 7(4c)	1H115b^1 ♀ 45 q 58.57. 2(3e)
2H93	a^1 – p – – –(2f)	a^2 – q – – –(3c)	a^3 – q – – –(3e)	a^4 – p – – –(4f)	1H100b^2 ♂ 60 q55.46. 1(4c)	1H115b^2 ♀ – q – – –(4e)
	a^5 00 p – – –(alb.)	b^1♂ 100zp35.35.28(5b)	b^2 100z 48.46.30(5d*)	b^3 85 49.39. 1(4b–c*)	... ...	... ...
	b^4 35 49. 1. 1(2d*)	b^5 35 49. 1. 1(2d*)	... ...	... ...	... ...	... ...
2H96	a^1♂ 100xq58.57.55(6e)	a^2♀ 50 q58.58. 2(3e)	a^3 100zq49.51.30(5c)	a^4 100zq52.51.29(5d)	1H115b^3 ♂ 65 q49.46. 3(4c)	1H142b^3 ♀ 100xq 58.57.55(6e)
	a^5 – q – – –(5d)	a^6 – q – – –(3g)	a^7♀ 00 p – – –(alb.)	... ...	... ...	... ...
2H99	a^1 65 q40.40. 1(4b–c)	a^2 80 q37.36. 4(4b–c)	a^3 – q – – –(5c)	a^4 00 p – – –(alb.)	1H142b^1 ♂ 80 q48.46. 2(5c)	1H103a^3 ♀ – q – – –(4b–c)
	a^5 00 p – – –(alb.)	... ...	... ...	... ...	... ...	... ...
2H100	a^1 00 p – – –(alb.)	a^2 00 p – – –(alb.)	a^3 00 p – – –(alb.)	... ...	1H123b^4 ♂ 80 q46.46. 4(5c–d)	1H168a^6 ♀ 100zq 33.33.28(5a)
2H101	a^1 100xq33.30.28(5a)	a^2 100xq48.46.47(6c)	a^3 100xq49.46.47(6c)	a^4 – q – – –(6c)	1H123b^3 ♂ 100xq57.56.55(6e)	1H168a^7 ♀ 100zq 35.30.28(5a)
	a^5 – q – – –(6c)	a^6 00 p – – –(alb.)	a^7 00 p – – –(alb.)	... ...	... ...	... ...
2H102	a^1♂ 100xq58.57.55(6e)	a^2 100xq33.30.28(5a)	a^3 – q – – –(6e)	a^4 – q – – –(5a)	1H123b^5 ♂ 100zp35.33.28(5–6b)	1H168a^4 ♀ 00 p – – –(alb.)
	a^5 – q – – –(5a)	a^6 – q – – –(5c)	a^7 – q – – –(5c)	... ...	... ...	... ...
2H103	a^1♀ – q – – –(4–5e)	a^2♂ 70 q58.55. 1(4e)	a^3 70 q49.46. 4(5c)	a^4 40 q49. 1. 3(3c)	1H123b^1 ♂ 60 q58.55. 1(3e)	1H158b^5 ♀ 100zq 49.48.14(5c)
	a^5 80 q48.46.11(5c)	... ...	... ...	... ...	... ...	... ...
2H104	a^1 100zq48.46. 9(5c)	a^2 100zq46.36. 7(5c)	... ...	... ...	1H130b^1 ♂ 30 p37. 1. 1(2a–b)	1H169a^1 ♀ 00 p – – –(alb.)
2H105	a^1 70 q48.34. 3(4c)	a^2 – q – – –(3c)	a^3 – q – – –(3c)	a^4 – q – – –(3c)	1H130b^3 ♂ 35 p39.35. 1(2a–b)	1H169a^2 ♀ 00 p – – –(alb.)
2H106	a^1 45 q58. 1. 1(2e)	a^2 90 q48.46. 2(4d)	a^3 00 p – – –(alb.)	a^4 00 p – – –(alb.)	1H124b^3 ♂ 65 q58.54. 2(4e)	1H169a^4 ♀ 100zq 49.48.47(5c–d)
2H107	a^1 70 q49.48. 1(3c)	a^2 100zq49.48. 7(5d)	a^3 45 p50.37. 1(3f)	a^4 55 p48.36. 1(3f)	1H112b^2 ♂ 35 q48. 1. 3(2c)	1H169a^6 ♀ 100zq 48.46. 4(6c)
	a^5 80 q50. 1. 1(3c)	a^6 100zp26.35.30(5b)	... ...	... ...	... ...	... ...
2H108	a^1♀ 100xq58.57.56(6e)	a^2♂ 100xq58.56.56(6ew)	a^3♀ 55 q58.55. 3(3–4e)	a^4 100zq48.46. 9(5c)	1H139a^2 ♂ – p – – –(1–2f)	1H171a^2 ♀ 00 p – – –(alb.)
	a^5 – q – – –(3cw)	a^6 – q – – –(4e)	a^7 – q – – –(5c)	a^8 – q – – –(6e)	... ...	... ...
2H109	a^1 90 q58.58.55(4e)	a^2 90 q48.46.12(4c)	a^3 75 q48.46. 1(4c)	a^4 60 q48. 1. 1(4c)	1H105a^4 ♂ 40 p37. 1. 1(2b)	1H171a^5 ♀ 00 p – – –(alb.)
	a^5 60 q46.46. 1(4c)	a^6 – q – – –(4c)	... ...	... ...	... ...	... ...
2H110	a^1 – q – – –(6d)	a^2 – q – – –(6d)	a^3 – q – – –(6dw)	... ...	1H149b^3 ♂ 100zq49.46.20(5d)	1H167a^2 ♀ 100xq 58.58.55(6e)
2H111	a^1♀ 60 q58. 1. 3(4e)	a^2 90 q35.33. 3(4a)	a^3 100yp35.35.28(5b)	a^4 – q – – –(4a)	1H158b^3 ♂ 100zq33.33. 4(5a)	1H167a^3 ♀ 100zq 49.46.21(6c)
2H112	a^1 100xq39.35.33(5a)	a^2 80 q58. 1. 2(4e)	a^3 85 q58.55. 2(4e)	a^4 95 q35.35.33(5a)	1H158b^4 ♂ 100zq35.30.28(5a)	1H143b^1 ♀ 95 q 46.33. 1(4c)
	a^5 95 q49.35. 2(4d)	a^6 100xq58.58.55(6e)	a^7 – q – – –(4a)	a^8 – q – – –(4e)	... ...	... ...
2H113	a^1 100zq48.46. 9(5c)	a^2 – q – – –(3c)	a^3 00 p – – –(alb.)	a^4 00 p – – –(alb.)	1H158b^1 ♂ 100zq46.46. 9(5c)	1H151b^1 ♀ 100zq 49.48.47(6c)

	a^5 00 p– – –(alb.)	...	...	...	...	...	...
2H114	a^1 ♂ 100zq58.58.55(6e)	a^2 85 q48.34. 1(4d)	a^3 60 q49.46. 1(4d)	a^4 – q– – –(6d)	1H167a^4 ♂ 100zq48.46.47(6c)	1H146b^1 ♀ 100zq	52.46.20(5c–d)
2H115	a^1 ♀ – q– – –(4e)	a^2 – q– – –(4e)	a^3 – q– – –(5d)	a^4 00 p– – –(alb.)	1H167a^5 ♂ 100zq48.46.12(6d)	1H146b^2 ♀ 100zq	49.46.20(5c)
2H116	a^1 ♂ 100zp39.35.28(5b)	a^2 ♀ 100xp35.35.30(5–6b)	a^3 ♂ – p– – –(5b)	a^4 – p– – –(5b)	1H165a^1 ♂ 80 q49.46.11(5c)	1H149b^6 ♀ 100zq	49.48.12(5d)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	...	...	...	...	...
2H117	a^1 ♂ – q– – –(6e)	a^2 ♀ 80 p39.39. 2(5b)	a^3 ♂ 90 p23.15. 2(4–5f)	a^4 55 q48.46. 1(4c)	1H164a^1 ♂ 100xq58.58.55(6e)	1H122b^2 ♀ 70 q	49.46. 4(4c)
	a^5 65 q49.46. 1(4c)	a^6 00 p– – –(alb.)	...	...	...	...	...
2H118	a^1 85 q46.51. 4(5c)	a^2 100zq48.46.43(5c)	a^3 100zq48.46.12(5c)	a^4 55 q37. 1. 3(3b)	1H164a^2 ♂ 100zq36.36. 9(5c)	1H122b^1 ♀ 25 q	49. 1. 1(1c)
	a^5 100zq37.37.30(5b)	...	...	...	...	...	...
2H119	a^1 ♀ 100xq58.58.55(6e)	a^2 ♀ 100xq58.58.55(6e)	a^3 65 q58. 1. 1(4e)	a^4 – q– – –(5d)	1H169a^3 ♂ 100xq58.58.55(6e)	1H119a^2 ♀ 100zq	52.46.43(5c)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	...	...	...	...	...
2H120	a^1 – q– – –(5c)	a^2 – q– – –(5c)	a^3 – p– – –(5b)	...	1H128b^1 ♂ 100zp35.35. 4(5–6b)	1H146b^3 ♀ 00 p	– – –(alb.)
2H123	a^1 ♀ 100xq58.58.56(6e)	a^2 100zq48.46.34(5c)	a^3 100zq49.46. 7(5c)	a^4 – q– – –(2e)	1H102b^1 ♂ 25 p37. 1. 1(1–2b)	1H167a^7 ♀ 00 p	– – –(alb.)
	a^5 – q– – –(4c)	a^6 – q– – –(4e)	...	...	...	...	...
2H124	a^1 ♀ 100zq58.57.56(6e)	a^2 ♂ 85 q58.55. 2(4e*)	a^3 80 q46.46. 3(4cw)	a^4 100z 44.47.14(5ch*)	1H153a^3 ♂ 00 p– – –(alb.)	1H121b^2 ♀ 75 p	39.34. 1(4a–b)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	...	...	...	...	...
2H125	a^1 100xq48.46.47(5c)	a^2 – q– – –(2c)	a^3 – q– – –(3b–c)	a^4 – q– – –(4c)	1H111b^1 ♂ 50 p37. 1. 2(3b)	1H149b^5 ♀ 00 p	– – –(alb.)
	a^5 – q– – –(5c)	a^6 – q– – –(5c)	...	...	...	...	...
2H126	a^1 55 q46.46. 2(3c)	a^2 50 q49. 1. 2(3c)	a^3 30 q47. 1. 1(3c)	a^4 95 q35.33. 1(4a)	1H164a^3 ♂ 00 p– – –(alb.)	1H136b^1 ♀ 90 p	39.35. 2(4a)
	a^5 – q– – –(5d)	...	...	...	...	...	...
2H127	a^1 ♀ 100xq58.57.55(6e)	a^2 ♂ 100xq56.55.55(5-6d-e)	a^3 80 q48.35. 7(5c)	a^4 75 q48.46. 2(4e)	1H165a^2 ♂ 00 p– – –(alb.)	1H136b^3 ♀ 100zp	37.35.28(5b)
	a^5 80 q49.46. 7(5c)	...	...	...	...	...	...
2H129	a^1 – q– – –(5c)	a^2 – q– – –(2c)	...	...	1H144b^2 ♂ – q– – –(5c)	1H37a^3 ♀ – q	– – –(3c-dw)
2H135	a^1 – p– – –(5b)	a^2 – p– – –(5b)	a^3 – p– – –(5b)	a^4 – p– – –(5b)	1H99a^1 ♂ 100zp39.35.28(6b)	1H129a^3 ♀ 100xp	24.23.15(6f)
	a^5 – p– – –(5b)	a^6 – p– – –(5b)	...	...	...	...	...
2H137	a^1 ♀ 35 p37. 1. 1(2b)	a^2 ♀ 55 p39.33. 1(3b)	a^3 35 p37. 1. 1(2b)	a^4 – p– – –(2b)	1H70a^2 ♂ 55 q37.33.30(3a)	1H136a^4 ♀ 35 p	39. 1. 1(2b)
2H139	a^1 100zq39.33.28(5a ?)	a^2 100zq39.33.28(5b ?)	a^3 100zq48.46.12(5cw)	a^4 100xq58.55.55(6e)	1H98a^3 ♂ 100zq49.46. 4(6c)	1H136a^3 ♀ – q	– – –(5a)
	a^5 100xp23.23.17(6fw)	...	...	...	...	...	...
2H142	a^1 ♀ 100zp39.36.28(5bw)	a^2 100zp39.39.28(5b–c)	a^3 – q– – –(5b–c)	a^4 – q– – –(5b–c)	1H148a^2 ♂ 80 q49.46. 4(5c)	1H98a^5 ♀ 100zq	48.46. 4(5–6c)
	a^5 – q– – –(5c)	a^6 – q– – –(5c)	a^7 – q– – –(5c)	a^8 – q– – –(5cw)	...	...	...
2H143	a^1 100xq58.58.55(6e)	a^2 30 q61. 1. 1(3e)	a^3 100xq58.58.57(6e)	a^4 35 p26. 1. 1(2fw)	1H60a^1 ♂ 100xq57.57.55(6e)	1H65b^1 ♀ – q	– – –(6e)
	a^5 100xq58.57.55(6e)	a^6 100xp23.23.18(6fw)	a^7 100xq58.58.55(6e)	...	...	...	...
2H144	a^1 ♀ 60 q58.56. 1(3e)	a^2 ♂ 80 q58.57. 1(4e)	a^3 95 q41.41.50(6g)	a^4 – q– – –(6g)	1H16a^1 ♂ 95 q41.50. 4(4g)	1H157a^2 ♀ – q	– – –(5a)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	...	...	...	...	...
2H145	a^1 100zq48.46.11(5c)	a^2 80 q48.46.11(4c)	a^3 75 q48.46. 3(3c)	a^4 00 p– – –(alb.)	1H111a^3 ♂ 100zq48.46. 7(5c)	1H156a^1 ♀ 100xq	58.57.55(6e)
	a^5 00 p– – –(alb.)	...	...	...	...	...	...
2H146	a^1 60 q49.48. 1(3c)	a^2 100zq48.36.12(5c)	a^3 95 p39.35.33(4b)	a^4 100zp39.35.28(5b)	1H143a^1 ♂ 100zq48.46. 2(5c)	1H158a^3 ♀ 100zq	49.48.34(5c)
	a^5 100xp23.23.18(6f)	a^6 100xq58.58.55(6e)	a^7 00 p– – –(alb.)	...	...	...	...
2H147	a^1 55 q?39. 1. 2(3a)	a^2 100xq41.41.50(6g)	a^3 55 p?39. 1. 1(3a)	a^4 – q– – –(5a)	1H92a^3 ♂ 100xq57.57.55(6e)	1H157a^1 ♀ – q	– – –(5a)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	a^8 00 p– – –(alb.)	...	...	...
2H148	a^1 100xp37.36.36(5a)	a^2 100xq48.47.46(5c)	a^3 100zq49.46. 9(3c)	a^4 90 p48.34. 2(4b–c)	1H161a^2 ♂ 100zq48.46. 7(5c)	1H151a^4 ♀ 80 q	49.48.34(5c)
	a^5 100zq48.46.30(5c)	a^6 80 p36.36. 4(4b)	a^7 100zp37.37. 2(5a)	a^8 80 p37.37. 2(4a)	...	...	...
	a^9 – q– – –(3c)	...	...	...	...	...	...
2H151	a^1 100zp?35.33.28(5a)	a^2 100zq46.46. 2(5c)	a^3 100zq?33.33.28(5a)	a^4 100zq?33. 2. 2(5a)	1H99a^1 ♂ 100zp39.35.28(6b)	1H97a^1 ♀ 100zq	33.30.28(5–6bl)
	a^5 95 q48.46.12(4c)	a^6 00 p– – –(alb.)	a^7 00 p– – –(alb.)	...	...	...	...
2H152	a^1 ♂ 60 q58. 1. 1(2–3e)	a^2 ♂ 65 q55. 1. 2(3–4e)	a^3 65 q41. 1. 1(3g)	a^4 – q– – –(4c)	1H102b^1 ♂ 25 p37. 1. 1(1–2b)	A43a^2 ♀ 00 p	– – –(A2 ?)
	a^5 – q– – –(3f–e)	...	...	...	...	...	...
2H153	a^1 100zq46.46.30(5c)	...	...	...	1H128b^1 ♂ 100zp35.35. 4(5–6b)	1H24a^1 ♀ 00 p	– – –(alb.)
2H154	a^1 50 q48.46. 2(3c)	...	...	...	1H111b^1 ♂ 50 p37. 1. 2(3b)	1H32a^1 ♀ 00 p	– – –(alb.)
2H155	a^1 ♂ 100xq58.57.55(5–6e)	a^2 ♀ 100xq58.57.56(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 100zq48.46.12(5c)	1H105a^4 ♂ 40 p37. 1. 1(2b)	A58a^4 ♀ 00 p	– – –(A × 3)

MATING	OFFSPRING				PARENTS	
					Father	Mother
2H79	a^{1} ♀ 85 q49.51. 4(3c)	a^{2} ♀ - q- - -(4c)	a^{3} ♀ - q- - -(4c)	a^{4} ♂ - q- - -(4c)	1H70a^{2} ♂ 55 q37.33.30(3a)	A159a^{3} ♀ 00 p - - -(A1)
	a^{5} ♀ - q- - -(3e)	a^{6} ♂ - q- - -(3e)	a^{7} ♀ - q- - -(3c)	a^{8} ♂ - q- - -(6e)	... ...	... ...
	a^{9} ♀ - q- - -(6e)	... ...	... ...	... ...	... ...	... ...
2H80	a^{1} ♀ 100xq58.57.56(6e)	a^{2} ♀ 60 q58. 1. 1(3-4e)	... ...	... ...	1H122a^{4} ♂ 55 p23. 1. 1(3f)	A159a^{4} ♀ 00 p (A1)
2H82	a^{1} ♀ 95 q41.41. 1(4g)	a^{2} ♀ 85 q41.50. 3(4g)	a^{3} ♀ q- - 4g	a^{4} ♂ 00 p- - -(alb.)	1H16a^{1} ♂ 95 q41.50. 4(4g)	A158a^{3} ♀ 00 p - - (A1)
	a^{5} ♂ 00 p- - -(alb.)	a^{6} ♂ 00 p- - -(alb.)	... ...	... ...	... ...	... ...
2H83	a^{1} ♀ q- - -(4e)	a^{2} ♂ - q- - -(4c)	a^{3} ♀ - q- - -(6e)	a^{4} ♀ - q- - -(6e)	1H60a^{1} ♂ 100xq57.57.55(6e)	A91a^{2} ♀ 00 p - - -(A1½)
	a^{5} ♂ 00 p- - -(alb.)	a^{6} ♂ 00 p- - -(alb.)	a^{7} ♀ 00 p- - -(alb.)	a^{8} ♂ 00 p- - -(alb.)	... ...	... ...
	a^{9} 00 p- - -(alb.)	... ...	... ...	... ...	... ...	... ...
2H85	a^{1} ♀ p 3b	a^{2} ♀ p - 3b	a^{3} ♂ 50 p37. 1. 1(2-3b)	a^{4} ♂ 45 p37. 1. 1(2-3b)	1H74b^{1} ♂ 55 p23. 1. 1(2a)	1H117b^{2} ♀ p - (3f)
	a^{5} ♂ 30 p36. 1. 2(2-3b)	a^{6} - p- - -(3b)	... ...	... ...	... ...	... ...
2H87	a^{1} 95 q46.46. 4(4b-c)	a^{2} 00 p- - -(alb.)	a^{3} 00 p- - -(alb.)	a^{4} 00 p- - -(alb.)	A153a^{4} ♂ 00 p- - -(A1½)	1H103b^{1} ♀ 60r? 40.39. 1(4b-c)
2H88	a^{1} 95 q40.37. 9(4b-c)	a^{2} 90 q48.39.30(4b-c)	a^{3} 85 q37.36. 3(4b-c)	a^{4} 00 p - (alb.)	A158a^{2} ♂ 00 p - (A1)	1H103b^{2} ♀ 55 p or r 40.35. 1(4b)
2H90	a^{1} - q- - -(5b)	a^{2} - q- - -(5c)	a^{3} - q- - -(5c)	a^{4} - q- - -(5c)	A147a^{1} ♂ 00 p- - -(A2×)	1H96b^{3} ♀ 100zp 35.33.28(5a-b)
	a^{5} - q- - -(5c)	a^{6} - q- - -(5c)	... ...	... ...	... ...	... ...
2H92	a^{1} 60 q48.46. 1(3c)	a^{2} - q (3c)	a^{3} q - 4c	... ...	1H150a^{2} ♂ 100zq48.46. 7(4c)	1H115b^{1} ♀ 45 q 58.57. 2(3c)
2H93	a^{1} - p- - -(2f)	a^{2} - q- - -(3c)	a^{3} - q- - -(3e)	a^{4} - p- - -(4f)	1H100b^{2} ♂ 60 q55.46. 1(4c)	1H115b^{2} ♀ - q - -(4c)
	a^{5} 00 p (alb.	b^{1} ♂ 100xp35.35.28(5b)	b^{2} 100z 48.46.30(5d*)	b^{3} 85 49.39. 1(4b-c*)	... ...	... ...
	b^{4} 35 49. 1. 1(2d*)	b^{5} 35 49. 1. 1(2d*)	... ...	... ...	... ...	... ...
2H96	a^{1} ♂ 100xp58.57.55(6)	a^{2} ♀ 70 q58.58. 2(3c)	a^{3} 100zq49.51.30(5c)	a^{4} 100xq52.51.29(5d)	1H115b^{3} ♂ 65 q49.46. 3(4)	1H142b^{2} ♀ 100xq 58.57.55(6e)
	a^{5} - q- - -(5d)	a^{6} - q- - -(3g)	a^{7} ♀ 00 p- - -(alb.)	... ...	... ...	... ...
2H99	a^{1} 65 q40.40. 1(4b-c)	a^{2} 80 q37.36. 4(4b-c)	a^{3} - q- - -(5c)	a^{4} 00 p- - -(alb.)	1H142b^{1} ♂ 80 q48.46. 2(5c)	1H103a^{3} ♀ q (4b-c)
	a^{5} 00 p- - -(alb.)	... ...	... ...	... ...	... ...	... ...
2H100	a^{1} 00 p- - -(alb.)	a^{2} 00 p- - -(alb.)	a^{3} 00 p- - -(alb.)	... ...	1H123b^{4} ♂ 80 q46.46. 4(5c-d)	1H168a^{6} ♀ 100zq 33.33.28(5a)
2H101	a^{1} 100xq33.30.28(5a)	a^{2} 100xq48.46.47(6c)	a^{3} 100xq49.46.47(6c)	a^{4} q (6c)	1H123b^{3} ♂ 100xq57.56.55(6e)	1H168a^{7} ♀ 100zq 35.30.28(5a)
	a^{5} - q- - -(6c)	a^{6} 00 p- - -(alb.)	a^{7} 00 p- - -(alb.)	... ...	... ...	... ...
2H102	a^{1} ♂ 100xq58.57.55(6e)	a^{2} 100xq33.30.28(5a)	a^{3} - q- - -(6e)	a^{4} - q- - -(5a)	1H123b^{5} ♂ 100zp35.33.28(5-6b)	1H168a^{4} ♀ 00 p - - -(alb.)
	a^{5} - q- - -(5a)	a^{6} - q- - -(5c)	a^{7} - q- - -(5c)	... ...	... ...	... ...
2H103	a^{1} ♀ - q 4(5c)	a^{2} ♂ 70 q58.55. 1(4e)	a^{3} 70 q49.46. 4(5c)	a^{4} 40 q49. 1. 3(3c)	1H123b^{2} ♂ 60 q58.55. 1(3c)	1H158b^{2} ♀ 100zq 49.48.14(5
	a^{5} 80 q48.46.11(5c)	... ...	... ...	... ...	... ...	... ...
2H104	a^{1} 100zq48.46. 9(5c)	a^{2} 100zq46.36. 7(5c)	... ...	... ...	1H130b^{1} ♂ 30 p37. 1. 1(2a-b)	1H169a^{1} ♀ 00 p - - -(alb.)
2H105	a^{1} 70 q48.34. 3(4c)	a^{2} q -(3c)	a^{3} - q- (3c)	a^{4} q (3c)	1H130b^{3} ♂ 35 p39.35. 1(2a-b)	1H169a^{2} ♀ 00 p (alb.)
2H106	a^{1} 55 q58. 1. 1(2e)	a^{2} 90 q48.46. 2(4d)	a^{3} 00 p alb.	a^{4} 00 p- - - alb.	1H124b^{2} ♂ 65 q58.54. 2(4c)	1H169a^{4} ♀ 100zq 49.48.47(5c-d)
2H107	a^{1} 70 q49.48. 1(3)	a^{2} 100zq49.48. 7(5d)	a^{3} 45 p30.37. 1(3f)	a^{4} 55 p48.36. 1(3f)	1H112b^{2} ♂ 30 q48. 1. 3(2e)	1H169a^{5} ♀ 100zq 48.46. 4(6)
	a^{5} 80 q50. 1. 1(3c)	a^{6} 100xp26.35.30(5b)	... ...	... ...	... ...	... ...
2H108	a^{1} ♀ 100xq58.57.56(6e)	a^{2} ♂ 100xq58.56.56(6cw)	a^{3} ♀ 55 q58.55. 3(3-4c)	a^{4} 100zq48.46. 9(5c)	1H139a^{2} ♂ p- (1-2f)	1H171a^{2} ♀ 00 p - -(alb.)
	a^{5} q- (3cw)	a^{6} - q- - -(4e)	a^{7} - q- - -(5c)	a^{8} - q- - -(6e)	... ...	... ...
2H109	a^{1} 90 q58.58.55(4e)	a^{2} 90 q48.46.12(4c)	a^{3} 75 q48.46. 1(4c)	a^{4} 60 q48. 1. 1(4c)	1H105a^{1} ♂ 40 p37. 1. 1(2b)	1H171a^{5} ♀ 00 p - - (alb.)
	a^{5} 60 q46.46. 1(4c)	a^{6} - q- - -(4c)	... ...	... ...	... ...	... ...
2H110	a^{1} - q- - -(6d)	a^{2} - q- - -(6d)	a^{3} - q- - -(6dw)	... ...	1H149b^{3} ♂ 100zq49.46.20(5d)	1H167a^{2} ♀ 100xq 58.58.55(6e)
2H111	a^{1} ♀ 60 q58. 1. 3(4c)	a^{2} 90 q35.33. 3(4a)	a^{3} 100yp35.35.28(5b)	a^{4} - q - -(4a)	1H158b^{2} ♂ 100zq33.33. 4(5a)	1H167a^{1} ♀ 100zq 49.46.21(6e)
2H112	a^{1} 100xp39.35.33(5a)	a^{2} 80 q58. 1. 2(4c)	a^{3} 85 q58.55. 2(4c)	a^{4} 95 q35.35.33(5a)	1H158b^{4} ♂ 100zq35.30.28(5a)	1H143b^{3} ♀ 95 q 46.33. 1(4c)
	a^{5} 95 q49.35. 2(4d)	a^{6} 100xq58.58.55(6e)	a^{7} - q- - -(4a)	a^{8} - q- - -(4e)	... ...	... ...
2H113	a^{1} 100zq48.46. 9(5c)	a^{2} - q- - -(3c)	a^{3} 00 p- - -(alb.)	a^{4} 00 p- - -(alb.)	1H158b^{1} ♂ 100zq46.46. 9(5c)	1H151b^{1} ♀ 100zq 49.48.47(6c)
	a^{5} 00 p- - -(alb.)	... ...	... ...	... ...	... ...	... ...
2H114	a^{1} ♂ 100zq58.58.55(6e)	a^{2} 85 q48.34. 1(4d)	a^{3} 60 q49.46. 1(4d)	a^{4} - q- - -(6d)	1H167a^{4} ♂ 100zq48.46.47(6c)	1H146b^{1} ♀ 100zq 52.46.20(5c-d)
2H115	a^{1} ♀ q (4c)	a^{2} q -(4c)	a^{3} q- - -(5d)	a^{4} 00 p - (alb.)	1H167a^{5} ♂ 100zq48.46.12(6d)	1H146b^{2} ♀ 100zq 49.46.20(5c)
2H116	a^{1} ♂ 100zp39.35.28(5b)	a^{2} ♀ 100xp35.35.30(5-6b)	a^{3} ♂ - p- - -(5b)	a^{4} - p- - -(5b)	1H165a^{1} ♂ 80 q49.46.11(5c)	1H149b^{6} ♀ 100zq 49.48.12(5d)
	a^{5} 00 p- - -(alb.)	a^{6} 00 p- - -(alb.)	... ...	... ...	... ...	... ...
2H117	a^{1} ♂ q - - (4e)	a^{2} ♀ 80 p39.39. 2(5b)	a^{3} ♂ 90 p23.15. 2(4-5f)	a^{4} 55 q48.46. 1(4c)	1H164a^{1} ♂ 100xq58.58.55(6e)	1H122b^{2} ♀ 70 q 49.46. 4(4c)
	a^{5} 65 q49.46. 1(4c)	a^{6} 00 p- - -(alb.)	... ...	... ...	... ...	... ...
2H118	a^{1} 85 q46.51. 4(5c)	a^{2} 100zq48.46.43(5c)	a^{3} 100zq48.46.12(5c)	a^{4} 55 q37. 1. 3(3b)	1H164a^{2} ♂ 100zq36.36. 9(5c)	1H122b^{1} ♀ 25 q 49. 1. 1(1c)
	a^{5} 100zp37.37.30(5b)	... ...	... ...	... ...	... ...	... ...
2H119	a^{1} ♀ 100xq58.58.55(6e)	a^{2} ♀ 100xq58.58.55(6e)	a^{3} 65 q58. 1. 1(4e)	a^{4} - q -(5d)	1H169a^{3} ♂ 100xq58.58.55(6e)	1H119a^{2} ♀ 100zq 52.46.43(5c)
	a^{5} 00 p alb.	a^{6} 00 p- - -(alb.)	... ...	... ...	... ...	... ...
2H120	a^{1} q- - (5c)	a^{2} q (5c)	a^{3} - p- (5b)	... ...	1H128b^{1} ♂ 100zp35.35. 4(5-6b)	1H146b^{2} ♀ 00 p - (alb.)
2H123	a^{1} ♀ 100xq58.58.56(6e)	a^{2} 100zq48.46.34(5c)	a^{3} 100zq49.46. 7(5c)	a^{4} q - (2e)	1H102b^{3} ♂ 25 p37. 1. 1(1-2b)	1H167a^{5} ♀ 00 p - - alb.)
	a^{5} - q- - -(4c)	a^{6} - q- - -(4e)	... ...	... ...	... ...	... ...
2H124	a^{1} ♀ 100zq58.57.56(6e)	a^{2} ♂ 85 q58.55. 2(4e*)	a^{3} 80 q46.46. 3(4cw)	a^{4} 100z 44.47 14(5ch*)	1H153a^{3} ♂ 00 p (alb.)	1H121b^{2} ♀ 75 p 39.34. 1(4a-b)
	a^{5} 00 p- - -(alb.)	a^{6} 00 p- - -(alb.)	... ...	... ...	... ...	... ...
2H125	a^{1} 100xq48.46.47(5c)	a^{2} - q- - -(2c)	a^{3} - q- - -(3b-c)	a^{4} - q- - -(4c)	1H111b^{1} ♂ 50 p37. 1. 2(3b)	1H149b^{5} ♀ 00 p - - -(alb.)
	a^{5} - q- - -(5c)	a^{6} - q- - -(5c)	... ...	... ...	... ...	... ...
2H126	a^{1} 55 q46.46. 2(3c)	a^{2} 50 q49. 1. 2(3c)	a^{3} 30 q47. 1. 1(3c)	a^{4} 95 q35.33. 1(4a)	1H164a^{3} ♂ 00 p- - -(alb.)	1H136b^{1} ♀ 90 p 39.35. 2(4a)
	a^{5} - q- - -(5d)	... ...	... ...	... ...	... ...	... ...
2H127	a^{1} ♀ 100xq58.57.55(6e)	a^{2} ♂ 100xq56.55.55(5-6d-e)	a^{3} 80 q48.35. 7(5c)	a^{4} 75 q48.46. 2(4c)	1H165a^{2} ♂ 00 p - (alb.)	1H136b^{3} ♀ 100zp 37.35.28(5b)
	a^{5} 80 q49.46. 7(5c)	... ...	... ...	... ...	... ...	... ...
2H129	a^{1} - q- - -(5c)	a^{2} - q- - -(2c)	... ...	... ...	1H144b^{2} ♂ - q- - -(5c)	1H37a^{3} ♀ - q - - -(3c-dw)
2H135	a^{1} - p- - -(5b)	a^{2} - p- - -(5b)	a^{3} - p- - -(5b)	a^{4} - p- - -(5b)	1H99a^{1} ♂ 100zp39.35.28(6b)	1H129a^{3} ♀ 100xp 24.23.15(6f)
	a^{5} - p- - -(5b)	a^{6} - p- - -(5b)	... ...	... ...	... ...	... ...
2H137	a^{1} ♀ 35 p37. 1. 1(2b)	a^{2} ♀ 55 p39.33. 1(3b)	a^{3} 35 p37. 1. 1(2b)	a^{4} - p- - -(2b)	1H70a^{2} ♂ 55 q37.33.30(3a)	1H136a^{4} ♀ 35 p 39. 1. 1(2b)
2H139	a^{1} 100zq39.33.28(5b)	a^{2} 100zq39.33.28(5b)	a^{3} 100zq48.46.12(5cw)	a^{4} 100xq58.55.55(6e)	1H98a^{1} ♂ 100zq49.46. 4(6e)	1H136a^{3} ♀ q - - (5a)
	a^{5} 100xp23.23.17(6fw)	... ...	... ...	... ...	... ...	... ...
2H142	a^{1} ♀ 100zp39.36.28(5bc)	a^{2} 100zp39.39.28(5b-c)	a^{3} - q- - -(5b-c)	a^{4} - q - (5b-c)	1H148a^{2} ♂ 80 q49.46. 4(5c)	1H98a^{5} ♀ 100zq 48.46. 4(5-6c)
	a^{5} - q- - -(5c)	a^{6} - q- - -(5c)	a^{7} - q- - -(5c)	a^{8} - q- - -(5cw)	... ...	... ...
2H143	a^{1} 100xq58.58.55(6e)	a^{2} 30 q61. 1. 1(3c)	a^{3} 100xq58.58.57(6e)	a^{4} 35 p26. 1. 1(2fw)	1H60a^{1} ♂ 100xq57.57.55(6e)	1H65b^{1} ♀ - q - (6e)
	a^{5} 100xq58.57.55(6e)	a^{6} 100xp23.23.18(6fw)	a^{7} 100xq58.58.55(6e)	... ...	... ...	... ...
2H144	a^{1} ♀ 60 q58.56. 1(3c)	a^{2} ♂ 80 q58.57. 1(4c)	a^{3} 95 q41.41.50(6g)	a^{4} q - (6g)	1H16a^{1} ♂ 95 q41.50. 4(4g)	1H157a^{2} ♀ q - (5a)
	a^{5} 00 p- - -(alb.)	a^{6} 00 p- - -(alb.)	... ...	... ...	... ...	... ...
2H145	a^{1} 100zq48.46.11(5c)	a^{2} 80 q48.46.11(4c)	a^{3} 75 q48.46. 3(3c)	a^{4} 00 p- - -(alb.)	1H111a^{3} ♂ 100zq48.46. 7(5c)	1H156a^{1} ♀ 100xq 58.57.55(6e)
	a^{5} 00 p- - -(alb.)	... ...	... ...	... ...	... ...	... ...
2H146	a^{1} 60 q49.48. 1(3c)	a^{2} 100zq48.36.12(5c)	a^{3} 95 p39.35.33(4b)	a^{4} 100zp39.35.28(5b)	1H143a^{1} ♂ 100zq48.46. 2(5c)	1H158a^{4} ♀ 100zq 49.48.34(5c)
	a^{5} 100xp23.23.18(6f)	a^{6} 100xq58.58.55(6e)	a^{7} 00 p- - -(alb.)	... ...	... ...	... ...
2H147	a^{1} 55 q?39. 1. 2(3c)	a^{2} 100xq41.41.50(6g)	a^{3} 55 p?39. 1. 1(3a)	a^{4} q (5a)	1H92a^{3} ♂ 100xq57.57.55(6e)	1H157a^{1} ♀ - q - - (5a)
	a^{5} 00 p- - -(alb.)	a^{6} 00 p- - -(alb.)	a^{7} 00 p- - -(alb.)	a^{8} 00 p- - -(alb.)	... ...	... ...
2H148	a^{1} 100xp37.36.36(5a)	a^{2} 100xq48.47.46(5c)	a^{3} 100zq49.46. 9(3c)	a^{4} 90 p48.34. 2(4b-c)	1H161a^{2} ♂ 100zq48.46. 7(5c)	1H151a^{4} ♀ 80 q 49.48.34(5c)
	a^{5} 100zq48.46.30(5c)	a^{6} 80 p36.36. 4(4b)	a^{7} 100zp37.37. 2(5a)	a^{8} 80 p37.37. 2(4a)	... ...	... ...
	a^{9} - q- - -(3c)	... ...	... ...	... ...	... ...	... ...
2H151	a^{1} 100zp?35.33.28(5a)	a^{2} 100zq46.46. 2(5c)	a^{3} 100zq?33.33.28(5a)	a^{4} 100zq?33. 2. 2(5a)	1H99a^{1} ♂ 100zp39.35.28(6b)	1H97a^{1} ♀ 100zq 33.30.28(5-6bl)
	a^{5} 95 q48.46.12(4c)	a^{6} 00 p- - -(alb.)	a^{7} 00 p- - -(alb.)	... ...	... ...	... ...
2H152	a^{1} ♂ 60 q58. 1. 1(2-3e)	a^{2} ♂ 65 q55. 1. 2(3-4e)	a^{3} 65 q41. 1. 1(3g)	a^{4} - q - -(4c)	1H102b^{1} ♂ 25 p37. 1. 1(1-2b)	A13a^{2} ♀ 00 p - (A12)
	a^{5} - q- - -(3f-e)	... ...	... ...	... ...	... ...	... ...
2H153	a^{1} 100zq46.46.30(5c)	... ...	... ...	... ...	1H128b^{1} ♂ 100zp35.35. 4(5-6b)	1H24a^{1} ♀ 00 p - - -(alb.)
2H154	a^{1} 50 q48.46. 2(3c)	... ...	... ...	... ...	1H111b^{1} ♂ 50 p37. 1. 2(3b)	1H32a^{1} ♀ 00 p - - -(alb.)
2H155	a^{1} ♂ 100xq58.57.55(5-6e)	a^{2} ♀ 100xq58.57.56(6e)	a^{3} ♀ 100xq58.58.55(6e)	a^{4} 100zq48.46.12(5c)	1H105a^{4} ♂ 40 p37. 1. 1(2b)	A58a^{4} ♀ 00 p - (A×3)

MATING	OFFSPRING				PARENTS	
					Father	Mother
2H155	a^5 – q – – –(5c)	... ...	... ...	... ...	... ...	... ...
2H156	a^1 ♀ 100xq58.56.56(6e)	a^2 ♂ 100xq56.56.56(6e)	a^3 ♂ 100xq58.57.55(6e)	a^4 ♂ 100xq57.56.56(6e)	1H139a^2 ♂ – p – – –(1–2f)	1H28b^4 ♀ 00 p – – –(alb.)
	a^5 – q – – –(6g)	... ...	... ...	... ...	... ...	... ...
2H157	a^1 90 q48.46. 2(4c)	a^2 100zq48.46.20(5c)	a^3 100zq49.35.20(5c)	a^4 95 q48.48.28(4c)	1H130b^3 ♂ 35 p39.35. 1(2a–b)	A30a^2 ♀ 00 p – – –(A2 ?)
	a^5 100zq48.46.20(5c)	a^6 100zq48.46. 7(5c)	a^7 100zq48.46. 9(5c)	a^8 100zq48.46.20(4c)	... ...	... ...
2H159	a^1 100xp24.24.24(6f)	a^2 100yp23.19. 9(5bw)	a^3 100zp23.36.28(5bw)	a^4 – p – – –(3f)	1H111b^2 ♂ 60 q58.55. 1(3e)	1H3a^2 ♀ – q – – –(6d)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb. w)	a^7 00 p – – –(alb.)	... ...	... ...	... ...
2H300	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 100xq58.57.56(6e)	a^3 ♂ 45 q61. 1. 1(2–3e)	b^1 ♀ 00 p – – –(alb.)	1H139b^1 ♂ 100xq58.57.55(6e)	1H270a^1 ♀ 100xq58.58.55(6e)
	b^2 ♀ 00 p – – –(alb.)	b^3 ♀ 100xq58.56.56(6e)	b^4 ♂ 00 p – – –(alb.)	b^5 ♂ 00 p – – –(alb.)	... ...	... ...
	c^1 ♂ 00 p – – –(alb.)	c^2 ♀ 00 p – – –(alb.)	c^3 ♀ 100xq58.56.56(6e)	c^4 ♀ 100xq57.56.55(6e)	... ...	... ...
	c^5 ♀ 100xq58.57.55(6e)	c^6 – p – – – –	c^7 – p – – – –	... ...	... ...	... ...
2H301	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 100xq58.57.55(6d–e)	a^3 ♂ 95 q58.55. 1(4e)	a^4 ♂ 100xp26.22.18(4–5f)	2H114a^1 ♂ 100zq58.58.55(6e)	1H274a^1 ♀ 100xq57.57.55(6e)
	a^5 ♀ 100xq58.57.55(6e)	a^6 ♀ 100xq58.57.56(5–6e)	a^7 ♀ 95 q58.55. 4(4–5e)	a^8 ♀ 100xp23.23. 8(5f)	... ...	... ...
2H302	a^1 – p – – –(f)	a^2 – q – – – –	a^3 – p – – – –	b^1 ♂ 100xp23.15.15(6f)	2H114a^1 ♂ 100zq58.58.55(6e)	1H274a^2 ♀ 100xq58.57.55(6e)
	b^2 ♂ 100xq58.57.55(6e)	b^3 ♂ – q – – –(6e)	b^4 ♀ 95 q58.57.55(5e)	b^5 ♀ 100xq58.58.55(6e)	... ...	... ...
	b^6 ♀ 100xq58.57.55(6e)	b^7 – q – – – –	b^8 – p – – – –	c^1 ♀ 100xp23.23.15(6f)	... ...	... ...
	c^2 ♀ – q – – –(5e)	c^3 ♀ 60 q58.57. 1(4e)	c^4 ♀ 100xq58.57.55(6e)	c^5 ♀ 100xq58.58.55(6e)	... ...	... ...
	c^6 ♀ 100xq58.57.56(6e)	c^7 – p – – – –	c^8 – q – – – –	... ...	... ...	... ...
2H303	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ – q – – –(4–5e)	a^3 ♂ 90 q58.55. 1(4e)	a^4 ♀ 100xq58.57.55(6e)	2H114a^1 ♂ 100zq58.58.55(6e)	1H274a^3 ♀ 100zq48.46.14(5-6c:e)
	b^1 ♂ – p – – –(6f)	b^2 ♂ 70 q58.55. 1(4e)	b^3 ♂ 90 q58.57. 1(5e)	b^4 ♂ 100xq58.58.55(5e)	... ...	... ...
	b^5 ♂ 100xq58.57.55(6e)	b^6 ♀ 100xq58.58.55(6e)	... ...	... ...	... ...	... ...
2H304	a^1 ♂ 100yp23.23.18(6f)	a^2 ♂ 80 p23.18. 4(4–5f)	a^3 ♂ 100xq58.58.55(6e)	a^4 ♀ 60 q58. 1. 1(3e)	2H114a^1 ♂ 100zq58.58.55(6e)	1H274a^4 ♀ 100xq58.58.55(6e)
	a^5 ♀ 85 q58.55. 2(4–5e)	a^6 ♀ 100xq58.58.55(6e)	a^7 – q – – – –	a^8 – q – – – –	... ...	... ...
	b^1 ♂ 100xq58.58.55(6e)	b^2 ♀ 100xq58.56.55(6e)	b^3 ♀ 100xq58.58.55(6e)	b^4 ♀ 85 q58.55. 2(5e)	... ...	... ...
	b^5 ♀ 100xp26.26.23(6f)	b^6 – p – – – –	b^7 – q – – – –	b^8 – q – – – –	... ...	... ...
	c^1 ♂ 100xp26.26.17(6f)	c^2 ♀ 55 q58. 1. 1(3e)	c^3 ♀ 60 q61.58. 1(3e)	... ...	... ...	... ...
2H305	a^1 ♂ 100xp26.23.23(6f)	a^2 ♂ 35 p23. 1. 1(3f)	a^3 ♀ 100zp26.18.17(6f)	a^4 ♀ 100xp22.22.18(6f)	3H9a^1 ♂ – p – – –(6f)	1H234a^3 ♀ 100xp22.15.15(5–6f)
	a^5 ♀ 100xp23.23.18(6f)	a^6 ♀ 25 p23. 1. 1(2f)	a^7 – p – – – –	b^1 ♂ 100xp26.23.18(5f)	... ...	... ...
	b^2 ♂ 100xp23.15.15(6f)	b^3 ♀ 100yp23.23.17(6f)	b^4 ♀ 30 p26. 1. 1(2f)	b^5 – p – – – –	... ...	... ...
	c^1 ♂ 100xp23.23.15(5f)	c^2 ♀ 100yp26.23. 1(5f)	c^3 ♀ 100xp26.23.18(5f)	... ...	... ...	... ...
2H306	a^1 ♀ 100yp26.23. 2(5f)	a^2 ♀ 100xp23.23.17(6f)	b^1 ♀ 35 p23. 1. 1(2f)	b^2 ♂ 100xp23.23.15(5f)	1H234a^2 ♂ 100xp37.19. 9(5–6f)	1H227a^3 ♀ 60 p22. 1. 1(4f)
	b^3 ♂ 100xp23.23.18(5f)	b^4 ♂ 25 p26. 1. 2(2–3f)	b^5 – p – – – –	b^6 – p – – – –	... ...	... ...
	b^7 – p – – –	c^1 ♂ 55 p26.23. 1(3f)	c^2 ♀ 100xp23.23.18(6f)	c^3 ♀ 30 p26. 1. 3(1–2f)	... ...	... ...
2H307	a^1 ♂ – p – – –(5b)	a^2 ♀ 100zp39.35.30(5b)	a^3 ♀ 100xp39.39.36(5b)	a^4 ♂ – p – – –(5b)	1H123b^5 ♂ 100zp35.33.28(5–6b)	1H276a^2 ♀ 55 p37.39. 4(4b)
	a^5 – p – – – –	a^6 – p – – – –	a^7 – p – – – –	a^8 – p – – – –	... ...	... ...
2H308	a^1 ♂ 100xp35.35.30(5b)	a^2 ♂ 100zp39.35. 9(5–6b)	a^3 ♀ 100zp39.35. 9(5b)	a^4 ♀ 100xp39.35.30(5b)	1H123b^5 ♂ 100zp35.33.28(5–6b)	1H276a^3 ♀ 100zp35.35.28(5b)
	a^5 ♀ 95 p39.35. 2(5b)	a^6 ♀ 100zp36.35.30(5b–f)	... ...	... ...	... ...	... ...
2H309	a^1 ♂ – p – – –(5b)	a^2 ♂ – p – – –(5b)	a^3 ♀ 100zp39.39.28(5b)	a^4 ♀ 100zp39.35.28(5b)	1H130b^1 ♂ 30 p37. 1. 1(2a–b)	1H281a^1 ♀ 100zp39.39. 6(5b)
	a^5 – p – – – –	a^6 – p – – – –	a^7 – p – – – –	a^8 – p – – – –	... ...	... ...
	b^1 ♀ 100zp39.35.30(5b)	b^2 ♂ 100zp39.39.28(5bw)	b^3 ♂ 100zp40.36.28(5b)	b^4 – p – – – –	... ...	... ...
	b^5 – p – – –	b^6 – p – – – –	b^7 – p – – – –	c^1 100zp40.37. 4(5b)	... ...	... ...
	c^2 100zp40.36. 4(5b)	c^3 100zp37.37. 3(5b)	c^4 100zp40.36. 4(5b)	c^5 – p – – – –	... ...	... ...
	c^6 – p – – – –	c^7 – p – – – –	c^8 – p – – – –	... ...	... ...	... ...
2H310	a^1 ♀ 60 p37.19. 2(4b)	a^2 ♀ 80 p39.36. 4(5b)	a^3 ♂ 10 p37. 1. 1(2b)	a^4 ♂ 55 p37. 1. 1(3b)	1H130b^3 ♂ 35 p39.35. 1(2a–b)	1H283a^1 ♀ 80 p39.35. 2(5b)
	a^5 60 39. 1. 1 –	b^1 ♂ – p – – –(1–2b)	b^2 ♂ 65 p39. 1. 3(4b)	b^3 ♀ 75 p36.35. 4(5a–b)	... ...	... ...

	c^1 ♂ 15 p37. 1. 1(1b)	...	...	...	...	...
2H312	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 85 q58.57. 2(4e)	a^4 ♀ 100xq58.57.55(6e)	1H139b^1 ♂ 100xq58.57.55(6e)	1H156a^1 ♀ 100xq58.57.55(6e)
	a^5 ♀ 100xq58.56.55(5–6e)	a^6 ♀ – q – – –(4e)	a^7 – p – – – –	b^1 ♂ – q – – –(6e)	...	...
	b^2 ♂ 00 p – – –(alb.)	b^3 ♂ 100xq58.57.55(6e)	b^4 ♀ 100xp15.15.10(6f)	b^5 ♀ 00 p – – –(alb.)	...	...
	b^6 ♀ – q – – –(6e)	b^7 – q – – – –	c^1 ♂ 00 p – – –(alb.)	c^2 ♂ 100xq58.58.55(6e)	...	...
	c^3 ♂ 100xq56.58.55(6e)	c^4 ♀ 90 q58.57. 2(5e)	c^5 – p – – – –	c^6 – p – – – –	...	...
2H313	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 35 q58. 1. 1(3e)	b^1 ♂ 00 p – – –(alb.)	b^2 ♀ 00 p – – –(alb.)	1H225a^2 ♂ 85 q57.55. 1(4e)	1H227a^6 ♀ 25 q58. 1. 1(2e)
	b^3 ♀ 40 q58. 1. 2(3e)	b^4 ♀ 00 p – – –(alb.)	...	...	...	...
2H314	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 35 p37. 1. 1(3b)	a^3 ♂ 40 q58. 1. 1(3e)	a^4 ♀ 35 q58. 1. 1(2e)	1H109b^1 ♂ 25 q58. 1. 2(2e)	1H187b^1 ♀ 55 q58.57. 1(3e)
	b^1 ♂ 00 p – – –(alb.)	b^2 ♂ – q – – –(2e)	b^3 ♀ 00 p – – –(alb.)	c^1 ♂ 00 p – – –(alb. w)	...	...
	c^2 ♀ 70 r42.50. 1(3g)	...	...	...	...	...
2H315	a^1 ♀ 90 q58.58. 2(4–5e)	b^1 ♂ 90 q58.58. 1(4e)	b^2 ♂ – q – – –(4e)	b^3 ♀ 85 q58.57. 1(3–4e)	1H232a^2 ♂ 65 q58.57. 1(4e)	1H270a^2 ♀ 90 q58.55. 2(4–5e)
	b^4 ♀ 85 q58.55. 1(4e)	b^5 ♀ 00 p – – –(alb.)	b^6 ♀ – q – – –(5e)	b^7 – q – – – –	...	...
	c^1 ♀ 70 q58. 1. 4(3–4e)	c^2 ♀ 85 q58.57. 3(4e)	c^3 ♂ 65 q58.57. 1(3e)	c^4 ♂ – q – – –(4e)	...	...
	c^5 ♂ 65 q58.58. 1(3e)	...	...	...	...	...
2H317	a^1 65 p26. 1. 1(3–4f*)	a^2 30 p37. 1. 1(2b–f*)	...	...	1H102b^1 ♂ 25 p37. 1. 1(1–2b)	1H232a^3 ♀ 55 p39.39. 2(3b)
3H1	a^1 ♂ 60 q58.57. 1(3e)	a^2 ♂ 50 r42.50. 1(3g)	a^3 ♀ 55 p26.26. 1(3f)	a^4 ♀ 100xp26.26.23(6f)	2H114a^1 ♂ 100zq58.58.55(6e)	2H124a^1 ♀ 100zq58.57.56(6e)
	a^5 ♀ 100xp26.23.18(6f)	a^6 ♀ 45 q58.55. 1(3e)	a^7 ♂ 55 p26.17. 3(3f)	a^8 ♀ 100xq58.57.55(6e)	...	...
	b^1 ♂ 65dr41.50. 3(4g)	b^2 ♂ 55 q58. 1. 1(4e)	b^3 ♂ 100xq57.57.56(5–6e)	b^4 ♂ 100xq58.58.56(6e)	...	...
	b^5 ♀ 45 q'55.56. 3(3e)	b^6 ♀ 60d.r41.41. 3(4g)	b^7 ♀ – q – – –(5–6e)	b^8 – q – – – –	...	...
	b^9 – q – – – –	b^{10} – p – – – –	c^1 ♂ 100xp22.22.17(6f)	c^2 ♂ 70 q58.58. 2(4e)	...	...
	c^3 ♂ 100xr41.41.50(6g)	c^4 ♂ 100xr 41.41.50(6g) or q	c^5 ♂ 100xq58.58.55(6e)	c^6 ♀ 65 q58.55. 2(4e)	...	...
	c^7 ♀ 100xq58.58.56(6e)	c^8 – q – – – –	c^9 – q – – – –	c^{10} – q – – – –	...	...
	c^{11} – p – – – –	c^{12} – p – – – –	...	...	...	...
3H2	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 85 q58.55. 2(4e)	a^3 ♂ 100xq57.57.55(5e)	a^4 ♂ 100xq58.58.55(5–6e)	2H155a^1 ♂ 100xq58.57.55(5–6e)	2H156a^1 ♀ 100xq58.56.56(6e)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♀ 75 q55. 1. 1(4e)	a^7 ♀ 100xq56.57.55(5–6e)	b^1 ♂ 00 p – – –(alb.)	...	...
	b^2 ♂ 00 p – – –(alb. w)	b^3 ♂ 100xp22.17.18(6f)	b^4 ♂ 100xq57.56.55(5–6e)	b^5 ♀ 100xp22.15.18(6f)	...	...
	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ 00 p – – –(alb.)	c^3 ♀ 00 p – – –(alb.)	...	...	...
3H3	a^1 ♂ 55 q58.56. 1(4ew)	a^2 ♂ 60 p23. 1. 1(3f)	a^3 ♀ 100xq58.58.55(5–6e)	a^4 ♂ 100xq58.58.55(6e)	2H156a^2 ♂ 100xq56.56.56(6e)	2H155a^2 ♀ 100xq58.57.56(6e)
	a^5 ♀ 100xq56.56.55(6e)	a^6 ♀ 00 p – – –(alb.)	a^7 ♀ 00 p – – –(alb.)	a^8 ♀ 100xp15.15.15(6f)	...	...
	a^9 – p – – – –	b^1 ♂ 00 p – – –(alb.)	b^2 ♂ 75 q58.55.55(4e)	b^3 ♂ – q – – –(4e)	...	...
	b^4 ♂ 100xq58.57.55(6e)	b^5 ♂ 100xq58.57.55(6e)	b^6 ♂ 100xq57.57.56(5e)	b^7 ♂ 100xq58.58.55(6e)	...	...
	b^8 ♀ 100yp23.23.18(6fw)	b^9 ♀ 100xq58.58.55(6e)	c^1 ♂ – p – – –(6f)	c^2 ♂ 100xq57.58.56(6e)	...	...
	c^3 ♂ 00 p – – –(alb. w)	c^4 ♂ 00 p – – –(alb.)	c^5 ♀ 60 p26.22. 2(3f)	c^6 ♀ 00 p – – –(alb.)	...	...
	c^7 ♀ 00 p – – –(alb.)	c^8 – q – – –(6e)	c^9 – p – – – –	c^{10} – p – – – –	...	...
3H4	a^1 ♂ – p – – –(6f)	a^2 ♂ – q – – –(6e)	a^3 ♀ 100xp23.22.18(6f)	a^4 ♀ 100xp23.23.17(6f)	2H156a^3 ♂ 100xq58.57.55(6e)	2H155a^3 ♀ 100xq58.58.55(6e)
	a^5 ♀ 100xq57.57.56(6e)	a^6 ♀ 100xq58.57.55(6e)	b^1 ♂ – q – – –(5e)	b^2 ♂ 100xq58.58.55(6e)	...	...
	b^3 ♂ 100xq58.58.55(6e)	b^4 ♂ 00 p – – –(alb.)	b^5 ♀ 85 q57.57. 2(4e)	b^6 ♀ 00 p – – –(alb.)	...	...
	b^7 – p – – – –	c^1 ♂ 00 p – – –(alb.)	c^2 ♂ 00 p – – –(alb.)	c^3 ♂ 100xp23.22.17(5a–f)	...	...
	c^4 ♂ 45 q58.58. 1(2–3e)	c^5 ♂ 95 q58.55. 2(5e)	c^6 – p – – – –	c^7 ♀ – q – – –(6e)	...	...
	c^8 ♀ 100xq58.57.55(6e)	c^9 ♀ 80 q58.57. 2(4e)	...	...	...	...
3H5	a^1 ♂ 100xq41.41.50(6g)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♂ 100xq57.57.55(6e)	a^4 ♂ 100xq58.58.55(6e)	2H49a^1 ♂ – q – – –(6e)	2H108a^1 ♀ 100xq58.57.56(6e)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♀ 00 p – – –(alb.)	b^1 ♂ 00 p – – –(alb. w)	b^2 ♂ 100xq41.50.26(6gw)	...	...
	b^3 ♂ 100xq58.58.55(6e)	b^4 ♂ – q – – –(6e)	b^5 ♀ 75 r41.41. 3(4gw)	b^6 ♀ 90 q58.58. 2(4e)	...	...
	b^7 ♀ 100xq56.57.55(6ew)	c^1 ♀ 100xq58.58.55(5e)	c^2 ♂ – q – – –(6e)	c^3 ♂ 00 p – – –(alb.)	...	...
	c^4 ♀ – q – – –(5g)	c^5 ♀ 100xq57.57.56(6e)	...	...	...	...
3H6	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 100yq58.57. 2(5e)	a^3 ♂ 100xq57.57.56(5–6e)	a^4 ♀ 100yq58.58.55(5e)	2H96a^1 ♂ 100xq58.57.55(6e)	2H119a^1 ♀ 100xq58.58.55(6e)

MATING	OFFSPRING				PARENTS	
					Father	Mother
2*H*155	a^5 − q − − −(5c)				... ...	... ...
2*H*156	a^1 ♀ 100xq58.56.56(6e)	a^2 ♂ 100xq56.56.56(6e)	a^3 ♂ 100xq58.57.55(6e)	a^4 ♂ 100xq57.56.56(6e)	1*H*139a^2 ♂ − p − − −(1–2f)	1*H*28b^1 ♀ 00 p − − −(alb.)
	a^5 − q − − −(6g)					
2*H*157	a^1 90 q48.46. 2(4c)	a^2 100zq48.46.20(5c)	a^3 100zq49.35.20(5c)	a^4 95 q48.48.28(4c)	1*H*130b^3 ♂ 35 p39.35. 1(2a–b)	A30a^2 ♀ 00 p − [illegible]
	a^5 100zq48.46.20(5c)	a^6 100zq48.46. 7(5c)	a^7 100zq48.46. 9(5c)	a^8 100zq48.46.20(4c)		
2*H*159	a^1 100xp24.24.24(6f)	a^2 100yp23.19. 9(5bw)	a^3 100zp23.36.28(5bw)	a^4 − p − − −(3f)	1*H*111b^2 ♂ 60 q58.55. 1(3e)	1*H*3a^2 ♀ q − − −(6d
	a^5 00 p − − −(alb.)	a^6 00 p − − (alb. w)	a^7 00 p − (alb.)			
2*H*300	a^1 ♀ 00 p − − −(alb.)	a^2 ♀ 100xq58.57.56(6e)	a^3 ♂ 45 q61. 1. 1(2–3e)	b^1 ♀ 00 p − − −(alb.)	1*H*139b^1 ♂ 100xq58.57.55(6e)	1*H*270a^1 ♀ 100rq58.58.55(6e)
	b^2 ♀ 00 p − − −(alb.)	b^3 ♀ 100xq58.56.56(6e)	b^4 ♂ 00 p − − −(alb.)	b^6 ♂ 00 p − − −(alb.)	... ...	... ...
	c^1 ♂ 00 p − − −(alb.)	c^2 ♀ 00 p − − −(alb.)	c^3 ♀ 100xq58.56.56(6e)	c^4 ♀ 100xq57.56.55(6e)	... ...	... ...
	c^5 ♀ 100rq58.57.55(6e)	c^6 p −	c^7 p −			
2*H*301	a^1 ♂ 100rq58.58.55(6e)	a^2 ♂ 100rq58.57.55(6d–e)	a^3 ♂ 95 q58.55. 1(4e)	a^4 ♂ 100xp26.22.18(4–5f)	2*H*114a^1 ♂ 100zq58.58.55(6e)	1*H*274a^1 ♀ 100rq57.57.55(6e)
	a^5 ♀ 100rq58.57.55(6e)	a^6 ♀ 100rq58.57.56(5–6e)	a^7 ♀ 95 q58.55. 4(4–5e)	a^8 ♀ 100xp23.23. 8(5f)		
2*H*302	a^1 − p − − −(f)	a^2 − q − − − −	a^3 − p − − − −	b^1 ♂ 100xp23.15.15(6f)	2*H*114a^1 ♂ 100zq58.58.55(6e)	1*H*274a^2 ♀ 100xq58.57.55(6e)
	b^2 ♂ 100xq58.57.55(6e)	b^3 ♂ q − (6e)	b^4 ♀ 95 q58.57.55(5e)	b^5 ♀ 100xq58.58.55(6e)	... ...	... ...
	b^6 ♀ 100xq58.57.55(6e)	b^7 − q − − − −	b^8 − p − − − −	c^1 ♀ 100xp23.23.15(6f)	... ...	... ...
	c^2 ♀ − q − − −(5e)	c^3 ♀ 60 q58.57. 1(4e)	c^4 ♀ 100xq58.57.55(6e)	c^5 ♀ 100xq58.58.55(6e)	... ...	... ...
	c^6 ♀ 100xq58.57.56(6e)	c^7 − p − − − −	c^8 − q − − − −		... ...	... ...
2*H*303	a^1 ♂ 100rq58.57.55(6e)	a^2 ♂ − q − (4–5e)	a^3 ♂ 90 q58.55. 1(4e)	a^4 ♀ 100rq58.57.55(6e)	2*H*114a^1 ♂ 100zq58.58.55(6e)	1*H*274a^3 ♀ 100zq48.46.14(5–6e)
	b^1 ♂ − p (6f)	b^2 ♂ 70 q58.55. 1(4e)	b^3 ♂ 90 q58.57. 1(5e)	b^4 ♂ 100rq58.58.55(5e)	... ...	... ...
	b^5 ♂ 100rq58.57.55(6e)	b^6 ♀ 100xq58.58.55(6e)				
2*H*304	a^1 ♂ 100yp23.23.18(6f)	a^2 ♂ 80 p23.18. 4(4–5f)	a^3 ♂ 100xq58.58.55(6e)	a^4 ♀ 60 q58. 1. 1(3e)	2*H*114a^1 ♂ 100zq58.58.55(6e)	1*H*274a^4 ♀ 100xq58.58.55(6e)
	a^5 ♀ 85 q58.55. 2(4–5e)	a^6 ♀ 100rq58.58.55(6e)	a^7 q −	a^8 − q − − − −	... ...	... ...
	b^1 ♂ 100rq58.58.55(6e)	b^2 ♀ 100rq58.56.55(6e)	b^3 ♀ 100rq58.58.55(6e)	b^4 ♀ 85 q58.55. 2(5e)	... ...	... ...
	b^5 ♀ 100rp26.26.23(6f)	b^6 p	b^7 − q − − − −	b^8 q	... ...	... ...
	c^1 ♂ 100rp26.26.17(6f)	c^2 ♀ 55 q58. 1. 1(3e)	c^3 ♀ 60 q61.58. 1(3e)		... ...	... ...
2*H*305	a^1 ♂ 100rp26.23.23(6f)	a^2 ♂ 55 p23. 1. 1(3f)	a^3 ♀ 100zp26.18.17(6f)	a^4 ♀ 100rp22.22.18(6f)	3*H*9a^1 ♂ − p (6f)	1*H*234a ♀ 100rp22.15.15(5–6f)
	a^5 ♀ 100rp23.23.18(6f)	a^6 ♀ 25 p23. 1. 1(2f)	a^7 p −	b^1 ♂ 100rp26.23.18(5f)	... ...	... ...
	b^2 ♂ 100rp23.15.15(6f)	b^3 ♀ 100yp23.23.17(6f)	b^4 ♀ 30 p26. 1. 1(2f)	b^5 − p	... ...	... ...
	c^1 ♂ 100rp23.23.15(5f)	c^2 ♀ 100yp26.23. 1(5f)	c^3 ♀ 100rp26.23.18(5f)			
2*H*306	a^1 ♀ 100yp26.23. 2(5f)	a^2 ♀ 100rp23.23.17(6f)	b^1 ♀ 35 p23. 1. 1(2f)	b^2 ♂ 100xp23.23.15(5f)	1*H*234a^2 ♂ 100rp37.19. 9(5–6f)	1*H*227a ♀ 60 p22. 1. 1(4f)
	b^3 ♂ 100rp23.23.18(5f)	b^4 ♂ 25 p26. 1. 2(2–3f)	b^5 p −	b^6 − p − − − −	... ...	... ...
	b^5 p	c^1 ♂ 55 p26.23. 1(3f)	c^2 ♀ 100rp23.23.18(6f)	c^3 ♀ 30 p26. 1. 3(1–2f)		
2*H*307	a^1 ♂ − p − − −(5b)	a^2 ♀ 100zp39.35.30(5b)	a^3 ♀ 100rp39.39.36(5b)	a^4 ♂ − p − − −(5b)	1*H*123b^5 ♂ 100zp35.33.28(5–6b)	1*H*276a^2 ♀ 55 p37.39. 4(4b)
	a^5 − p	a^6 p − − − −	a^7 − p − −	a^8 − p − − − −		
2*H*308	a^1 ♂ 100xp35.35.30(5b)	a^2 ♂ 100zp39.35. 9(5–6b)	a^3 ♀ 100zp39.35. 9(5b)	a^4 ♀ 100xp39.35.30(5b)	1*H*123b^5 ♂ 100zp35.33.28(5–6b)	1*H*276a^3 ♀ 100zp35.35.28(5b)
	a^5 ♀ 95 p39.35. 2(5b)	a^6 ♀ 100zp36.35.30(5b–f)				
2*H*309	a^1 ♂ − p − − −(5b)	a^2 ♂ − p − − −(5b)	a^3 ♀ 100zp39.39.28(5b)	a^4 ♀ 100zp39.35.28(5b)	1*H*130b^1 ♂ 30 p37. 1. 1(2a–b)	1*H*281a^1 ♀ 100zp39.39. 6(5b)
	a^5 − p − − − −	a^6 − p − − − −	a^7 − p − − −	a^8 p	... ...	... ...
	b^1 ♀ 100zp39.35.30(5b)	b^2 ♂ 100zp39.39.28(5bw)	b^3 ♂ 100zp40.36.28(5b)	b^4 p − − −	... ...	... ...
	b^5 − p − − −	b^6 − p − − − −	b^7 − p − − −		... ...	... ...
	c^2 100zp40.36. 4(5b)	c^3 100zp37.37. 3(5b)	c^4 100zp40.36. 4(5b)	c^5 100zp40.37. 4(5b)	... ...	... ...
	c^6 − p − − − −	c^7 − p − − − −	c^8 − p − −	c^9 p	... ...	... ...
2*H*310	a^1 ♀ 60 p37.19. 2(4b)	a^2 ♀ 80 p39.36. 4(5b)	a^3 ♂ 10 p37. 1. 1(2b)	a^4 ♂ 55 p37. 1. 1(3b)	1*H*130b^3 ♂ 35 p39.35. 1(2a–b)	1*H*283a^1 ♀ 80 p39.35. 2(5b)
	a^5 60 39. 1. 1 −	b^1 ♂ − p − − −(1–2b)	b^2 ♂ 65 p39. 1. 3(4b)	b^3 ♀ 75 p36.35. 4(5a–b)	... ...	... ...
	c^1 ♂ 15 p37. 1. 1(1b)					
2*H*312	a^1 ♂ 00 p − − −(alb.)	a^2 ♀ 00 p − − −(alb.)	a^3 ♀ 85 q58.57. 2(4e)	a^4 ♀ 100xq58.57.55(6e)	1*H*139b^5 ♂ 100rq58.57.55(6e)	1*H*156a^1 ♀ 100rq58.57.55(6e)
	a^5 ♀ 100xq58.56.55(5–6e)	a^6 ♀ − q − − −(4e)	a^7 − p − − − −	b^1 ♂ − q − − −(6e)	... ...	... ...
	b^2 ♂ 00 p − − −(alb.)	b^3 ♂ 100xq58.57.55(6e)	b^4 ♀ 100xp15.15.10(6f)	b^5 ♀ 00 p − − −(alb.)	... ...	... ...
	b^6 ♀ − q − − −(6e)	b^7 − q − − − −	c^1 ♂ 00 p − − −(alb.)	c^2 ♂ 100rq58.58.55(6e)	... ...	... ...
	c^3 ♂ 100xq56.58.55(6e)	c^4 ♀ 90 q58.57. 2(5e)	c^5 − p − − − −	c^6 − p − − − −		
2*H*313	a^1 ♂ 00 p − − −(alb.)	a^2 ♀ 35 q58. 1. 1(3e)	b^1 ♂ 00 p − − −(alb.)	b^2 ♀ 00 p − − −(alb.)	1*H*225a^1 ♂ 85 q57.55. 1(4e)	1*H*227a^6 ♀ 25 p58. 1. 1(2e)
	b^3 ♀ 40 q58. 1. 2(3e)	b^4 ♀ 00 p − − −(alb.)				
2*H*314	a^1 ♂ 00 p − − −(alb.)	a^2 ♂ 35 p37. 1. 1(3b)	a^3 ♂ 40 q58. 1. 1(3e)	a^4 ♀ 35 q58. 1. 1(2e)	1*H*109b^1 ♂ 25 q58. 1. 2(2e)	1*H*187b^1 ♀ 55 q58.57. 1(3e)
	b^1 ♂ 00 p − − −(alb.)	b^2 ♂ q − − −(2e)	b^3 ♀ 00 p − − −(alb.)	c^1 ♂ 00 p − − −(alb. w)	... ...	... ...
	c^2 ♀ 70 r42.50. 1(3g)					
2*H*315	a^1 ♀ 90 q58.58. 2(4–5e)	b^1 ♂ 90 q58.58. 1(4e)	b^2 ♂ − q − − −(4e)	b^3 ♀ 85 q58.57. 1(3–4e)	1*H*232a^2 ♂ 65 q58.57. 1(4e)	1*H*270a^2 ♀ 90 q58.55. 2(4–5e)
	b^4 ♀ 85 q58.55. 1(4e)	b^5 ♀ 00 p − − −(alb.)	b^6 ♀ − q − − −(5e)	b^7 − q − − − −	... ...	... ...
	c^1 ♀ 70 q58. 1. 1(3–4e)	c^2 ♀ 85 q58.57. 3(4e)	c^3 ♂ 65 q58.57. 1(3e)	c^4 ♂ − q − − −(4e)	... ...	... ...
	c^5 ♂ 65 q58.58. 1(3e)	... ...				
2*H*317	a^1 65 p26. 1. 1(3–4f*)	a^2 30 p37. 1. 1(2b–f*)	... ...		1*H*102b^1 ♂ 25 p37. 1. 1(1–2b)	1*H*232a^3 ♀ 55 p39.39. 2(3b)
3*H*1	a^1 ♂ 60 q58.57. 1(3e)	a^2 ♂ 50 r42.50. 1(3g)	a^3 ♀ 55 p26.26. 1(3f)	a^4 ♀ 100rp26.26.23(6f)	2*H*114a^1 ♂ 100zq58.58.55(6e)	2*H*124a^1 ♀ 100zq58.57.56(6e)
	a^5 ♀ 100xp26.23.18(6f)	a^6 ♀ 45 q58.55. 1(3e)	a^7 ♂ 75 p26.17. 3(3f)	a^8 ♀ 100xq58.57.55(6e)	... ...	... ...
	b^1 ♂ 65dr41.50. 3(4g)	b^2 ♂ 55 q58. 1. 1(4e)	b^3 ♂ 100xq57.57.56(5–6e)	b^4 ♂ 100xq58.58.56(6e)	... ...	... ...
	b^5 ♀ 45 q55.56. 3(3e)	b^6 ♀ 60d.r41.41. 3(4g)	b^7 ♀ q (5–6e)	b^8 q	... ...	... ...
	b^9 q	b^{10} p	c^1 ♂ 100rp22.22.17(6f)	c^2 ♂ 70 q58.58. 2(4e)	... ...	... ...
	c^3 ♂ 100rr41.41.50(6g)	c^4 ♂ 100xr 41.41.50(6g) or q	c^5 ♂ 100rq58.58.55(6e)	c^6 ♀ 65 q58.55. 2(4e)		
	c^7 ♀ 100xq58.58.56(6e)	c^8 − q	c^9 q	c^{10} q	... ...	... ...
	c^{11} − p − − − −	c^{12} p −			... ...	... ...
3*H*2	a^1 ♂ 00 p − − −(alb.)	a^2 ♂ 85 q58.55. 2(4e)	a^3 ♂ 100rq57.57.55(5e)	a^4 ♂ 100xq58.58.55(5–6e)	2*H*155a^1 ♂ 100rq58.57.55(5–6e)	2*H*156a^1 ♀ 100rq58.56.56(6e)
	a^5 ♀ 00 p − − −(alb.)	a^6 ♀ 75 q55. 1. 1(4e)	a^7 ♀ 100rq56.57.55(5–6e)	b^1 ♂ 00 p − − −(alb.)	... ...	... ...
	b^2 ♂ 00 p − − −(alb. w)	b^3 ♂ 100xp22.17.18(6f)	b^4 ♂ 100xq57.56.55(5–6e)	b^5 ♀ 100xp22.15.18(6f)	... ...	... ...
	c^1 ♂ 100rq58.58.55(6e)	c^2 ♂ 00 p − − −(alb.)	c^3 ♀ 00 p − − −(alb.)			
3*H*3	a^1 ♂ 55 q58.56. 1(4ew)	a^2 ♂ 60 p23. 1. 1(3f)	a^3 ♀ 100xq58.58.55(5–6e)	a^4 ♂ 100rq58.58.55(6e)	2*H*156a^2 ♂ 100rq56.56.56(6e)	2*H*155a^2 ♀ 100rq58.57.56(6e)
	a^5 ♀ 100rq56.56.55(6e)	a^6 ♀ 00 p − − −(alb.)	a^7 ♀ 00 p − − −(alb.)	a^8 ♀ 100xp15.15.15(6f)	... ...	... ...
	a^9 p	b^1 ♂ 00 p − − −(alb.)	b^2 ♂ 75 q58.55.55(4e)	b^3 ♂ − q − − −(4e)	... ...	... ...
	b^4 ♂ 100rq58.57.55(6e)	b^5 ♂ 100xq58.57.55(6e)	b^6 ♂ 100xq57.57.56(5e)	b^7 ♂ 100xq58.58.55(6e)	... ...	... ...
	b^8 ♀ 100yp23.23.18(6fw)	b^9 ♀ 100xq58.58.55(6e)	c^1 ♂ − p − − −(6f)	c^2 ♂ 100xq57.58.56(6e)	... ...	... ...
	c^3 ♂ 00 p − − −(alb. w)	c^4 ♂ 00 p − − −(alb.)	c^5 ♀ 60 p26.22. 2(3f)	c^6 ♀ 00 p − − −(alb.)	... ...	... ...
	c^7 ♀ 00 p − − −(alb.)	c^8 − q − − −(6e)	c^9 − p − −	c^{10} p		
3*H*4	a^1 ♂ − p − − −(6f)	a^2 ♂ − q − − −(6e)	a^3 ♀ 100rp23.22.18(6f)	a^4 ♀ 100rp23.23.17(6f)	2*H*156a^3 ♂ 100xq58.57.55(6e)	2*H*155a^3 ♀ 100rq58.58.55(6e)
	a^5 ♀ 100xq57.57.56(6e)	a^6 ♀ 100xq58.57.55(6e)	b^1 ♂ q − − −(5e)	b^2 ♂ 100xq58.58.55(6e)	... ...	... ...
	b^3 ♂ 100rq58.58.55(6e)	b^4 ♂ 00 p − − −(alb.)	b^5 ♀ 85 q57.57. 2(4e)	b^6 ♀ 00 p − − −(alb.)	... ...	... ...
	b^7 − p	c^1 ♂ 00 p − − −(alb.)	c^2 ♂ 00 p − − −(alb.)	c^3 ♂ 100xp23.22.17(5e–f)	... ...	... ...
	c^4 ♂ 45 q58.58. 1(2–3e)	c^5 ♂ 95 q58.55. 2(5e)	c^6 − p − − − −	c^7 ♀ q − − −(6e)	... ...	... ...
	c^8 ♀ 100rq58.57.55(6e)	c^9 ♀ 80 q58.57. 2(4e)				
3*H*5	a^1 ♂ 100xq41.41.50(6g)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♂ 100xq57.57.55(6e)	a^4 ♂ 100rq58.58.55(6e)	2*H*49a^1 ♂ − q − − −(6e)	2*H*108a^1 ♀ 100xq58.57.56(6e)
	a^5 ♀ 00 p − − −(alb.)	a^6 ♀ 00 p − − −(alb.)	b^1 ♂ 00 p − − −(alb. w)	b^2 ♂ 100rq41.50.26(6gw)	... ...	... ...
	b^3 ♂ 100rq58.58.55(6e)	b^4 ♂ − q − − −(6e)	b^5 ♀ 75 r41.41. 3(4gw)	b^6 ♀ 90 q58.58. 2(4e)	... ...	... ...
	b^7 ♀ 100rq56.57.55(4ew)	c^1 ♀ 100xq58.58.55(5e)	c^2 ♂ − q − − −(6e)	c^3 ♂ 00 p − − −(alb.)	... ...	... ...
	c^4 ♀ − q − − −(5g)	c^5 ♀ 100xq57.57.56(6e)			... ...	... ...
3*H*6	a^1 ♂ 00 p − − −(alb.)	a^2 ♂ 100yq58.57. 2(5e)	a^3 ♂ 100xq57.57.56(5–6e)	a^4 ♀ 100yq58.58.55(5e)	2*H*96a^1 ♂ 100xq58.57.55(6e)	2*H*119a^1 ♀ 100xq58.58.55(6e)

W. F. R. WELDON'S *Records of Mice Matings* (continued).

MATING	OFFSPRING				PARENTS	
					Father	Mother
3H6	b^1 ♀ 100xq58.58.55(6e)	b^2 ♀ 100xq58.58.55(5–6e)	b^3 ♀ 100xq56.57.55(5–6e)	b^4 ♂ 100xq58.57.55(5–6e)	... ...	... ...
	b^5 ♂ 100xq57.57.56(5e)	b^6 ♂ 00 p – – – (alb.)	b^7 – q – – – –	c^1 ♂ 100xq58.58.55(5e)	... ...	... ...
	c^2 ♂ 100xq58.57.55(5e)	c^3 ♂ 100xq58.58.55(5e)	c^4 ♀ – q – – – (5e)	c^5 ♀ 00 p – – – (alb.)	... ...	... ...
	c^6 – p – – – –	... ...	... ...	... ...	... ...	... ...
3H7	a^1 ♂ – q – – – (6e)	a^2 ♀ 35 q61. 1. 3(3e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq58.58.55(6e)	2H96a^1 ♂ 100xq58.57.55(6e)	2H119a^2 ♀ 100xq58.58.55(6e)
	a^5 ♀ 100xq58.58.55(6e)	b^1 ♂ – q – – – (5–6e)	b^2 ♂ 100xq58.58.55(5–6e)	b^3 ♂ – q – – – (4e)	... ...	... ...
	b^4 ♂ 100xp23.23.17(6f)	b^5 ♀ 100xq58.57.55(5e)	b^6 ♀ 90 p26.23. 4(5f)	b^7 – q – – – –	... ...	... ...
3H8	a^1 ♂ 00 p – – – (alb.)	a^2 ♀ 35 r41.41. 3(2–3g)	a^3 ♀ 100xq57.58.55(5e)	a^4 – q – – – –	2H49a^2 ♂ 100xq58.57.55(6e)	2H123a^1 ♀ 100xq58.58.56(6e)
	a^5 – q – – – –	... ...	... ...	... ...	... ...	... ...
3H9	a^1 ♂ – p – – – (6f)	a^2 ♂ 100xq58.58.55(5e)	a^3 ♂ 100xq58.58.55(6e)	a^4 ♀ 00 p – – – (alb.)	2H102a^1 ♂ 100xq58.57.55(6e)	2H80a^1 ♀ 100xq58.57.56(6e)
	a^5 ♀ – q – – – (4e)	b^1 ♂ 00 p – – – (alb.)	b^2 ♂ 00 p – – – (alb.)	b^3 ♂ 00 p – – – (alb.)	... ...	... ...
	b^4 ♂ 40 q58. 1. 3(3e)	b^5 ♀ 00 p – – – (alb.)	b^6 ♀ 00 p – – – (alb.)	b^7 – p – – – –	... ...	... ...
	c^1 ♀ 100xq58.58.56(6e)	c^2 ♀ 100xq58.58.55(5e)	... ...	... ...	... ...	... ...
3H10	a^1 100xq41.41.50(6f)	a^2 100xq41.41.50(6gw)	a^3 – q – – – (6e)	a^4 – q – – – (3e)	2H108a^2 ♂ 100xq58.56.56(6ew)	2H49a^3 ♀ 100xq58.57.55(6e)
	a^5 00 p – – – (alb.)	a^6 00 p – – – (alb. w)	a^7 – q – – – –	b^1 00 p – – – (alb. w)	... ...	... ...
	b^2 00 p – – – (alb.)	b^3 100xq58.58.55(6ew)	b^4 00 p – – – (alb. w)	b^5 100xq58.58.55(5e)	... ...	... ...
	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ 100xq41.41.26(6g)	c^3 ♀ 100xq58.58.54(6e)	c^4 ♀ – q – – – (3e)	... ...	... ...
	c^5 ♀ 00 p – – – (alb.)	c^6 – p – – – –	... ...	... ...	... ...	... ...
3H11	a^1 ♂ 100xq57.57.55(5–6e)	a^2 ♀ 100xp23.22.17(5f)	a^3 ♀ 00 p – – – (alb.)	a^4 ♀ 100xq58.58.55(6e)	2H117a^1 ♂ – q – – – (6e)	2H127a^1 ♀ 100xq58.57.55(6e)
	a^5 ♀ 100xq57.57.56(5–6e)	a^6 ♀ 100xq58.58.55(5–6e)	a^7 ♀ 100xq57.57.56(5–6e)	a^8 – p – – – –	... ...	... ...
	b^1 ♂ 100xp22.22.15(6f)	b^2 ♀ 00 p – – – (alb.)	... ...	... ...	... ...	... ...
3H12	a^1 100xq58.58.55 –	a^2 85 q58.55. 2 –	a^3 15 p23. 1. 1 –	a^4 40 p23.18. 2 –	2H127a^2 ♂ 100xq56.55.55(5–6d–e)	2H103a^1 ♀ – q – – – (4–5e)
	a^5 85 q58.57. 2 –	a^6 – q – – – –	a^7 – q – – – –	a^8 – p – – – –	... ...	... ...
	a^9 – p – – – –	a^{10} – p – – – –	b^1 – q – – – –	b^2 – p – – – –	... ...	... ...
	c^1 – q – – – –	c^2 – q – – – –	c^3 – q – – – –	c^4 – p – – – –	... ...	... ...
	c^5 – p – – – –	... ...	... ...	... ...	... ...	... ...
3H13	a^1 ♂ 75 q58.55. 2(3e)	a^2 ♀ 80 r41.50. 1(4g)	a^3 ♀ 90 p37.19. 1(5b)	a^4 ♀ 60 q58.58. 1(3–4e)	2H124a^2 ♂ 85 q58.55. 2(4e)	2H96a^2 ♀ 50 q58.58. 2(3e)
	b^1 ♂ 60 q58.55. 2(3e)	b^2 ♀ 70 q58. 1. 1(4e)	b^3 ♀ – r – – – (4g)	b^4 ♀ 00 p – – – (alb.)	... ...	... ...
	b^5 – p – – – –	... ...	... ...	... ...	... ...	... ...
3H14	a^1 ♂ 00 p – – – (alb.)	a^2 ♂ 00 p – – – (alb.)	a^3 ♀ 00 p – – – (alb.)	a^4 ♂ 70 q45. 1. 2(4ch)	2H152a^2 ♂ 65 q55. 1. 2(3–4e)	2H144a^1 ♀ 60 q58.56. 1(3e)
	a^5 ♀ – q – – – (5ch)	b^1 ♂ 65 q58.58. 2(4e)	b^2 ♂ 00 p – – – (alb.)	b^3 ♀ 50 q58. 1. 2(3–4e)	... ...	... ...
	b^4 ♀ 65 q57. 1. 1(4ch)	b^5 ♀ 00 p – – – (alb.)	b^6 ♀ 00 p – – – (alb.)	c^1 ♂ 40 q45. 1. 2(3ch)	... ...	... ...
	c^2 – p – – – –	c^3 – q – – – –	c^4 – q – – – –	c^5 – q – – – –	... ...	... ...
	c^6 – q – – – –	c^7 – q – – – –	c^8 – q – – – –	... ...	... ...	... ...
3H15	a^1 ♂ 00 p – – – (alb.)	a^2 ♂ 00 p – – – (alb.)	a^3 ♂ 35 q58.55. 2(2–3e)	a^4 ♂ 65 q58. 1. 2(3e)	2H144a^2 ♂ 80 q58.57. 1(4e)	2H111a^1 ♀ 60 q58. 1. 3(4e)
	a^5 ♂ 55 q58.58. 3(3–4e)	a^6 ♂ 65 q57.55. 3(4e)	a^7 ♀ 55 q58. 1. 3(3–4e)	a^8 ♀ 65 q58.58. 2(4e)	... ...	... ...
	b^1 ♂ 75 q58.58. 1(4e)	b^2 ♂ 75 q58.55. 4(4e)	b^3 ♂ – q – – – (4e)	b^4 ♂ 00 p – – – (alb.)	... ...	... ...
	b^5 ♀ 75 q58.58. 2(4–5e)	b^6 ♀ 00 p – – – (alb.)	c^1 ♂ 60 q58. 1. 1(4e)	c^2 ♂ 65 q58.58. 2(4e)	... ...	... ...
	c^3 ♂ 50 q58. 1. 2(3e)	c^4 ♂ 30 q58. 1. 1(2–3e)	c^5 ♀ 75 q58.58. 3(4e)	c^6 ♀ 60 q58. 1. 3(4e)	... ...	... ...
	c^7 ♀ – q – – – (3e)	c^8 – q – – – –	... ...	... ...	... ...	... ...
3H16	a^1 ♂ 80 q58.57. 2(4e)	a^2 ♂ 70 q58.58. 2(4e)	a^3 ♀ 65 p26. 1. 2(3f)	a^4 ♀ 40 p26.17. 1(3f)	2H103a^2 ♂ 70 q58.55. 1(4e)	2H108a^3 ♀ 55 q58.55. 3(3–4e)
	a^5 ♀ 40 q61. 1. 3(3e)	a^6 ♀ 80 q57.57. 1(4e)	a^7 – p – – – –	b^1 ♀ 85 q56.55. 3(4–5e)	... ...	... ...
	b^2 ♀ 65 q56.58. 1(4e)	b^3 ♀ 00 p – – – (alb.)	b^4 – q – – – –	... ...	... ...	... ...
3H17	a^1 ♀ 85 q58.58. 4(4–5e)	... ...	... ...	... ...	2H152a^1 ♂ 60 q58. 1. 1(2–3e)	2H115a^1 ♀ – q – – – (4e)
3H19	a^1 ♀ 60 p39.39. 3(3b)	b^1 ♀ 100xp39.39.34(6b)	b^2 ♀ – p – – – (5b)	b^3 ♀ – p – – – (5b)	2H116a^1 ♂ 100zp39.35.28(5b)	2H12a^3 ♀ 100zp39.39.34(5b–c)

3H20	a^1 ♂ 75 p36.35. 4(5b)	a^2 ♂ 80 p36.36. 4(5b)	a^3 ♀ – p – – –(5b)	a^4 ♀ 100zp39.35.28(5b)	2H93b^1 ♂ 100zp35.35.28(5b)	2H116a^2 ♀ 100xp35.35.30(5–6b)
	a^5 ♀ 100zp35.35.28(6b)	a^6 – p – – – –	a^7 – p – – – –	... ...	... ...	... ...
3H21	a^1 ♂ 80 p39.35. 2(5b)	a^2 ♂ 80 p39.35. 2(5b)	a^3 ♀ 75 p39.39. 4(5b)	a^4 ♀ 80 p39.39. 4(5b)	2H116a^3 ♂ – p – – –(5b)	2H117a^2 ♀ 80 p39.39. 2(5b)
	a^5 ♀ 100zp39.39.28(5b)	a^6 ♀ 100zp39.39.30(5–6b)	a^7 – p – – – –	a^8 – p – – – –	... ...	... ...
	a^9 – p – – – –	... ...	... ...	... ...	... ...	... ...
3H24	a^1 ♀ – p – – –(2bw)	a^2 ♀ 35 p24. 1. 3(3f)	a^3 – p – – – –	a^4 – p – – – –	2H85a^3 ♂ 30 p37. 1. 1(2–3b)	2H137a^1 ♀ 35 p37. 1. 1(2b)
3H25	a^1 ♂ – p – – –(2b)	a^2 ♂ 25 p37. 1. 3(2b)	a^3 ♂ 40 p37. 1. 1(2b)	a^4 ♂ 50 p37. 1. 3(3b)	2H85a^3 ♂ 30 p37. 1. 1(2–3b)	2H137a^2 ♀ 55 p39.33. 1(3b)
	a^5 ♂ 55 p37.35.30(3b)	a^6 ♀ 30 p19. 1. 1(2b)	a^7 ♀ – p – – –(2b)	a^8 ♀ 45 p39. 1. 4(2b)	... ...	... ...
	b^1 ♂ 45 p39. 1. 3(3b)	b^2 ♂ 25 p38. 1. 2(3b)	b^3 ♂ 70 p37. 1. 1(4b)	b^4 ♂ – p – – –(1b)	... ...	... ...
	b^5 ♂ 25 p19. 1. 1(2b)	b^6 ♀ – p – – –(3b)	b^7 ♀ 40 p39.39.28(3b)	b^8 ♀ 25 p37. 1. 3(2b)	... ...	... ...
	b^9 ♀ 10 p35. 1. 2(1b)	c^1 ♂ 35 p37. 1. 4(2b)	c^2 ♀ 25 p39. 1. 2(1–2b)	... ...	... ...	... ...
3H26	a^1 ♂ 15 p37. 1. 4(2b)	a^2 ♂ 20 p37. 1. 2(2b)	a^3 ♂ 30 p39. 1. 1(3b)	... ...	2H85a^4 ♂ 45 p37. 1. 1(2–3b)	2H55a^1 ♀ 25 p39. 1. 4(1–2a–b)
3H27	a^1 ♂ 25 p36. 1. 2(2b)	a^2 ♀ 30 p39. 1. 2(2b)	b^1 ♂ 30 p39. 1. 1(2b)	b^2 ♂ 20 p37. 1. 1(2b)	2H85a^4 ♂ 45 p37. 1. 1(2–3b)	2H55a^2 ♀ 35 p39.39. 3(2a–b)
3H28	a^1 ♀ 45 p39.39. 3(3b)	a^2 ♀ 15 p39. 1. 2(1b)	a^3 – p – – – –	a^4 – p – – – –	2H55a^3 ♂ 10 p37. 1. 1(1a–b)	2H15a^4 ♀ 40 p39.35. 1(3b)
	a^5 – p – – – –	b^1 – p – – – –	b^2 – p – – – –	b^3 – p – – – –	... ...	... ...
3H29	a^1 ♂ 35 p39. 1. 1(2b)	a^2 ♂ 70 p37.39. 3(3–4b)	a^3 ♂ – p – – –(3–4f)	a^4 ♀ 40 p37.39. 3(2–3b)	2H55a^3 ♂ 10 p37. 1. 1(1a–b)	2H15a^1 ♀ 80 p36. 1. 3(4b)
	b^1 ♂ 40 p36.36. 1(3b)	b^2 ♂ 55 p39.39. 1(4b)	b^3 ♀ 50 p39.39. 3(3b)	b^4 ♀ 55 p37.35. 3(3b)	... ...	... ...
	b^5 ♀ 40 p39. 1. 1(2b)	c^1 ♂ 20 p23. 1. 2(2f)	c^2 ♀ 70 p26. 1. 2(4f)	c^3 ♀ 60 p39. 1. 2(4b)	... ...	... ...
	c^4 ♀ 45 p37. 1. 2(3b)	c^5 ♀ – p – – –(2–3b)	... ...	... ...	... ...	... ...
3H30	a^1 ♂ 45 p23. 1. 2(2f)	a^2 ♂ 40 p40. 1. 2(2b)	a^3 ♀ 55 p35.35. 2(2b)	... ...	2H85a^5 ♂ 30 p36. 1. 2(2–3b)	2H11a^2 ♀ – p – – –(2–3b)
3H31	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ – q – – –(6c)	a^3 ♂ 100xq51.46.47(6c)	a^4 ♂ 85 p36.36. 1(4b)	2H13a^1 ♂ 100xp26.23.18(6f)	Z139 ♀ 00 p – – –(A1)
	a^5 ♀ 100xq49.48.46(6c)	a^6 ♀ 100xq49.48.46(6c)	a^7 ♀ 80 q56.57. 1(4e)	a^8 ♀ 100xp39.39.38(6b)	... ...	... ...
	b^1 ♀ 100xp10.10.10(6f)	b^2 ♀ – q – – –(3e)	b^3 ♀ 75 q49. 1. 1(4–5c)	b^4 ♀ 70 q49.46. 4(4–5c)	... ...	... ...
	b^5 ♂ 100xp39.35.34(6b)	b^6 ♂ 100zq49.48.46(6c)	b^7 ♂ 100zq49.48.46(6c)	b^8 ♂ 100xq49.48.46(6c)	... ...	... ...
	c^1 ♂ – q – – –(6c)	c^2 ♂ 95 q49.47. 2(5c)	c^3 ♂ 100zq49.48.46(6c)	c^4 ♂ 100zq49.48.46(6c)	... ...	... ...
	c^5 ♂ 100xp39.35.30(6b)	c^6 ♂ 90 q58.55. 2(4e)	c^7 ♀ 100xp17.17.18(6f)	c^8 ♀ 100xq58.58.55(6e)	... ...	... ...
3H33	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♂ 100xq58.57.55(6e)	a^4 ♂ 100xq58.57.55(6e)	2H13a^3 ♂ 100xp22.23.17(6f)	Z173 ♀ 00 p – – –(A1)
	a^5 ♀ 100xq58.58.57(6e)	a^6 ♀ 100xq58.58.55(6e)	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ 85 q58.57. 1(4e)	... ...	... ...
	b^3 ♂ 60 q58.57. 1(4e)	b^4 ♀ 70 q58.57. 2(4e)	b^5 ♀ 80 q58.56. 1(4e)	b^6 ♀ 100xq58.58.55(6e)	... ...	... ...
	b^7 ♀ 100xq58.58.55(6e)	b^8 ♀ 100xq58.58.55(6e)	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ 100xq58.58.55(6e)	... ...	... ...
	c^3 ♂ 100xq58.58.55(6e)	c^4 ♀ 100xq58.58.55(6e)	c^5 ♀ 100xq58.58.55(6e)	c^6 ♀ 100xq58.58.55(6e)	... ...	... ...
	c^7 ♂ 100xq57.57.56(6e)	c^8 ♂ 100xq58.58.56(6e)	... ...	... ...	... ...	... ...
3H37	a^1 ♂ – p – – –(5b)	a^2 ♂ 85 p35.35. 4(5b)	a^3 ♂ 80 p36.35. 3(5b)	a^4 ♀ 100zp35.33.28(5a)	2H116a^3 ♂ – p – – –(5b)	1H269a^2 ♀ 100zp33.30.30(5a–b)
	a^5 ♀ 80 p39.39.28(5b)	a^6 ♀ 80 p35.35.28(5b)	... ...	... ...	... ...	... ...
3H38	a^1 85 q58.55. 4(4e)	a^2 ♂ 75 q61.58. 2(4e)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ 00 p – – –(alb.)	2H144a^2 ♂ 80 q58.57. 1(4e)	2H80a^2 ♀ 60 q58. 1. 1(3–4e)
	a^5 ♀ 00 p – – –(alb.)	b^1 ♂ 00 p – – –(alb.)	b^2 ♂ 00 p – – –(alb.)	b^3 ♀ 75 q58.57. 3(4e)	... ...	... ...
	b^4 ♀ 75 q58.57. 2(4e)	b^5 ♀ 85 q57.55. 4(4e)	b^6 ♀ 80 q57.55. 6(4e)	b^7 ♀ – q – – –(4e)	... ...	... ...
	c^1 ♂ – q – – –(3e)	c^2 ♂ 40 q58. 1. 1(3e)	c^3 ♂ 90 q58.55. 1(5e)	c^4 ♂ 00 p – – –(alb.)	... ...	... ...
	c^5 ♀ 00 p – – –(alb.)	c^6 ♀ 00 p – – –(alb.)	c^7 – p – – – –	... ...	... ...	... ...
3H39	a^1 ♂ – p – – –(5b)	a^2 ♂ – p – – –(2b)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ 00 p – – –(alb.)	2H127a^2 ♂ 100xq56.55.55(5–6d–e)	1H115b^2 ♀ – q – – –(4e)
	b^1 ♂ – q – – –(1e)	b^2 ♂ 00 p – – –(alb.)	b^3 ♀ 00 p – – –(alb.)	... ...	... ...	... ...
3H40	a^1 ♂ 70 p50.50. 1(4f)	a^2 ♂ – q – – –(4e)	a^3 ♀ 70 q58.57. 1(4e)	a^4 ♀ 90 q58.58. 1(5e)	2H124a^2 ♂ 85 q58.55. 2(4e)	1H231a^7 ♀ 85 q57.55. 1(4–5e)
	a^5 85 p23.23. 2(5f)	b^1 ♂ 90 q58.55. 2(4e)	b^2 ♂ 80 q58.58.55(4–5e)	b^3 ♂ 95 q58.55. 2(4–5e)	... ...	... ...
	b^4 ♀ 85 q57.55. 1(4e)	b^5 ♀ 90 q58.45. 1(4e)	b^6 ♀ 85 q58.56. 2(4–5e)	b^7 ♀ 65 q58.57. 2(3e)	... ...	... ...
3H42	a^1 – p – – – –	a^2 ♂ 55 p23.23. 1(2–3f)	b^1 ♀ 100zp23.35.28(5b)	b^2 ♀ 100xp15.15.17(6f)	2H93b^1 ♂ 100zp35.35.28(5b)	1H121b^2 ♀ 75 p39.34. 1(4a–b)
	b^3 – p – – – –	c^1 ♂ 50 p35. 1. 3(3–4b)	c^2 – p – – – –	... ...	... ...	... ...
3H44	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ – q – – –(6e)	a^3 ♂ – q – – –(6e)	a^4 ♀ 100xq58.58.55(6e)	2H301a^1 ♂ 100xq58.58.55(6e)	2H303a^4 ♀ 100xq58.57.55(6e)
	a^5 ♀ 100xq58.58.57(6e)	b^1 ♂ 100xq58.58.57(6e)	b^2 ♂ 100xq58.58.55(6e)	b^3 ♀ 100xq58.58.55(6e)	... ...	... ...
	b^4 ♀ 100xq58.58.57(6e)	b^5 – q – – – –	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ – q – – –(6e)	... ...	... ...
	c^3 ♂ 100xq58.58.55(6e)	c^4 ♀ – q – – –(6e)	c^5 ♀ 100xq58.58.55(6e)	c^6 – p – – – –	... ...	... ...

Mating	Offspring				Parents	
					Father	Mother
3H6	b^1 ♀ 100xq58.58.55(6e)	b^2 ♀ 100xq58.58.55(5–6e)	b^3 ♀ 100xq56.57.55(5–6e)	b^4 ♂ 100xq58.57.55(5–6e)		
	b^5 ♂ 100xq57.57.56(5e)	b^6 ♂ 00 p– – –(alb.)	b^7 – q– – – –	c^1 ♂ 100xq58.58.55(5e)		
	c^2 ♂ 100xq58.57.55(5e)	c^3 ♂ 100xq58.58.55(5e)	c^4 ♀ – q– – –(5e)	c^5 ♀ 00 p– – –(alb.)		
	c^6 – p– – – –					
3H7	a^1 ♂ – q– – –(6e)	a^2 ♀ 35 q61. 1. 3(3e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq58.58.55(6e)	2H96a^1 ♂ 100xq58.57.55(6e)	2H119a^2 ♀ 100xq58.58.55(6e)
	a^6 ♀ 100xq58.58.55(6e)	b^1 ♂ – q– – –(5–6e)	b^2 ♂ 100xq58.58.55(5–6e)	b^3 ♂ – q– – –(4e)		
	b^4 ♂ 100xp23.23.17(6f)	b^5 ♀ 100xq58.57.55(5e)	b^6 ♀ 00 p26.23. 4(5f)	b^7 – q– – – –		
3H8	a^1 ♂ 00 p– – –(alb.)	a^2 ♀ 35 r41.41. 3(2–3g)	a^3 ♀ 100xq57.58.55(5e)	a^4 – q– – – –	2H49a^2 ♂ 100xq58.57.55(6e)	2H123a^1 ♀ 100xq58.58.56(6e)
	a^5 – q– – – –					
3H9	a^1 ♂ – p– – –(6f)	a^2 ♂ 100xq58.58.55(5e)	a^3 ♂ 100xq58.58.55(6e)	a^4 ♀ 00 p– – –(alb.)	2H102a^1 ♂ 100xq58.57.55(6e)	2H80a^1 ♀ 100xq58.57.56(6e)
	a^5 ♀ – q– – –(4e)	b^1 ♂ 00 p– – –(alb.)	b^2 ♂ 00 p– – –(alb.)	b^3 ♂ 00 p– – –(alb.)		
	b^4 ♂ 40 q58. 1. 3(3e)	b^5 ♀ 00 p– – –(alb.)	b^6 ♀ 00 p– – –(alb.)	b^7 – p– – – –		
	c^1 ♀ 100xq58.58.56(6e)	c^2 ♀ 100xq58.58.55(5e)				
3H10	a^1 100xq41.41.50(6g)	a^2 100xq41.41.50(6gw)	a^3 – q– – –(6e)	a^4 – q– – –(3e)	2H108a^2 ♂ 100xq58.56.56(6ew)	2H49a^3 ♀ 100xq58.57.55(6e)
	a^5 00 p– – –(alb.)	a^6 00 p– – –(alb. w)	a^7 – q– – – –	b^1 00 p– – –(alb. w)		
	b^2 00 p– – –(alb.)	b^3 100xq58.58.55(6ew)	b^4 00 p– – –(alb. w)	b^5 100xq58.58.55(5e)		
	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ 100xq41.41.26(6g)	c^3 ♀ 100xq58.58.54(6e)	c^4 ♀ – q– – –(3e)		
	c^5 ♀ 00 p– – –(alb.)	c^6 – p– – – –				
3H11	a^1 ♂ 100xq57.57.55(5–6e)	a^2 ♀ 100xp23.22.17(5f)	a^3 ♀ 00 p– – –(alb.)	a^4 ♀ 100xq58.58.55(6e)	2H117a^1 ♂ – q– – –(6e)	2H127a^1 ♀ 100xq58.57.55(6e)
	a^5 ♀ 100xq57.57.56(5–6e)	a^6 ♀ 100xq58.58.55(5–6e)	a^7 ♀ 100xq57.57.56(5–6e)	a^8 – p– – – –		
	b^1 ♂ 100xp22.22.15(6f)	b^2 ♀ 00 p– – –(alb.)				
3H12	a^1 100xq58.58.55	a^2 85 q58.55. 2 –	a^3 15 p23. 1. 1	a^4 40 p23.18. 2 –	2H127a^2 ♂ 100xq56.55.55(5–6d–e)	2H103a^1 ♀ – q– – –(4–5e)
	a^5 85 q58.57. 2	a^6 – q– – – –	a^7 – q– – – –	a^8 – p– – – –		
	a^9 – p– – – –	a^{10} – p– – – –	b^1 – q– – – –	b^2 – p– – – –		
	c^1 – q– – – –	c^2 – q– – – –	c^3 – q– – – –	c^4 – p– – – –		
	c^5 – p– – – –					
3H13	a^1 ♂ 75 q58.55. 2(3e)	a^2 ♀ 80 r41.50. 1(4g)	a^3 ♀ 90 p37.19. 1(5b)	a^4 ♀ 60 q58.58. 1(3–4e)	2H124a^2 ♂ 85 q58.55. 2(4e)	2H96a^2 ♀ 50 q58.58. 2(3e)
	b^1 ♂ 60 q58.55. 2(3e)	b^2 ♀ 70 q58. 1. 1(4e)	b^3 ♀ – r– – –(4g)	b^4 ♀ 00 p– – –(alb.)		
	b^5 – p– – – –					
3H14	a^1 ♂ 00 p– – –(alb.)	a^2 ♂ 00 p– – –(alb.)	a^3 ♀ 00 p– – –(alb.)	a^4 ♂ 70 q45. 1. 2(4ch)	2H152a^2 ♂ 65 q55. 1. 2(3–4e)	2H144a^1 ♀ 60 q58.56. 1(3e)
	a^5 ♀ – q– – –(5ch)	b^1 ♂ 65 q58.58. 2(4e)	b^2 ♂ 00 p– – –(alb.)	b^3 ♀ 50 q58. 1. 2(3–4e)		
	b^4 ♀ 65 q57. 1. 1(4ch)	b^5 ♀ 00 p– – –(alb.)	b^6 ♀ 00 p– – –(alb.)	c^1 ♂ 40 q45. 1. 2(3ch)		
	c^2 – p– – – –	c^3 – q– – – –	c^4 – q– – – –	c^5 – q– – – –		
	c^6 – q– – – –	c^7 – q– – – –	c^8 – q– – – –			
3H15	a^1 ♂ 00 p– – –(alb.)	a^2 ♂ 00 p– – –(alb.)	a^3 ♂ 35 q58.55. 2(2–3e)	a^4 ♂ 65 q58. 1. 2(3e)	2H144a^2 ♂ 80 q58.57. 1(4e)	2H111a^1 ♀ 60 q58. 1. 3(4e)
	a^5 ♂ 75 q58.58. 3(3–4e)	a^6 ♂ 65 q57.55. 3(4e)	a^7 ♀ 55 q58. 1. 3(3–4e)	a^8 ♀ 65 q58.58. 2(4e)		
	b^1 ♂ 75 q58.58. 1(4e)	b^2 ♂ 75 q58.55. 4(4e)	b^3 ♂ – q– – –(4e)	b^4 ♂ 00 p– – –(alb.)		
	b^5 ♀ 75 q58.58. 2(4–5e)	b^6 ♀ 00 p– – –(alb.)	c^1 ♂ 60 q58. 1. 1(4e)	c^2 ♂ 65 q58.58. 2(4e)		
	c^3 ♂ 70 q58. 1. 2(3e)	c^4 ♂ 30 q58. 1. 1(2–3e)	c^5 ♀ 75 q58.58. 3(4e)	c^6 ♀ 60 q58. 1. 3(4e)		
	c^7 ♀ – q– – –(3e)	c^8 – q– – – –				
3H16	a^1 ♂ 80 q58.57. 2(4e)	a^2 ♂ 70 q58.58. 2(4e)	a^3 ♀ 65 p26. 1. 2(3f)	a^4 ♀ 40 p26.17. 1(3f)	2H103a^2 ♂ 70 q58.55. 1(4e)	2H108a^3 ♀ 55 q58.55. 3(3–4e)
	a^5 ♀ 40 q61. 1. 3(3e)	a^6 ♀ 80 q57.57. 1(4e)	a^7 – p– – –	b^1 ♀ 85 q56.55. 3(4–5e)		
	b^2 ♀ 65 q56.58. 1(4e)	b^3 ♀ 00 p– – –(alb.)	b^4 – q– –			
3H17	a^1 ♀ 85 q58.58. 4(4–5e)				2H152a^1 ♂ 60 q58. 1. 1(2–3e)	2H115a^1 ♀ – q– – –(4e)
3H19	a^1 ♀ 60 p39.39. 3(3b)	b^1 ♀ 100xp39.39.34(6b)	b^2 ♀ – p– – –(5b)	b^3 ♀ – p– – –(5b)	2H116a^1 ♂ 100zp39.35.28(5b)	2H12a^3 ♀ 100zp39.39.34(5b–c)

Mating	Offspring				Father	Mother
3H20	a^1 ♂ 75 p36.35. 4(5b)	a^2 ♂ 80 p36.36. 4(5b)	a^3 ♀ – p– – –(5b)	a^4 ♀ 100zp39.37.28(5b)	2H93b^1 ♂ 100zp35.35.28(5b)	2H116a^2 ♀ 100zp35.35.30(5–6b)
	a^5 ♀ 100zp35.35.28(6b)	a^6 – p– – –	a^7 – p– – – –			
3H21	a^1 ♂ 80 p34.34. 2(5b)	a^2 ♂ 80 p34.35. 2(5b)	a^3 ♀ 75 p39.39. 4(5b)	a^4 ♀ 80 p39.39. 4(5b)	2H116a^3 ♂ – p– – –(5b)	2H117a^2 ♀ 80 p39.39. 2(5b)
	a^5 ♀ 100zp39.39.28(5b)	a^6 ♀ 100zp34.39.30(5–6b)	a^7 – p– – – –	a^8 – p– – –		
	a^9 – p– – –					
3H24	a^1 ♀ – p– – –(2bw)	a^2 ♀ 35 p24. 1. 3(3f)	a^3 – p– –	a^4 – p– – – –	2H85a^3 ♂ 30 p37. 1. 1(2–3b)	2H137a^1 ♀ 35 p37. 1. 1(2b)
3H25	a^1 ♂ – p– – –(2b)	a^2 ♂ 25 p37. 1. 3(2b)	a^3 ♂ 40 p37. 1. 1(2b)	a^4 ♂ 40 p37. 1. 3(3b)	2H85a^3 ♂ 30 p37. 1. 1(2–3b)	2H137a^2 ♀ 55 p39.33. 1(3b)
	a^5 ♂ 45 p37.35.30(3b)	a^6 ♀ 30 p19. 1. 1(2b)	a^7 ♀ – p– – –(2b)	a^8 ♀ 45 p39. 1. 4(2b)		
	b^1 ♂ 45 p39. 1. 3(3b)	b^2 ♂ 25 p38. 1. 2(3b)	b^3 ♂ 70 p37. 1. 1(4b)	b^4 ♂ – p– – –(4b)		
	b^5 ♂ 25 p19. 1. 1(2b)	b^6 ♀ – p– – –(3b)	b^7 ♀ 40 p39.39.28(3b)	b^8 ♀ 25 p37. 1. 3(2b)		
	b^9 ♀ 10 p35. 1. 2(1b)	c^1 ♂ 35 p37. 1. 4(2b)	c^2 ♀ 25 p39. 1. 2(1–2b)			
3H26	a^1 ♂ 15 p37. 1. 1(2b)	a^2 ♂ 20 p37. 1. 2(2b)	a^3 ♂ 30 p39. 1. 1(3b)		2H85a^4 ♂ 45 p37. 1. 1(2–3b)	2H55a^1 ♀ 25 p39. 1. 4(1–2a–b)
3H27	a^1 ♂ 25 p36. 1. 2(2b)	a^2 ♀ 30 p39. 1. 2(2b)	b^1 ♂ 30 p39. 1. 1(2b)	b^2 ♂ 20 p37. 1. 1(2b)	2H85a^4 ♂ 45 p37. 1. 1(2–3b)	2H55a^2 ♀ 35 p39.39. 3(2a–b)
3H28	a^1 ♀ 45 p39.39. 3(3b)	a^2 ♀ 15 p39. 1. 2(1b)	a^3 – p–	a^4 – p– – –	2H55a^3 ♂ 10 p37. 1. 1(1a–b)	2H15a^4 ♀ 40 p39.35. 1(3b)
	a^5 – p– –	b^1 – p–	b^2 – p– –	b^3 – p– –		
3H29	a^1 ♂ 55 p39. 1. 1(2b)	a^2 ♂ 70 p37.39. 3(3–4b)	a^3 ♂ – p– – –(3–4f)	a^4 ♀ 40 p37.39. 3(2–3b)	2H55a^1 ♂ 10 p37. 1. 1(1a–b)	2H15a^1 ♀ 80 p36. 1. 3(4b)
	b^1 ♂ 40 p36.36. 1(3b)	b^2 ♂ 55 p39.39. 1(4b)	b^3 ♀ 40 p39.39. 3(3b)	b^4 ♀ 55 p37.35. 3(3b)		
	b^5 ♀ 40 p39. 1. 1(2b)	c^1 ♂ 20 p23. 1. 2(2f)	c^2 ♀ 70 p26. 1. 2(4f)	c^3 ♀ 60 p39. 1. 2(4b)		
	c^4 ♀ 45 p37. 1. 2(3b)	c^5 ♀ – p– – –(2–3b)				
3H30	a^1 ♂ 45 p23. 1. 2(2f)	a^2 ♂ 40 p40. 1. 2(2b)	a^3 ♀ 45 p35.35. 2(2b)		2H85a^5 ♂ 30 p36. 1. 2(2–3b)	2H11a^2 ♀ – p– – –(2–3b)
3H31	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ – q– – –(6e)	a^3 ♂ 100xp51.46.47(6e)	a^4 ♂ 85 p36.36. 1(4b)	2H13a^1 ♂ 100xp26.23.18(6f)	Z139 ♀ 00 p– – –(A1)
	a^5 ♀ 100xq49.48.46(6e)	a^6 ♀ 100xq49.48.46(6e)	a^7 ♀ 80 q56.57. 1(4e)	a^8 ♀ 100xp39.39.38(6b)		
	b^1 ♀ 100xp10.10.10(6f)	b^2 ♀ – q– – –(3e)	b^3 ♀ 75 q49. 1. 1(4–5e)	b^4 ♀ 70 q49.46. 4(4–5e)		
	b^5 ♂ 100xp39.35.31(6b)	b^6 ♂ 100zq49.48.46(6e)	b^7 ♂ 100zq49.48.46(6e)	b^8 ♂ 100xq49.48.46(6e)		
	c^1 ♂ – p– – –(6b)	c^2 ♂ 95 q49.47. 2(5e)	c^3 ♂ 100zq49.48.46(6e)	c^4 ♂ 100xq49.48.46(6e)		
	c^5 ♂ 100xp39.35.30(6b)	c^6 ♂ 90 q58.55. 2(4e)	c^7 ♀ 100xp17.17.18(6f)	c^8 ♀ 100xq58.58.55(6e)		
3H33	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♂ 100xq58.57.55(6e)	a^4 ♂ 100xq58.57.55(6e)	2H13a^3 ♂ 100xp22.23.17(6f)	Z173 ♀ 00 p– – –(A1)
	a^5 ♀ 100xq58.58.57(6e)	a^6 ♀ 100xq58.58.55(6e)	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ 85 q58.57. 1(4e)		
	b^3 ♂ 60 q58.57. 1(4e)	b^4 ♀ 70 q58.57. 2(4e)	b^5 ♀ 80 q58.56. 1(4e)	b^6 ♀ 100xq58.58.55(6e)		
	b^7 ♀ 100xq58.58.55(6e)	b^8 ♀ 100xq58.58.55(6e)	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ 100xq58.58.55(6e)		
	c^3 ♂ 100xq58.58.55(6e)	c^4 ♀ 100xq58.58.55(6e)	c^5 ♀ 100xq58.58.55(6e)	c^6 ♀ 100xq58.58.55(6e)		
	c^7 ♂ 100xq57.57.56(6e)	c^8 ♂ 100xq58.58.56(6e)				
3H37	a^1 ♂ – p– – –(5b)	a^2 ♂ 85 p35.35. 4(5b)	a^3 ♂ 80 p36.35. 3(5b)	a^4 ♀ 100zp35.33.28(5a)	2H116a^3 ♂ – p– – –(5b)	1H269a^2 ♀ 100zp33.30.30(5a–b)
	a^5 ♀ 80 p39.39.28(5b)	a^6 ♀ 80 p35.35.28(5b)				
3H38	a^1 85 q58.55. 4(4e)	a^2 ♂ 75 q61.58. 2(4e)	a^3 ♀ 00 p– – –(alb.)	a^4 ♀ 00 p– – –(alb.)	2H144a^2 ♂ 80 q58.57. 1(4e)	2H80a^2 ♀ 60 q58. 1. 1(3–4e)
	a^5 ♀ 00 p– – –(alb.)	b^1 ♂ 00 p– – –(alb.)	b^2 ♂ 00 p– – –(alb.)	b^3 ♀ 75 q58.57. 3(4e)		
	b^4 ♀ 75 q58.57. 2(4e)	b^5 ♀ 85 q57.55. 4(4e)	b^6 ♀ 80 q57.55. 6(4e)	b^7 ♀ – q– – –(4e)		
	c^1 ♂ – q– – –(3e)	c^2 ♂ 40 q58. 1. 1(3e)	c^3 ♂ 90 q58.55. 1(5e)	c^4 ♂ 00 p– – –(alb.)		
	c^5 ♀ 00 p– – –(alb.)	c^6 ♀ 00 p– – –(alb.)	c^7 – p– – – –			
3H39	a^1 ♂ – p– – –(5b)	a^2 ♂ – p– – –(2b)	a^3 ♂ 00 p– – –(alb.)	a^4 ♀ 00 p– – –(alb.)	2H127a^2 ♂ 100xq56.55.55(5–6d–e)	1H115b^2 ♀ – q– – –(4e)
	b^1 ♂ – q– – –(4e)	b^2 ♂ 00 p– – –(alb.)	b^3 ♀ 00 p– – –(alb.)			
3H40	a^1 ♂ 70 p50.50. 1(4f)	a^2 ♂ – q– – –(4e)	a^3 ♀ 70 q58.57. 1(4e)	a^4 ♀ 90 q58.58. 1(5e)	2H124a^2 ♂ 85 q58.55. 2(4e)	1H231a^7 ♀ 85 q57.55. 1(4–5e)
	a^5 85 p23.23. 2(5f)	b^1 ♂ 90 q58.55. 2(4e)	b^2 ♂ 80 q58.58.55(4–5e)	b^3 ♂ 95 q58.55. 2(4–5e)		
	b^4 ♀ 85 q57.55. 1(4e)	b^5 ♀ 90 q58.45. 1(4e)	b^6 ♀ 85 q58.56. 2(4–5e)	b^7 ♀ 65 q58.57. 2(3e)		
3H42	a^1 – p– – – –	a^2 ♂ 55 p23.23. 1(2–3f)	b^1 ♀ 100zp23.35.28(5b)	b^2 ♀ 100xp15.15.17(6f)	2H93b^1 ♂ 100zp35.35.28(5b)	1H121b^2 ♀ 75 p39.34. 1(4a–b)
	b^3 – p– – – –	c^1 ♂ 50 p35. 1. 3(3–4b)	c^2 – p– – – –			
3H44	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ – q– – –(6e)	a^3 ♂ – q– – –(6e)	a^4 ♀ 100xq58.58.55(6e)	2H301a^1 ♂ 100xq58.58.55(6e)	2H303a^4 ♀ 100xq58.57.55(6e)
	a^5 ♀ 100xq58.58.57(6e)	b^1 ♂ 100xq58.58.57(6e)	b^2 ♂ 100xq58.58.55(6e)	b^3 ♀ 100xq58.58.55(6e)		
	b^4 ♀ 100xq58.58.57(6e)	b^5 – q– – – –	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ – q– – –(6e)		
	c^3 ♂ 100xq58.58.55(6e)	c^4 ♀ – q– – –(6e)	c^5 ♀ 100xq58.58.55(6e)	c^6 – p– – – –		

MATING	OFFSPRING				PARENTS	
					Father	Mother
3H44	c^7 – p – – – –	c^8 – p – – – –	...	...	...	...
3H45	a^1 ♂ 00 p – – – (alb. w)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq58.58.55(6e)	2H301a^2 ♂ 100xq 58.57.55(6d–e)	2H300a^2 ♀ 100xq58.57.56(6e)
	a^5 ♀ 100xq58.58.55(6e)	a^6 ♀ 100xp22.22.17(6f)	a^7 – q – – – –	a^8 – p – – – –	...	...
	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ 100xq58.58.55(6e)	b^3 ♂ 100xq58.57.55(6e)	b^4 ♂ 00 p – – – (alb.)	...	...
	b^5 ♀ 100xq58.58.55(6e)	b^6 ♀ 100xq58.58.55(6e)	b^7 ♀ 100xq57.57.55(6e)	c^1 ♂ 100xq58.58.55(6e)	...	...
	c^2 ♀ 100xq58.58.55(6e)	c^3 ♀ 100xp23.15.18(6f)	c^4 ♀ 100xp17.17.15(6f)	c^5 ♀ 00 p – – – (alb. w)	...	...
	c^6 ♂ 00 p – – – (alb.)	...	...	...	...	...
3H46	a^1 ♂ 45 p50. 1. 3(3f)	a^2 ♂ 55 p26. 1. 3(3f)	a^3 ♂ 100xp26.26.23(6f)	a^4 ♀ 85 p26. 1. 1(4–5f)	2H301a^4 ♂ 100xp 26.22.18(4–5f)	2H306a^1 ♀ 100yp26.23. 2(5f)
	a^5 ♀ 100xp50.26.22(6f)	a^6 ♀ 15 p26. 1. 1(1f)	a^7 – p – – – –	a^8 – p – – – –	...	...
	a^9 – p – – – –	b^1 ♂ 100xp23.23.17(6f)	b^2 ♂ 90 p26.23. 2(4–5f)	b^3 ♂ 80 p26. 1. 1(4f)	...	...
	b^4 ♀ 100xp26.26.23(6f)	b^5 – p – – – –	...	...	...	...
3H48	a^1 ♂ 85 p41.41. 1(4f–g)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♂ 100xq58.58.55(6e)	a^4 ♀ 100xq58.58.55(6e)	2H303a^1 ♂ 100xq 58.57.55(6e)	2H301a^5 ♀ 100xq58.57.55(6e)
	a^5 ♀ – q – – – (6e)	...	...	...	...	...
3H49	a^1 ♂ 100xq58.58.55(5e)	a^2 ♀ 90 q58.60. 2(4e)	a^3 ♀ 100xq58.58.55(6e)	a^4 – p – – – –	2H303a^1 ♂ 100xq 58.57.55(6e)	2H301a^6 ♀ 100xq58.57.56(5–6e)
3H50	a^1 ♂ 100xp22.12.17(5f)	a^2 ♂ 100yp23.17. 1(5f)	a^3 ♂ – p – – – (5f)	a^4 ♀ 100xp22.22.15(6f)	2H305a^1 ♂ 100xp 26.23.23(6f)	2H305a^3 ♀ 100zp26.18.17(6f)
	a^5 ♀ 100xp26.26.22(5f)	a^6 ♀ 100xp23.22.17(6f)	a^7 ♀ – p – – – (5f)	a^8 – p – – – –	...	...
	a^9 – p – – – –	b^1 ♂ 100xp23.23.17(5f)	b^2 ♂ 95 p23.17. 1(4–5f)	b^3 ♀ 100xp37.23.23(5f)	...	...
	b^4 ♀ 100xp26.23.23(6f)	b^5 ♀ – p – – – (6f)	b^6 – p – – – –	b^7 – p – – – –	...	...
	b^8 – p – – – –	b^9 – p – – – –	b^{10} – p – – – –	c^1 ♂ 100xp26.23.17(6f)	...	...
	c^2 ♂ 100xp19.19. 9(6b)	c^3 ♀ 100xp26.21.22(5f)	c^4 ♀ 100xp25.25.17(6f)	c^5 ♀ 100xp19.19. 9(5b)	...	...
3H51	a^1 ♂ 100xp19.19. 9(6a)	a^2 ♂ – p – – – (5f)	a^3 ♂ 100xp26.23.17(5f)	a^4 ♂ 100xp26.23.15(5f)	2H305a^1 ♂ 100xp 26.23.23(6f)	2H305a^4 ♀ 100xp22.22.18(6f)
	a^5 ♀ 100xp37.19. 9(5a)	a^6 ♀ 100xp26.26.23(6f)	a^7 – p – – – –	a^8 – p – – – –	...	...
	b^1 ♂ 100xp19.19. 9(6a–b)	b^2 ♂ 35 p50. 1. 1(3f)	b^3 ♀ 100xp26.23.23(5f)	b^4 ♀ 100xp25.25.15(5f)	...	...
	b^5 – p – – – –	b^6 – p – – – –	b^7 – p – – – –	b^8 – p – – – –	...	...
	b^9 – p – – – –	...	...	...	...	...
3H52	a^1 ♂ 45 p37. 1. 1(3b)	a^2 ♀ 100zp39.35. 3(5b)	a^3 ♀ 100zp39.35. 3(5b)	a^4 ♀ 50 p37. 1. 1(3b)	2H310b^2 ♂ 65 p 39. 1. 3(4b)	2H309b^1 ♀ 100zp39.35.30(5b)
	b^1 ♀ – p – – – (4b)	b^2 ♀ 75 p39.39. 4(5b)	b^3 ♂ 45 p40.36. 3(3b)	b^4 ♂ – p – – – (5b)	...	...
	b^5 ♂ 80 p26.23. 2(4b–f)	b^6 ♂ 100xp26.23.23(6f)	c^1 ♂ – p – – – (4b)	c^2 ♂ 45 p39. 1. 2(2b)	...	...
	c^3 ♀ 100zp39.39.28(5b)	c^4 ♀ 100zp39.35. 2(5b)	c^5 ♀ 75 p39.35. 2(4b)	c^6 – p – – – –	...	...
	c^7 – p – – – –	...	...	...	...	...
3H53	a^1 ♂ – p – – – (6b)	a^2 ♂ 100zp39.39.29(5b)	a^3 ♀ 100xp39.35.31(6b)	a^4 ♀ 100xp35.35.30(6b)	2H308a^2 ♂ 100zp 39.35. 9(5–6b)	2H307a^2 ♀ 100zp39.35.30(5b)
	a^5 ♀ 100zp39.35.30(5b)	b^1 ♂ 100xp39.35.30(6b)	b^2 ♂ 100zp35.35.28(6b)	b^3 ♂ 100zp35.35.30(6b)	...	...
	b^4 ♂ 100zp35.35.28(5b)	b^5 ♀ 100zp39.35. 9(5b)	b^6 ♀ 100zp39.35.31(6b)	b^7 – p – – – –	...	...
	b^8 – p – – – –	c^1 ♂ 100zp39.35. 3(5b)	c^2 ♂ 80 p36.34. 4(5b)	c^3 ♀ 100zp39.35.28(6b)	...	...
	c^4 ♀ 100zp39.35.34(6b)	c^5 ♀ 100zp39.35.30(5b)	c^6 ♀ 100zp36.36. 4(5b)	c^7 – p – – – –	...	...
3H54	a^1 ♂ 100xp23.17.17(6f)	a^2 ♂ 100xp39.39.30(6bw)	a^3 ♀ 35 p37. 1. 3(2–3b)	a^4 ♀ 100xp35.35.30(6bw)	3H308a^1 ♂ 100xp 35.35.30(5b)	2H307a^3 ♀ 100xp39.39.36(5b)
	b^1 ♂ 100xp37.19.19(5b)	b^2 ♂ 100xp26.26.26(5b)	b^3 ♂ 30 p37. 1. 3(2b)	b^4 ♀ 100xp15.10.10(5f)	...	...
	c^1 ♂ 35 p36. 1. 1(2b)	c^2 ♂ 35 p37. 1. 1(2b)	c^3 ♀ 100zp35.35. 9(5b)	c^4 ♀ 100yp23.15.10(5f)	...	...
	c^5 – p – – – –	...	...	...	...	...
3H57	a^1 ♂ 100zp39.35.28(5b)	a^2 ♂ 25 p39. 1. 1(2b)	a^3 ♀ 20 p39. 1. 1(2b)	a^4 ♀ 55 p39. 1. 1(3–4b)	2H309a^2 ♂ – p – – – (5b)	2H308a^3 ♀ 100zp39.35. 9(5b)
	a^5 ♀ 80 p39.39. 4(5b)	...	...	...	...	...
3H59	a^1 ♂ 100xq41.41.50(6e–g)	a^2 ♀ 80 r41. 1. 1(4gw)	a^3 ♀ 60 r?41.37. 1(4g)	a^4 ♀ 45 r41. 1. 2(3–4g)	3H1c^4 ♂ 100xr or q 41.41.50(6g)	2H314c^2 ♀ 70 q42.50. 1(3g)
	a^5 ♀ 55 p36. 9. 3(4bw)	a^6 – p – – – –	a^7 – p – – – –	b^1 ♂ 90 r41.41. 1(4g)	...	...
	b^2 ♀ 100xr41.41.50(6g)	b^3 ♀ 100xr41.41.50(5g)	b^4 ♀ 85 r41.50. 1(4g)	b^5 ♀ 100xp19. 6. 6(6b)	...	...

	b^6 - p - - - -	b^7 - q - - - -	c^1 ♂ 100xr41.41.50(5g)	c^2 ♂ - r - - -(3g)	... ...	... ...
	c^3 ♂ 85 p37.34. 3(5b)	c^4 ♀ 80 r41.50. 1(4g)	c^5 ♀ 100xr41.41.50(5–6g)	c^6 ♀ 60 p19. 9. 1(4b)	... ...	... ...
3H60	a^1 ♂ 50 p23.23. 1(3f)	a^2 ♂ 60 p26. 1. 1(4f)	a^3 ♂ 65 p23.15. 1(4f)	a^4 ♀ 100xp22.18.18(6f)	2H302b^1 ♂ 100xp 23.15.15(6f)	2H305b^3 ♀ 100yp23.23.17(6f)
3H61	a^1 ♂ 100xp23.17.18(6f)	a^2 ♀ 100xp23.23.15(5f)	a^3 ♀ 100yp23.15.10(5f)	a^4 ♀ 50 p26. 1. 1(3f)	2H303b^1 ♂ - p - - -(6f)	2H305b^4 ♀ 30 p26. 1. 1(2f)
	a^5 ♀ 40 p50. 1. 1(3f)	a^6 ♀ - p - - -(3f)	a^7 - p - - - -	a^8 - p - - - -	... ...	... ...
	a^9 - p - - - -	a^{10} - p - - - -	a^{11} - p - - - -	b^1 ♂ 50 p26. 1. 3(3f)	... ...	... ...
	b^2 ♀ 55 p26. 1. 2(3f)	b^3 ♀ 60 p26. 1. 3(3f)	b^4 ♀ 55 p50. 1. 1(3f)	b^5 ♀ 100xp26.26.23(6f)	... ...	... ...
	b^6 - p - - - -	... ...	... ...	... ...	... ...	... ...
3H64	a^1 ♂ 100xp15.15.18(6f)	a^2 ♂ - p - - -(5f)	a^3 ♂ - p - - -(6f)	a^4 ♂ 100xp19.19.18(6f)	2H306b^3 ♂ 100xp 23.23.18(5f)	2H312b^4 ♀ 100xp15.15.10(6f)
	a^5 - p - - - -	a^6 - p - - - -	b^1 ♂ - p - - -(6f)	b^2 ♂ 100xp22.17.15(6f)	... ...	... ...
	b^3 ♀ 100xp23.15.10(6f)	b^4 ♀ 100xp22.17.15(6f)	b^5 - p - - - -	b^6 - p - - - -	... ...	... ...
	b^7 - p - - - -	c^1 ♂ - p - - -(5f)	c^2 ♂ 100xp23.15.18(6f)	c^3 ♀ 100xp23.23.17(6f)	... ...	... ...
	c^4 ♀ 100xp13.10.10(6f)	c^5 - p - - - -	c^6 - p - - - -	... ...	... ...	... ...
3H65	a^1 ♂ 100xp23.22.17(6f)	a^2 ♂ 100xp22.22.18(5f)	a^3 ♂ 100yp19. 6. 2(5f)	a^4 ♂ 95 p22.18. 1(5f)	2H306b^2 ♂ 100xp 23.23.15(5f)	2H305a^5 ♀ 100xp23.23.18(6f)
	a^5 ♀ 100yp22.17.10(4–5f)	a^6 - p - - - -	a^7 - p - - - -	b^1 ♂ 100xp22.18.18(5f)	... ...	... ...
	b^2 ♂ 100xp22.22.15(5f)	b^3 ♂ 100xp23.17.17(5f)	c^1 ♂ 100yp23.23.17(6f)	c^2 ♂ 100yp26.23.17(5f)	... ...	... ...
	c^3 ♂ 100yp26.26. 2(5f)	c^4 ♂ 100yp22.18.10(5f)	c^5 ♂ 100xp23.17.15(5f)	... ...	... ...	... ...
3H66	a^1 ♂ 55 p26. 1. 1(3f)	a^2 ♂ - p - - -(2–3f)	a^3 ♂ - p - - -(2f)	a^4 ♂ - p - - -(2f)	2H306b^4 ♂ 25 p 26. 1. 2(2–3f)	2H305a^6 ♀ 25 p23. 1. 1(2f)
	a^5 ♂ 15 p23. 1. 2(1f)	a^6 ♀ 15 p23. 1. 1(1–2f)	a^7 ♀ 30 p25. 1. 2(2f)	b^1 ♂ - p - - -(2f)	... ...	... ...
	b^2 ♂ 30 p26. 1. 1(2–3f)	b^3 ♀ 25 p23. 1. 1(2f)	b^4 ♀ 25 p26. 1. 1(2f)	b^5 ♀ 55 p26.26. 3(3f)	... ...	... ...
	b^6 ♀ 20 p26. 1. 2(1f)	b^7 ♀ 25 p26. 1. 1(1–2f)	b^8 - p - - - -	b^9 - p - - - -	... ...	... ...
	c^1 ♂ - p - - -(1f)	c^2 ♂ - p - - -(1f)	c^3 ♀ 10 p23. 1. 1(1f)	c^4 ♀ 30 p23. 1. 3(2f)	... ...	... ...
	c^5 ♀ 25 p23. 1. 1(2f)	c^6 ♀ 25 p26. 1. 2(2f)	c^7 - p - - - -	c^8 - p - - - -	... ...	... ...
	c^9 - p - - - -	c^{10} - p - - - -	c^{11} - p - - - -	... ...	... ...	... ...
3H67	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ - q - - -(6e)	a^3 ♀ 00 p - - -(alb.)	a^4 ♀ 100xq58.58.55(6e)	2H312c^2 ♂ 100xq 58.58.55(6e)	2H302b^5 ♀ 100xq58.58.55(6e)
	a^5 ♀ 100xq58.58.56(6e)	b^1 ♂ 00 p - - -(alb.)	b^2 ♂ 100xp23.23.17(6f)	b^3 ♂ 100xq58.58.55(6e)	... ...	... ...
	b^4 ♂ 100xq58.58.55(6e)	b^5 ♀ 100xp23.23.23(6f)	b^6 - q - - - -	b^7 - q - - - -	... ...	... ...
	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ 90 q61.52. 6(4e)	c^3 ♀ 100xq58.58.55(6e)	c^4 ♀ 100xp24.24.15(6f)	... ...	... ...
	c^5 - p - - - -	... ...	... ...	... ...	... ...	... ...
3H69	a^1 ♂ 00 p - - -(alb.)	a^2 ♂ 100xp23.23.17(6f)	a^3 ♂ 100xq58.58.57(6e)	a^4 ♂ 100xq58.58.57(6e)	2H312b^3 ♂ 100xq 58.57.55(6e)	2H300b^3 ♀ 100xq58.56.56(6e)
	a^5 - q - - - -	b^1 ♂ 100xq58.58.55(6e)	b^2 ♀ 100xq58.58.57(6e)	b^3 ♀ 00 p - - -(alb.)	... ...	... ...
	b^4 - p - - - -	b^5 - p - - - -	b^6 - q - - - -	b^7 - q - - - -	... ...	... ...
	c^1 ♂ 00 p - - -(alb.)	c^2 ♂ 00 p - - -(alb.)	c^3 ♂ 00 p - - -(alb.)	c^4 ♀ 100xq58.58.55(6e)	... ...	... ...
	c^5 - q - - - -	c^6 - q - - - -	c^7 - q - - - -	... ...	... ...	... ...
3H70	a^1 ♀ 85 q58.58. 2(4e)	a^2 ♀ 100xq58.58.55(6e)	a^3 ♂ 100xq58.58.55(6e)	a^4 ♂ 100xq58.58.55(6e)	2H303b^4 ♂ 100xq 58.58.55(5e)	2H300c^3 ♀ 100xq58.56.56(6e)
	a^5 - q - - - -	a^6 - p - - - -	b^1 ♂ 100xp15.15.18(6f)	b^2 ♂ 100xq57.57.55(5e)	... ...	... ...
	b^3 ♀ 65 q58.58. 1(4e)	b^4 ♀ 100xq58.58.55(6e)	b^5 - p - - - -	b^6 - q - - - -	... ...	... ...
	c^1 ♂ 65 q58. 1. 1(3e)	c^2 ♀ 100xq58.58.56(6e)	c^3 ♀ 100xq58.58.55(6e)	c^4 ♀ 100xq58.58.55(6e)	... ...	... ...
	c^5 - q - - - -	c^6 - q - - - -	... ...	... ...	... ...	... ...
3H71	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♂ - q - - -(6e)	a^4 ♂ - q - - -(6e)	2H303b^5 ♂ 100xq 58.57.55(6e)	2H300c^4 ♀ 100xq57.56.55(6e)
	a^5 ♀ - q - - -(5e)	a^6 - q - - - -	a^7 - q - - - -	b^1 ♂ 100xq58.57.55(6e)	... ...	... ...
	b^2 ♂ 100xq58.58.55(6e)	c^1 ♂ 100xq58.57.55(6e)	c^2 ♂ 100xq58.58.55(6e)	c^3 ♂ 100xq57.57.52(5e)	... ...	... ...
	c^4 ♀ 100xq58.55.55(6e)	c^5 - q - - - -	c^6 - q - - - -	... ...	... ...	... ...
4H1	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♂ 100xq58.57.55(6e)	a^4 ♀ 45 q58. 1. 1(3e)	3H3a^4 ♂ 100xq 58.58.55(6e)	3H1a^8 ♀ 100xq58.57.55(6e)
	a^5 ♀ 80 r41. 1. 1(4g)	a^6 ♀ 100xr41.41.50(6g)	a^7 ♀ 100xr41.41.50(5g)	... ...	... ...	... ...
4H2	a^1 ♂ 70 q58.55. 2(4e)	a^2 ♂ 100xq58.57.56(6e)	a^3 ♂ 100xq58.57.56(6e)	a^4 ♂ 00 p - - -(alb.)	3H9a^3 ♂ 100xq 58.58.55(6e)	3H2a^7 ♀ 100xq56.57.55(5–6e)
	a^5 ♀ 00 p - - -(alb.)	a^6 ♀ 90 q57.55. 1(5e)	a^7 ♀ 100xq58.58.57(6e)	b^1 ♂ 00 p - - -(alb.)	... ...	... ...

Mating	Offspring				Parents	
					Father	Mother
3H44	c^7 – p – – – –	c^8 – p – – – –	... ...	... ...	... ...	... ...
3H45	a^1 ♂ 00 p – (alb. w)	a^2 ♂ 100rq58.57.55(6e)	a^3 ♀ 100rq58.58.55(6e)	a^4 ♀ 100xq58.58.55(6e)	2H301a^2 ♂ 100xq 58.57.55(6d–e)	2H300a^2 ♀ 100xq58.57.56(6e)
	a^5 ♀ 100xq58.58.55(6e)	a^6 ♀ 100xp22.22.17(6f)	a^7 q – –	a^8 p – –	... ...	... ...
	b^1 ♂ 100rq58.58.55(6e)	b^2 ♂ 100rq58.58.55(6e)	b^3 ♂ 100rq58.57.55(6e)	b^4 ♂ 00 p – – (alb.)	... ...	... ...
	b^5 ♀ 100rq58.58.55(6e)	b^6 ♀ 100xq58.58.55(6e)	b^7 ♀ 100rq57.57.55(6e)	c^1 ♂ 100rq58.58.55(6e)	... ...	... ...
	c^2 ♀ 100xq58.58.55(6e)	c^3 ♀ 100rp23.15.18(6f)	c^4 ♀ 100rp17.17.15(6f)	c^5 ♀ 00 p – – (alb. w)	... ...	... ...
	c^6 ♂ 00 p – – – (alb.)	... ...	... ...	... ...	... ...	... ...
3H46	a^1 ♂ 45 p50. 1. 3(3f)	a^2 ♂ 55 p26. 1. 3(3f)	a^3 ♂ 100xp26.26.23(6f)	a^4 ♀ 85 p26. 1. 1(4–5f)	2H301a^1 ♂ 100rp 26.22.18(4–5f)	2H306a^1 ♀ 100yp26.23. 2(5f)
	a^5 ♀ 100rp50.26.22(6f)	a^6 ♀ 15 p26. 1. 1(1f)	a^7 p –	a^8 – p – – – –	... ...	... ...
	a^9 p – – –	b^1 ♂ 100rp23.23.17(6f)	b^2 ♂ 90 p26.23. 2(4–5f)	b^3 ♂ 80 p26. 1. 1(4f)	... ...	... ...
	b^4 ♀ 100rp26.26.23(6f)	b^5 – p –	... ...	... ...	... ...	... ...
3H48	a^1 ♂ 85 p41.41. 1(4f–g)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♂ 100rp58.58.55(6e)	a^4 ♀ 100xq58.58.55(6e)	2H303a^1 ♂ 100rq 58.57.55(6e)	2H301a^6 ♀ 100xq58.57.55(6e
	a^5 ♀ q 6e)	... ...	... ...	... ...	... ...	... ...
3H49	a^1 ♂ 100rq58.58.55(5e)	a^2 ♀ 90 q58.60. 2(4e)	a^3 ♀ 100xq58.58.55(6e)	a^4 – p	2H303a^1 ♂ 100xq 58.57.55(6e)	2H301a^6 ♀ 100xq58.57.56(5–6e)
3H50	a^1 ♂ 100xp22.12.17(5f)	a^2 ♂ 100yp23.17. 1(5f)	a^3 ♂ – p – – – (5f)	a^4 ♀ 100rp22.22.15(6f)	2H305a^1 ♂ 100xp 26.23.23(6f)	2H305a^3 ♀ 100zp26.18.17(6f)
	a^5 ♀ 100xp26.26.22(5f)	a^6 ♀ 100xp23.22.17(6f)	a^7 ♀ – p – – – (5f)	a^8 – p –	... ...	... ...
	a^9 – p	b^1 ♂ 100rp23.23.17(5f)	b^2 ♂ 95 p23.17. 1(4–5f)	b^3 ♀ 100xp37.23.23(5f)	... ...	... ...
	b^4 ♀ 100rp26.23.23(6f)	b^5 ♀ p 6f	b^6 p – –	b^7 – p – – – –	... ...	... ...
	b^8 p – – – –	b^9 – p –	b^{10} p –	c^1 ♂ 100xp26.23.17(6f)	... ...	... ...
	c^2 ♂ 100xp19.19. 9(6b)	c^3 ♀ 100xp26.21.22(5f)	c^4 ♀ 100xp25.25.17(6f)	c^5 ♀ 100xp19.19. 9(5b)	... ...	... ...
3H51	a^1 ♂ 100xp19.19. 9(6a)	a^2 ♂ – p – – – (5f)	a^3 ♂ 100rp26.23.17(5f)	a^4 ♂ 100xp26.23.15(5f)	2H305a^1 ♂ 100xp 26.23.23(6f)	2H305a^4 ♀ 100xp22.22.18(6f)
	a^5 ♀ 100xp37.19. 9(5a)	a^6 ♀ 100xp26.26.23(6f)	a^7 p –	a^8 – p – –	... ...	... ...
	b^1 ♂ 100xp19.19. 9(6a–b)	b^2 ♂ 35 p50. 1. 1(3f)	b^3 ♀ 100xp26.23.23(5f)	b^4 ♀ 100xp25.25.15(5f)	... ...	... ...
	b^5 p – – – –	b^6 p	b^7 p – –	b^8 – p – –	... ...	... ...
	b^9 p		... ...	... ...	... ...	... ...
3H52	a^1 ♂ 45 p37. 1. 1(3b)	a^2 ♀ 100zp39.35. 3(5b)	a^3 ♀ 100zp39.35. 3(5b)	a^4 ♀ 50 p37. 1. 1(3b)	2H310b^3 ♂ 65 p 39. 1. 3(4b)	2H309b^1 ♀ 100zp39.35.30(5b)
	b^1 ♀ – p – – – (4b)	b^2 ♀ 75 p39.39. 4(5b)	b^3 ♂ 45 p40.36. 3(3b)	b^4 ♂ – p 5b	... ...	... ...
	b^5 ♂ 80 p26.23. 2(4b–f)	b^6 ♂ 100xp26.23.23(6f)	c^1 ♂ p – 4b	c^2 ♂ 45 p39. 1. 2(2b)	... ...	... ...
	c^3 ♀ 100zp39.39.28(5b)	c^4 ♀ 100zp39.35. 2(5b)	c^5 ♀ 65 p39.35. 2(4b)	c^6 p – –	... ...	... ...
	c^7 – p – – – –	... ...	... ...	... ...	... ...	... ...
3H53	a^1 ♂ – p – – – (6b)	a^2 ♂ 100zp39.34.29(5b)	a^3 ♀ 100zp39.35.31(6b)	a^4 ♀ 100xp35.35.30(6b)	2H308a^2 ♂ 100zp 39.35. 9(5–6b)	2H307a^2 ♀ 100zp39.35.30(5b)
	a^5 ♀ 100zp39.35.30(5b)	b^1 ♂ 100zp39.35.30(6b)	b^2 ♂ 100zp35.35.28(6b)	b^3 ♂ 100zp35.35.30(6b)	... ...	... ...
	b^4 ♂ 100zp35.35.28(5b)	b^5 ♀ 100zp39.35. 9(5b)	b^6 ♀ 100zp39.35.31(6b)	b^7 p	... ...	... ...
	b^8 – p – – – –	c^1 ♂ 100zp39.35. 3(5b)	c^2 ♂ 80 p36.34. 4(5b)	c^3 ♀ 100zp39.35.28(6b)	... ...	... ...
	c^4 ♀ 100zp39.35.31(6b)	c^5 ♀ 100zp39.35.30(5b)	c^6 ♀ 100zp36.36. 4(5b)	c^7 p	... ...	... ...
3H54	a^1 ♂ 100xp23.17.17(6f)	a^2 ♂ 100xp39.39.30(6bw)	a^3 ♀ 35 p37. 1. 3(2–3b)	a^4 ♀ 100xp35.35.30(6bw)	3H308a^1 ♂ 100xp 35.35.30(5b)	2H307a^3 ♀ 100xp39.39.36(5b)
	b^1 ♂ 100xp37.19.19(5b)	b^2 ♂ 100xp26.26.26(5b)	b^3 ♂ 30 p37. 1. 3(2b)	b^4 ♀ 100xp15.10.10(5f)	... ...	... ...
	c^1 ♂ 35 p36. 1. 1(2b)	c^2 ♂ 15 p37. 1. 1(2b)	c^3 ♀ 100zp35.35. 9(5b)	c^4 ♀ 100yp23.15.10(5f)	... ...	... ...
	c^5 – p –	... ...	... ... —	... ...	... ...	... ...
3H57	a^1 ♂ 100zp39.35.28(5b)	a^2 ♂ 25 p39. 1. 1(2b)	a^3 ♀ 20 p39. 1. 1(2b)	a^4 ♀ 55 p39. 1. 1(3–4b)	2H309a^2 ♂ – p – – – (5b)	2H308a^3 ♀ 100zp39.35. 9(5b)
	a^5 ♀ 80 p39.39. 4(5b)	... ...	... ...	... ...	... ...	... ...
3H59	a^1 ♂ 100xq41.41.50(6e–g)	a^2 ♀ 80 r41. 1. 1(4gw)	a^3 ♀ 60 r?41.37. 1(4g)	a^4 ♀ 45 r41. 1. 2(3–4g)	3H1c^1 ♂ 100xr (or q) 41.41.50(6g)	2H314c^2 ♀ 70 q42.50. 1(3g)
	a^5 ♀ 55 p36. 9. 3(4bw)	a^6 – p – – – –	a^7 – p – – – –	b^1 ♂ 90 r41.41. 1(4g)	... ...	... ...
	b^2 ♀ 100xr41.41.50(6g)	b^3 ♀ 100xr41.41.50(5g)	b^4 ♀ 85 r41.50. 1(4g)	b^5 ♀ 100xp19. 6. 6(6b)	... ...	... ...
	b^6 – p – – – –	b^7 – q	c^1 ♂ 100xr41.41.50(5g)	c^2 ♂ r 3g)	... ...	... ...
	c^3 ♂ 85 p37.34. 3(5b)	c^4 ♀ 80 r41.50. 1(4g)	c^5 ♀ 100xr41.41.50(5–6g)	c^6 ♀ 60 p19. 9. 1(4b)	... ...	... ...
3H60	a^1 ♂ 50 p23.23. 1(3f)	a^2 ♂ 60 p26. 1. 1(4f)	a^3 ♂ 65 p23.15. 1(4f)	a^4 ♀ 100rp22.18.18(6f)	2H302b^1 ♂ 100xp 23.15.15(6f)	2H305b^3 ♀ 100yp23.23.17(6f)
3H61	a^1 ♂ 100xp23.17.18(6f)	a^2 ♀ 100rp23.23.15(5f)	a^3 ♀ 100yp23.15.10(5f)	a^4 ♀ 50 p26. 1. 1(3f)	2H303b^1 ♂ – p – – – (6f)	2H305b^4 ♀ 30 p26. 1. 1(2f)
	a^5 ♀ 40 p50. 1. 1(3f)	a^6 ♀ p – – – 3f)	a^7 – p	a^8 – p – – – –	... ...	... ...
	a^9 – p – – – –	a^{10} p	a^{11} – p –	b^1 ♂ 50 p26. 1. 3(3f)	... ...	... ...
	b^2 ♀ 55 p26. 1. 2(3f)	b^3 ♀ 60 p26. 1. 3(3f)	b^4 ♀ 55 p50. 1. 1(3f)	b^5 ♀ 100xp26.26.23(6f)	... ...	... ...
	b^6 – p – – – –	... ...	... ...	... ...	... ...	... ...
3H64	a^1 ♂ 100rp15.15.18(6f)	a^2 ♂ – p – (5f)	a^3 ♂ – p – – – (6f)	a^4 ♂ 100xp19.19.18(6f)	2H306b^1 ♂ 100rp 23.23.18(5f)	2H312b^4 ♀ 100xp15.15.10(6f)
	a^5 – p – – – –	a^6 p – –	b^1 ♂ – p – – – (6f)	b^2 ♂ 100xp22.17.15(6f)	... ...	... ...
	b^3 ♀ 100xp23.15.10(6f)	b^4 ♀ 100rp22.17.15(6f)	b^5 – p – – – –	b^6 – p	... ...	... ...
	b^7 – p – – – –	c^1 ♂ – p – 5f	c^2 ♂ 100xp23.15.18(6f)	c^3 ♀ 100xp23.23.17(6f)	... ...	... ...
	c^4 ♀ 100xp13.10.10(6f)	c^5 p – –	c^6 – p – – – –	... ...	... ...	... ...
3H65	a^1 ♂ 100rp23.22.17(6f)	a^2 ♂ 100rp22.22.18(5f)	a^3 ♂ 100yp19. 6. 2(5f)	a^4 ♂ 95 p22.18. 1(5f)	2H306b^2 ♂ 100xp 23.23.15(5f)	2H305a^5 ♀ 100xp23.23.18(6f)
	a^5 ♀ 100yp22.17.10(4–5f)	a^6 – p – – – –	a^7 p	b^1 ♂ 100rp22.18.18(5f)	... ...	... ...
	b^2 ♂ 100rp22.22.15(5f)	b^3 ♂ 100xp23.17.17(5f)	c^1 ♂ 100yp23.23.17(6f)	c^2 ♂ 100yp26.23.17(5f)	... ...	... ...
	c^3 ♂ 100yp26.26. 2(5f)	c^4 ♂ 100yp22.18.10(5f)	c^5 ♂ 100rp23.17.15(5f)	... ...	... ...	... ...
3H66	a^1 ♂ 75 p26. 1. 1(3f)	a^2 ♂ – p – – – (2–3f)	a^3 ♂ – p – – 2f)	a^4 ♂ p – (2f)	2H306b^4 ♂ 25 p 26. 1. 2(2–3f)	2H305a^6 ♀ 25 p23. 1. 1(2f)
	a^5 ♂ 15 p23. 1. 2(1f)	a^6 ♀ 15 p23. 1. 1(1–2f)	a^7 ♀ 30 p25. 1. 2(2f)	b^1 ♂ p – – 2f	... ...	... ...
	b^2 ♂ 30 p26. 1. 1(2–3f)	b^3 ♀ 25 p23. 1. 1(2f)	b^4 ♀ 25 p26. 1. 1(2f)	b^5 ♀ 55 p26.26. 3(3f)	... ...	... ...
	b^6 ♀ 20 p26. 1. 2(1f)	b^7 ♀ 20 p26. 1. 1(1–2f)	b^8 – p – – – –	b^9 p – – –	... ...	... ...
	c^1 ♂ p 1f	c^2 ♂ p 1f)	c^3 ♀ 10 p23. 1. 1(1f)	c^4 ♀ 30 p23. 1. 3(2f)	... ...	... ...
	c^5 ♀ 25 p23. 1. 1(2f)	c^6 ♀ 25 p26. 1. 2(2f)	c^7 – p – – – –	c^8 – p – –	... ...	... ...
	c^9 p	c^{10} p –	c^{11} – p – – – –	... ...	... ...	... ...
3H67	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ q – – (6e)	a^3 ♀ 00 p – – – (alb.)	a^4 ♀ 100xq58.58.55(6e)	2H312c^2 ♂ 100xq 58.58.55(6e	2H302b^5 ♀ 100xq58.58.55(6e)
	a^5 ♀ 100rq58.58.56(6e)	b^1 ♂ 00 p – alb.)	b^2 ♂ 100xp23.23.17(6f)	b^3 ♂ 100xq58.58.55(6e)	... ...	... ...
	b^4 ♂ 100rq58.58.55(6e)	b^5 ♀ 100rp23.23.23(6f)	b^6 – q – – – –	b^7 – q – – – –	... ...	... ...
	c^1 ♂ 100rq58.58.55(6e)	c^2 ♂ 90 q61.52. 6(4e)	c^3 ♀ 100xq58.58.55(6e)	c^4 ♀ 100xp24.24.15(6f)	... ...	... ...
	c^5 – p – – – –	... ...	... ...	... ...	... ...	... ...
3H69	a^1 ♂ 00 p – – – (alb.)	a^2 ♂ 100rp23.23.17(6f)	a^3 ♂ 100xq58.58.57(6e)	a^4 ♂ 100xq58.58.57(6e)	2H312b^3 ♂ 100xq 58.57.55(6e)	2H300b^3 ♀ 100xq58.56.56(6e)
	a^5 – q – – – –	b^1 ♂ 100rq58.58.55(6e)	b^2 ♀ 100xq58.58.57(6e)	b^3 ♀ 00 p – – – (alb.)	... ...	... ...
	b^4 – p – – – –	b^5 – p – – – –	b^6 – q – – – –	b^7 q – – – –	... ...	... ...
	c^1 ♂ 00 p – – – (alb.)	c^2 ♂ 00 p – – – (alb.)	c^3 ♂ 00 p – – – (alb.)	c^4 ♀ 100xq58.58.55(6e)	... ...	... ...
	c^5 – q – – – –	c^6 – q – – – –	c^7 – q – – – –	... ...	... ...	... ...
3H70	a^1 ♀ 85 q58.58. 2(4e)	a^2 ♀ 100xq58.58.55(6e)	a^3 ♂ 100xq58.58.55(6e)	a^4 ♂ 100xq58.58.55(6e)	2H303b^4 ♂ 100xq 58.58.55(5e)	2H300c^3 ♀ 100xq58.56.56(6e
	a^5 – q – – – –	a^6 – p – – – –	b^1 ♂ 100xp15.15.18(6f)	b^2 ♂ 100rq57.57.55(5e)	... ...	... ...
	b^3 ♀ 65 q58.58. 1(4e)	b^4 ♀ 100xq58.58.55(6e)	b^5 – p – – – –	b^6 – q – – – –	... ...	... ...
	c^1 ♂ 65 q58. 1. 1(3e)	c^2 ♀ 100xq58.58.56(6e)	c^3 ♀ 100xq58.58.55(6e)	c^4 ♀ 100xq58.58.55(6e)	... ...	... ...
	c^5 – q – – – –	c^6 – q – – – –	... ...	... ...	... ...	... ...
3H71	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♂ q – – (6e)	a^4 ♂ – q – – – (6e)	2H303b^5 ♂ 100xq 58.57.55(6e)	2H300c^4 ♀ 100xq57.56.55(6e)
	a^5 ♀ – q – – – (5e)	a^6 – q – – – –	a^7 – q – – – –	b^1 ♂ 100xq58.57.55(6e)	... ...	... ...
	b^2 ♂ 100rq58.58.55(6e)	c^1 ♂ 100xq58.57.55(6e)	c^2 ♂ 100xq58.58.55(6e)	c^3 ♂ 100xq57.57.52(5e)	... ...	... ...
	c^4 ♀ 100xq58.55.55(6e)	c^5 – q – – – –	c^6 – q – – – –	... ...	... ...	... ...
4H1	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♂ 100xq58.57.55(6e)	a^4 ♀ 45 q58. 1. 1(3e)	3H3a^4 ♂ 100xq 58.58.55(6e)	3H1a^5 ♀ 100xq58.57.55(6e)
	a^5 ♀ 80 r41. 1. 1(4g)	a^6 ♀ 100xr41.41.50(6g)	a^7 ♀ 100xr41.41.50(5g)	... ...	... ...	... ...
4H2	a^1 ♂ 70 q58.55. 2(4e)	a^2 ♂ 100xq58.57.56(6e)	a^3 ♂ 100xq58.57.56(6e)	a^4 ♂ 00 p – – – (alb.)	3H9a^1 ♂ 100xq 58.58.55(6e)	3H2a^7 ♀ 100xq56.57.55(5–6e)
	a^5 ♀ 00 p – – – (alb.)	a^6 ♀ 90 q57.55. 1(5e)	a^7 ♀ 100xq58.58.57(6e)	b^1 ♂ 00 p – – – (alb.)	... ...	... ...

MATING	OFFSPRING				PARENTS	
					Father	Mother
4H2	b^{2} ♂ 100xq58.57.56(6e)	b^{3} ♂ 100xq58.57.55(6e)	b^{4} ♂ 70 q58.57. 2(3–4e)	b^{5} ♂ 90 q58.55. 2(4e)	... ...	... ...
	b^{6} – p – – – –	b^{7} – p – – – –	c^{1} ♂ 100xq58.57.55(6e)	c^{2} ♂ 100xq58.57.55(6e)	... ...	... ...
	c^{3} ♂ 100xq58.57.55(6e)	c^{4} ♂ 85 q58.55. 1(4e)	c^{5} ♂ 00 p – – – (alb.)	c^{6} ♀ 100xq58.57.56(6e)	... ...	... ...
	c^{7} ♀ 60 q58. 1. 2(4e)	c^{8} ♀ 70 q58.55. 2(4e)	... ...	... ...	... ...	... ...
4H3	a^{1} ♂ 100xq58.57.55(6ew)	a^{2} ♂ – q – – – (6e)	a^{3} ♀ 00 p – – – (alb.)	a^{4} ♀ 55 q58. 1. 2(3e)	3H2a^{4} ♂ 100xq58.58.55(5–6e)	3H3a^{3} ♀ 100xq58.58.55(5–6e)
	a^{5} ♀ 00 p – – – (alb.)	a^{6} – p – – – –	b^{1} ♂ 100xq58.58.55(6e)	b^{2} ♂ 100xq58.57.56(6e)	... ...	... ...
	b^{3} ♂ 100xq58.58.55(6e)	b^{4} ♀ 70 q58. 1. 2(4e)	b^{5} ♀ 100xq58.57.56(6e)	b^{6} ♀ 100xq58.57.56(6e)	... ...	... ...
	b^{7} – p – – – –	c^{1} ♂ – q – – – (6ew)	c^{2} ♂ 100xq57.57.56(6e)	c^{3} ♂ 45 q58. 1. 2(4e)	... ...	... ...
	c^{4} ♂ 00 p – – – (alb. w)	c^{5} ♀ 60 q58. 1. 4(4ew)	c^{6} ♀ 100xq58.58.55(6e)	c^{7} ♀ 100xq58.57.55(6e)	... ...	... ...
	c^{8} – p – – – –	c^{9} – p – – – –	c^{10} – p – – – –	... ...	... ...	... ...
4H4	a^{1} ♂ 100xp23.23.18(6f)	a^{2} ♂ 100xq57.57.56(5–6e)	a^{3} ♀ 100xp22.18. 2(4–5f)	a^{4} ♀ 100xq58.57.55(5e)	3H2a^{3} ♂ 100xq57.57.55(5e)	3H6a^{4} ♀ 100yq58.58.55(5e)
	a^{5} – q – – – –	b^{1} ♂ 100xq58.57.55(5e)	b^{2} ♂ 100xq58.58.55(5e)	b^{3} ♂ 100xq58.57.55(6e)	... ...	... ...
	b^{4} ♀ 100xq58.57.55(6e)	b^{5} ♀ 45 q58. 1. 1(3e)	b^{6} – p – – – –	b^{7} – p – – – –	... ...	... ...
	c^{1} ♂ 100xq58.58.55(6e)	c^{2} ♂ – p – – – (3fw)	c^{3} ♂ – p – – – (6f)	c^{4} ♀ 100yq58.55. 1(5e)	... ...	... ...
	c^{5} ♀ 85 q58.55. 1(4e)	c^{6} ♀ 35 q58. 1. 1(3e)	c^{7} ♀ 100yp23.18. 4(5f)	c^{8} – q – – – –	... ...	... ...
	c^{9} – q – – – –	c^{10} – q – – – –	... ...	... ...	... ...	... ...
4H6	a^{1} ♀ 00 p – – – (alb.)	a^{2} ♀ 95 q58.57.55(5e)	b^{1} ♂ 100yq58.55. 2(5e)	b^{2} ♂ 100yq57.56.55(5e)	3H6a^{2} ♂ 100yq58.57. 2(5e)	3H7a^{4} ♀ 100xq58.58.55(6e)
	b^{3} ♂ 100yq58.57.56(5e)	b^{4} ♀ 95 q58.57. 2(5e)	b^{5} ♀ 100yq57.57.55(5e)	c^{1} ♂ 100xq58.57.55(5e)	... ...	... ...
	c^{2} ♀ 100yq57.57.56(5e)	c^{3} ♀ 100yq58.56. 2(5e)	c^{4} ♀ 100xq57.57.56(6e)	c^{5} ♀ 100xq58.57.57(6e)	... ...	... ...
4H7	a^{1} ♂ 100xq58.57.55(5e)	a^{2} ♀ 100xq58.57.56(5–6e)	... ...	... ...	3H6a^{3} ♂ 100xq57.57.56(5–6e)	3H7a^{3} ♀ 100xq58.58.55(6e)
4H8	a^{1} ♀ 80 p26.23. 1(4–5f)	a^{2} ♂ 65 p26.23. 1(4f)	a^{3} ♀ 55 p26.22. 1(3–4f)	a^{4} ♂ 65 p26.23. 2(3–4f)	3H3a^{2} ♂ 60 p23. 1. 1(3f)	3H1a^{3} ♀ 55 p26.26. 1(3f)
	a^{5} ♂ 50 p19. 9. 3(3f)	a^{6} ♂ 90 p26. 3. 3(5f)	a^{7} ♂ 55 p23.22. 2(3f)	a^{8} – p – – – –	... ...	... ...
4H9	a^{1} ♂ 95 p26.22.18(4–5f)	a^{2} ♂ – p – – – (4f)	a^{3} ♂ 45 p37. 1. 2(3f)	a^{4} ♂ 00 p – – – (alb.)	3H1a^{1} ♂ 60 q58.57. 1(3e)	3H2a^{6} ♀ 75 q55. 1. 1(4e)
	a^{5} ♀ 55 p37.37. 1(4f)	a^{6} ♀ 60 p37.37. 4(4f)	a^{7} ♀ 30 q58. 1. 1(3e)	a^{8} ♀ 00 p – – – (alb.)	... ...	... ...
	b^{1} ♀ 70 p26.23. 1(4f)	b^{2} ♀ 60 p26. 1. 2(4f)	c^{1} ♂ 75 q58. 1. 2(4e)	c^{2} ♂ 45 q58. 1. 1(3e)	... ...	... ...
	c^{3} ♀ 80 p26. 1. 2(4f)	c^{4} ♀ 65 p23. 1. 3(4f)	c^{5} – p – – – –	... ...	... ...	... ...
4H10	a^{1} ♂ 75 q58.55. 2(4e)	a^{2} ♂ 65 q58. 1. 2(4ew)	a^{3} ♂ 00 p – – – (alb.)	a^{4} ♀ 70 p26.26. 1(4f)	3H2a^{2} ♂ 85 q58.55. 2(4e)	3H9a^{5} ♀ – q – – – (4e)
	a^{5} ♀ 00 p – – – (alb.)	a^{6} ♀ 90 q58.55. 4(4–5e)	a^{7} ♀ 55 q58.57. 1(4e)	a^{8} ♀ 55 q58. 1. 2(3ew)	... ...	... ...
	a^{9} – q – – – –	a^{10} – q – – – –	b^{1} ♂ 90 p26.18. 2(4fw)	b^{2} ♂ 55 q58. 1. 2(3ew)	... ...	... ...
	b^{3} ♂ – p – – – (3fw)	b^{4} ♀ 95 q58.55. 2(5e)	b^{5} ♀ 45 p26. 1. 1(3f)	b^{6} ♂ – q – – – (4e)	... ...	... ...
	b^{7} ♂ 90 q 58.55. 2(4e)	b^{8} ♂ 95 q58.55. 2(4e)	... ...	... ...	... ...	... ...
4H11	a^{1} ♀ 95 q 58.56. 2(4–5e)	a^{2} ♀ 95 q57.56. 1(4–5e)	a^{3} ♀ 60 q58. 1. 1(4e)	a^{4} ♀ 80 q57.56. 1(4e)	3H15a^{6} ♂ 65 q57.55. 3(4e)	3H13a^{4} ♀ 60 q58.58. 1(3–4e)
	a^{5} ♂ – q – – – –	a^{6} ♂ – q – – – –	a^{7} – q – – – –	a^{8} ♂ – p – – – –	... ...	... ...
	b^{1} ♂ 00 p – – – (alb.)	b^{2} ♂ 60 q58.57. 1(4e)	b^{3} ♀ 75 q58.57. 1(4e)	b^{4} ♀ 60 q58.57. 2(3–4e)	... ...	... ...
	b^{5} – q – – – –	b^{6} – q – – – –	b^{7} – q – – – –	b^{8} – q – – – –	... ...	... ...
4H12	a^{1} ♂ 55 r42.50. 2(4g)	a^{2} ♂ 65 r42.50. 1(4g)	a^{3} ♂ 00 p – – – (alb.)	a^{4} – ? – – – ?	3H1a^{2} ♂ 50 r42.50. 1(3g)	3H13a^{2} ♀ 80 r41.50. 1(4g)
	a^{5} ♀ 30 r42.50. 2(3g)	a^{6} ♀ 60 r26.26. 2(4g)	b^{1} ♂ 65 r41.50. 2(3–4g)	b^{2} ♂ 00 p – – – (alb.)	... ...	... ...
	b^{3} ♀ 95 r41.50. 3(4–5g)	b^{4} ♀ 45 r41.41. 2(3g)	b^{5} ♀ 00 p – – – (alb.)	c^{1} ♂ 70 r41.50. 2(4g)	... ...	... ...
	c^{2} ♀ 25 q41. 1. 2(2g)	c^{3} ♀ 85 r41.26. 2(4g)	c^{4} ♀ 60 r50.50. 1(4g)	c^{5} ♀ 55 r41.50. 1(3–4g)	... ...	... ...
	c^{6} ♀ 00 p – – – (alb.)	c^{7} – q – – – –	c^{8} – q – – – –	... ...	... ...	... ...
4H13	a^{1} ♂ 60 q58.58. 3(4e)	a^{2} ♂ 90 q58.56. 2(4–5e)	a^{3} ♂ 90 q58.55. 2(4–5e)	a^{4} ♂ – q – – – (3-4ew)	3H15a^{4} ♂ 65 q58. 1. 2(3e)	3H1a^{6} ♀ 45 q58.55. 1(3e)
	a^{5} ♂ 40 q58. 1. 2(3e)	a^{6} ♀ 00 p – – – (alb.)	a^{7} ♀ 00 p – – – (alb.)	b^{1} ♂ 85 q58.55. 1(4–5e)	... ...	... ...
	b^{2} ♂ 80 q58.57. 2(4e)	b^{3} ♂ 60 q58.55. 2(3–4e)	b^{4} ♂ 50 q58. 1. 1(3e)	b^{5} ♂ 55 q57.55. 2(3ew)	... ...	... ...
	b^{6} ♀ 90 q58.58. 4(5e)	b^{7} ♀ 75 q58. 1. 2(3–4e)	b^{8} ♀ 00 p – – – (alb.)	b^{9} – p – – – –	... ...	... ...
	c^{1} ♂ 00 p – – – (alb. w)	c^{2} ♂ 00 p – – – (alb.)	c^{3} ♂ 75 q58.55. 2(4e)	c^{4} ♂ 60 q58. 1. 2(3–4e)	... ...	... ...

	c^5 ♀ 35 q58. 1. 2(3ew)	c^6 ♀ 65 q58. 1. 3(3–4e)	c^7 – q– – – –	c^8 – q– – – –	... ...	... ...
4H14	a^1 ♀ 75 p35.35. 2(5b)	a^2 ♀ 75 p35.35. 2(5b)	a^3 ♀ 75 p35.35. 2(5b)	a^4 ♂ 75 p35.35. 3(5b)	3H20a^1 ♂ 75 p36.35. 4(5b)	3H21a^3 ♀ 75 p39.39. 4(5b)
	a^5 ♂ 75 p39.35.28(5b)	a^6 ♂ 75 p35.35. 2(5b)	a^7 – p– – – –	b^1 ♂ 75 p39.39.28(5b)	... ...	... ...
	b^2 ♂ 100zp39.35.28(5b)	b^3 ♀ 75 p39.39.28(5b)	b^4 ♀ 75 p39.39.28(5b)	b^5 ♀ 80 p39.39. 2(5a)	... ...	... ...
	b^6 – p– – – –	b^7 – p– – – –	c^1 ♂ 75 p36.35. 4(5b)	c^2 ♂ 75 p39.39. 2(5b)	... ...	... ...
	c^3 ♂ 75 p39.39. 4(4–5b)	c^4 ♂ 100zp39.39.29(5b)	c^5 ♀ 100zp35.33.28(5b)	c^6 ♀ 100zp35.35.28(5b)	... ...	... ...
	c^7 – p– – – –	... ...	... ...	... ...	... ...	... ...
4H15	a^1 ♂ 75 p36.23. 2(5a–b)	a^2 ♂ 100xp23.23.18(6f)	a^3 ♂ 100xp10.10.10(6f)	a^4 ♂ 100zp15.15. 5(6f)	3H20a^1 ♂ 75 p36.35. 4(5b)	3H21a^4 ♀ 80 p39.39. 4(5b)
	a^5 ♀ 75 p36.36. 2(5b)	b^1 ♂ 100zp39.35.28(5a–f)	b^2 ♂ – p– – –(5b)	b^3 ♀ – p– – –(6f)	... ...	... ...
	b^4 ♀ 100zp35.35.28(5b)	b^5 ♀ – p– – –(5b)	b^6 – p– – – –	b^7 – p– – – –	... ...	... ...
	b^8 – p– – – –	... ...	... ...	... ...	... ...	... ...
4H16	a^1 ♀ 80 p39.39. 4(4–5b)	a^2 ♀ – p– – –(5b)	a^3 ♀ 100zp39.39.28(5b)	a^4 ♀ 80 p39.39.28(5b)	3H20a^2 ♂ 80 p36.36. 4(5b)	3H21a^5 ♀ 100zp39.39.28(5b)
	a^5 ♂ 100zp35.35.28(5b)	a^6 ♂ 100zp39.35.28(5b)	a^7 ♂ 80 p35.35. 2(5b)	a^8 ♂ 75 p39.35. 2(5b)	... ...	... ...
	b^1 ♂ 100zp39.39.30(5b)	b^2 ♂ 100zp39.39.28(4-5a-b)	b^3 ♂ 100zp39.39.28(5b)	b^4 ♂ 75 p39.35. 2(5b)	... ...	... ...
	b^5 ♂ 100zp39.35.30(5a)	b^6 ♂ 60 p26.26. 1(3b)	b^7 – p– – – –	b^8 – p– – – –	... ...	... ...
	c^1 ♂ 100zp39.35.29(5b)	c^2 ♀ 100zp40.40.31(5b)	c^3 ♀ 75 p39.39. 4(5b)	c^4 ♀ 75 p39.39.28(4–5b)	... ...	... ...
	c^5 ♀ 65 p39.39. 2(4b)	... ...	... ...	... ...	... ...	... ...
4H17	a^1 ♂ 100xp23.23.18(6f)	a^2 ♂ 100xp26.26.17(6f)	a^3 ♀ 100xp23.23.18(6f)	a^4 ♀ 100zp39.39.30(5a–b)	3H20a^2 ♂ 80 p36.36. 4(5b)	3H21a^6 ♀ 100zp39.39.30(5–6b)
	a^5 ♀ 75 p39.39.28(5b)	a^6 ♀ 80 p35.35.28(5a)	b^1 ♂ 100yp23.23.18(6f)	b^2 ♂ – p– – –(5a)	... ...	... ...
	b^3 ♂ – p– – –(5b)	b^4 ♀ 100xp23.23.18(6f)	b^5 ♀ 100xp23.23.23(6f)	b^6 ♀ 75 p35.35.28(5a)	... ...	... ...
	b^7 ♀ 45 p39.39. 2(3b)	... ...	... ...	... ...	... ...	... ...
4H19	a^1 ♂ 30 p39. 1. 2(2–3b)	a^2 ♂ 55 p39. 1. 4(3b)	a^3 ♂ 40 p26. 1. 4(3f)	a^4 ♀ 45 p39. 1. 3(3b)	3H30a^2 ♂ 40 p40. 1. 2(2b)	3H29a^4 ♀ 40 p37.39. 3(2–3b)
	a^5 ♀ 30 p23. 1. 2(2–3f)	b^1 ♂ 25 p39. 1. 2(2b)	b^2 ♂ 20 p37. 1. 1(2b)	b^3 ♂ 25 p39. 1. 2(2b)	... ...	... ...
	b^4 ♂ 30 p37. 1. 1(3b)	b^5 ♂ 40 p37. 1. 2(3b)	b^6 ♂ 30 p39. 1. 3(3b)	b^7 ♀ 45 p39. 1. 3(3b)	... ...	... ...
	c^1 ♀ 65 p39. 1. 2(4b)	... ...	... ...	... ...	... ...	... ...
4H20	a^1 ♀ 100zp36.36.30(6b)	a^2 ♀ 100xp35.35.30(6b)	a^3 ♀ 100zp39.39.28(5b)	a^4 ♀ 100xp23.23.15(6f)	3H21a^1 ♂ 80 p39.35. 2(5b)	3H20a^3 ♀ – p– – –(5b)
	a^5 ♀ 100xp10.23.15(6f)	b^1 ♂ 100xp23.23.17(6f)	b^2 ♂ 100zp39.39.28(5b)	b^3 ♀ 100zp39.35.30(5–6b)	... ...	... ...
	b^4 ♀ 100zp35.35.28(5b)	b^5 ♀ 100zp39.35.31(6b)	b^6 ♀ 100xp39.35.31(6b)	... ...	... ...	... ...
4H21	a^1 ♂ 75 p35.35.28(5b)	a^2 ♂ 100zp39.35.34(6b)	a^3 ♂ – p– – –(5b)	a^4 ♂ – p– – –(5b)	3H21a^1 ♂ 80 p39.35. 2(5b)	3H20a^4 ♀ 100zp39.35.28(5b)
	a^5 ♀ 100xp39.39.36(6b)	a^6 ♀ 75 p40.35. 2(5b)	... ...	... ...	... ...	... ...
4H22	a^1 ♂ 80 p39.39. 2(5b)	a^2 ♂ 85 p35.35. 2(5b)	a^3 ♂ 85 p35.35.28(5b)	a^4 ♂ 100zp35.35.30(5b)	3H21a^2 ♂ 80 p39.35. 2(5b)	3H20a^5 ♀ 100zp35.35.28(6b)
	a^5 ♀ 80 p35.35. 3(5b)	a^6 ♀ 100xp23.23.18(6f)	b^1 ♂ 85 p39.35.28(5b)	b^2 ♂ 80 p35.35.28(5b)	... ...	... ...
	b^3 ♀ 75 p39.39. 3(5b)	b^4 ♀ 80 p35.35.28(5b)	b^5 ♀ 100zp39.39.28(5b)	b^6 ♀ 75 p35.35. 2(5b)	... ...	... ...
	b^7 ♀ 100zp39.39.28(5b)	c^1 ♂ 75 p39.35. 2(5b)	c^2 ♂ 100xp15.15.10(6f)	c^3 ♂ 100xp15.15.10(6f)	... ...	... ...
	c^4 ♀ 80 p35.35. 2(5b)	c^5 ♀ 80 p35.35. 2(5b)	c^6 – p– – – –	... ...	... ...	... ...
4H23	a^1 – q– – – –	a^2 – p– – – –	b^1 ♂ 100xp37.19.31(6a–f)	b^2 ♂ 100xp37.19.9(5-6a–f)	3H5a^1 ♂ 100xq41.41.50(6g)	3H8a^2 ♀ 35 r41.41. 3(2–3g)
	b^3 ♂ 100xp37.19.31(6a–f)	b^4 ♀ 00 p– – –(alb.)	b^5 ♀ 00 p– – –(alb.)	b^6 ♀ 100xp19.19.31(6a–f)	... ...	... ...
	c^1 ♂ 100xr41.41.50(5–6g)	c^2 ♂ 100xr41.41.26(5–6g)	c^3 ♂ 100xp19.19. 9(5b)	c^4 ♂ 00 p– – –(alb.)	... ...	... ...
	c^5 ♂ 00 p– – –(alb.)	c^6 ♀ 100xq58.58.44(6e)	c^7 ♀ 00 p– – –(alb.)	... ...	... ...	... ...
4H24	a^1 ♂ 100xq58.57.55(5–6e)	a^2 ♀ – q– – –(5–6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 00 p– – –(alb.)	3H5a^3 ♂ 100xq57.57.55(6e)	3H8a^3 ♀ 100xq57.58.55(5e)
	a^5 – p– – – –	a^6 – q– – – –	a^7 – q– – – –	b^1 ♂ 100yq57.57.55(5e)	... ...	... ...
	b^2 ♂ 100xq58.57.55(5e)	b^3 ♂ 00 p– – –(alb.)	b^4 ♀ 100xq58.57.57(6e)	b^5 ♀ 100yq57.57.55(5e)	... ...	... ...
	b^6 – q– – – –	b^7 – q– – – –	b^8 – q– – – –	b^9 – q– – – –	... ...	... ...
	c^1 ♀ 100yq58.58.55(5e)	c^2 ♀ 00 p– – –(alb.)	c^3 – p– – – –	c^4 – q– – – –	... ...	... ...
4H25	x – p– – – –	a^1 ♂ 100xq57.57.56(6e)	a^2 ♀ 100xq58.58.55(6e)	b^1 ♂ 100xq58.58.55(6e)	3H5a^4 ♂ 100xq58.58.55(6e)	3H4a^5 ♀ 100xq57.57.56(6e)
	b^2 ♂ 100xq58.57.55(6e)	b^3 ♀ 100xq58.58.55(6e)	b^4 ♀ 100xq58.57.55(6e)	b^5 – p– – – –	... ...	... ...
	c^1 ♂ 100xq57.57.56(6e)	c^2 ♂ 100xp23.23.18(6f)	c^3 ♀ 100xq58.57.55(6e)	... ...	... ...	... ...
4H26	a^1 ♂ – q– – –(6e)	a^2 ♂ – q– – –(6e)	a^3 ♂ 85 q58.55. 2(4–5e)	a^4 ♀ 100xp26.23.23(6a–f)	3H5a^2 ♂ 100xq58.58.55(6e)	3H4a^6 ♀ 100xq58.57.55(6e)
	a^5 ♀ 100xq58.58.56(6e)	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ 100xp22.22.18(6f)	b^3 ♂ 100xp23.23.18(6f)	... ...	... ...
	b^4 – q– – – –	c^1 ♂ 100xq58.57.55(6e)	c^2 ♂ 100xq58.57.55(6e)	c^3 ♂ 75 p26.19. 2(4f)	... ...	... ...

MATING	OFFSPRING				PARENTS	
					Father	Mother
4H2	b^2 ♂ 100xq58.57.56(6e)	b^3 ♂ 100xq58.57.55(6e)	b^4 ♂ 70 q58.57. 2(3–4e)	b^5 ♂ 90 q58.55. 2(4e)	...	...
	b^6 – p– – – –	b^7 – p– – – –	c^1 ♂ 100xq58.57.55(6e)	c^2 ♂ 100xq58.57.55(6e)	...	...
	c^3 ♂ 100xq58.57.55(6e)	c^4 ♂ 85 q58.55. 1(4e)	c^5 ♂ 00 p– – –(alb.)	c^6 ♀ 100xq58.57.56(6e)	...	...
	c^7 ♀ 60 q58. 1. 2(4e)	c^8 ♀ 70 q58.55. 2(4e)	...	...	...	...
4H3	a^1 ♂ 100xq58.57.55(6ew)	a^2 ♂ – q – – –(6e)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ 55 q58. 1. 2(3e)	3H2a^4 ♂ 100xq58.58.55(5–6e)	3H3a^3 ♀ 100xq58.58.55(5–6e)
	a^5 ♀ 00 p– – –(alb.)	a^6 – p– – – –	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ 100xq58.57.56(6e)	...	...
	b^3 ♂ 100xq58.58.55(6e)	b^4 ♀ 70 q58. 1. 2(4e)	b^5 ♀ 100xq58.57.56(6e)	b^6 ♀ 100xq58.57.56(6e)	...	...
	b^7 – p– – – –	c^1 ♂ – q– – –(6ew)	c^2 ♂ 100xq57.57.56(6e)	c^3 ♂ 45 q58. 1. 2(4e)	...	...
	c^4 ♂ 00 p – – –(alb.w)	c^5 ♀ 60 q58. 1. 4(4ew)	c^6 ♀ 100xq58.58.55(6e)	c^7 ♀ 100xq58.57.55(6e)	...	...
	c^8 – p– – – –	c^9 – p– – – –	c^{10} – p– – – –	...	...	...
4H4	a^1 ♂ 100xp23.23.18(6f)	a^2 ♂ 100xq57.57.56(5–6e)	a^3 ♀ 100xp22.18. 2(4–5f)	a^4 ♀ 100xq58.57.55(5e)	3H2a^3 ♂ 100xq57.57.55(5e)	3H6a^4 ♀ 100yq58.58.55(5e,
	a^5 – q– – – –	b^1 ♂ 100xq58.57.55(5e)	b^2 ♂ 100xq58.58.55(5e)	b^3 ♂ 100xq58.57.55(6e)	...	...
	b^4 ♀ 100xq58.57.55(6e)	b^5 ♀ 45 q58. 1. 1(3e)	b^6 – p– – – –	b^7 – p– – – –	...	...
	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ – p– – –(3fw)	c^3 ♂ – p– – –(6f)	c^4 ♀ 100yq58.55. 1(5e)	...	...
	c^5 ♀ 85 q58.55. 1(4e)	c^6 ♀ 35 q58. 1. 1(3e)	c^7 ♀ 100yp23.18. 4(5f)	c^8 – q– – – –	...	...
	c^9 – q– – – –	c^{10} – q– – – –	...	...	...	...
4H6	a^1 ♀ 00 p – – –(alb.)	a^2 ♀ 95 q58.57.55(5e)	b^1 ♂ 100yq58.55. 2(5e)	b^2 ♂ 100yq57.56.55(5e)	3H6a^2 ♂ 100yq58.57. 2(5e)	3H7a^1 ♀ 100xq58.58.55(6e)
	b^3 ♂ 100yq58.57.56(5e)	b^4 ♀ 95 q58.57. 2(5e)	b^5 ♀ 100yq57.57.55(5e)	c^1 ♂ 100xq58.57.55(5e)	...	...
	c^2 ♀ 100yq57.57.56(5e)	c^3 ♀ 100yq58.56. 2(5e)	c^4 ♀ 100xq57.57.56(6e)	c^5 ♀ 100xq58.57.57(6e)	...	...
4H7	a^1 ♂ 100xq58.57.55(5e)	a^2 ♀ 100xq58.57.56(5–6e)	...	...	3H6a^1 ♂ 100xq57.57.56(5–6e)	3H7a^1 ♀ 100xq58.58.55(6e)
4H8	a^1 ♀ 80 p26.23. 1(4–5f)	a^2 ♂ 65 p26.23. 1(4f)	a^3 ♀ 55 p26.22. 1(3–4f)	a^4 ♂ 65 p26.23. 2(3–4f)	3H3a^2 ♂ 60 p23. 1. 1(3f)	3H1a^3 ♀ 55 p26.26. 1(3f)
	a^5 ♂ 50 p19. 9. 3(3f)	a^6 ♂ 90 p26. 3. 3(5f)	a^7 ♂ 55 p23.22. 2(3f)	a^8 – p– – – –	...	...
4H9	a^1 ♂ 95 p26.22.18(4–5f)	a^2 ♂ – p – – –(4f)	a^3 ♂ 45 p37. 1. 2(3f)	a^4 ♂ 00 p – – –(alb.)	3H1a^1 ♂ 60 q58.57. 1(3e)	3H2a^6 ♀ 75 q55. 1. 1(4e)
	a^5 ♀ 55 p37.37. 1(4f)	a^6 ♀ 60 p37.37. 4(4f)	a^7 ♀ 30 q58. 1. 1(3e)	a^8 ♀ 00 p– – –(alb.)	...	...
	b^1 ♀ 70 p26.23. 1(4f)	b^2 ♀ 60 p26. 1. 2(4f)	c^1 ♂ 75 q58. 1. 2(4e)	c^2 ♂ 45 q58. 1. 1(3e)	...	...
	c^3 ♀ 80 p26. 1. 2(4f)	c^4 ♀ 65 p23. 1. 3(4f)	c^5 – p– – – –	...	...	...
4H10	a^1 ♂ 75 q58.55. 2(4e)	a^2 ♂ 65 q58. 1. 2(4ew)	a^3 ♂ 00 p– – –(alb.)	a^4 ♀ 70 p26.26. 1(4f)	3H2a^2 ♂ 85 q58.55. 2(4e)	3H9a^5 ♀ – q– – –(4e)
	a^5 ♀ 00 p– – –(alb.)	a^6 ♀ 90 q58.55. 4(4–5e)	a^7 ♀ 55 q58.57. 1(4e)	a^8 ♀ 55 q58. 1. 2(3ew)	...	...
	a^9 – q– – – –	a^{10} – q– – – –	b^1 ♂ 90 p26.18. 2(4fw)	b^2 ♂ 55 q58. 1. 2(3ew)	...	...
	b^3 ♂ – p– – –(3fw)	b^4 ♀ 95 q58.55. 2(5e)	b^5 ♀ 45 p26. 1. 1(3f)	b^6 ♂ – q– – –(4e)	...	...
	b^7 ♂ 90 q 58.55. 2(4e)	b^8 ♂ 95 q58.55. 2(4e)	...	...	...	...
4H11	a^1 ♀ 95 q 58.56. 2(4–5e)	a^2 ♀ 95 q57.56. 1(4–5e)	a^3 ♀ 60 q58. 1. 1(4e)	a^4 ♀ 80 q57.56. 1(4e)	3H15a^6 ♂ 65 q57.55. 3(4e)	3H13a^4 ♀ 60 q58.58. 1(3–4e)
	a^5 ♂ – q– – – –	a^6 ♂ – q– – – –	a^7 – q– – – –	a^8 ♂ – p– – – –	...	...
	b^1 ♂ 00 p – – –(alb.)	b^2 ♂ 60 q58.57. 1(4e)	b^3 ♀ 75 q58.57. 1(4e)	b^4 ♀ 60 q58.57. 2(3–4e)	...	...
	b^5 – q– – – –	b^6 – q– – – –	b^7 – q– – – –	b^8 – q– – – –	...	...
4H12	a^1 ♂ 55 r42.50. 2(4g)	a^2 ♂ 65 r42.50. 1(4g)	a^3 ♂ 00 p– – –(alb.)	a^4 – ?– – – ?	3H1a^2 ♂ 50 r42.50. 1(3g)	3H13a^2 ♀ 80 r41.50. 1(4g)
	a^5 ♀ 30 r42.50. 2(3g)	a^6 ♀ 60 r26.26. 2(4g)	b^1 ♂ 65 r41.50. 2(3–4g)	b^2 ♂ 00 p– – –(alb.)	...	...
	b^3 ♀ 95 r41.50. 3(4–5g)	b^4 ♀ 45 r41.41. 2(3g)	b^5 ♀ 00 p– – –(alb.)	c^1 ♂ 70 r41.50. 2(4g)	...	...
	c^2 ♀ 25 q41. 1. 2(2g)	c^3 ♀ 85 r41.26. 2(4g)	c^4 ♀ 60 r50.50. 1(4g)	c^5 ♀ 75 r41.50. 1(3–4g)	...	...
	c^6 ♀ 00 p– – –(alb.)	c^7 – q– – – –	c^8 – q– – – –	...	...	...
4H13	a^1 ♂ 60 q58.58. 3(4e)	a^2 ♂ 90 q58.56. 2(4–5e)	a^3 ♂ 90 q58.55. 2(4–5e)	a^4 ♂ – q – –(3–4ew)	3H15a^4 ♂ 65 q58. 1. 2(3e)	3H1a^6 ♀ 45 q58.55. 1(3e)
	a^5 ♂ 40 q58. 1. 2(3e)	a^6 ♀ 00 p– – –(alb.)	a^7 ♀ 00 p– – –(alb.)	b^1 ♂ 85 q58.55. 1(4–5e)	...	...
	b^2 ♂ 80 q58.57. 2(4e)	b^3 ♂ 60 q58.55. 2(3–4e)	b^4 ♂ 50 q58. 1. 1(3e)	b^5 ♂ 55 q57.55. 2(3ew)	...	...
	b^6 ♀ 90 q58.58. 4(5e)	b^7 ♀ 75 q58. 1. 2(3–4e)	b^8 ♀ 00 p– – –(alb.)	b^9 – p– – – –	...	...
	c^1 ♂ 00 p– – –(alb.w)	c^2 ♂ 00 p– – –(alb.)	c^3 ♂ 75 q58.55. 2(4e)	c^4 ♂ 60 q58. 1. 2(3–4e)	...	...
	c^5 ♀ 35 q58. 1. 2(3ew)	c^6 ♀ 65 q58. 1. 3(3–4e)	c^7 – q– – – –	c^8 – q– – – –	...	...
4H14	a^1 ♀ 75 p35.35. 2(5b)	a^2 ♀ 75 p35.35. 2(5b)	a^3 ♀ 75 p35.35. 2(5b)	a^4 ♂ 75 p35.35. 3(5b)	3H20a^1 ♂ 75 p36.35. 4(5b)	3H21a^3 ♀ 75 p39.39. 4(5b)
	a^5 ♂ 75 p39.35.28(5b)	a^6 ♂ 75 p35.35. 2(5b)	a^7 – p– – – –	b^1 ♂ 75 p39.39.28(5b)	...	...
	b^2 ♂ 100zp39.35.28(5b)	b^3 ♀ 75 p39.39.28(5b)	b^4 ♀ 75 p39.39.28(5b)	b^5 ♀ 80 p39.39. 2(5a)	...	...
	b^6 – p– – – –	b^7 – p– – – –	c^1 ♂ 75 p36.35. 4(5b)	c^2 ♂ 75 p39.39. 2(5b)	...	...
	c^3 ♂ 75 p39.39. 4(4–5b)	c^4 ♂ 100zp39.39.29(5b)	c^5 ♀ 100zp35.33.28(5b)	c^6 ♀ 100zp35.35.28(5b)	...	...
	c^7 – p– – – –	...	...	...	...	...
4H15	a^1 ♂ 75 p36.23. 2(5a–b)	a^2 ♂ 100xp23.23.18(6f)	a^3 ♂ 100xp10.10.10(6f)	a^4 ♂ 100zp15.15. 5(6f)	3H20a^1 ♂ 75 p36.35. 4(5b)	3H21a^4 ♀ 80 p39.39. 4(5b)
	a^6 ♀ 75 p36.36. 2(5b)	b^1 ♂ 100zp39.35.28(5a–f)	b^2 ♂ – p – – –(5b)	b^3 ♀ – p– – –(6f)	...	...
	b^4 ♀ 100zp35.35.28(5b)	b^5 ♀ – p – – –(5b)	b^6 – p – – – –	b^7 – p– – – –	...	...
	b^8 – p–	...	...	...	...	...
4H16	a^1 ♀ 80 p39.39. 4(4–5b)	a^2 ♀ – p – – –(5b)	a^3 ♀ 100zp39.39.28(5b)	a^4 ♀ 80 p39.39.28(5b)	3H20a^2 ♂ 80 p36.36. 4(5b)	3H21a^5 ♀ 100zp39.39.28(5b)
	a^5 ♂ 100zp35.35.28(5b)	a^6 ♂ 100zp39.35.28(5b)	a^7 ♂ 80 p35.35. 2(5b)	a^8 ♂ 75 p39.35. 2(5b)	...	...
	b^1 ♂ 100zp39.39.30(5b)	b^2 ♂ 100zp39.39.28(4–5a–b)	b^3 ♂ 100zp39.39.28(5b)	b^4 ♂ 75 p39.35. 2(5b)	...	...
	b^5 ♂ 100zp39.35.30(5a)	b^6 ♂ 60 p26.26. 1(3b)	b^7 – p– – –	b^8 – p – – – –	...	...
	c^1 ♂ 100zp39.35.29(5b)	c^2 ♀ 100zp40.40.31(5b)	c^3 ♀ 75 p39.39. 4(5b)	c^4 ♀ 75 p39.39.28(4–5b)	...	...
	c^5 ♀ 65 p39.39. 2(4b)	...	...	...	...	...
4H17	a^1 ♂ 100xp23.23.18(6f)	a^2 ♂ 100xp26.26.17(6f)	a^3 ♀ 100xp23.23.18(6f)	a^4 ♀ 100zp39.39.30(5a–b)	3H20a^2 ♂ 80 p36.36. 4(5b)	3H21a^6 ♀ 100zp39.39.30(5–6b)
	a^5 ♀ 75 p39.39.28(5b)	a^6 ♀ 80 p35.35.28(5a)	b^1 ♂ 100yp23.23.18(6f)	b^2 ♂ – p– – –(5a)	...	...
	b^3 ♂ – p – – –(5b)	b^4 ♀ 100xp23.23.18(6f)	b^5 ♀ 100xp23.23.23(6f)	b^6 ♀ 75 p35.35.28(5a)	...	...
	b^7 ♀ 45 p39.39. 2(3b)	...	...	...	...	...
4H19	a^1 ♂ 50 p39. 1. 2(2–3b)	a^2 ♂ 55 p39. 1. 4(3b)	a^3 ♂ 40 p26. 1. 4(3f)	a^4 ♀ 45 p39. 1. 3(3b)	3H30a^2 ♂ 40 p40. 1. 2(2b)	3H29a^4 ♀ 40 p37.39. 3(2–3b)
	a^5 ♀ 30 p23. 1. 2(2–3f)	b^1 ♂ 25 p39. 1. 2(2b)	b^2 ♂ 20 p37. 1. 1(2b)	b^3 ♂ 25 p39. 1. 2(2b)	...	...
	b^4 ♂ 30 p37. 1. 1(3b)	b^5 ♂ 40 p37. 1. 2(3b)	b^6 ♂ 30 p39. 1. 3(3b)	b^7 ♀ 45 p39. 1. 3(3b)	...	...
	c^1 ♀ 65 p39. 1. 2(4b)	...	...	...	...	...
4H20	a^1 ♀ 100zp36.36.30(6b)	a^2 ♀ 100xp35.35.30(6b)	a^3 ♀ 100zp39.39.28(5b)	a^4 ♀ 100xp23.23.15(6f)	3H21a^1 ♂ 80 p39.35. 2(5b)	3H20a^3 ♀ – p– – –(5b)
	a^5 ♀ 100xp10.23.15(6f)	b^1 ♂ 100xp23.23.17(6f)	b^2 ♂ 100zp39.39.28(5b)	b^3 ♀ 100zp39.35.30(5–6b)	...	...
	b^4 ♀ 100zp35.35.28(5b)	b^5 ♀ 100zp39.35.31(6b)	b^6 ♀ 100xp39.35.31(6b)	...	...	...
4H21	a^1 ♂ 75 p35.35.28(5b)	a^2 ♂ 100zp39.35.34(6b)	a^3 ♂ – p– – –(5b)	a^4 ♂ – p– – –(5b)	3H21a^1 ♂ 80 p39.35. 2(5b)	3H20a^4 ♀ 100zp39.35.28(5b)
	a^5 ♀ 100zp39.39.36(6b)	a^6 ♀ 75 p40.35. 2(5b)	...	...	...	...
4H22	a^1 ♂ 80 p39.39. 2(5b)	a^2 ♂ 85 p35.35. 2(5b)	a^3 ♂ 85 p35.35.28(5b)	a^4 ♂ 100zp35.35.30(5b)	3H21a^2 ♂ 80 p39.35. 2(5b)	3H20a^5 ♀ 100zp35.35.28(6b)
	a^5 ♀ 80 p35.35. 3(5b)	a^6 ♀ 100xp23.23.18(6f)	b^1 ♂ 85 p39.35.28(5b)	b^2 ♂ 80 p35.35.28(5b)	...	...
	b^3 ♀ 75 p39.39. 3(5b)	b^4 ♀ 80 p35.35.28(5b)	b^5 ♀ 100zp39.39.28(5b)	b^6 ♀ 75 p35.35. 2(5b)	...	...
	b^7 ♀ 100zp39.39.28(5b)	c^1 ♂ 75 p39.35. 2(5b)	c^2 ♂ 100xp15.15.10(6f)	c^3 ♂ 100xp15.15.10(6f)	...	...
	c^4 ♀ 80 p35.35. 2(5b)	c^5 ♀ 80 p35.35. 2(5b)	c^6 – p– – – –	...	...	...
4H23	a^1 – q – – – –	a^2 – p – – – –	b^1 ♂ 100xp37.19.31(6a–f)	b^2 ♂ 100xp37.19.9(5–6a–f)	3H5a^1 ♂ 100xq41.41.50(6g)	3H8a^2 ♀ 35 r41.41. 3(2–3g)
	b^3 ♂ 100xp37.19.31(6a–f)	b^4 ♀ 00 p– – –(alb.)	b^5 ♀ 00 p– – –(alb.)	b^6 ♀ 100xp19.19.31(6a–f)	...	...
	c^1 ♂ 100xr41.41.50(5–6g)	c^2 ♂ 100xr41.41.26(5–6g)	c^3 ♂ 100xp19.19. 9(5b)	c^4 ♂ 00 p– – –(alb.)	...	...
	c^5 ♂ 00 p– – –(alb.)	c^6 ♀ 100xq58.58.44(6e)	c^7 ♀ 00 p– – –(alb.)	...	...	...
4H24	a^1 ♂ 100xq58.57.55(5–6e)	a^2 ♀ – q – – –(5–6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 00 p– – –(alb.)	3H5a^1 ♂ 100xq57.57.55(6e)	3H8a^1 ♀ 100xq57.58.55(5e)
	a^5 – p – – – –	a^6 – q – – – –	a^7 – q– – – –	b^1 ♂ 100yq57.57.55(5e)	...	...
	b^2 ♂ 100xq58.57.55(5e)	b^3 ♂ 00 p– – –(alb.)	b^4 ♀ 100xq58.57.57(6e)	b^5 ♀ 100yq57.57.55(5e)	...	...
	b^6 – q– – – –	b^7 – q–	b^8 – q– – –	b^9 – q– – – –	...	...
	c^1 ♀ 100yq58.58.55(5e)	c^2 ♀ 00 p– – –(alb.)	c^3 – p – –	c^4 – q– – – –	...	...
4H25	x – p– – – –	a^1 ♂ 100xq57.57.56(6e)	a^2 ♀ 100xq58.58.55(6e)	b^1 ♂ 100xq58.58.55(6e)	3H5a^4 ♂ 100xq58.58.55(6e)	3H4a^5 ♀ 100xq57.57.56(6e)
	b^2 ♂ 100xq58.57.55(6e)	b^3 ♀ 100xq58.58.55(6e)	b^4 ♀ 100xq58.57.55(6e)	b^5 – p– – – –	...	...
	c^1 ♂ 100xq57.57.56(6e)	c^2 ♂ 100xp23.23.18(6f)	c^3 ♀ 100xq58.57.55(6e)	...	...	...
4H26	a^1 ♂ – q– – –(6e)	a^2 ♂ – q– – –(6e)	a^3 ♂ 85 q58.55. 2(4–5e)	a^4 ♀ 100xp26.23.23(6a–f)	3H5a^2 ♂ 100xq58.58.55(6e)	3H4a^6 ♀ 100xq58.57.55(6e)
	a^5 ♀ 100xq58.58.56(6e)	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ 100xp22.22.18(6f)	b^3 ♂ 100xp23.23.18(6f)	...	...
	b^4 – q– – –	c^1 ♂ 100xq58.57.55(6e)	c^2 ♂ 100xq58.57.55(6e)	c^3 ♂ 75 p26.19. 2(4f)	...	...

W. F. R. Weldon's *Records of Mice Matings* (continued).

Mating	Offspring				Parents			
					Father		Mother	
4H26	c^4 ♀ 100xq58.57.55(5e)	...	...	...	...	...	...	...
4H27	a^1 ♂ 100xp23.23.18(6f)	a^2 ♂ 100xp23.23.18(6f)	a^3 ♀ – p – – –(4f)	a^4 ♀ 100xp23.23.18(5f)	?	♂ – p – – –(6f)	3H4a^3	♀ 100xp23.22.18(6f)
	a^5 ♀ 100xp19.19. 9(6a)	a^6 ♀ 100xp23.23.17(6f)	a^7 ♀ 100xp19.19. 9(6a)	a^8 – p – – – –	...	...	...	...
	b^1 ♂ 100xp26.23.17(6f)	b^2 ♀ 100xp19.19. 6(6a)	b^3 ♀ 100xp19.19. 6(6a–f)	b^4 ♀ 100xp23.23.15(6f)	...	...	...	...
	b^5 ♀ 90 p37.19. 1(4a)	b^6 ♀ 45 p26.26. 3(3–4f)	b^7 – p – – – –	b^8 – p – – – –	...	...	...	...
	b^9 – p – – – –	b^{10} – p – – – –	c^1 ♂ 100xp23.23.15(6f)	c^2 ♂ 100xp26.23.18(6f)	...	...	...	...
	c^3 ♂ 100xp23.23.17(6f)	c^4 ♀ 100xp23.23.18(6f)	c^5 ♀ 100zp26.23.18(6f)	c^6 ♀ 70 p26.23. 2(4f)	...	...	...	...
	c^7 – p – – – –	...	...	...	...	...	...	...
4H28	a^1 ♂ 100xp23.23.17(5f)	a^2 ♂ 100xp19.19. 9(5a–f)	a^3 ♀ 90 p26.26. 2(4a–f)	a^4 ♀ 100xp19.19.31(6a–f)	?	♂ – p – – –(6f)	3H4a^4	♀ 100xp23.23.17(6f)
	a^5 ♀ 90 p19.19. 1(5a–f)	a^6 ♀ 50 p23. 1. 2(3f)	b^1 ♂ 100xp26.23.18(6f)	b^2 ♂ 100xp26.23.17(6f)	...	...	...	...
	b^3 ♂ 100zp19.19. 9(6b)	b^4 ♀ 100xp19. 9. 9(6b)	b^5 ♀ 100xp23.23.17(6f)	b^6 ♀ 100zp19.19. 9(5b)	...	...	...	...
	b^7 ♀ 50 p23. 1. 2(3f)	c^1 ♂ 100yp26.23. 2(4–5f)	c^2 ♂ 100xp19.19. 9(6a)	c^3 ♂ 100xp26.23.17(6f)	...	...	...	...
	c^4 ♂ 100xp15.15.17(6f)	c^5 ♀ 65 p26. 1. 2(4f)	c^6 ♀ 100xp19.19. 9(6a)	c^7 ♀ 100xp23.23.15(6f)	...	...	...	...
	c^8 ♀ 100xp23.23.17(6f)	c^9 – p – – – –	...	...	...	...	...	...
4H29	a^1 ♂ 90 p19.23. 2(4f)	a^2 ♂ 75 p26.23. 2(4f)	a^3 ♂ 65 p23.23. 2(4f)	a^4 ♂ 65 p26. 9. 1(4f)	3H42a^2	♂ 55 p23.23. 1(2–3f)	3H40a^1	♀ 70 p50.50. 1(4f)
	a^5 ♀ 75 p15.15. 2(4f)	a^6 ♀ 65 p50. 1. 2(4f)	a^7 ♀ 65 p23.23. 1(4f)	...	...	...	...	...
4H30	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♂ 100xq58.57.55(6e)	a^4 ♀ 100xq58.57.55(6e)	3H1b^4	♂ 100xq58.58.56(6e)	3H3a^5	♀ 100xq56.56.55(6e)
	a^5 ♀ 100xq58.57.55(6e)	a^6 ♀ 100xq58.57.56(6e)	a^7 – q – – –(6e)	a^8 – q – – –(6e)	...	...	...	...
4H31	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♀ 100xq58.58.55(6ew)	a^4 ♀ 100xq58.58.57(6ew)	3H1b^3	♂ 100xq57.57.56(5–6e)	3H3b^9	♀ 100xq58.58.55(6e)
	a^5 ♀ 100xq58.58.55(6e)	a^6 – p – – – –	a^7 – p – – – –	b^1 ♂ 100xq57.57.55(6ew)	...	...	...	...
	b^2 ♂ 100xq41.41.26(6gw)	b^3 ♂ 100xq?41.41.26(6g)	b^4 ♂ 100xp23.23.18(6f)	b^5 ♀ 100xp23.23.18(6f)	...	...	...	...
	c^1 ♂ 100xq58.58.55(6ew)	c^2 ♀ 100xp19.19. 9(6b)	c^3 ♀ 100xp23.23. 8(6f)	c^4 ♀ 100xq58.58.55(6e)	...	...	...	...
	c^5 ♀ 100xr40.40.50(6g)	...	...	...	...	...	...	...
4H33	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♀ 100xp15.15.15(6f)	a^4 ♀ 100xp15.15.18(6f)	3H3b^5	♂ 100xq58.57.55(6e)	3H11a^5	♀ 100xq57.57.56(5–6e)
	b^1 ♀ 100xq58.58.55(6ew)	b^2 ♂ 00 p – – –(alb. w)	c^1 ♂ – q – – –(6e)	c^2 ♂ – q – – –(6e)	...	...	...	...
	c^3 ♂ 00 p – – –(alb.)	c^4 ♀ 100xq58.58.55(5e)	c^5 ♀ 00 p – – –(alb.)	c^6 – q – – – –	...	...	...	...
	c^7 – q – – – –	c^8 – p – – – –	c^9 – p – – – –	...	...	...	...	...
4H36	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ – p – – –(6f)	3H4b^3	♂ 100xq58.58.55(6e)	3H6b^2	♀ 100xq58.58.55(5–6e)
	a^5 – q – – – –	b^1 ♂ – q – – –(6e)	b^2 ♂ 100xq58.58.55(6e)	b^3 ♂ 00 p – – –(alb.)	...	...	...	...
	b^4 ♀ – q – – –(6e)	b^5 ♀ 100xq58.58.55(6e)	b^6 – q – – – –	b^7 – p – – – –	...	...	...	...
4H37	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ – p – – –(6f)	a^3 ♀ 100xq58.57.56(6e)	a^4 ♀ 100xp26.26.17(6f)	3H4b^2	♂ 100xq58.58.55(6e)	3H6b^3	♀ 100xq56.57.55(5–6e)
	a^5 ♀ 100xq58.58.55(6e)	b^1 ♂ – q – – –(6e)	b^2 ♂ 100xq58.57.56(6e)	b^3 ♂ 100xq58.58.56(6e)	...	...	...	...
	b^4 ♀ 100xq58.58.55(6e)	b^5 ♀ 100xq58.58.55(6e)	b^6 ♀ 100xq57.57.56(6e)	c^1 ♂ 00 p – – –(alb.)	...	...	...	...
	c^2 ♂ 00 p – – –(alb.)	c^3 ♂ 00 p – – –(alb.)	c^4 ♀ 00 p – – –(alb.)	...	...	...	...	...
4H38	a^1 ♂ 100xp22.22.18(5f)	a^2 ♂ 100xp26.26.18(6f)	a^3 ♂ 100xp26.23.17(6f)	a^4 ♂ – p – – –(3–4f)	3H2b^3	♂ 100xp22.17.18(6f)	3H1a^4	♀ 100xp26.26.23(6f)
	a^5 ♂ 45 p37. 1. 2(3f)	a^6 ♂ 50 p26.23. 2(3f)	a^7 ♂ 55 p26.23. 2(3f)	a^8 – p – – – –	...	...	...	...
	a^9 – p – – – –	b^1 ♂ 40 p26.19. 2(2f)	b^2 ♂ 70 p26.23. 2(4f)	b^3 ♂ 100xp23.23.18(6f)	...	...	...	...
	b^4 ♂ 100xp26.26.18(6f)	b^5 ♂ 100xp26.23.18(6f)	b^6 ♂ 100xp26.26.23(6f)	b^7 ♀ 100xp26.26.23(6f)	...	...	...	...
	b^8 ♀ – p – – –(6f)	b^9 – p – – – –	b^{10} – p – – – –	b^{11} – p – – – –	...	...	...	...
	b^{12} – p – – – –	c^1 ♂ 100xp22.22.15(6f)	c^2 ♂ 100xp23.23.18(6f)	c^3 ♂ 100xp23.23.18(6f)	...	...	...	...
	c^4 ♂ 100xp26.23.18(6f)	c^5 ♂ 100yp23.23.15(6f)	c^6 ♂ 40 p26. 1. 2(2f)	c^7 ♀ 100xp26.26.23(6f)	...	...	...	...
	c^8 ♀ 100xp23.23.18(6f)	...	...	...	...	...	...	...
4H39	a^1 ♂ 60 p26. 1. 2(3f)	a^2 ♀ 65 p26.23. 2(3f)	a^3 ♀ 75 p26.26. 1(4f)	a^4 ♀ 100xp23.23.18(6f)	3H2b^3	♂ 100xp22.17.18(6f)	3H1a^5	♀ 100xp26.23.18(6f)
	b^1 ♂ 50 p26. 1. 2(3f)	b^2 ♂ 55 p26.26. 1(3f)	b^3 ♀ 100xp23.23.18(6f)	b^4 ♀ 100xp17.15.15(6f)	...	...	...	...
	c^1 ♀ 100xp23.23.17(6f)	c^2 ♀ 45 p26. 1. 1(3f)	...	...	...	...	...	...
4H40	a^1 ♂ 100xp15.15.18(6f)	a^2 ♂ 100xp23.23.18(6f)	a^3 ♀ 100xp23.23.17(6f)	b^1 ♂ 100xp23.23.17(6f)	3H7b^4	♂ 100xp23.23.17(6f)	3H3a^8	♀ 100xp15.15.15(6f)

	b^2 ♀ 100xp15.15.15(6f)	b^3 ♀ 100xp15.15.18(6f)	c^1 ♀ 100xp23.23.15(6f)	c^2 ♀ 100xp15.15.15(6f)	...	... ...	...	... ...
4H41	a^1 ♂ 100xp23.23.18(6f)	a^2 ♂ 100xp23.23.18(6f)	a^3 ♂ 100xp23.23.18(6f)	a^4 ♂ – p – – – (5f)	3H11b^1	♂ 100xp22.22.15(6f)	3H11a^2	♀ 100xp23.22.17(5f)
	a^5 ♂ 100xp23.23.18(5f)	a^6 ♀ 100xp23.23.17(6f)	a^7 – p – – – –	a^8 – p – – – –	...	... ...	...	... ...
	b^1 ♂ 100xp23.23.18(6f)	b^2 ♀ 100xp26.26.17(6f)	c^1 ♂ 100xp23.23.18(6f)	c^2 ♀ 100xp26.26.23(6f)	...	... ...	...	... ...
	c^3 ♀ – p – – – (6f)	... ...	... ...	... ...	...	... ...	...	... ...
4H43	a^1 ♂ 55 r41. 1. 2(3g)	a^2 ♂ – r – – – (4g)	a^3 ♂ 80 ? 41.50. 2(4g)	a^4 ♂ 100xp19.19. 9(4–5b)	3H1b^1	♂ 65 r41.50. 3(4g)	3H13b^3	♀ – r – – – (4g)
	a^5 ♀ 60 r41.41. 1(3–4g)	a^6 ♀ 80 p37. 1. 1(4b)	a^7 – q – – – –	a^8 – q – – – –	...	... ...	...	... ...
4H45	a^1 ♂ 100xp23.23.17(6f)	a^2 ♀ 100xp23.23.18(6f)	a^3 ♀ 100xp23.23.18(6fw)	a^4 – p – – – –	3H1c^1	♂ 100xp22.22.17(6f)	3H2b^5	♀ 100xp22.15.18(6f)
	a^5 – p – – – –	a^6 – p – – – –	a^7 – p – – – –	b^1 ♂ 100xp18. 3. 3(6f)	...	... ...	...	... ...
	b^2 ♂ 100xp26.23.18(6f)	b^3 ♀ – p – – – (6f)	b^4 ♀ 100xp23.23.18(6f)	b^5 ♀ 100xporr23.23.18(6f)	...	... ...	...	... ...
	b^6 – p – – – –	b^7 – p – – – –	b^8 – p – – – –	b^9 – p – – – –	...	... ...	...	... ...
	c^1 ♂ 100xp23.23.15(6f)	c^2 ♀ 100xp15.15.15(6f)	c^3 ♀ 100xp26.23.18(6f)	c^4 – p – – – –	...	... ...	...	... ...
	c^5 – p – – – –	c^6 – p – – – –	c^7 – p – – – –	... ...	...	... ...	...	... ...
4H46	a^1 ♂ 00 p – – – (alb.)	a^2 ♂ 00 p – – – (alb.)	a^3 ♂ 100xq58.57.56(5e)	a^4 ♀ 100xq58.58.57(6e)	3H2b^1	♂ 100xq57.56.55(5–6e)	3H11a^6	♀ 100xq58.58.55(5–6e)
	b^1 ♂ 60 q58. 1. 1(4e)	b^2 ♂ 100yq57.55. 1(5e)	b^3 ♂ 100xq58.57.55(6e)	b^4 ♀ 100xp17.17.17(6a–f)	...	... ...	...	... ...
	b^5 – p – – – –	b^6 – p – – – –	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ 100xq58.57.60(6e)	...	... ...	...	... ...
	c^3 ♂ 00 p – – – (alb.)	c^4 ♂ 00 p – – – (alb.)	c^5 ♀ 100xq57.57.55(5e)	c^6 ♀ 100xq58.57.56(6e)	...	... ...	...	... ...
4H47	a^1 ♂ 100xp22.22.18(6f)	a^2 ♂ 100xp22.22.15(6f)	a^3 ♀ 100xq57.57.55(6e)	a^4 ♀ 100xq58.58.56(6e)	3H11a^1	♂ 100xq57.57.55(5–6e)	3H9c^2	♀ 100xq58.58.55(5e)
	a^5 ♀ 00 p – – – (alb.)	a^6 ♀ 00 p – – – (alb.)	a^7 – p – – – –	b^1 ♂ 100xp23.23.18(6f)	...	... ...	...	... ...
	b^2 ♀ 100xp19.19. 8(6f)	c^1 ♂ 100xq58.57.55(6e)	c^2 ♂ 100xq58.57.55(6e)	c^3 ♂ 100xq57.57.55(6e)	...	... ...	...	... ...
	c^4 ♂ 100xq57.57.56(5e)	c^5 ♂ 100xp15.15.18(6f)	c^6 ♀ 100xq58.58.55(6e)	c^7 – p – – – –	...	... ...	...	... ...
4H48	a^1 ♂ 00 p – – – (alb.)	a^2 ♂ 00 p – – – (alb.)	a^3 ♂ 00 p – – – (alb.w)	a^4 ♀ 00 p – – – (alb.)	3H2c^1	♂ 100xq58.58.55(6e)	3H9c^1	♀ 100xq58.58.56(6e)
	a^5 ♀ 100xp23.23.18(6f)	a^6 ♀ – p – – – (6f)	a^7 ♀ 100xq58.58.56(6e)	b^1 ♂ 85 q58.55. 1(4e)	...	... ...	...	... ...
	b^2 ♂ 100xp22.22.18(6f)	b^3 ♂ 100xp23.23.17(6f)	b^4 ♀ 100xp23.15.18(6fw)	c^1 ♂ 100xq58.57.55(6e)	...	... ...	...	... ...
	c^2 ♂ 100xp23.23.18(6f)	c^3 ♂ 100xp17.17. 4(6fw)	c^4 ♂ 00 p – – – (alb.)	c^5 ♀ 100xq58.58.55(6e)	...	... ...	...	... ...
4H49	a^1 ♂ 100xp26.26.23(6f)	a^2 ♂ 100xq57.57.55(5e)	a^3 ♀ 100xq58.58.55(5e)	a^4 – q – – – –	3H3c^2	♂ 100xq57.58.56(6e)	3H7b^5	♀ 100xq58.57.55(5e)
	b^1 ♂ 100xq58.57.55(6e)	b^2 ♀ – q – – (6e)	b^3 ♀ 100xq58.58.55(5e)	b^4 ♀ 100xp26.26.23(6f)	...	... ...	...	... ...
	b^5 – q – – – –	b^6 – q – – – –	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ – q – – – (6e)	...	... ...	...	... ...
	c^3 ♀ 100xq57.57.55(6e)	c^4 ♀ 100xq58.58.55(5e)	... ...	... ...	...	... ...	...	... ...
4H50	a^1 ♂ – q – – – (4–5e)	a^2 ♂ 70 q57.57. 1(4e)	a^3 ♂ 100xp19.19.18(6f)	a^4 ♀ 100xq58.56.56(6e)	3H4c^5	♂ 95 q58.55. 2(5e)	3H1c^7	♀ 100xq58.58.56(6e)
	a^5 – q – – – –	b^1 ♂ 100xq58.58.55(6e)	c^1 ♂ 80 q58.55. 1(3e)	c^2 ♀ 80 q58.58. 1(4e)	...	... ...	...	... ...
	c^3 ♀ 70 p23.23. 1(4f)	... ...	... ...	... ...	...	... ...	...	... ...
4H51	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xq57.57.55(5–6e)	a^3 ♂ 100xq58.58.55(6e)	a^4 ♀ 100xq58.57.56(6e)	3H1c^5	♂ 100xq58.58.55(6e)	3H4c^8	♀ 100xq58.57.55(6e)
	a^5 ♀ 100xq57.57.56(5–6e)	a^6 ♀ 100xp23.23.18(6f)	a^7 – p – – – –	a^8 – p – – – –	...	... ...	...	... ...
	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ 100xq58.58.55(6e)	b^3 ♂ 100xq58.57.55(6e)	b^4 ♂ 100xq58.58.55(6e)	...	... ...	...	... ...
	b^5 ♀ 100xq58.57.55(6e)	b^6 ♀ 100xp23.23.18(6f)	b^7 ♀ 100xr40.40.37(6g)	b^8 ♀ 100xr40.40.37(6g)	...	... ...	...	... ...
	c^1 ♂ 100xp26.23.15(6f)	c^2 ♀ 100xq58 58.55(6e)	c^3 ♀ 100xp23.23.18(6f)	c^4 ♀ 100xd.r40.40.26(6g)	...	... ...	...	... ...
4H52	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ – q – – – (6e)	a^3 ♂ 100xp23.23.18(5f)	a^4 ♀ 100xq58.57.56(6e)	3H6b^4	♂ 100xq58.57.55(5–6e)	3H5c^1	♀ 100xq58.58.55(5e)
	b^1 ♂ 100xp26.23.15(6f)	b^2 ♀ 100yq58.58.56(5e)	b^3 – q – – – –	b^4 – q – – – –	...	... ...	...	... ...
	b^5 – q – – – –	c^1 ♂ 100yq58.58.56(5e)	... ...	... ...	...	... ...	...	... ...
4H54	a^1 ♀ 50 q58. 1. 3(4e)	a^2 ♀ – q – – – (4e)	a^3 ♀ 00 p – – – (alb.)	b^1 ♂ 95 q57.55. 1(5e)	3H16a^2	♂ 70 q58.58. 2(4e)	3H15c^6	♀ 60 q58. 1. 3(4e)
	b^2 ♂ 75 q57.56. 3(4e)	b^3 ♂ 55 q58.55. 1(3–4e)	b^4 ♂ 65 q57. 1. 1(4e)	b^5 ♀ 65 q57.55. 1(4e)	...	... ...	...	... ...
	b^6 ♀ 75 q58.57. 3(4e)	b^7 ♀ 00 p – – – (alb.)	c^1 ♂ 60 q57. 1. 1(4ew)	c^2 ♂ 45 q57. 1. 1(3e)	...	... ...	...	... ...
4H55	a^1 ♂ 80 q58.54. 2(4e)	a^2 ♂ – q – – – (3e)	a^3 ♂ 45 q61. 1. 2(3e)	a^4 ♀ 60 q58.55. 2(4e)	3H38a^2	♂ 75 q61.58. 2(4e)	3H15c^7	♀ – q – – – (3e)
	a^5 ♀ 70 q58. 1. 4(4e)	a^6 ♀ 00 p – – – (alb.)	... ...	... ...	...	... ...	...	... ...
4H56	a^1 ♂ 85 q57.55. 1(4e)	a^2 ♂ 80 q57. 1. 1(4e)	a^3 ♂ 75 q58. 1. 1(4e)	a^4 ♂ 85 q57.55. 2(4e)	3H15b^2	♂ 75 q58.55. 4(4e)	3H13b^2	♀ 70 q58. 1. 1(4e)
	a^5 ♂ 60 q58. 1. 2(4e)	a^6 – q – – – –	a^7 – q – – – –	a^8 – q – – – –	...	... ...	...	... ...
	b^1 ♂ 80 q58.57.52(4e)	b^2 ♂ 60 q58. 1. 1(4e)	b^3 ♂ 80 q57.55. 1(4e)	b^4 ♀ 80 q58.55. 3(4e)	...	... ...	...	... ...
	b^5 ♀ 90 q58.55. 1(4e)	b^6 – q – – – –	b^7 – q – – – –	b^8 – q – – – –	...	... ...	...	... ...
4H57	a^1 ♀ 80 q58.57. 2(4–5e)	b^1 ♂ – q – – – (6e)	b^2 ♂ 00 p – – – (alb.)	b^3 ♀ 00 p – – – (alb.)	3H15b^3	♂ – q – – – (4e)	3H16b^1	♀ 85 q56.55. 3(4–5e)

MATING	OFFSPRING				PARENTS	
					Father	Mother
4H26	c^4 ♀ 100xq58.57.55(5e)					
4H27	a^1 ♂ 100xp23.23.18(6f)	a^2 ♂ 100xp23.23.18(6f)	a^3 ♀ – p – – –(4f)	a^4 ♀ 100xp23.23.18(5f)	? ♂ – p – – –(6f)	3H4a^3 ♀ 100xp23.22.18(6f)
	a^5 ♀ 100xp19.19. 9(6a)	a^6 ♀ 100xp23.23.17(6f)	a^7 ♀ 100xp19.19. 9(6a)	a^8 – p – – – –		
	b^1 ♂ 100xp26.23.17(6f)	b^2 ♀ 100xp19.19. 6(6a)	b^3 ♀ 100xp19.19. 6(6a–f)	b^4 ♀ 100xp23.23.15(6f)		
	b^5 ♀ 90 p37.19. 1(4a)	b^6 ♀ 45 p26.26. 3(3–4f)	b^7 – p – – – –	b^8 – p – – – –		
	b^9 – p – – – –	b^{10} – p – – – –	c^1 ♂ 100xp23.23.15(6f)	c^2 ♂ 100xp26.23.18(6f)		
	c^3 ♂ 100xp23.23.17(6f)	c^4 ♀ 100xp23.23.18(6f)	c^5 ♀ 100xp26.23.18(6f)	c^6 ♀ 70 p26.23. 2(4f)		
	c^7 – p – – – –					
4H28	a^1 ♂ 100xp23.23.17(5f)	a^2 ♂ 100xp19.19. 9(5a–f)	a^3 ♀ 90 p26.26. 2(4a–f)	a^4 ♀ 100xp19.19.31(6a–f)	? ♂ – p – – –(6f)	3H4a^4 ♀ 100xp23.23.17(6f)
	a^5 ♀ 90 p19.19. 1(5a–f)	a^6 ♀ 50 p23. 1. 2(3f)	b^1 ♂ 100xp26.23.18(6f)	b^2 ♂ 100xp26.23.17(6f)		
	b^3 ♂ 100xp19.19. 9(6b)	b^4 ♀ 100xp19. 9. 9(6b)	b^5 ♀ 100xp23.23.17(6f)	b^6 ♀ 100xp19.19. 9(5b)		
	b^7 ♀ 50 p23. 1. 2(3f)	c^1 ♂ 100yp26.23. 2(4–5f)	c^2 ♂ 100xp19.19. 9(6a)	c^3 ♂ 100xp26.23.17(6f)		
	c^4 ♂ 100xp15.15.17(6f)	c^5 ♀ 65 p26. 1. 2(4f)	c^6 ♀ 100xp19.19. 9(6a)	c^7 ♀ 100xp23.23.15(6f)		
	c^8 ♀ 100xp23.23.17(6f)	c^9 – p – – –				
4H29	a^1 ♂ 90 p19.23. 2(4f)	a^2 ♂ 75 p26.23. 2(4f)	a^3 ♂ 65 p23.23. 2(4f)	a^4 ♂ 65 p26. 9. 1(4f)	3H42a^2 ♂ 55 p23.23. 1(2–3f)	3H40a^1 ♀ 70 p50.50. 1(4f)
	a^5 ♀ 75 p15.15. 2(4f)	a^6 ♀ 65 p50. 1. 2(4f)	a^7 ♀ 65 p23.23. 1(4f)			
4H30	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♂ 100xq58.57.55(6e)	a^4 ♀ 100xq58.57.55(6e)	3H1b^4 ♂ 100xq58.58.56(6e)	3H3a^5 ♀ 100xq56.56.55(6e)
	a^5 ♀ 100xq58.57.55(6e)	a^6 ♀ 100xq58.57.56(6e)	a^7 – q – – –(6e)	a^8 – q – – –(6e)		
4H31	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♀ 100xq58.58.55(6ew)	a^4 ♀ 100xq58.58.57(6ew)	3H1b^3 ♂ 100xq57.57.56(5–6e)	3H3b^9 ♀ 100xq58.58.55(6e)
	a^5 ♀ 100xq58.58.55(6e)	a^6 – p – – –	a^7 – p – – – –	b^1 ♂ 100xq57.57.55(6ew)		
	b^2 ♂ 100xq41.41.26(6gw)	b^3 ♂ 100xq?41.41.26(6g)	b^4 ♂ 100xp23.23.18(6f)	b^5 ♀ 100xp23.23.18(6f)		
	c^1 ♂ 100xq58.58.55(6ew)	c^2 ♀ 100xp19.19. 9(6b)	c^3 ♀ 100xp23.23. 8(6f)	c^4 ♀ 100xq58.58.55(6e)		
	c^5 ♀ 100xr40.40.50(6g)					
4H33	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♀ 100xp15.15.15(6f)	a^4 ♀ 100xp15.15.18(6f)	3H3b^5 ♂ 100xq58.57.55(6e)	3H11a^6 ♀ 100xq57.57.56(5–6e)
	b^1 ♀ 100xq58.58.55(6ew)	b^2 ♂ 00 p – – –(alb. w)	c^1 ♂ – q – – –(6e)	c^2 ♂ – q – – –(6e)		
	c^3 ♂ 00 p – – –(alb.)	c^4 ♀ 100xq58.58.55(5e)	c^5 ♀ 00 p – – –(alb.)	c^6 – q – – – –		
	c^7 – q – – – –	c^8 – p – – – –	c^9 – p – – – –			
4H36	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ – p – – –(6f)	3H4b^3 ♂ 100xq58.58.55(6e)	3H6b^2 ♀ 100xq58.58.55(5–6e)
	a^5 – q – – – –	b^1 ♂ – q – – –(6e)	b^2 ♂ 100xq58.58.55(6e)	b^3 ♂ 00 p – – –(alb.)		
	b^4 ♀ – q – – –(6e)	b^5 ♀ 100xq58.58.55(6e)	b^6 – q – – –	b^7 – p – – –		
4H37	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ – p – – –(6f)	a^3 ♀ 100xq58.57.56(6e)	a^4 ♀ 100xp26.26.17(6f)	3H4b^2 ♂ 100xq58.58.55(6e)	3H6b^3 ♀ 100xq56.57.55(5–6e)
	a^5 ♀ 100xq58.58.55(6e)	b^1 ♂ – q – – –(6e)	b^2 ♂ 100xq58.57.56(6e)	b^3 ♂ 100xq58.58.56(6e)		
	b^4 ♀ 100xq58.58.55(6e)	b^5 ♀ 100xq58.58.55(6e)	b^6 ♀ 100xq57.57.56(6e)	c^1 ♂ 00 p – – –(alb.)		
	c^2 ♂ 00 p – – –(alb.)	c^3 ♂ 00 p – – –(alb.)	c^4 ♀ 00 p – – –(alb.)			
4H38	a^1 ♂ 100xp22.22.18(5f)	a^2 ♂ 100xp26.26.18(6f)	a^3 ♂ 100xp26.23.17(6f)	a^4 ♂ – p – – –(3–4f)	3H2b^3 ♂ 100xp22.17.18(6f)	3H1a^4 ♀ 100xp26.26.23(6f)
	a^5 ♂ 45 p37. 1. 2(3f)	a^6 ♂ 50 p26.23. 2(3f)	a^7 ♂ 75 p26.23. 2(3f)	a^8 – p – – –		
	a^9 – p – – –	b^1 ♂ 40 p26.19. 2(2f)	b^2 ♂ 70 p26.23. 2(4f)	b^3 ♂ 100xp23.23.18(6f)		
	b^4 ♂ 100xp26.26.18(6f)	b^5 ♂ 100xp26.23.18(6f)	b^6 ♂ 100xp26.26.23(6f)	b^7 ♀ 100xp26.26.23(6f)		
	b^8 ♀ – p – – –(6f)	b^9 – p – – –	b^{10} – p – – –	b^{11} – p – – –		
	b^{12} – p – – –	c^1 ♂ 100xp22.22.15(6f)	c^2 ♂ 100xp23.23.18(6f)	c^3 ♂ 100xp23.23.18(6f)		
	c^4 ♂ 100xp26.23.18(6f)	c^5 ♂ 100yp23.23.15(6f)	c^6 ♂ 40 p26. 1. 2(2f)	c^7 ♀ 100xp26.26.23(6f)		
	c^8 ♀ 100xp23.23.18(6f)					
4H39	a^1 ♂ 60 p26. 1. 2(3f)	a^2 ♀ 65 p26.23. 2(3f)	a^3 ♀ 75 p26.26. 1(4f)	a^4 ♀ 100xp23.23.18(6f)	3H2b^3 ♂ 100xp22.17.18(6f)	3H1a^5 ♀ 100xp26.23.18(6f)
	b^1 ♂ 50 p26. 1. 2(3f)	b^2 ♂ 55 p26.26. 1(3f)	b^3 ♀ 100xp23.23.18(6f)	b^4 ♀ 100xp17.15.15(6f)		
	c^1 ♀ 100xp23.23.17(6f)	c^2 ♀ 45 p26. 1. 1(3f)				
4H40	a^1 ♂ 100xp15.15.18(6f)	a^2 ♂ 100xp23.23.18(6f)	a^3 ♀ 100xp23.23.17(6f)	b^1 ♂ 100xp23.23.17(6f)	3H7b^4 ♂ 100xp23.23.17(6f)	3H3a^8 ♀ 100xp15.15.15(6f)
	b^2 ♀ 100xp15.15.15(6f)	b^3 ♀ 100xp15.15.18(6f)	c^1 ♀ 100xp23.23.15(6f)	c^2 ♀ 100xp15.15.15(6f)		
4H41	a^1 ♂ 100xp23.23.18(6f)	a^2 ♂ 100xp23.23.18(6f)	a^3 ♂ 100xp23.23.18(6f)	a^4 ♂ – p – – –(5f)	3H11b^1 ♂ 100xp22.22.15(6f)	3H11a^2 ♀ 100xp23.22.17(5f)
	a^5 ♂ 100xp23.23.18(5f)	a^6 ♀ 100xp23.23.17(6f)	a^7 – p – – –	a^8 – p – – – –		
	b^1 ♂ 100xp23.23.18(6f)	b^2 ♀ 100xp26.26.17(6f)	c^1 ♂ 100xp23.23.18(6f)	c^2 ♀ 100xp26.26.23(6f)		
	c^3 ♀ – p – – –(6f)					
4H43	a^1 ♂ 55 r41. 1. 2(3g)	a^2 ♂ – r – – –(4g)	a^3 ♂ 80 r41.50. 2(4g)	a^4 ♂ 100xp19.19. 9(4–5b)	3H1b^1 ♂ 65 r41.50. 3(4g)	3H13b^3 ♀ – r – – –(4g)
	a^5 ♀ 60 r41.41. 1(3–4g)	a^6 ♀ 80 p37. 1. 1(4b)	a^7 – q – – –	a^8 – q		
4H45	a^1 ♂ 100xp23.23.17(6f)	a^2 ♀ 100xp23.23.18(6f)	a^3 ♀ 100xp23.23.18(6fw)	a^4 – p – – – –	3H1c^1 ♂ 100xp22.22.17(6f)	3H2b^5 ♀ 100xp22.15.18(6f)
	a^5 – p – – –	a^6 – p – – – –	a^7 – p – –	b^1 ♂ 100xp18. 3. 3(6f)		
	b^2 ♂ 100xp26.23.18(6f)	b^3 ♀ – p – – –(6f)	b^4 ♀ 100xp23.23.18(6f)	b^5 ♀ 100xporr23.23.18(6f)		
	b^6 – p	b^7 – p	b^8 – p	b^9 – p		
	c^1 ♂ 100xp23.23.15(6f)	c^2 ♀ 100xp15.15.15(6f)	c^3 ♀ 100xp26.23.18(6f)	c^4 – p		
	c^5 – p – – – –	c^6 – p – – – –	c^7 – p			
4H46	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 100xq58.57.56(5e)	a^4 ♀ 100xq58.58.57(6e)	3H2b^1 ♂ 100xq57.56.55(5–6e)	3H11a^6 ♀ 100xq58.58.55(5–6e)
	b^1 ♂ 60 q58. 1. 1(4e)	b^2 ♂ 100yq57.55. 1(5e)	b^3 ♂ 100xq58.57.55(6e)	b^4 ♀ 100xp17.17.17(6a–f)		
	b^5 – p – – – –	b^6 – p – –	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ 100xq58.57.60(6e)		
	c^3 ♂ 00 p – – –(alb.)	c^4 ♂ 00 p – – –(alb.)	c^5 ♀ 100xq57.57.55(5e)	c^6 ♀ 100xq58.57.56(6e)		
4H47	a^1 ♂ 100xp22.22.18(6f)	a^2 ♂ 100xp22.22.15(6f)	a^3 ♀ 100xq57.57.55(6e)	a^4 ♀ 100xq58.58.56(6e)	3H11a^1 ♂ 100xq57.57.55(5–6e)	3H9c^2 ♀ 100xq58.58.55(5e)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♀ 00 p – – –(alb.)	a^7 – p	b^1 ♂ 100xp23.23.18(6f)		
	b^2 ♀ 100xp19.19. 8(6f)	c^1 ♂ 100xq58.57.55(6e)	c^2 ♂ 100xq58.57.55(6e)	c^3 ♂ 100xq57.57.55(6e)		
	c^4 ♂ 100xq57.57.56(5e)	c^5 ♂ 100xp15.15.18(6f)	c^6 ♀ 100xq58.58.55(6e)	c^7 – p		
4H48	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb. w)	a^4 ♀ 00 p – – –(alb.)	3H2c^1 ♂ 100xq58.58.55(6e)	3H9c^1 ♀ 100xq58.58.56(6e)
	a^5 ♀ 100xp23.23.18(6f)	a^6 ♀ – p – – –(6f)	a^7 ♀ 100xq58.58.56(6e)	b^1 ♂ 85 q58.55. 1(4e)		
	b^2 ♂ 100xp22.22.18(6f)	b^3 ♂ 100xp23.23.17(6f)	b^4 ♀ 100xp23.15.18(6fw)	c^1 ♂ 100xq58.57.55(6e)		
	c^2 ♂ 100xp23.23.18(6f)	c^3 ♂ 100xp17.17. 4(6fw)	c^4 ♂ 00 p – – –(alb.)	c^5 ♀ 100xq58.58.55(6e)		
4H49	a^1 ♂ 100xp26.26.23(6f)	a^2 ♂ 100xq57.57.55(5e)	a^3 ♀ 100xq58.58.55(5e)	a^4 – q – – –	3H3c^2 ♂ 100xq57.58.56(6e)	3H7b^5 ♀ 100xq58.57.55(5e)
	b^1 ♂ 100xq58.57.55(6e)	b^2 ♀ – q – – –(6e)	b^3 ♀ 100xq58.58.55(5e)	b^4 ♀ 100xp26.26.23(6f)		
	b^5 – q – – – –	b^6 – q	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ – q – – –(6e)		
	c^3 ♀ 100xq57.57.55(6e)	c^4 ♀ 100xq58.58.55(5e)				
4H50	a^1 ♂ – q – – –(4–5e)	a^2 ♂ 70 q57.57. 1(4e)	a^3 ♂ 100xp19.19.18(6f)	a^4 ♀ 100xq58.56.56(6e)	3H4c^5 ♂ 95 q58.55. 2(5e)	3H1c^7 ♀ 100xq58.58.56(6e)
	a^5 – q – – – –	b^1 ♂ 100xq58.58.55(6e)	c^1 ♂ 80 q58.55. 1(3e)	c^2 ♀ 80 q58.58. 1(4e)		
	c^3 ♀ 70 p23.23. 1(4f)					
4H51	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xq57.57.55(5–6e)	a^3 ♂ 100xq58.58.55(6e)	a^4 ♀ 100xq58.57.56(6e)	3H1c^5 ♂ 100xq58.58.55(6e)	3H4c^8 ♀ 100xq58.57.55(6e)
	a^5 ♀ 100xq57.57.56(5–6e)	a^6 ♀ 100xp23.23.18(6f)	a^7 – p – – – –	a^8 – p		
	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ 100xq58.58.55(6e)	b^3 ♂ 100xq58.57.55(6e)	b^4 ♂ 100xq58.58.55(6e)		
	b^5 ♀ 100xq58.57.55(6e)	b^6 ♀ 100xp23.23.18(6f)	b^7 ♀ 100xr40.40.37(6g)	b^8 ♀ 100xr40.40.37(6g)		
	c^1 ♂ 100xp26.23.15(6f)	c^2 ♀ 100xq58.58.55(6e)	c^3 ♀ 100xp23.23.18(6f)	c^4 ♀ 100xd.r40.40.26(6g)		
4H52	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ – q – – –(6e)	a^3 ♂ 100xp23.23.18(5f)	a^4 ♀ 100xq58.57.56(6e)	3H6b^4 ♂ 100xq58.57.55(5–6e)	3H5c^1 ♀ 100xq58.58.55(5e)
	b^1 ♂ 100xp26.23.15(6f)	b^2 ♀ 100yq58.58.56(5e)	b^3 – q – –	b^4 – q		
	b^5 – q – – – –	c^1 ♂ 100yq58.58.56(5e)				
4H54	a^1 ♀ 70 q58. 1. 3(4e)	a^2 ♀ – q – – –(4e)	a^3 ♀ 00 p – – –(alb.)	b^1 ♂ 95 q57.55. 1(5e)	3H16a^2 ♂ 70 q58.58. 2(4e)	3H15c^6 ♀ 60 q58. 1. 3(4e)
	b^2 ♂ 75 q57.56. 3(4e)	b^3 ♀ 55 q58.55. 1(3–4e)	b^4 ♂ 65 q57. 1. 1(4e)	b^5 ♀ 65 q57.55. 1(4e)		
	b^6 ♀ 75 q58.57. 3(4e)	b^7 ♀ 00 p – – –(alb.)	c^1 ♂ 60 q57. 1. 1(4ew)	c^2 ♂ 45 q57. 1. 1(3e)		
4H55	a^1 ♂ 80 q58.54. 2(4e)	a^2 ♂ – q – – –(3e)	a^3 ♂ 45 q61. 1. 2(3e)	a^4 ♀ 60 q58.55. 2(4e)	3H38a^2 ♂ 75 q61.58. 2(4e)	3H15c^7 ♀ – q – – –(3e)
	a^5 ♀ 70 q58. 1. 1(4e)	a^6 ♀ 00 p – – –(alb.)				
4H56	a^1 ♂ 85 q57.55. 1(4e)	a^2 ♂ 80 q57. 1. 1(4e)	a^3 ♂ 75 q58. 1. 1(4e)	a^4 ♂ 85 q57.55. 2(4e)	3H15b^2 ♂ 75 q58.55. 4(4e)	3H13b^2 ♀ 70 q58. 1. 1(4e)
	a^5 ♂ 60 q58. 1. 2(4e)	a^6 – q – – – –	a^7 – q – – – –	a^8 – q – – – –		
	b^1 ♂ 80 q58.57.52(4e)	b^2 ♂ 60 q58. 1. 1(4e)	b^3 ♂ 80 q57.55. 1(4e)	b^4 ♀ 80 q58.55. 3(4e)		
	b^5 ♀ 90 q58.55. 1(4e)	b^6 – q – – – –	b^7 – q – – – –	b^8 – q – – – –		
4H57	a^1 ♀ 80 q58.57. 2(4–5e)	b^1 ♂ – q – – –(6e)	b^2 ♂ 00 p – – –(alb.)	b^3 ♀ 00 p – – –(alb.)	3H15b^3 ♂ – q – – –(4e)	3H16b^1 ♀ 85 q56.55. 3(4–5e)

MATING	OFFSPRING				PARENTS	
					Father	Mother
4*H*57	b^4 – q– – – –	b^5 – q– – – –	b^6 – q– – – –	b^7 – q– – – –	... ...	... ...
	b^8 – q– – – –	b^9 – q– – – –	... ...	... ...	... ...	... ...
4*H*58	a^1 ♂ 35 p23. 1. 2(2f)	a^2 ♀ 65 q58. 1. 2(4e)	a^3 55 p23. 1. 1(3f)	b^1 ♂ 00 p– – –(alb.)	3*H*1c^2 ♂ 70 q58.58. 2(4e)	3*H*16b^2 ♀ 65 q56.58. 1(4e)
	b^2 ♂ – p– – –(3f)	b^3 ♂ 60 p26. 1. 1(3f–g)	b^4 – q– – – –	b^5 – p– – – –	... ...	... ...
	b^6 – p– – – –	c^1 ♂ 00 p– – –(alb.)	c^2 ♂ 00 p– – –(alb.)	c^3 ♀ 85 q58. 1. 4(4e)	... ...	... ...
	c^4 – p– – – –	c^5 – q– – – –	c^6 – q– – –	... ...	... ...	... ...
4*H*59	a^1 ♂ 70 q58.52. 2(4e)	a^2 ♂ 80 q58. 4. 4(4e)	a^3 ♂ 80 q58.57. 1(4e)	a^4 ♂ 70 q57. 1. 1(3–4e)	3*H*15c^3 ♂ 50 q58. 1. 2(3e)	3*H*1c^6 ♀ 65 q58.55. 2(4e)
	a^5 ♂ 55 q57.57. 1(3e)	a^6 ♀ 80 q58.55. 1(4–5e)	a^7 – q– – – –	a^8 – q– – – –	... ...	... ...
	b^1 ♂ 55 q58. 1. 1(3e)	b^2 ♂ 55 q58. 1. 1(3e)	b^3 ♂ – q– – –(4e)	b^4 ♂ – q– – –(4e)	... ...	... ...
	b^5 – q– – – –	b^6 – q– – – –	b^7 – q– – – –	b^8 – q– – – –	... ...	... ...
	b^9 – q– – – –	b^{10} – q– – – –	c^1 ♂ 45 q55. 1. 3(3e)	c^2 ♂ 75 q58.55. 2(3–4e)	... ...	... ...
	c^3 ♂ 60 q58. 1. 1(4e)	c^4 ♀ 45 q58. 1. 2(3e)	c^5 ♀ 60 q58. 1. 1(3–4e)	c^6 ♀ 65 q58. 1. 3(3–4e)	... ...	... ...
	c^7 ♀ 75 q58.55. 3(4–5e)	c^8 – q– – – –	c^9 – q– – – –	... ...	... ...	... ...
4*H*60	a^1 ♂ 35 q58. 1. 1(2–3e)	a^2 ♂ 35 q58. 1. 1(2–3e)	a^3 ♂ 55 q58.57. 1(3e)	a^4 ♀ 25 q61. 1. 1(2e)	3*H*15c^4 ♂ 30 q58. 1. 1(2–3e)	3*H*1b^5 ♀ 45 q55.56. 3(3e)
	a^5 ♀ 40 q58. 1. 1(3e)	a^6 ♀ 45 q58. 1. 1(3e)	a^7 – q– – – –	b^1 ♂ 25 q58. 1. 1(1e)	... ...	... ...
	b^2 ♂ 30 q58. 1. 1(2e)	b^3 ♂ – q– – –(3–4e)	b^4 ♀ 30 q61.61. 1(1–2e)	b^5 – q– – – –	... ...	... ...
	b^6 – q– – – –	b^7 – q– – – –	b^8 – q– – – –	b^9 – q– – – –	... ...	... ...
	c^1 ♂ 55 q58. 1. 2(3–4e)	c^2 ♂ 65 q58.55. 1(3–4e)	c^3 ♀ 85 q58.57. 4(4e)	c^4 ♂ – q– – –(2e)	... ...	... ...
	c^5 – q– – – –	c^6 – q– – – –	c^7 – q– – – –	... ...	... ...	... ...
4*H*61	a^1 ♂ 75 q58.57. 2(4e)	a^2 ♂ 35 q58. 1. 1(2e)	a^3 ♀ 70 q61.60. 1(4e)	a^4 ♀ 60 q58.57. 3(3e)	3*H*15c^2 ♂ 65 q58.58. 2(4e)	3*H*40b^6 ♀ 85 q58.56. 2(4–5e)
	a^5 ♀ 70 q58.57. 1(4e)	a^6 ♀ 95 q58.58.55(4–5e)	a^7 – q– – – –	a^8 – q– – – –	... ...	... ...
	b^1 ♂ 60 q58.55. 1(4e)	b^2 ♂ 90 q58.57. 2(4–5e)	b^3 ♀ – q– – –(2e)	c^1 ♂ 90 q58.55. 2(4e)	... ...	... ...
	c^2 ♂ – q– – –(4e)	c^3 ♂ – q– – –(4e)	c^4 ♂ 85 q57.55. 2(4e)	c^5 ♂ 60 q58.55. 2(4e)	... ...	... ...
	c^6 ♂ 85 q58.57.55(4e)	c^7 ♂ 60 q58.55. 2(3e)	c^8 ♂ 35 q58. 1. 2(2e)	c^9 – q– – – –	... ...	... ...
4*H*62	a^1 ♂ 90 q58.55. 2(4e)	a^2 ♂ 85 q58.57. 3(4e)	a^3 ♂ 95 q58.58.55(4e)	a^4 ♀ 85 q57. 1. 1(4e)	3*H*15c^1 ♂ 60 q58. 1. 1(4e)	3*H*40b^4 ♀ 85 q57.55. 1(4e)
	a^5 ♀ 85 q57. 1. 1(4e)	a^6 ♀ 00 p– – –(alb.)	a^7 – q– – – –	b^1 ♂ 90 q58.57. 4(4–5e)	... ...	... ...
	b^2 ♂ 85 q58.58. 3(4–5e)	b^3 ♂ 90 q57.57. 1(4–5e)	b^4 ♂ 00 p– – –(alb.)	b^5 ♀ 90 q58.58. 2(5e)	... ...	... ...
	b^6 – q– – – –	b^7 – q– – – –	... ...	... ...	... ...	... ...
4*H*63	a^1 ♂ 80 q57.56. 1(3–4e)	a^2 ♂ 60 q57.55. 3(3–4e)	a^3 ♂ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	3*H*4c^4 ♂ 45 q58.58. 1(2–3e)	3*H*40b^7 ♀ 65 q58.57. 2(3e)
	a^5 ♀ 50 q58.57. 3(3–4e)	a^6 ♀ 60 q58. 1. 3(3–4e)	a^7 ♀ 00 p– – –(alb.)	a^8 – q– – – –	... ...	... ...
	b^1 ♂ 45 q58. 1. 1(2e)	b^2 ♂ 55 q58.55. 2(3e)	b^3 ♀ 60 q58.57. 2(3e)	b^4 ♀ 60 q58. 1. 1(3e)	... ...	... ...
	b^5 ♀ – q– – –(3–4e)	b^6 ♀ 90 q58.57. 1(4e)	b^7 ♀ 00 p– – –(alb.)	b^8 ♀ 00 p– – –(alb.)	... ...	... ...
	c^1 ♂ 00 p– – –(alb.)	c^2 ♂ – q– – –(2–3e)	c^3 ♂ 55 q58. 3. 3(3e)	c^4 ♂ 60 q58. 1. 2(3e)	... ...	... ...
	c^5 ♀ 55 q58. 1. 2(3e)	c^6 ♀ 65 q58. 1. 4(4e)	c^7 – q– – – –	c^8 – q– – – –	... ...	... ...
	c^9 – q– – – –	... ...	... ...	... ...	... ...	... ...
4*H*64	a^1 ♂ 55 q58. 1. 2(3e)	a^2 ♀ 90 q57.55. 2(4–5e)	a^3 ♀ 65 q58. 1. 1(4–5e)	a^4 ♀ 65 q58.57. 3(4e)	3*H*40b^1 ♂ 90 q58.55. 2(4e)	3*H*38b^3 ♀ 75 q58.57. 3(4e)
	a^5 ♀ 50 q58. 1. 2(3e)	a^6 ♀ 85 q58. 1. 3(4e)	b^1 ♂ 85 q58.55. 3(4e)	b^2 ♀ 60 q58.57. 3(4e)	... ...	... ...
	b^3 ♀ 80 q57.57. 3(4e)	b^4 ♀ – q– – –(4e)	b^5 – q– – – –	b^6 – q– – – –	... ...	... ...
	b^7 – q– – – –	b^8 – q– – – –	c^1 ♂ 85 q57.55. 1(4e)	c^2 ♂ 75 q58.55. 1(4e)	... ...	... ...
	c^3 ♀ 75 q58.55. 1(3–4e)	... ...	... ...	... ...	... ...	... ...
4*H*65	a^1 ♂ 55 q58. 1. 1(3e)	a^2 ♀ 90 q58.55. 2(4e)	a^3 ♀ 95 q58.55. 3(4e)	a^4 ♀ 90 q58.55. 2(4e)	3*H*14b^1 ♂ 65 q58.58. 2(4e)	3*H*38b^4 ♀ 75 q58.57. 2(4e)
	a^5 ♀ 75 q45.45. 1(4ch)	a^6 ♀ 90 q55.44. 4(4–5ch)	a^7 ♀ 00 p– – –(alb.)	a^8 ♀ 00 p– – –(alb.)	... ...	... ...
	a^9 – p– – – –	b^1 ♀ 55 q57.44. 4(4ch)	b^2 ♀ 70 q57.58. 1(4ch)	b^3 ♂ 55 q58.58. 1(4e)	... ...	... ...
	b^4 ♂ 80 q58.55. 3(4e)	b^5 ♂ – q– – –(3e)	b^6 ♂ 00 p– – –(alb.)	b^7 ♂ 95 q45.45.45(5ch)	... ...	... ...
	b^8 ♂ 00 p– – –(alb.)	b^9 – q– – – –	c^1 ♂ 65 q58.57. 3(4e)	c^2 ♂ 00 p– – –(alb.)	... ...	... ...
	c^3 ♀ 95 q58.55. 4(4e)	c^4 ♀ 95 q58.58.55(4e)	c^5 – p– – – –	... ...	... ...	... ...

4H66	a^1 ♂ 100xq57.57.55(5e)	a^2 ♂ 100xq58.57.55(5–6e)	a^3 ♂ 100yq58.57.55(5e)	a^4 ♂ 55 q58.55. 3(3–4e)	3H6b^5 ♂ 100xq57.57.56(5e)	3H38b^5 ♀ 85 q57.55. 4(4e)
	a^5 ♀ – q– – –(6e)	a^6 ♀ 100yq58.58.55(5e)	a^7 ♀ 80 q58.57. 1(4e)	a^8 ♀ 45 q58. 1. 4(3e)	… … … …	… … … …
	a^9 – q– – – –	b^1 ♂ 40 q61. 1. 1(2–3e)	b^2 ♂ 55 q61. 1. 1(4e)	b^3 ♀ 100xq58.58.57(5e)	… … … …	… … … …
	b^4 ♀ 100xq57.58.55(6e)	b^5 – q– – – –	… … …	… … …	… … … …	… … … …
4H67	a^1 ♂ 75 q58.57. 2(3–4e)	a^2 ♂ 60 r41. 1. 2(3–4g)	a^3 ♀ 65 q61.61. 1(4e)	a^4 ♀ 85 q58.58. 2(4e)	3H13b^1 ♂ 60 q58.55. 2(3e)	3H38b^6 ♀ 80 q57.55. 6(4e)
	a^5 ♀ 55 q61. 1. 1(3e)	a^6 – q– – – –	b^1 ♂ – ?– – –(4g)	b^2 ♂ 75 p23. 1.18(3–4f)	… … … …	… … … …
	b^3 ♀ 95 q58.58.55(4–5e)	b^4 – p– – – –	b^5 – q– – – –	b^6 – q– – – –	… … … …	… … … …
	b^7 – q– – – –	b^8 – q– – – –	b^9 – q– – – –	b^{10} – q– – – –	… … … …	… … … …
	c^1 ♀ – q– – –(4e)	c^2 ♀ 65 q58.57. 3(4e)	c^3 ♀ 65 q57. 1. 1(4e)	c^4 ♀ 80 q58.57. 2(4e)	… … … …	… … … …
	c^5 ♀ 85 q58.58. 4(4–5e)	c^6 ♀ 90 q58.58.55(4–5e)	c^7 ♀ 55 r41. 1. 1(3g)	c^8 – q– – – –	… … … …	… … … …
4H69	a^1 ♂ 100zp39.35. 9(5b)	a^2 ♂ 100xp39.39.36(5b)	a^3 ♂ 100xp39.35.30(5b)	a^4 ♀ – p– – –(?b)	3H27b^1 ♂ 30 p39. 1. 1(2b)	3H19b^1 ♀ 100xp39.39.34(6b)
	a^5 ♀ 80 p39.39.28(5b)	a^6 ♀ 75 p39.39.28(5b)	a^7 ♀ 100zp39.39.30(5b)	a^8 – p– – – –	… … … …	… … … …
	a^9 – p– – – –	b^1 ♂ 100xp39.39.30(5b)	b^2 ♂ 100zp39.39.28(5b)	b^3 ♀ 100yp22.15.18(5f)	… … … …	… … … …
	c^1 ♀ 80 p39.39.30(5b)	c^2 ♂ 100xp39.39.31(6b)	c^3 ♂ 100zp39.33.30(5b)	c^4 ♂ 80 p39.35. 6(5b)	… … … …	… … … …
	c^5 ♂ 75 p39.35. 3(5b)	c^6 ♂ 100zp39.39.28(5b)	c^7 – p– – – –	… … …	… … … …	… … … …
4H71	a^1 ♂ 30 p23. 1. 1(2fw)	a^2 ♂ 70 p26.23. 1(4fw)	a^3 ♀ 95 p23.23.18(4fw)	a^4 ♀ 65 p26. 1. 1(4fw)	4H10b^1 ♂ 90 p26.18. 2(4fw)	3H3b^8 ♀ 100yp23.23.18(6fw)
	a^5 ♀ – p– – –(4fw)	b^1 ♂ 90 p26.18. 1(3–4fw)	b^2 ♀ 100xp23.23.18(6fw)	b^3 ♀ 100xp19.23.18(6fw)	… … … …	… … … …
	b^4 ♀ 100xp23.23.18(6fw)	… … … …	… … …	… … …	… … … …	… … … …
4H73	a^1 ♂ 25 q45. 1. 3(1ch)	a^2 ♂ – q– – –(2ch)	a^3 ♂ – q– – –(4ch)	a^4 ♂ 00 p– – –(alb.)	3H14c^1 ♂ 40 q45. 1. 2(3ch)	3H14a^5 ♀ – q– – –(5ch)
	a^5 – q– – – –	b^1 ♂ 50 q44. 1. 3(4ch)	b^2 ♂ 00 p– – –(alb.)	b^3 ♀ 30 q44. 1. 3(3ch)	… … … …	… … … …
	b^4 ♀ 80 q44.44. 3(4ch)	b^5 ♀ 00 p– – –(alb.)	b^6 00 p– – –(alb.)	… … …	… … … …	… … … …
4H74	a^1 ♂ 80 p39.39.30(5b)	a^2 ♂ – p– – –(5b)	a^3 ♂ 75 p39.35. 3(5b)	a^4 ♀ 100zp39.39.28(5b)	3H53a^2 ♂ 100zp39.39.29(5b)	3H57a^5 ♀ 80 p39.39. 4(5b)
	a^5 ♀ 80 p39.35.28(5b)	b^1 ♂ 75 p39.35. 3(5b)	b^2 ♂ – p– – –(5b)	b^3 ♂ 100zp39.35.30(5b)	… … … …	… … … …
	b^4 ♀ 100zp39.35. 3(5b)	b^5 – p– – – –	b^6 – p– – – –	b^7 – p– – – –	… … … …	… … … …
	b^8 – p– – – –	c^1 ♀ 95 p35.35.30(5b)	c^2 ♀ 80 p35.35. 3(5b)	c^3 ♀ 100zp39.35.30(5b)	… … … …	… … … …
4H75	a^1 ♂ 75 p36.36. 4(5b)	a^3 ♀ – p– – –(5b)	a^4 ♀ 100zp35.36.30(5b)	a^5 ♀ 100zp39.35.30(5b)	3H57a^1 ♂ 100zp39.35.28(5b)	3H52a^2 ♀ 100zp39.35. 3(5b)
	a^6 – p– – – –	b^1 ♂ – p– – –(5b)	b^2 ♂ 100zp39.35.28(5b)	b^3 ♀ 80 p40.36.28(5b)	… … … …	… … … …
	b^4 ♀ 75 p39.35.28(5b)	b^5 ♀ 75 p39.39. 4(5b)	b^6 – p– – – –	b^7 – p– – – –	… … … …	… … … …
	b^8 – p– – – –	c^1 ♂ – p– – –(5b)	c^2 ♂ 75 p36.36. 3(5b)	c^3 ♀ 80 p39.39.28(5b)	… … … …	… … … …
	c^4 ♀ 75 p35.35. 3(5b)	c^5 ♀ 80 p35.35. 3(5b)	… … …	… … …	… … … …	… … … …
4H76	a^1 ♂ 45 p37. 1. 1(2b)	a^2 ♂ 75 p35.35. 4(5b)	a^3 ♂ 55 p35.30. 3(4b)	a^4 ♂ 100xp35.35.30(5b)	3H53b^3 ♂ 100zp35.35.30(6b)	3H54c^3 ♀ 100zp35.35. 9(5b)
	a^5 ♂ 25 p37. 1. 1(2b)	a^6 ♀ 100zp35.35.28(5b)	a^7 ♀ 80 p35.35. 3(5bw)	a^8 ♀ 15 p39. 1. 1(1bw)	… … … …	… … … …
4H77	a^1 ♂ – p– – –(6f)	a^2 ♂ 100xp15.15.10(6f)	a^3 ♀ 100xp26.23.15(6f)	a^4 ♀ 100xp15.15.15(6f)	3H70b^1 ♂ 100xp15.15.18(6f)	3H45c^3 ♀ 100xp23.15.18(6f)
	a^5 ♀ 100xp15.15.18(6f)	b^1 ♂ 100xp23.23.15(6f)	b^2 ♂ 100xp23.23.17(6f)	b^3 ♂ 100xp23.23.18(6f)	… … … …	… … … …
	b^4 ♀ 100xp23.23.15(6f)	b^5 ♀ 100xp15.15.15(6f)	… … …	… … …	… … … …	… … … …
4H79	a^1 ♂ 100xp23.23.15(5f)	a^2 ♂ 100xp26.26.15(5f)	a^3 ♀ 100xp26.26.23(6f)	a^4 – p– – – –	3H60a^3 ♂ 65 p23.15. 1(4f)	3H64c^3 ♀ 100xp23.23.17(6f)
	a^5 – p– – – –	a^6 – p– – – –	… … …	… … …	… … … …	… … … …
4H80	a^1 ♂ 100xp23.23.15(5f)	a^2 ♂ 100xp15.15.15(5f)	a^3 ♀ 100xp23.23.15(6f)	a^4 ♀ 100xp23.23.15(6f)	3H60a^1 ♂ 50 p23.23. 1(3f)	3H64c^4 ♀ 100xp13.10.10(6f)
	a^5 – p– – – –	b^1 ♂ 100xp23.23.18(6f)	b^2 ♂ 100xp26.26.15(6f)	b^3 ♀ 100xp26.26.23(6f)	… … … …	… … … …
	c^1 ♂ 100xp15.15.10(6f)	c^2 ♂ 100xp15.15.10(6f)	c^3 ♀ 100xp13.13.15(6f)	c^4 ♀ 100xp13.13.15(5f)	… … … …	… … … …
4H82	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 65 q61.60. 1(3e)	a^3 ♀ 85 q61.60. 2(4e)	b^1 ♂ 00 p– – –(alb.)	3H71c^1 ♂ 100xq58.57.55(6e)	3H67c^3 ♀ 100xq58.58.55(6e)
	b^2 ♀ 85 q61.60. 3(4–5e)	b^3 ♀ 00 p– – –(alb.)	c^1 ♂ 90 q58.60. 2(5e)	c^2 ♂ 00 p– – –(alb.)	… … … …	… … … …
	c^3 ♀ 100xq60.57.55(6e)	c^4 ♀ 100xq58.60.55(6e)	c^5 ♀ 00 p– – –(alb.)	… … …	… … … …	… … … …
4H84	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♂ 100xq57.55.56(6e)	a^4 ♂ 90 q58.57.55(5e)	3H71c^3 ♂ 100xq57.57.52(5e)	3H70c^2 ♀ 100xq58.58.56(6e)
	a^5 ♀ 100xq58.60.55(5e)	a^6 ♀ 100xp23.23.15(6f)	b^1 ♂ 60 q61. 1. 1(3e)	b^2 ♀ 100xq58.57.55(6e)	… … … …	… … … …
	b^3 ♀ 100xq58.57.55(6e)	b^4 ♀ 100xq58.58.55(6e)	b^5 ♀ 100xq58.58.55(5–6e)	c^1 ♂ 100xq58.60.55(6e)	… … … …	… … … …
	c^2 ♂ 60 q61. 1. 2(3e)	c^3 ♀ 100xq58.58.60(6e)	c^4 ♀ 100xq58.60.55(5–6e)	c^5 ♀ 80 q58.60.60(3e)	… … … …	… … … …
4H85	a^1 ♂ 100xq57.60.55(6e)	a^2 ♂ 100xq57.60.60(6e)	a^3 ♂ 100xp18.15.15(6f)	a^4 ♀ 100xp23.23 18(6f)	3H70c^1 ♂ 65 q58. 1. 1(3e)	3H45c^2 ♀ 100xq58.58.55(6e)
	a^5 ♀ 100xp22.22.17(6f)	b^1 ♀ 100xq61.61.60(6e)	b^2 ♀ 100xq61.60.60(6e)	… … …	… … … …	… … … …
4H87	a^1 ♀ 70 r41.41. 4(3 4g)	… … … …	… … …	… … …	3H59a^1 ♂ 100xq41.41.50(6e–g)	3H59c^4 ♀ 80 r41.50. 1(4g)

MATING	OFFSPRING				PARENTS Father	PARENTS Mother
4H57	b^4 – q – – – –	b^5 – q – – – –	b^6 – q – – – –	b^7 – q – – – –		
	b^8 – q – – – –	b^9 – q – – – –	...	...		
4H58	a^1 ♂ 85 p23. 1. 2(2f)	a^2 ♀ 65 q58. 1. 2(4e)	a^3 [illegible] p23. 1. 1(3f)	b^1 ♂ 00 p – – – (alb.)	3H1c^2 ♂ 70 q58.58. 2(4e)	3H16b^2 ♀ 65 q56.58. 1(4e)
	b^2 ♂ – p – – – (3f)	b^3 ♂ 60 p26. 1. 1(3f–g)	b^4 – q – – – –	b^5 – p – – – –		
	b^6 – p – – – –	c^1 ♂ 00 p – – – (alb.)	c^2 ♂ 00 p – – – (alb.)	c^3 ♀ 85 q58. 1. 4(4e)		
	c^4 – p – – – –	c^5 – q – – – –	c^6 – q – – – –	...		
4H59	a^1 ♂ 70 q58.52. 2(4e)	a^2 ♂ 80 q58. 4. 4(4e)	a^3 ♂ 80 q58.57. 1(4e)	a^4 ♂ 70 q57. 1. 1(3–4e)	3H15c^3 ♂ 50 q58. 1. 2(3e)	3H1c^6 ♀ 65 q58.55. 2(4e)
	a^5 ♂ 55 q57.57. 1(3e)	a^6 ♀ 80 q58.55. 1(4–5e)	a^7 – q – – – –	a^8 – q – – – –		
	b^1 ♂ 55 q58. 1. 1(3e)	b^2 ♂ [illegible] q58. 1. 1(3e)	b^3 ♂ – q – – – (4e)	b^4 ♂ – q – – – (4e)		
	b^5 – q – – – –	b^6 – q – – – –	b^7 – q – – – –	b^8 – q – – – –		
	b^9 – q – – – –	b^{10} – q – – – –	c^1 ♂ 45 q55. 1. 3(3e)	c^2 ♂ 75 q58.55. 2(3–4e)		
	c^3 ♂ 60 q58. 1. 1(4e)	c^4 ♀ 45 q58. 1. 2(3e)	c^5 ♀ 60 q58. 1. 1(3–4e)	c^6 ♀ 65 q58. 1. 3(3–4e)		
	c^7 ♀ 75 q58.55. 3(4–5e)	c^8 – q – – – –	c^9 – q – – – –	...		
4H60	a^1 ♂ 35 q58. 1. 1(2–3e)	a^2 ♂ 35 q58. 1. 1(2–3e)	a^3 ♂ 55 q58.57. 1(3e)	a^4 ♀ 25 q61. 1. 1(2e)	3H15c^4 ♂ 30 q58. 1. 1(2–3e)	3H1b^6 ♀ 45 q55.56. 3(3e)
	a^5 ♀ 40 q58. 1. 1(3e)	a^6 ♀ 45 q58. 1. 1(3e)	a^7 – q – – – –	b^1 ♂ 25 q58. 1. 1(1e)		
	b^2 ♂ 30 q58. 1. 1(2e)	b^3 ♂ – q – – – (3–4e)	b^4 ♀ 30 q61.61. 1(1–2e)	b^5 – q – – – –		
	b^6 – q – – – –	b^7 – q – – – –	b^8 – q – – – –	b^9 – q – – – –		
	c^1 ♂ 55 q58. 1. 2(3–4e)	c^2 ♂ 65 q58.55. 1(3–4e)	c^3 ♀ 85 q58.57. 4(4e)	c^4 ♂ – q – – – (2e)		
	c^5 – q – – – –	c^6 – q – – – –	c^7 – q – – – –	...		
4H61	a^1 ♂ 75 q58.57. 2(4e)	a^2 ♂ 35 q58. 1. 1(2e)	a^3 ♀ 70 q61.60. 1(4e)	a^4 ♀ 60 q58.57. 3(3e)	3H15c^2 ♂ 65 q58.58. 2(4e)	3H40b^6 ♀ 85 q58.56. 2(4–5e)
	a^5 ♀ 70 q58.57. 1(4e)	a^6 ♀ 95 q58.58.55(4–5e)	a^7 – q – – – –	a^8 – q – – – –		
	b^1 ♂ 60 q58.55. 1(4e)	b^2 ♂ 90 q58.57. 2(4–5e)	b^3 ♀ – q – – – (2e)	c^1 ♂ 90 q58.55. 2(4e)		
	c^2 ♂ – q – – – (4e)	c^3 ♂ – q – – – (4e)	c^4 ♂ 85 q57.55. 2(4e)	c^5 ♂ 60 q58.55. 2(4e)		
	c^6 ♂ 85 q58.57.55(4e)	c^7 ♂ 60 q58.55. 2(3e)	c^8 ♂ 35 q58. 1. 2(2e)	c^9 – q – – – –		
4H62	a^1 ♂ 90 q58.55. 2(4e)	a^2 ♂ 85 q58.57. 3(4e)	a^3 ♂ 95 q58.58.55(4e)	a^4 ♀ 85 q57. 1. 1(4e)	3H15c^1 ♂ 60 q58. 1. 1(4e)	3H40b^4 ♀ 85 q57.55. 1(4e)
	a^5 ♀ 85 q57. 1. 1(4e)	a^6 ♀ 00 p – – – (alb.)	a^7 – q – – – –	b^1 ♂ 90 q58.57. 4(4–5e)		
	b^2 ♂ 85 q58.58. 3(4–5e)	b^3 ♂ 90 q57.57. 1(4–5e)	b^4 ♂ 00 p – – – (alb.)	b^5 ♀ 90 q58.58. 2(5e)		
	b^6 – q	b^7 – q	...	...		
4H63	a^1 ♂ 80 q57.56. 1(3–4e)	a^2 ♂ 60 q57.55. 3(3–4e)	a^3 ♂ 00 p – – – (alb.)	a^4 ♂ 00 p – – – (alb.)	3H4c^4 ♂ 45 q58.58. 1(2–3e)	3H40b^7 ♀ 65 q58.57. 2(3e)
	a^5 ♀ 50 q58.57. 3(3–4e)	a^6 ♀ 60 q58. 1. 3(3–4e)	a^7 ♀ 00 p – – – (alb.)	a^8 – q – – – –		
	b^1 ♂ 45 q58. 1. 1(2e)	b^2 ♂ 55 q58.55. 2(3e)	b^3 ♀ 60 q58.57. 2(3e)	b^4 ♀ 60 q58. 1. 1(3e)		
	b^5 ♀ – q – – – (3–4e)	b^6 ♀ 90 q58.57. 1(4e)	b^7 ♀ 00 p – – – (alb.)	b^8 ♀ 00 p – – – (alb.)		
	c^1 ♂ 00 p – – – (alb.)	c^2 ♂ – q – – – (2–3e)	c^3 ♂ 55 q58. 3. 3(3e)	c^4 ♂ 60 q58. 1. 2(3e)		
	c^5 ♀ 55 q58. 1. 2(3e)	c^6 ♀ 65 q58. 1. 4(4e)	c^7 – q – – – –	c^8 – q – – – –		
	c^9 – q –	...	...	...		
4H64	a^1 ♂ 55 q58. 1. 2(3e)	a^2 ♀ 90 q57.55. 2(4–5e)	a^3 ♀ 65 q58. 1. 1(4–5e)	a^4 ♀ 65 q58.57. 3(4e)	3H40b^1 ♂ 90 q58.55. 2(4e)	3H38b^3 ♀ 75 q58.57. 3(4e)
	a^5 ♀ 50 q58. 1. 2(3e)	a^6 ♀ 85 q58. 1. 3(4e)	b^1 ♂ 85 q58.55. 3(4e)	b^2 ♀ 60 q58.57. 3(4e)		
	b^3 ♀ 80 q57.57. 3(4e)	b^4 ♀ – q – – – (4e)	b^5 – q – – – –	b^6 – q – – – –		
	b^7 – q – – – –	b^8 – q – – – –	c^1 ♂ 85 q57.55. 1(4e)	c^2 ♂ 75 q58.55. 1(4e)		
	c^3 ♀ 75 q58.55. 1(3–4e)	...	...	...		
4H65	a^1 ♂ 55 q58. 1. 1(3e)	a^2 ♀ 90 q58.55. 2(4e)	a^3 ♀ 95 q58.55. 3(4e)	a^4 ♀ 90 q58.55. 2(4e)	3H14b^1 ♂ 65 q58.58. 2(4e)	3H38b^1 ♀ 75 q58.57. 2(4e)
	a^5 ♀ 55 q45.45. 1(4ch)	a^6 ♀ 90 q55.44. 4(4–5ch)	a^7 ♀ 00 p – – – (alb.)	a^8 ♀ 00 p – – – (alb.)		
	a^9 – p – – – –	b^1 ♀ 55 q57.44. 4(4ch)	b^2 ♀ 70 q57.58. 1(4ch)	b^3 ♂ 55 q58.58. 1(4e)		
	b^4 ♂ 80 q58.55. 3(4e)	b^5 ♂ – q – – – (3e)	b^6 ♂ 00 p – – – (alb.)	b^7 ♂ 95 q45.45.45(5ch)		
	b^8 ♂ 00 p – – – (alb.)	b^9 – q – – – –	c^1 ♂ 65 q58.57. 3(4e)	c^2 ♂ 00 p – – – (alb.)		
	c^3 ♀ 95 q58.55. 4(4e)	c^4 ♀ 95 q58.58.55(4e)	c^5 – p – – – –	...		
4H66	a^1 ♂ 100xq57.57.55(5e)	a^2 ♂ 100xq58.57.55(5–6e)	a^3 ♂ 100yq58.57.55(5e)	a^4 ♂ 55 q58.55. 3(3–4e)	3H6b^5 ♂ 100xq57.57.56(5e)	3H38b^6 ♀ 85 q57.55. 4(4e)
	a^5 ♀ – q – – – (6e)	a^6 ♀ 100xq58.58.[illegible]	a^7 ♀ 80 q58.57. 1(4e)	a^8 ♀ [illegible] q58. 1. 1(3e)		
	a^9 – q	b^1 ♂ 70 q61. 1. 1(2–3e)	b^2 ♂ 55 q61. 1. 1(4e)	b^3 ♀ 100xq58.58.57(5e)		
	b^4 ♀ 100xq57.58.55(6e)	b^5 – q	...			
4H67	a^1 ♂ [illegible] q58.57. 2(3–4e)	a^2 ♂ 60 r41. 1. 2(3–4g)	a^3 ♀ 65 q61.61. 1(4e)	a^4 ♀ 85 q58.58. 2(4e)	3H13b^1 ♂ 60 q58.55. 2(3e)	3H38$b^{[illegible]}$ ♀ 80 q57.55. 6(4e)
	a^5 ♀ 55 q61. 1. 1(3e)	a^6 – q	b^1 ♂ – r – – – (4g)	b^2 ♂ 75 p23. 1.18(3–4f)		
	b^3 ♀ 95 q58.58.55(4–5e)	b^4 – p	b^5 – q – – – –	b^6 – q		
	b^7 – q	b^8 – q	b^9 – q – – – –	b^{10} – q		
	c^1 ♀ – q – – – (4e)	c^2 ♀ 65 q58.57. 3(4e)	c^3 ♀ 65 q57. 1. 1(4e)	c^4 ♀ 80 q58.57. 2(4e)		
	c^5 ♀ 85 q58.58. 4(4–5e)	c^6 ♀ 90 q58.58.55(4–5e)	c^7 ♀ [illegible] r41. 1. 1(3g)	c^8 – q		
4H69	a^1 ♂ 100zp39.39.29(5b)	a^2 ♂ 100zp39.39.36(5b)	a^3 ♂ 100zp39.35.30(5b)	a^4 ♀ – p – – – (1b)	3H27b^1 ♂ 30 p39. 1. 1(2b)	3H19b^3 ♀ 100zp39.39.34(6b)
	a^5 ♀ 80 p39.39.28(5b)	a^6 ♀ 75 p34.39.28(5b)	a^7 ♀ 100 p39.39.30(5b)	a^8 – p – – – –		
	a^9 – p	b^1 ♂ 100zp39.39.30(5b)	b^2 ♂ 100zp39.39.28(5b)	b^3 ♀ 100yp23.15.18(5f)		
	c^1 ♀ 80 p39.39.30(5b)	c^2 ♂ 100zp39.39.31(6b)	c^3 ♂ 100zp39.33.30(5b)	c^4 ♂ 80 p39.35. 6(5b)		
	c^5 ♂ 75 p39.35. 3(5b)	c^6 ♂ 100zp39.39.28(5b)	c^7 – p			
4H71	a^1 ♂ 50 p23. 1. 1(2fw)	a^2 ♂ 70 p26.23. 1(4fw)	a^3 ♀ 95 p23.23.18(4fw)	a^4 ♀ 65 p26. 1. 1(4fw)	4H10b^1 ♂ 90 p26.18. 2(4fw)	3H3b^8 ♀ 100yp23.23.18(6fw)
	a^5 ♀ – p – – – (1fw)	b^1 ♂ 80 p26.18. 1(3–4fw)	b^2 ♀ 100xp23.23.18(6fw)	b^3 ♀ 100xp19.23.18(6fw)		
	b^4 ♀ 100xp23.23.18(6fw)	...	...			
4H73	a^1 ♂ 45 q45. 1. 3(4ch)	a^2 ♂ – q – – – (2ch)	a^3 ♂ – q – – – (4ch)	a^4 ♂ 00 p – – – (alb.)	3H14c^1 ♂ 40 q45. 1. 2(3ch)	3H14a^6 ♀ – q – – – (5ch)
	a^5 – q	b^1 ♂ 50 q44. 1. 3(4ch)	b^2 ♂ 00 p – – – (alb.)	b^3 ♀ 30 q44. 1. 3(3ch)		
	b^4 ♀ 80 q44.44. 3(4ch)	b^5 ♀ 00 p – – – (alb.)	b^6 00 p – – – (alb.)	...		
4H74	a^1 ♂ 80 p39.39.30(5b)	a^2 ♂ – p – – – (5b)	a^3 ♂ 75 p39.35. 3(5b)	a^4 ♀ 100zp39.39.28(5b)	3H53a^2 ♂ 100zp39.39.29(5b)	3H57a^6 ♀ 80 p39.39. 4(5b)
	a^5 ♀ 80 p39.35.28(5b)	b^1 ♂ 75 p39.35. 3(5b)	b^2 ♂ – p – – – (5b)	b^3 ♂ 100zp39.35.30(5b)		
	b^4 ♀ 100zp39.35. 3(5b)	b^5 – p –	b^6 – p	b^7 – p		
	b^8 – p	c^1 ♀ 90 p35.35.30(5b)	c^2 ♀ 80 p35.35. 3(5b)	c^3 ♀ 100 p39.35.30(5b)		
4H75	a^1 ♂ [illegible] p36.36. 4(5b)	a^2 ♀ – p – – – (5b)	a^3 ♀ 100 p35.36.30(5b)	a^4 ♀ 100zp39.35.30(5b)	3H57a^1 ♂ 100zp39.35.28(5b)	3H52a^2 ♀ 100zp39.35. 3(5b)
	a^5 – p	b^1 ♂ – p – – – (5b)	b^2 ♂ 100zp39.35.28(5b)	b^3 ♀ 80 p40.36.28(5b)		
	b^4 ♀ [illegible] p39.35.28(5b)	b^5 ♀ 75 p39.39. 4(5b)	b^6 – p	b^7 – p – – – –		
	b^8 – p – – – –	c^1 ♂ – p – – – (5b)	c^2 ♂ 75 p36.36. 3(5b)	c^3 ♀ 80 p39.39.28(5b)		
	c^4 ♀ 75 p35.35. 3(5b)	c^5 ♀ 80 p35.35. 3(5b)	...	...		
4H76	a^1 ♂ 45 p37. 1. 1(2b)	a^2 ♂ 75 p35.35. 4(5b)	a^3 ♂ 55 p35.30. 3(4b)	a^4 ♂ 100xp35.35.30(5b)	3H53b^3 ♂ 100zp35.35.30(6b)	3H54c^3 ♀ 100zp35.35. 9(5b)
	a^5 ♂ 25 p37. 1. 1(2b)	a^6 ♀ 100zp35.35.28(5b)	a^7 ♀ 80 p35.35. 3(5bw)	a^8 ♀ 15 p39. 1. 1(1bw)		
4H77	a^1 ♂ – p – – – (6f)	a^2 ♂ 100xp15.15.10(6f)	a^3 ♀ 100xp26.23.15(6f)	a^4 ♀ 100xp15.15.15(6f)	3H70b^1 ♂ 100xp15.15.18(6f)	3H45c^3 ♀ 100xp23.15.18(6f)
	a^5 ♀ 100xp15.15.18(6f)	b^1 ♂ 100xp23.23.15(6f)	b^2 ♂ 100xp23.23.17(6f)	b^3 ♂ 100xp23.23.18(6f)		
	b^4 ♀ 100xp23.23.15(6f)	b^5 ♀ 100xp15.15.15(6f)	...	...		
4H79	a^1 ♂ 100xp23.23.15(5f)	a^2 ♂ 100xp26.26.15(5f)	a^3 ♀ 100xp26.26.23(6f)	a^4 – p – – – –	3H60a^3 ♂ 65 p23.15. 1(4f)	3H64c^1 ♀ 100xp23.23.17(6f)
	a^5 – p	a^6 – p				
4H80	a^1 ♂ 100xp23.23.15(5f)	a^2 ♂ 100xp15.15.15(5f)	a^3 ♀ 100xp23.23.15(6f)	a^4 ♀ 100xp23.23.15(6f)	3H60a^1 ♂ 50 p23.23. 1(3f)	3H64c^4 ♀ 100xp13.10.10(6f)
	a^5 – p	b^1 ♂ 100xp23.23.18(6f)	b^2 ♂ 100xp26.26.15(6f)	b^3 ♀ 100xp26.26.23(6f)		
	c^1 ♂ 100xp15.15.10(6f)	c^2 ♂ 100xp15.15.10(6f)	c^3 ♀ 100xp13.13.15(6f)	c^4 ♀ 100xp13.13.15(5f)		
4H82	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 65 q61.60. 1(3e)	a^3 ♀ 85 q61.60. 2(4e)	b^1 ♂ 00 p – – – (alb.)	3H71c^1 ♂ 100xq58.57.55(6e)	3H67c^3 ♀ 100xq58.58.55(6e)
	b^2 ♀ 85 q61.60. 3(4–5e)	b^3 ♀ 00 p – – – (alb.)	c^1 ♂ 90 q58.60. 2(5e)	c^2 ♂ 00 p – – – (alb.)		
	c^3 ♀ 100xq60.57.55(6e)	c^4 ♀ 100xq58.60.55(6e)	c^5 ♀ 00 p – – – (alb.)	...		
4H84	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♂ 100xq57.55.56(6e)	a^4 ♂ 90 q58.57.55(5e)	3H71c^3 ♂ 100xq57.57.52(5e)	3H70c^2 ♀ 100xq58.58.56(6e)
	a^5 ♀ 100xq58.60.55(6e)	a^6 ♀ 100xp23.23.15(6f)	b^1 ♂ 60 q61. 1. 1(3e)	b^2 ♀ 100xq58.57.55(6e)		
	b^3 ♀ 100xq58.57.55(6e)	b^4 ♀ 100xq58.58.55(6e)	b^5 ♀ 100xq58.58.55(5–6e)	c^1 ♂ 100xq58.60.55(6e)		
	c^2 ♂ 60 q61. 1. 2(3e)	c^3 ♀ 100xq58.58.60(6e)	c^4 ♀ 100xq58.60.55(5–6e)	c^5 ♀ 80 q58.60.60(3e)		
4H85	a^1 ♂ 100xq57.60.55(6e)	a^2 ♂ 100xq57.60.60(6e)	a^3 ♂ 100xp18.15.15(6f)	a^4 ♀ 100xp23.23.18(6f)	3H70c^1 ♂ 65 q58. 1. 1(3e)	3H45c^2 ♀ 100xq58.58.55(6e)
	a^5 ♀ 100xp23.22.17(6f)	b^1 ♀ 100xq61.61.60(6e)	b^2 ♀ 100xq61.60.60(6e)			
4H87	a^1 ♀ [illegible] r41.41. 1(3–4g)		...		3H59a^1 ♂ 100xq41.41.50(6e–g)	3H59c^4 ♀ 80 r41.50. 1(4g)

W. F. R. WELDON'S *Records of Mice Matings (continued)*.

MATING	OFFSPRING				PARENTS	
					Father	Mother
5*H*1	a^1 ♀ 100*xp*26.23.15(6*f*)	a^2 ♀ 100*xp*23.23.22(6*f*)	a^3 ♀ 100*xp*26.26.23(6*f*)	a^4 ♀ 100*xp*23.23.17(6*f*)	4*H*17 {a^1 ♂ 100*xp*23.23.18(6*f*)} {a^2 ♂ 100*xp*26.26.17(6*f*)}	4*H*17a^3 ♀ 100*xp*23.23.18(6*f*)
	a^5 – *p* – – –(6*f*)	... ...	... ...	... ...	... ...	... ...
5*H*3	? – *p* – – – –	a^1 ♂ 100*xp*13.13.15(6*f*)	a^2 ♂ 100*xp*26.26 23(6*f*)	a^3 ♂ 100*xp*24.24.17(6*f*)	4*H*27a^2 ♂ 100*xp*23.23.18(6*f*)	4*H*22a^6 ♀ 100*xp*23.23.18(6*f*)
	a^4 ♂ 100*xp*23.23.17(6*f*)	a^5 ♀ 100*xp*13.13.18(6*f*)	a^6 ♀ 100*xp*23.23.17(6*f*)	a^7 ♀ – *p* – – –(6*f*)	... ...	... ...
5*H*4	a^1 ♀ 100*xp*15.15.15(6*f*)	a^2 ♀ 100*xp*23.23.15(6*f*)	a^3 ♀ 100*xp*23.23.17(6*f*)	a^4 ♀ 100*xp*15.15.15(6*f*)	4*H*17a^1 ♂ 100*xp*23.23.18(6*f*)	4*H*20a^4 ♀ 100*xp*23.23.15(6*f*)
	a^5 – *p* – – – –	b^1 ♂ 100*xp*22.15.15(6*f*)	b^2 ♂ 100*xp*22.22.17(6*f*)	b^3 ♀ 100*xp*26.23.15(6*f*)	... ...	... ...
	c^1 ♂ 100*xp*23.23.15(6*f*)	c^2 ♂ 100*xp*17.17.15(6*f*)	c^3 ♂ 100*xp*22.22.15(6*f*)	c^4 ♀ 100*xp*23.23.17(6*f*)	... ...	... ...
	c^5 ♀ 00 *p* – – –(alb.)	... ...	... ...	... ...	... ...	... ...
5*H*5	a^1 ♂ 100*xp*15.15.10(6*f*)	a^2 ♂ 100*xp*15.15.10(6*f*)	a^3 ♂ 100*xp*23.23.15(6*f*)	a^4 ♂ 100*xp*23.23.15(6*f*)	4*H*28a^1 ♂ 100*xp*23.23.17(5*f*)	4*H*20a^5 ♀ 100*xp*10.23.15(6*f*)
5*H*6	a^1 ♀ 100*xp*23.23.15(6*f*)	a^2 ♀ 100*xp*23.23.15(6*f*)	a^3 ♀ 100*xp*26.23.17(6*f*)	a^4 – *p* – – – –	4*H*15a^2 ♂ 100*xp*23.23.18(6*f*)	4*H*27a^4 ♀ 100*xp*23.23.18(5*f*)
	b^1 ♂ 100*xp*23.23.15(6*f*)	b^2 ♂ 100*xp*23.23.17(6*f*)	b^3 ♀ 100*xp*23.23.15(6*f*)	b^4 ♀ 100*xp*26.23.10(6*f*)	... ...	... ...
	b^5 ♀ 100*xp*26.23.15(5*f*)	... ...	... ...	... ...	... ...	... ...
5*H*7	a^1 ♂ 100*xp*23.23.17(6*fw*)	a^2 ♂ 100*xp*23.23.15(6*fw*)	a^3 ♂ 100*xp*23.15.17(6*f*)	a^4 ♂ 100*xp*26.26.23(5*f*)	4*H*4a^1 ♂ 100*xp*23.23.18(6*f*)	4*H*27a^6 ♀ 100*xp*23.23.17(6*f*)
	a^5 ♀ 100*xp*23.23.18(6*f*)	a^6 ♀ 100*xp*23.23.18(6*fw*)	b^1 ♂ 100*xp*23.23.17(6*f*)	b^2 ♂ 100*xp*23.23.17(6*f*)	... ...	... ...
	b^3 ♂ 95 *p*23.23.17(4–5*f*)	b^4 ♀ 100*xp*23.23.17(6*f*)	b^5 ♀ 100*xp*26.23.17(6*f*)	... ...	... ...	... ...
5*H*8	a^1 ♂ 100*xp*26.23.15(5*f*)	a^2 ♀ 100*xp*23.23.17(6*f*)	a^3 ♀ 95 *p*15.15.18(5*f*)	a^4 ♀ 100*xp*22.22.18(5*f*)	4*H*17a^2 ♂ 100*xp*26.26.17(6*f*)	4*H*19a^5 ♀ 30 *p*23. 1. 2(2–3*f*)
	a^5 – *p* – – – –	b^1 ♂ 100*xp*26.26.17(6*f*)	b^2 ♂ 100*xp*23.23.15(6*f*)	b^3 ♂ 100*xp*26.23.17(5*f*)	... ...	... ...
	b^4 ♀ 100*xp*26.23.17(6*f*)	b^5 ♀ 100*xp*23.23.18(5*f*)	b^6 – *p* – – – –	c^1 ♂ – *p* – – –(6*f*)	... ...	... ...
	c^2 ♂ – *p* – – –(6*f*)	c^3 ♂ 100*yp*26.26.23(5*f*)	c^4 ♂ 100*xp*26.23.15(5*f*)	c^5 ♀ 100*xp*23.23.18(6*f*)	... ...	... ...
	c^6 ♀ 100*xp*26.23.15(6*f*)	c^7 – *p* – – – –	c^8 – *p* – – – –	... ...	... ...	... ...
5*H*10	a^1 ♂ 100*xq*58.55.55(6*e*)	a^2 ♂ 100*xq*58.57.55(6*e*)	a^3 ♂ 100*xq*57.55.55(5*e*)	a^4 ♀ 100*xq*58.58.55(6*e*)	4*H*1a^1 ♂ 100*xq*58.57.55(6*e*)	4*H*4a^4 ♀ 100*xq*58.57.55(5*e*)
	a^5 ♀ 100*xq*58.58.55(5*e*)	a^6 – *q* – – – –	a^7 – *q* – – – –	... ...	... ...	... ...
5*H*11	a^1 ♂ 100*xq*58.57.55(6*e*)	a^2 ♂ 100*xq*58.57.56(6*e*)	a^3 ♂ 100*xq*58.57.56(6*e*)	a^4 ♂ 100*xq*58.57.55(6*e*)	4*H*1a^2 ♂ 100*xq*58.57.55(6*e*)	4*H*2a^7 ♀ 100*xq*58.58.57(6*e*)
	a^5 ♂ – *q* – – –(6*e*)	a^6 – *q* – – – –	b^1 ♂ 100*xq*57.56.55(6*e*)	b^2 ♂ 100*xq*57.56.55(6*e*)	... ...	... ...
	b^3 ♀ 100*xq*58.57.56(6*e*)	b^4 ♀ 100*xq*58.57.55(6*e*)	b^5 ♀ 100*xq*58.58.56(6*e*)	c^1 ♂ 100*xq*58.57.55(6*e*)	... ...	... ...
	c^2 ♂ 100*xq*58.57.55(6*e*)	c^3 ♂ 100*xq*57.57.56(6*e*)	c^4 ♂ 100*xq*58.57.55(6*e*)	c^5 ♂ 100*xq*58.57.56(6*e*)	... ...	... ...
	c^6 ♀ 100*xq*58.58.55(6*e*)	c^7 – *q* – – – –	... ...	... ...	... ...	... ...
5*H*12	a^1 ♂ 100*xq*58.57.55(6*e*)	a^2 ♂ 100*xq*57.57.55(6*e*)	a^3 ♂ 100*xq*57.57.55(6*e*)	a^4 ♀ 100*xq*58.58.55(6*e*)	4*H*1a^3 ♂ 100*xq*58.57.55(6*e*)	4*H*7a^2 ♀ 100*xq*58.57.56(5–6*e*)
	b^1 ♂ 100*xq*58.58.55(5–6*e*)	b^2 ♀ 100*xq*58.57.55(6*e*)	b^3 – *q* – – – –	b^4 – *q* – – – –	... ...	... ...
	b^5 – *q* – – – –	b^6 – *q* – – – –	b^7 – *q* – – – –	... ...	... ...	... ...
5*H*14	a^1 ♂ 100*xr*41.41.26(6*g*)	a^2 ♀ 100*xq*58.57.56(6*e*)	a^3 ♀ 100*xq*58.57.56(6*e*)	a^4 ♀ 100*xq*58.57.56(5–6*e*)	4*H*7a^1 ♂ 100*xq*58.57.55(5*e*)	4*H*24a^2 ♀ – *q* – – –(5–6*e*)
	a^5 ♀ 00 *p* – – –(alb.)	a^6 ♀ 00 *p* – – –(alb.)	a^7 – *p* – – – –	... ...	... ...	... ...
5*H*15	a^1 ♂ 100*xq*58.57.55(6*e*)	a^2 ♂ 100*xq*57.57.55(6*e*)	a^3 ♂ 100*xq*58.57.55(6*e*)	a^4 – *q* – – – –	4*H*24a^1 ♂ 100*xq*58.57.55(5–6*e*)	4*H*26a^5 ♀ 100*xq*58.58.56(6*e*)
	b^1 ♂ 100*xq*58.57.55(6*e*)	b^2 ♂ 100*xq*57.57.55(5*e*)	b^3 ♀ 100*xq*58.58.55(6*e*)	b^4 ♀ 100*yq*58.57. 2(5*e*)	... ...	... ...
	b^5 ♀ 100*xq*58.58.56(6*e*)	c^1 ♂ 100*yq*58.58. 2(5*e*)	c^2 ♀ 100*xq*58.58.55(5*e*)	... ...	... ...	... ...
5*H*17	a^1 ♂ 100*xq*58.57.56(6*e*)	a^2 ♂ 100*xq*58.57.56(6*e*)	a^3 ♂ 100*xq*58.57.56(6*e*)	a^4 ♂ 100*xp*37.37.18(6*f*)	4*H*25a^1 ♂ 100*xq*57.57.56(6*e*)	4*H*25a^2 ♀ 100*xq*58.58.55(6*e*)
	a^5 ♀ 100*xq*58.57.55(6*e*)	a^6 ♀ 100*xq*58.57.55(6*e*)	a^7 ♀ 100*xp*15.15.18(6*f*)	a^8 – *p* – – – –	... ...	... ...
	b^1 ♂ 100*xq*56.57.55(6*e*)	b^2 ♂ 100*xq*58.58.55(6*e*)	b^3 ♂ 100*xp*17.17.18(6*f*)	b^4 ♀ 100*xq*58.57.55(6*e*)	... ...	... ...
	b^5 ♀ 100*xp*23.23.15(6*f*)	b^6 ♀ 100*xp*23.23.18(6*f*)	c^1 ♂ 100*xq*58.58.55(6*e*)	c^2 ♂ 100*xq*57.57.56(6*e*)	... ...	... ...
	c^3 ♂ 100*xq*58.58.55(6*e*)	c^4 ♀ 100*xq*58.57.55(6*e*)	c^5 ♀ 100*xq*58.58.57(6*e*)	c^6 ♀ 100*xp*22.22.18(6*f*)	... ...	... ...
5*H*18	a^1 ♀ 70 *q*58. 1. 1(4*ew*)	a^2 ♀ 50 *q*58. 1. 1(3*ew*)	a^3 ♀ 45 *q*58. 1. 1(3*ew*)	a^4 ♀ 45 *q*58. 1. 1(2*ew*)	4*H*13a^4 ♂ – *q* – – –(3–4*ew*)	4*H*10a^8 ♀ 55 *q*58. 1. 2(3*ew*)
5*H*19	a^1 ♂ 100*xr*41.41.26(6*g*)	a^2 ♂ 100*xr*41.41.26(6*g*)	a^3 ♀ 100*xr*41.41.50(6*g*)	a^4 ♀ 100*xr*41.41.26(6*g*)	4*H*12b^1 ♂ 65 *r*41.50. 2(3–4*g*)	4*H*1a^6 ♀ 100*xr*41.41.50(6*g*)
	a^5 ♀ 100*xq*41.41.50(6*g*)	a^6 ♀ – *q* – – –(6*g*)	b^1 ♂ 100*xr*41.41.26(5*g*)	b^2 ♂ 100*xr*41.41.26(6*g*)	... ...	... ...
	b^3 ♀ 100*xr*41.26.26(6*g*)	b^4 ♀ – *r* – – –(5–6*g*)	b^5 ♀ 100*xr*41.41.26(5–6*g*)	b^6 ♀ 100*xq* or *r*41.41.26(6*g*)	... ...	... ...

	b^7 – q – – – –	b^8 – q – – – –	b^9 – q – – – –	c^1 ♀ 100xr41.41.26(6g)	… … … …	… … … …
	c^2 ♀ 100xr41.41.26(6g)	c^3 ♀ 100xr41.41.50(6g)	c^4 ♀ 100xr50.50.23(5g)	c^5 ♀ 100yr41.50.50(5g)	… … … …	… … … …
	c^6 ♀ 100xr41.50.50(5g)	c^7 ♀ – r – – –(5g)	c^8 – q – – – –	… … …	… … … …	… … … …
5H20	a^1 ♂ 100xq58.57.60(5e)	a^2 ♂ 60 q58.58. 1(4ew)	a^3 ♀ 100xq58.58.55(5e)	a^4 ♀ 100xq58.58.55(5e)	4H13a^2 ♂ 90 q58.56. 2(4–5e)	4H6a^2 ♀ 95 q58.57.55(5e)
	a^5 ♀ 45 q58. 1. 1(3e)	a^6 ♀ 45 q58.58. 1(3e)	a^7 – q – – – –	a^8 – q – – – –	… … … …	… … … …
	a^9 – q – – – –	b^1 ♂ – q – – –(5e)	b^2 ♂ 80 q58.57. 1(4e)	b^3 ♀ 100xq58.57.55(5e)	… … … …	… … … …
	b^4 – q – – – –	b^5 – q – – – –	b^6 – q – – – –	b^7 – q – – – –	… … … …	… … … …
	b^8 – q – – – –	b^9 – q – – – –	b^{10} – q – – – –	… … …	… … … …	… … … …
5H21	a^1 ♂ 90 q58.58.55(4e)	a^2 ♂ 75 q58.55. 2(4e)	a^3 ♂ 80 q58.55. 2(4e)	a^4 ♀ 40 q61. 1. 1(2–3e)	4H10a^1 ♂ 75 q58.55. 2(4e)	4H3a^4 ♀ 55 q58. 1. 2(3e)
	a^5 ♀ 60 q61.61. 1(3e)	a^6 ♀ 50 q61.61. 1(3e)	a^7 ♀ 55 q58. 2. 2(3–4e)	a^8 ♀ – q – – –(5e)	… … … …	… … … …
	a^9 ♀ – q – – –(5e)	b^1 ♂ 60 q61. 1. 1(4e)	b^2 ♂ 75 q58.55. 1(4e)	b^3 ♀ 50 q61. 1. 2(3e)	… … … …	… … … …
	b^4 ♀ 50 q58. 1. 2(3e)	b^5 ♀ 80 q58.58. 1(4e)	b^6 ♀ 95 q57.55. 1(4e)	b^7 ♀ 60 q58.58. 1(4e)	… … … …	… … … …
	b^8 – q – – – –	c^1 ♂ 85 q58.55. 1(4e)	c^2 ♂ 90 q58.55. 3(4e)	c^3 ♀ 60 q61.61. 1(3–4e)	… … … …	… … … …
	c^4 ♀ 60 q58.58. 1(3–4e)	c^5 ♀ 85 q58.57. 1(4e)	… … …	… … …	… … … …	… … … …
5H22	a^1 ♂ 80 q52.47. 2(4e)	a^2 ♂ 55 q58.47. 2(3e)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ 65 q56.55. 4(3e)	4H13a^3 ♂ 90 q58.55. 2(4–5e)	4H2a^6 ♀ 90 q57.55. 1(5e)
	a^5 ♀ 60 q58. 1. 4(3e)	a^6 – q – – – –	a^7 – q – – – –	b^1 ♂ – q – – –(4e)	… … … …	… … … …
	b^2 ♂ 85 q58.55. 1(4e)	b^3 ♂ 00 p – – –(alb.)	b^4 ♀ 00 p – – –(alb.)	b^5 ♀ 90 q58.55. 2(5e)	… … … …	… … … …
	b^6 ♀ 85 q57.55. 1(4e)	b^7 ♀ 95 q58.58. 2(5e)	b^8 ♂ 50 q45.45. 3(3e)	… … …	… … … …	… … … …
5H23	a^1 ♀ 20 r42. 1. 1(1gw)	b^1 ♂ 30 q58. 1. 1(2e)	b^2 ♂ 35 q41.26. 1(3g)	b^3 ♂ 00 p – – –(alb.)	4H13a^5 ♂ 40 q58. 1. 2(3e)	4H1a^4 ♀ 45 q58. 1. 1(3e)
	b^4 ♀ 40 q58. 1. 2(2ew)	b^5 ♀ 55 q61. 1. 1(2e)	b^6 ♀ 35 q42.50. 1(2g)	c^1 ♂ 35 q61. 1. 1(2e)	… … … …	… … … …
	c^2 ♂ – q – – –(3e)	c^3 ♂ 75 r41.41. 1(4g)	c^4 ♀ 40 q41.41. 2(3g)	c^5 ♀ 00 p – – –(alb.)	… … … …	… … … …
	c^6 ♀ 00 p – – –(alb.)	c^7 – q – – – –	… … …	… … …	… … … …	… … … …
5H24	a^1 ♂ 80 q58.57. 2(4e)	a^2 ♂ 75 q58.55. 2(4e)	a^3 ♂ 90 r41.26. 1(4g)	a^4 ♀ 00 p – – –(alb.)	4H2a^1 ♂ 70 q58.55. 2(4e)	4H11a^1 ♀ 95 q58.56. 2(4–5e)
	a^5 ♀ 55 q58. 1. 1(2–3e)	a^6 – p – – – –	a^7 – q – – – –	b^1 ♂ 95 q58.55. 2(4e)	… … … …	… … … …
	b^2 ♀ 45 q58.58. 1(3e)	b^3 ♀ 60 q58.57. 1(3e)	c^1 ♂ 90 q57.55. 1(4e)	c^2 ♂ 55 q58. 1. 2(3e)	… … … …	… … … …
	c^3 ♂ 45 p23. 1. 1(3f)	c^4 ♂ 90 p22.15. 1(4f)	c^5 ♂ 00 p – – –(alb.)	c^6 ♀ 00 p – – –(alb.)	… … … …	… … … …
5H25	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq57.57.55(6e)	4H2b^2 ♂ 100xq58.57.56(6e)	4H3b^6 ♀ 100xq58.57.56(6e)
	a^5 – q – – – –	a^6 – q – – – –	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ – p – – –(6f)	… … … …	… … … …
	b^3 ♂ – p – – –(6f)	b^4 ♂ 40 p37. 1. 1(3f)	b^5 ♀ 100xq58.58.55(6e)	b^6 ♀ 100xp22.22.17(6f)	… … … …	… … … …
	c^1 ♂ 100xq58.57.55(6e)	c^2 ♂ – q – – –(6e)	c^3 ♂ – q – – –(6e)	c^4 ♂ 90 q58.55. 2(4e)	… … … …	… … … …
	c^5 ♀ 100xp26.26.23(6f)	c^6 – q – – – –	… … …	… … …	… … … …	… … … …
5H26	a^1 ♂ 80 q57.55. 1(3e)	a^2 ♂ 55 q58.58. 1(3–4e)	a^3 ♀ 75 q57.56. 1(3–4e)	b^1 ♂ 60 q57. 4. 4(4e)	4H67a^1 ♂ 75 q58.57. 2(3–4e)	4H65a^4 ♀ 90 q58.55. 2(4e)
	b^2 ♂ 90 q57.55. 1(4e)	b^3 ♀ – q – – –(5e)	b^4 ♀ 65 q58.55. 1(3–4e)	b^5 ♀ 45 q58. 1. 1(2–3e)	… … … …	… … … …
	c^1 ♂ 90 q58.57. 1(5e)	c^2 ♂ 65 q58.57. 1(4e)	c^3 ♂ 60 q58. 1. 1(3e)	c^4 ♀ 90 q58.55. 2(4e)	… … … …	… … … …
	c^5 ♀ 65 q58.57. 2(4e)	… … …	… … …	… … …	… … … …	… … … …
5H27	a^1 ♂ – q – – –(3e)	a^2 ♂ 55 q58. 1. 1(3e)	a^3 ♀ – q – – –(3e)	a^4 ♀ 50 q57.56. 1(3e)	4H65a^1 ♂ 55 q58. 1. 1(3e)	4H67a^5 ♀ 55 q61. 1. 1(3e)
	a^5 ♀ 50 q58.58. 1(3e)	a^6 ♀ – q – – –(3e)	b^1 ♂ 60 q58. 1. 1(4e)	b^2 ♀ 55 q58. 1. 2(4e)	… … … …	… … … …
	b^3 ♀ 85 q58.57. 1(4e)	b^4 ♀ 45 q58. 1. 1(3e)	b^5 ♀ 80 q57.57. 1(4ch)	b^6 ♀ – q – – –(4ch)	… … … …	… … … …
	b^7 ♀ 65 q58. 1. 1(4e)	b^8 ♀ 75 q58.57. 3(4e)	b^9 ♂ – q – – –(3e)	c^1 ♂ 85 q58.44. 1(4e)	… … … …	… … … …
	c^2 ♂ 35 q44. 1. 1(3ch)	c^3 ♂ 40 q52. 1. 1(3ch)	c^4 ♀ 60 q58. 1. 3(4e)	c^5 ♂ 50 q61.60. 1(3e)	… … … …	… … … …
	c^6 ♀ 75 q58.56. 1(4e)	c^7 ♀ 60 q58.58. 1(4e)	c^8 ♀ 80 q57.58.12(4e)	c^9 – q – – – –	… … … …	… … … …
	c^{10} – q – – – –	… … …	… … …	… … …	… … … …	… … … …
5H29	a^1 ♂ 100xq57.57.56(5e)	a^2 ♀ 100xq58.58.56(6e)	a^3 ♀ 100xq58.57.55(6e)	a^4 ♀ 100xq57.57.56(5e)	4H30a^1 ♂ 100xq58.57.55(6e)	4H37a^3 ♀ 100xq58.57.56(6e)
	a^5 – p – – – –	b^1 ♂ 100xq58.58.57(5e)	b^2 ♂ 100xq58.57.56(5e)	b^3 ♀ 100xq58.57.56(6e)	… … … …	… … … …
	b^4 ♀ 100xq58.58.55(6e)	b^5 ♀ 00 p – – –(alb.)	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ – q – – –(6e)	… … … …	… … … …
	c^3 ♀ 100xq58.58.56(6e)	c^4 ♂ 100xq58.58.55(6e)	c^5 ♀ 100xq57.57.56(5e)	… … …	… … … …	… … … …
5H30	a^1 ♂ 100xp22.22.18(6f)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 100xp23.23.18(6f)	a^4 ♀ 100xq58.58.55(6e)	4H30a^2 ♂ 100xq58.57.55(6e)	4H37a^5 ♀ 100xq58.58.55(6e)
	a^5 ♀ 100xq58.58.55(5e)	a^6 – p – – – –	a^7 – p – – – –	a^8 – p – – – –	… … … …	… … … …
	b^1 ♂ 00 p – – –(alb.)	b^2 ♂ 100xp23.23.15(6f)	b^3 ♀ 100xp15.15.18(6f)	b^4 ♀ 100xp23.23.15(6f)	… … … …	… … … …
	b^5 ♀ 100xq58.58.55(6e)	b^6 ♀ 100xq58.57.55(6e)	b^7 ♀ 00 p – –(alb.)	c^1 ♂ 100xp15.15.15(6f)	… … … …	… … … …

Mating	Offspring				Parents	
					Father	Mother
5H1	a^1 ♀ 100xp26.23.15(6f)	a^2 ♀ 100xp23.23.22(6f)	a^3 ♀ 100xp26.26.23(6f)	a^4 ♀ 100xp23.23.17(6f)	4H17 a^1 ♂ 100xp23.23.18(6f) a^2 ♂ 100xp26.26.17(6f)	4H17a^1 ♀ 100xp23.23.18(6f)
	a^5 – p – – –(6f)					
5H3	♀ – p – – – –	a^1 ♂ 100xp13.13.15(6f)	a^2 ♂ 100xp26.26.23(6f)	a^3 ♂ 100xp24.24.17(6f)	4H27a^2 ♂ 100xp23.23.18(6f)	4H22a^1 ♀ 100xp23.23.18(6f)
	a^4 ♂ 100xp23.23.17(6f)	a^5 ♀ 100xp13.13.18(6f)	a^6 ♀ 100xp23.23.17(6f)	a^7 ♀ – p – – –(6f)		
5H4	a^1 ♀ 100xp15.15.15(6f)	a^2 ♀ 100xp23.23.15(6f)	a^3 ♀ 100xp23.23.17(6f)	a^4 ♀ 100xp15.15.15(6f)	4H17a^1 ♂ 100xp23.23.18(6f)	4H20a^1 ♀ 100xp23.23.15(6f)
	a^5 – p – – – –	b^1 ♂ 100xp22.15.15(6f)	b^2 ♂ 100xp22.22.17(6f)	b^3 ♀ 100xp26.23.15(6f)		
	c^1 ♂ 100xp23.23.15(6f)	c^2 ♂ 100xp17.17.15(6f)	c^3 ♂ 100xp22.22.15(6f)	c^4 ♀ 100xp23.23.17(6f)		
	c^5 ♀ 00 p – – –(alb.)					
5H5	a^1 ♂ 100xp15.15.10(6f)	a^2 ♂ 100xp15.15.10(6f)	a^3 ♂ 100xp23.23.15(6f)	a^4 ♂ 100xp23.23.15(6f)	4H28a^1 ♂ 100xp23.23.17(5f)	4H20a^2 ♀ 100xp10.23.15(6f)
5H6	a^1 ♀ 100xp23.23.15(6f)	a^2 ♀ 100xp23.23.15(6f)	a^3 ♀ 100xp26.23.17(6f)	a^4 p –	4H15a^2 ♂ 100xp23.23.18(6f)	4H27a^4 ♀ 100xp23.23.18(5f)
	b^1 ♂ 100xp23.23.15(6f)	b^2 ♂ 100xp23.23.17(6f)	b^3 ♀ 100xp23.23.15(6f)	b^4 ♀ 100xp26.23.10(6f)		
	b^5 ♀ 100xp26.23.15(5f)					
5H7	a^1 ♂ 100xp23.23.17(6fw)	a^2 ♂ 100xp23.23.15(6fw)	a^3 ♂ 100xp23.15.17(6f)	a^4 ♂ 100xp26.26.23(5f)	4H4a^1 ♂ 100xp23.23.18(6f)	4H27a^5 ♀ 100xp23.23.17(6f)
	a^5 ♀ 100xp23.23.18(6f)	a^6 ♀ 100xp23.23.18(6fw)	b^1 ♂ 100xp23.23.17(6f)	b^2 ♂ 100xp23.23.17(6f)		
	b^3 ♂ 90 p23.23.17(4–5f)	b^4 ♀ 100xp23.23.17(6f)	b^5 ♀ 100xp26.23.17(6f)			
5H8	a^1 ♂ 100xp26.23.15(5f)	a^2 ♀ 100xp23.23.17(6f)	a^3 ♀ 90 p15.15.18(5f)	a^4 ♀ 100xp22.22.18(5f)	4H17a^2 ♂ 100xp26.26.17(6f)	4H19a^3 ♀ 50 p23. 1. 2(2–3f)
	a^5 p	b^1 ♂ 100xp26.26.17(6f)	b^2 ♂ 100xp23.23.15(6f)	b^3 ♂ 100xp26.23.17(5f)		
	b^4 ♀ 100xp26.23.17(6f)	b^5 ♀ 100xp23.23.18(5f)	b^6 p	c^1 ♂ – p – – (6f)		
	c^2 ♂ p – – (6f)	c^3 ♂ 100xp26.26.23(5f)	c^4 ♂ 100xp26.23.15(5f)	c^5 ♀ 100xp23.23.18(6f)		
	c^6 ♀ 100xp26.23.15(6f)	c^7 – p –	c^8 p –			
5H10	a^1 ♂ 100xq58.55.55(6e)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♂ 100xq57.55.55(5e)	a^4 ♀ 100xq58.58.55(6e)	4H1a^1 ♂ 100xq58.57.55(6e)	4H4a^1 ♀ 100xq58.57.55(5e)
	a^5 ♀ 100xq58.58.55(5e)	a^6 – q – – –	a^7 q –			
5H11	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xq58.57.56(6e)	a^3 ♂ 100xq58.57.56(6e)	a^4 ♂ 100xq58.57.55(6e)	4H1a^2 ♂ 100xq58.57.55(6e)	4H2a^7 ♀ 100xq58.58.57(6e)
	a^5 ♂ q – – (6e)	a^6 q –	b^1 ♂ 100xq57.56.55(6e)	b^2 ♂ 100xq57.56.55(6e)		
	b^3 ♀ 100xq58.57.56(6e)	b^4 ♀ 100xq58.57.55(6e)	b^5 ♀ 100xq58.58.56(6e)	c^1 ♂ 100xq58.57.55(6e)		
	c^2 ♂ 100xq58.57.55(6e)	c^3 ♂ 100xq57.57.56(6e)	c^4 ♂ 100xq58.57.55(6e)	c^5 ♂ 100xq58.57.56(6e)		
	c^6 ♀ 100xq58.58.55(6e)	c^7 q – – –				
5H12	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xq57.57.55(6e)	a^3 ♂ 100xq57.57.55(6e)	a^4 ♀ 100xq58.58.55(6e)	4H1a^3 ♂ 100xq58.57.55(6e)	4H7a^2 ♀ 100xq58.57.56(5–6e)
	b^1 ♂ 100xq58.58.55(5–6e)	b^2 ♀ 100xq58.57.55(6e)	b^3 – q – – – –	b^4 – q – – – –		
	b^5 q	b^6 – q – – –	b^7 q			
5H14	a^1 ♂ 100xr41.41.26(6g)	a^2 ♀ 100xq58.57.56(6e)	a^3 ♀ 100xq58.57.56(6e)	a^4 ♀ 100xq58.57.56(5–6e)	4H7a^1 ♂ 100xq58.57.55(5e)	4H24a^2 ♀ q – – (5–6e)
	a^5 ♀ 00 p (alb.)	a^6 ♀ 00 p (alb.)	a^7 p – –			
5H15	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xq57.57.55(6e)	a^3 ♂ 100xq58.57.55(6e)	a^4 – q – – – –	4H24a^1 ♂ 100xq58.57.55(5–6e)	4H26a^5 ♀ 100xq58.58.56(6e)
	b^1 ♂ 100xq58.57.55(6e)	b^2 ♂ 100xq57.57.55(5e)	b^3 ♀ 100xq58.58.55(6e)	b^4 ♀ 100yq58.57. 2(5e)		
	b^5 ♀ 100xq58.58.56(6e)	c^1 ♂ 100yq58.58. 2(5e)	c^2 ♀ 100xq58.58.55(5e)			
5H17	a^1 ♂ 100xq58.57.56(6e)	a^2 ♂ 100xq58.57.56(6e)	a^3 ♂ 100xq58.57.56(6e)	a^4 ♂ 100xp37.37.18(6f)	4H25a^1 ♂ 100xq57.57.56(6e)	4H25a^2 ♀ 100xq58.58.55(6e)
	a^5 ♀ 100xq58.57.55(6e)	a^6 ♀ 100xq58.57.55(6e)	a^7 ♀ 100xp15.15.18(6f)	a^8 – p – – – –		
	b^1 ♂ 100xq56.57.55(6e)	b^2 ♂ 100xq58.58.55(6e)	b^3 ♂ 100xp17.17.18(6f)	b^4 ♀ 100xq58.57.55(6e)		
	b^5 ♀ 100xp23.23.15(6f)	b^6 ♀ 100xp23.23.18(6f)	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ 100xq57.57.56(6e)		
	c^3 ♂ 100xq58.58.55(6e)	c^4 ♀ 100xq58.57.55(6e)	c^5 ♀ 100xq58.58.57(6e)	c^6 ♀ 100xp22.22.18(6f)		
5H18	a^1 ♀ 70 q58. 1. 1(4ew)	a^2 ♀ 50 q58. 1. 1(3ew)	a^3 ♀ 45 q58. 1. 1(3ew)	a^4 ♀ 45 q58. 1. 1(2ew)	4H13a^4 ♂ – q – – –(3–4ew)	4H10a^8 ♀ 55 q58. 1. 2(3ew)
5H19	a^1 ♂ 100xr41.41.26(6g)	a^2 ♂ 100xr41.41.26(6g)	a^3 ♀ 100xr41.41.50(6g)	a^4 ♀ 100xr41.41.26(6g)	4H12b^1 ♂ 65 r41.50. 2(3–4g)	4H1a^6 ♀ 100xr41.41.50(6g)
	a^5 ♀ 100xq41.41.50(6g)	a^6 ♀ q (6g)	b^1 ♂ 100xr41.41.26(5g)	b^2 ♂ 100xr41.41.26(6g)		
	b^3 ♀ 100xr41.26.26(6g)	b^4 ♀ r (5–6g)	b^5 ♀ 100xr41.41.26(5–6g)	b^6 ♀ 100xq or r41.41.26(6g)		
	b^7 – q – – –	b^8 q –	b^9 q –	c^1 ♀ 100xr41.41.26(6g)		
	c^2 ♀ 100xr41.41.26(6g)	c^3 ♀ 100xr41.41.50(6g)	c^4 ♀ 100xr50.50.23(5g)	c^5 ♀ 100yr41.50.50(5g)		
	c^6 ♀ 100xr41.50.50(5g)	c^7 ♀ – r – – –(5g)	c^8 q – – – –			
5H20	a^1 ♂ 100xq58.57.60(5e)	a^2 ♂ 60 q58.58. 1(4ew)	a^3 ♀ 100xq58.58.55(5e)	a^4 ♀ 100xq58.58.55(5e)	4H13a^2 ♂ 90 q58.56. 2(4–5e)	4H6a^2 ♀ 95 q58.57.55(5e)
	a^5 ♀ 45 q58. 1. 1(3e)	a^6 ♀ 45 q58.58. 1(3e)	a^7 – q – – – –	a^8 – q – – – –		
	a^9 q – – –	b^1 ♂ – q – – –(5e)	b^2 ♂ 80 q58.57. 1(4e)	b^3 ♀ 100xq58.57.55(5e)		
	b^4 q –	b^5 – q – – – –	b^6 – q – – – –	b^7 – q – – – –		
	b^8 q –	b^9 – q – – – –	b^{10} – q – – – –			
5H21	a^1 ♂ 90 q58.58.55(4e)	a^2 ♂ 75 q58.55. 2(4e)	a^3 ♂ 80 q58.55. 2(4e)	a^4 ♀ 40 q61. 1. 1(2–3e)	4H10a^1 ♂ 75 q58.55. 2(4e)	4H3a^4 ♀ 55 q58. 1. 2(3e)
	a^5 ♀ 60 q61.61. 1(3e)	a^6 ♀ 50 q61.61. 1(3e)	a^7 ♀ 55 q58. 2. 2(3–4e)	a^8 ♀ – q – – –(5e)		
	a^9 ♀ q – – –(5e)	b^1 ♂ 60 q61. 1. 1(4e)	b^2 ♂ 75 q58.55. 1(4e)	b^3 ♀ 50 q61. 1. 2(3e)		
	b^4 ♀ 50 q58. 1. 2(3e)	b^5 ♀ 80 q58.58. 1(4e)	b^6 ♀ 95 q57.55. 1(4e)	b^7 ♀ 60 q58.58. 1(4e)		
	b^8 q –	c^1 ♂ 85 q58.55. 1(4e)	c^2 ♂ 90 q58.55. 3(4e)	c^3 ♀ 60 q61.61. 1(3–4e)		
	c^4 ♀ 60 q58.58. 1(3–4e)	c^5 ♀ 85 q58.57. 1(4e)				
5H22	a^1 ♂ 80 q52.47. 2(4e)	a^2 ♂ 55 q58.47. 2(3e)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ 65 q56.55. 4(3e)	4H13a^3 ♂ 90 q58.55. 2(4–5e)	4H2a^6 ♀ 90 q57.55. 1(5e)
	a^5 ♀ 60 q58. 1. 4(3e)	a^6 – q – – – –	a^7 – q – – – –	b^1 ♂ – q – – –(4e)		
	b^2 ♂ 85 q58.55. 1(4e)	b^3 ♂ 00 p – – –(alb.)	b^4 ♀ 00 p – – –(alb.)	b^5 ♀ 90 q58.55. 2(5e)		
	b^6 ♀ 85 q57.55. 1(4e)	b^7 ♀ 95 q58.58. 2(5e)	b^8 ♂ 50 q45.45. 3(3e)			
5H23	a^1 ♀ 20 r42. 1. 1(1gw)	b^1 ♂ 30 q58. 1. 1(2e)	b^2 ♂ 35 q41.26. 1(3g)	b^3 ♂ 00 p – – –(alb.)	4H13a^1 ♂ 40 q58. 1. 2(3e)	4H1a^4 ♀ 45 q58. 1. 1(3e)
	b^4 ♀ 40 q58. 1. 2(2ew)	b^5 ♀ 55 q61. 1. 1(2e)	b^6 ♀ 35 q42.50. 1(2g)	c^1 ♂ 35 q61. 1. 1(2e)		
	c^2 ♂ q – – (3e)	c^3 ♂ 75 r41.41. 1(4g)	c^4 ♀ 40 q41.41. 2(3g)	c^5 ♀ 00 p – – –(alb.)		
	c^6 ♀ 00 p (alb.)	c^7 – q – – – –				
5H24	a^1 ♂ 80 q58.57. 2(4e)	a^2 ♂ 75 q58.55. 2(4e)	a^3 ♂ 90 r41.26. 1(4g)	a^4 ♀ 00 p – – –(alb.)	4H2a^1 ♂ 70 q58.55. 2(4e)	4H11a^1 ♀ 95 q58.56. 2(4–5e)
	a^5 ♀ 55 q58. 1. 1(2–3e)	a^6 p – –	a^7 – q – – – –	b^1 ♂ 95 q58.55. 2(4e)		
	b^2 ♀ 45 q58.58. 1(3e)	b^3 ♀ 60 q58.57. 1(3e)	c^1 ♂ 90 q57.55. 1(4e)	c^2 ♂ 55 q58. 1. 2(3e)		
	c^3 ♂ 45 p23. 1. 1(3f)	c^4 ♂ 90 p22.15. 1(4f)	c^5 ♂ 00 p – – –(alb.)	c^6 ♀ 00 p – – –(alb.)		
5H25	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 100xq58.57.55(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq57.57.55(6e)	4H2b^2 ♂ 100xq58.57.56(6e)	4H3b^6 ♀ 100xq58.57.56(6e)
	a^5 q – – –	a^6 – q – – – –	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ – p – – –(6f)		
	b^3 ♂ – p – – –(6f)	b^4 ♂ 40 p37. 1. 1(3f)	b^5 ♀ 100xq58.58.55(6e)	b^6 ♀ 100xp22.22.17(6f)		
	c^1 ♂ 100xq58.57.55(6e)	c^2 ♂ – q – – –(6e)	c^3 ♂ – q – – –(6e)	c^4 ♂ 90 q58.55. 2(4e)		
	c^5 ♀ 100xp26.26.23(6f)	c^6 – q – – – –				
5H26	a^1 ♂ 80 q57.55. 1(3e)	a^2 ♂ 55 q58.58. 1(3–4e)	a^3 ♀ 75 q57.56. 1(3–4e)	b^1 ♂ 60 q57. 4. 4(4e)	4H67a^1 ♂ 75 q58.57. 2(3–4e)	4H65a^4 ♀ 90 q58.55. 2(4e)
	b^2 ♂ 90 q57.55. 1(4e)	b^3 ♀ q – –(5e)	b^4 ♀ 65 q58.55. 1(3–4e)	b^5 ♀ 45 q58. 1. 1(2–3e)		
	c^1 ♂ 90 q58.57. 1(5e)	c^2 ♂ 65 q58.57. 1(4e)	c^3 ♂ 60 q58. 1. 1(3e)	c^4 ♀ 90 q58.55. 2(4e)		
	c^5 ♀ 65 q58.57. 2(4e)					
5H27	a^1 ♂ q (3e)	a^2 ♂ 55 q58. 1. 1(3e)	a^3 ♀ q (3e)	a^4 ♀ 50 q57.56. 1(3e)	4H65a^1 ♂ 55 q58. 1. 1(3e)	4H67a^5 ♀ 55 q61. 1. 1(3e)
	a^5 ♀ 50 q58.58. 1(3e)	a^6 ♀ q – – –(3e)	b^1 ♂ 60 q58. 1. 1(4e)	b^2 ♀ 55 q58. 1. 2(4e)		
	b^3 ♀ 85 q58.57. 1(4e)	b^4 ♀ 45 q58. 1. 1(3e)	b^5 ♀ 80 q57.57. 1(4ch)	b^6 ♀ – q – – –(4ch)		
	b^7 ♀ 65 q58. 1. 1(4e)	b^8 ♀ 75 q58.57. 3(4e)	b^9 ♂ – q – – –(3e)	c^1 ♂ 85 q58.44. 1(4e)		
	c^2 ♂ 75 q44. 1. 1(3ch)	c^3 ♂ 40 q52. 1. 1(3ch)	c^4 ♀ 60 q58. 1. 3(4e)	c^5 ♂ 50 q61.60. 1(3e)		
	c^6 ♀ 75 q58.56. 1(4e)	c^7 ♀ 60 q58.58. 1(4e)	c^8 ♀ 80 q57.58.12(4e)	c^9 – q – – – –		
	c^{10} – q – – – –					
5H29	a^1 ♂ 100xq57.57.56(5e)	a^2 ♀ 100xq58.58.56(6e)	a^3 ♀ 100xq58.57.55(6e)	a^4 ♀ 100xq57.57.56(5e)	4H30a^1 ♂ 100xq58.57.55(6e)	4H37a^1 ♀ 100xq58.57.56(6e)
	a^5 p	b^1 ♂ 100xq58.58.57(5e)	b^2 ♂ 100xq58.57.56(5e)	b^3 ♀ 100xq58.57.56(6e)		
	b^4 ♀ 100xq58.58.55(6e)	b^5 ♀ 00 p – – –(alb.)	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ – q – – –(6e)		
	c^3 ♀ 100xq58.58.56(6e)	c^4 ♂ 100xq58.58.55(6e)	c^5 ♀ 100xq57.57.56(5e)			
5H30	a^1 ♂ 100xp22.22.18(6f)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 100xp23.23.18(6f)	a^4 ♀ 100xq58.58.55(6e)	4H30a^2 ♂ 100xq58.57.55(6e)	4H37a^2 ♀ 100xq58.58.55(6e)
	a^5 ♀ 100xq58.58.55(5e)	a^6 – p – – – –	a^7 p – – – –	a^8 – p		
	b^1 ♂ 00 p – (alb.)	b^2 ♂ 100xp23.23.15(6f)	b^3 ♀ 100xp15.15.18(6f)	b^4 ♀ 100xp23.23.15(6f)		
	b^5 ♀ 100xq58.58.55(6e)	b^6 ♀ 100xq58.57.55(6e)	b^7 ♀ 00 p – – –(alb.)	c^1 ♂ 100xp15.15.15(6f)		

MATING	OFFSPRING				PARENTS	
					Father	Mother
5H30	c^2 ♀ 100xq58.58.55(6e)	c^3 ♀ 100xq58.56.55(6e)	c^4 ♀ 100xq58.58.55(6e)	c^5 ♀ 100xq58.58.55(6e)	... ...	... ...
	c^6 ♀ 00 p – – –(alb.)	c^7 – q – – – –	... ...	... ...	... ...	... ...
5H31	a^1 ♂ 100xq58.58.57(6e)	a^2 ♂ 100xp23.23.18(6f)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq58.57.55(6e)	4H30a^3 ♂ 100xq58.57.55(6e)	4H50a^4 ♀ 100xq58.56.56(6e)
	a^5 ♀ 100xq58.58.55(6e)	a^6 ♀ 100xq58.57.55(6e)	a^7 ♀ 100xp23.23.18(6f)	a^8 – q – – – –	... ...	... ...
	a^9 – q – – – –	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ 100xp26.23.15(6f)	b^3 ♂ 100xq58.58.55(5–6e)	... ...	... ...
	b^4 ♂ 100yq57.57.56(5–6e)	b^5 ♂ 100xq58.58.55(6e)	b^6 ♂ 100xq58.58.55(5e)	b^7 ♀ 100xq58.57.56(6e)	... ...	... ...
	b^8 ♀ 100xp22.22.18(6f)	b^9 ♀ 100xp15.15.18(6f)	b^{10} – q – – – –	c^1 ♂ 100xq58.58.57(6e)	... ...	... ...
	c^2 ♂ 100xq58.58.57(6e)	c^3 ♂ 100xq58.58.55(6e)	c^4 ♂ 100xq58.58.55(6e)	c^5 ♂ 100xp26.26.23(6f)	... ...	... ...
	c^6 ♀ 100xq58.58.55(6e)	c^7 ♀ 100xq58.58.57(6e)	c^8 ♀ – q – – –(6e)	c^9 ♀ – p – – –(6f)	... ...	... ...
	c^{10} ♀ – p – – –(6f)	... ...	... ...	... ...	... ...	... ...
5H32	a^1 ♂ – q – – –(6e)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♀ – q – – –(6e)	a^4 ♀ 100xq58.57.55(6e)	4H51a^1 ♂ 100xq58.57.55(6e)	4H30a^4 ♀ 100xq58.57.55(6e)
	a^5 ♀ 90 q58.58. 1(4e)	a^6 ♀ 00 p – – –(alb.)	b^1 ♂ 100xq58.57.55(5e)	b^2 ♀ 00 p – – –(alb.)	... ...	... ...
	b^3 ♀ 100xq58.58.55(6e)	b^4 ♀ 100xq58.57.55(6e)	b^5 – q – – – –	c^1 ♂ 100xq58.57.55(6e)	... ...	... ...
	c^2 ♀ 100xq58.58.56(6e)	c^3 ♀ 100xq58.57.55(6e)	c^4 ♀ 00 p – – –(alb.)	c^5 – q – – – –	... ...	... ...
	c^6 – q – – – –	... ...	... ...	... ...	... ...	... ...
5H33	a^1 ♂ 100xq57.57.55(6e)	a^2 ♂ 100xq58.57.56(6e)	a^3 ♀ 100xq55.55.54(6e)	a^4 ♀ 100xq58.57.55(6e)	4H50b^1 ♂ 100xq58.58.55(6e)	4H30a^5 ♀ 100xq58.57.55(6e)
	b^1 ♂ 100xp23.15.18(6f)	b^2 ♂ 100xq58.57.55(6e)	b^3 ♂ 100xq58.58.55(6e)	b^4 ♂ 100xq58.57.55(6e)	... ...	... ...
	b^5 ♀ 100xq58.57.56(6e)	b^6 ♀ 100xq58.57.55(6e)	b^7 ♀ 100xq58.57.55(5e)	... ...	... ...	... ...
5H34	a^1 ♂ 100xp23.23.15(6f)	a^2 ♀ 100xq58.57.55(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq58.58.56(6e)	4H51 { b^2 ♂ 100xq58.58.55(6e) } { b^3 ♂ 100xq58.57.55(6e) }	4H30a^6 ♀ 100xq58.57.56(6e)
	a^5 ♀ 100xq58.58.57(6e)	a^6 ♀ 100xp23.23.18(6f)	b^1 ♂ 100xq58.57.55(6e)	c^1 ♂ 100xq58.58.55(6e)	... ...	... ...
	c^2 ♂ 100xq58.57.55(6e)	c^3 ♀ 100xq58.58.55(6e)	c^4 ♀ 100xp23.23.17(6f)	c^5 – p – – – –	... ...	... ...
	c^6 – p – – – –	... ...	... ...	... ...	... ...	... ...
5H35	a^1 ♂ – p – – –(5b)	a^2 ♀ 100zp39.35.28(5b)	a^3 – p – – – –	a^4 – p – – – –	4H14b^1 ♂ 75 p39.39.28(5b)	4H15b^4 ♀ 100zp35.35.28(5b)
	a^5 – p – – – –	... ...	... ...	... ...	... ...	... ...
5H36	a^1 ♂ 100zp37.37. 3(5b)	a^2 ♂ 100zp35.35.30(5b)	a^3 ♀ 100xp22.22.18(6f)	a^4 ♀ 100zp35.35.28(5b)	4H14b^2 ♂ 100zp39.35.28(5b)	4H22b^3 ♀ 75 p39.39. 3(5b)
	a^5 ♂ 100zp26.35. 6(5b)	a^6 – p – – – –	a^7 – p – – – –	b^1 ♂ 100xp23.23.18(6f)	... ...	... ...
	b^2 ♂ 100xp23.23.18(6f)	b^3 ♀ 100xp23.23.18(6f)	b^4 ♀ 100xp22.22.18(6f)	b^5 ♀ 100zp39.35.28(5b)	... ...	... ...
	b^6 ♀ 75 p39.39. 3(5b)	... ...	... ...	... ...	... ...	... ...
5H37	a^1 ♂ 100zp35.35.28(5–6b)	a^2 ♀ – p – – –(5b)	a^3 ♀ 100zp39.35.30(5b)	b^1 ♂ 100xp23.23.22(6f)	4H16b^3 ♂ 100zp39.39.28(5b)	4H15b^5 ♀ – p – – –(5b)
	b^2 ♂ 100zp35.35. 6(5b)	b^3 ♀ 100xp13.13.18(6f)	c^1 ♂ 100yp18.18.18(5b)	c^2 ♂ 100xp39.39.30(5b)	... ...	... ...
	c^3 ♂ 85 p36.36. 3(5b)	c^4 ♀ – p – – –(5b)	... ...	... ...	... ...	... ...
5H40	a^1 ♂ 75 q46.46. 2(5–6c)	a^2 ♀ 100xq58.58.56(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100zq48.46.14(5c)	4H15b^2 ♂ – p – – –(5b)	4H47a^5 ♀ 00 p – – –(alb.)
	a^5 ♀ – q – – –(5c)	a^6 ♀ 00 p – – –(alb.)	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ 00 p – – –(alb.)	... ...	... ...
	b^3 ♀ 100xq58.58.55(5e)	b^4 ♀ 00 p – – –(alb.)	c^1 ♂ 80 q48.46.11(5c)	c^2 ♀ 00 p – – –(alb.)	... ...	... ...
	c^3 ♀ 00 p – – –(alb.)	... ...	... ...	... ...	... ...	... ...
5H44	a^1 ♂ 95 q58.55. 4(4e)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	A2b^1 ♂ 00 p – – –(alb. A1)	4H17b^4 ♀ 100xp23.23.18(6f)
	a^5 ♀ 100xq58.58.57(6e)	a^6 ♀ 100xq58.58.55(6e)	a^7 ♀ 00 p – – –(alb.)	a^8 ♀ 00 p – – –(alb.)	... ...	... ...
	b^1 ♂ 00 p – – –(alb.)	b^2 ♂ 00 p – – –(alb.)	b^3 ♀ 00 p – – –(alb.)	b^4 ♀ 100xq58.57.55(6e)	... ...	... ...
	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ 100xq58.58.55(6e)	c^3 ♂ 00 p – – –(alb.)	c^4 ♀ 100xq58.57.55(6e)	... ...	... ...
	c^5 ♀ 00 p – – –(alb.)	c^6 ♀ 00 p – – –(alb.)	... ...	... ...	... ...	... ...
5H45	a^1 ♂ 100xq58.57.55(6e)	a^2 ♀ 100xq58.57.55(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq57.57.56(6e)	A2b^1 ♂ 00 p – – –(alb. A1)	4H48a^6 ♀ – p – – –(6f)
	a^5 ♀ 100xq58.58.55(6e)	... ...	... ...	... ...	... ...	... ...
5H46	a^1 ♂ 75 q48.46.11(5c)	a^2 ♂ 75 q48.46.11(5c)	a^3 ♂ 100zq48.46.11(5c)	a^4 ♂ 100zq48.46.28(5c)	4H48a^1 ♂ 00 p – – –(alb.)	4H69a^5 ♀ 80 p39.39.28(5b)
	a^5 ♂ – q – – –(5d)	a^6 ♀ 100xq58.58.55(6e)	a^7 ♀ 100xq58.58.55(6e)	b^1 ♂ 100xq58.58.55(6e)	... ...	... ...

	b^2 ♂ 80 q48.46. 2(5c)	b^3 ♂ 75 q48.46. 2(5c)	b^4 ♂ 100zq48.46. 4(5c)	b^5 ♂ 75 q48.46. 4(5c)	... ...	... ...
	b^6 ♂ 75 q49.46. 4(5c)	b^7 ♂ 60 q61.58. 1(3-4e)	b^8 ♀ 100xq58.58.55(6e)	c^1 ♂ 100xq58.57.55(6e)	... ...	... ...
	c^2 ♂ 70 q61.55. 1(3e)	c^3 ♀ 100xq58.58.55(6e)	c^4 ♀ - q - - - (2-3e)	c^5 - q - - - -	... ...	... ...
5H47	a^1 ♂ 00 p - - - (alb.)	a^2 ♀ 00 p - - - (alb.)	a^3 ♀ 100xq58.58.57(6e)	a^4 ♀ 85 q58.57. 2(4e)	4H46a^1 ♂ 00 p - - - (alb.)	4H16c^2 ♀ 100zp40.40.31(5b)
	a^5 ♀ - q - - - (5c)	a^6 - q - - - -	a^7 - q - - - -	a^8 - p - - - -	... ...	... ...
	a^9 - p - - - -	b^1 ♂ 00 p - - - (alb.)	b^2 ♂ 00 p - - - (alb.)	b^3 ♂ 00 p - - - (alb.)	... ...	... ...
	b^4 ♂ 100zq48.46.11(5c)	b^5 ♀ 00 p - - - (alb.)	b^6 ♀ 70 q48.46. 1(4c)	b^7 - q - - - -	... ...	... ...
	b^8 - p - - - -	c^1 ♂ 00 p - - - (alb.)	c^2 ♂ 00 p - - - (alb.)	c^3 ♂ 00 p - - - (alb.)	... ...	... ...
	c^4 ♂ 00 p - - - (alb.)	c^5 ♀ 00 p - - - (alb.)	c^6 ♀ 100xq58.58.57(6e)	c^7 - q - - - -	... ...	... ...
5H50	a^1 ♀ 90 q48.46. 2(5c)	a^2 ♀ 100zq49.46.12(5c)	a^3 ♀ 75 q49.46. 7(5c)	a^4 ♀ 65 q48.46. 1(4c)	4H16b^1 ♂ 100zp39.39.30(5b)	4H55a^6 ♀ 00 p - - - (alb.)
	b^1 ♂ 75 q49.46.11(5c)	b^2 ♂ 75 q49.46.11(5c)	b^3 ♀ 90 q49.48. 1(4c)	b^4 ♀ 65 q49.48. 1(3-4c)	... ...	... ...
	b^5 ♂ - q - - - (5c)	c^1 ♂ 80 q49.46. 7(5c)	c^2 ♀ - q - - - (3-4c)	c^3 - q - - - -	... ...	... ...
5H51	a^1 ♂ 85 p35.35. 3(5b)	a^2 ♂ 00 p - - - (alb.)	a^3 ♂ 00 p - - - (alb.)	a^4 ♀ 80 q49.46.14(5c)	4H36b^3 ♂ 00 p - - - (alb.)	4H16c^3 ♀ 75 p39.39. 4(5b)
	a^5 ♀ 100yp35.35. 3(5b)	a^6 ♀ 00 p - - - (alb.)	a^7 ♀ 00 p - - - (alb.)	a^8 - p - - - -	... ...	... ...
	a^9 - p - - - -	b^1 - q - - - -	b^2 ♀ 00 p - - - (alb.)	b^3 ♀ 00 p - - - (alb.)	... ...	... ...
	b^4 ♀ 00 p - - - (alb.)	... ...	... ...	... ...	... ...	... ...
5H52	a^1 ♀ 55 q48.46. 4(2-3c)	a^2 ♀ 35 q49. 1. 1(2c)	a^3 ♂ - q - - - (-c)	b^1 ♂ 50 q48.46. 1(3c)	4H57b^2 ♂ 00 p - - - (alb.)	4H69a^6 ♀ 75 p39.39.28(5b)
	b^2 ♀ 100xq49.48.47(6c)	b^3 ♀ 100yq49.48. 1(5c)	b^4 ♀ 30 q48. 1. 2(2c)	b^5 ♀ 50 q49.46. 1(3c)	... ...	... ...
	b^6 ♀ 55 q49.46. 1(3c)	c^1 ♂ 100yq48.46. 2(5c)	c^2 ♂ 50 q48. 1. 1(3c)	c^3 ♀ 100zq48.46.12(5c)	... ...	... ...
5H53	a^1 ♂ 100xq57.57.56(6e)	a^2 ♂ 100xp15.15.15(6f)	a^3 ♂ 100xp22.17.17(6f)	a^4 ♂ 75 p36.35. 2(5b)	4H22c^1 ♂ 75 p39.35. 2(5b)	4H33c^5 ♀ 00 p - - - (alb.)
	a^5 ♀ - p - - - (5b)	a^6 ♀ 100xp15.15.15(6f)	a^7 ♀ 100xq58.58.55(6e)	a^8 ♀ 100zq48.46.43(5c)	... ...	... ...
5H54	a^1 ♂ 100xp15.15.15(6f)	a^2 ♂ 35 q61. 1. 1(2e)	b^1 ♂ 100xq58.58.56(5e)	b^2 ♂ 100xq58.57.56(5e)	4H58b^1 ♂ 00 p - - - (alb.)	4H69b^3 ♀ 100yp22.15.18(5f)
	b^3 ♀ 60 q58. 1. 1(3e)	b^4 ♀ 100xp10.10.10(6f)	b^5 ♀ 100xp15.15.15(6f)	c^1 ♀ 45 q61.61. 1(3e)	... ...	... ...
	c^2 - p - - - -	... ...	... ...	... ...	... ...	... ...
5H55	a^1 ♂ 90 q57.56. 2(4e)	a^2 ♂ 60 q57. 1. 1(3e)	a^3 ♂ 75 q58. 1. 1(3e)	a^4 ♂ 100yq58.57.55(4e)	4H45a^1 ♂ 100xp23.23.17(6f)	4H63a^7 ♀ 00 p - - - (alb.)
	a^5 ♀ 80 q58.58. 2(4e)	... ...	... ...	... ...	... ...	... ...
5H56	a^1 ♂ 100xq58.58.55(6e)	b^1 ♂ - q - - - (6e)	b^2 ♂ 90 q57.57.55(4e)	b^3 ♂ 80 q57.55. 1(4e)	4H65b^6 ♂ 00 p - - - (alb.)	4H41b^2 ♀ 100xp26.26.17(6f)
	b^4 ♀ 100xq58.58.55(6e)	b^5 ♀ 100xq58.57.55(6e)	c^1 ♂ 80 q58.55. 2(4e)	c^2 ♀ 100xq58.58.55(6e)	... ...	... ...
	c^3 ♀ 95 q58.55. 1(5e)	... ...	... ...	... ...	... ...	... ...
5H57	a^1 ♂ 100xq57.57.56(5e)	a^2 ♂ - q - - - (3-4e)	a^3 ♀ 100xq58.57.55(6e)	a^4 ♀ 100xq58.58.55(5e)	4H66a^1 ♂ 100xq57.57.55(5e)	4H52b^2 ♀ 100yq58.58.56(5e)
	a^5 ♀ - q - - - (5e)	b^1 ♂ 100yq57.57.56(5e)	b^2 ♂ 100xq58.57.55(5e)	b^3 ♂ 100xq57.57.56(5e)	... ...	... ...
	b^4 ♂ 75 q58.57. 4(4e)	b^5 ♀ 100xq58.58.56(5e)	b^6 - q - - - -	c^1 ♂ - q - - - (6e)	... ...	... ...
	c^2 ♂ 100xq58.57.55(6e)	c^3 ♀ 100xq58.58.56(6e)	c^4 ♀ 100xq58.58.55(6e)	c^5 - q - - - -	... ...	... ...
5H58	a^1 ♂ - q - - - (6e)	a^2 ♂ 60 q58. 1. 1(3e)	a^3 ♀ 00 p - - - (alb.)	a^4 ♀ 100yq58.58.55(5e)	4H66a^3 ♂ 100yq58.57.55(5e)	4H49a^3 ♀ 100xq58.58.55(5e)
	a^5 ♀ 100xq58.58.55(5e)	... ...	... ...	... ...	... ...	... ...
5H59	a^1 ♂ 100xq58.57.56(6e)	a^2 ♂ 100xq58.57.55(5e)	a^3 ♂ 40 q58.58. 1(2e)	a^4 ♀ - q - - - (6e)	4H52c^1 ♂ 100yq58.58.56(5e)	4H51a^4 ♀ 100xq58.57.56(6e)
	a^5 ♀ 100xq58.58.55(6e)	a^6 ♀ 75 q58.55. 1(4e)	a^7 ♀ 75 q58.57. 1(4e)	b^1 ♂ 100xq57.57.56(6e)	... ...	... ...
	b^2 ♂ 100xq58.58.56(6e)	b^3 ♂ 65 q58. 1. 1(4e)	b^4 ♂ - q - - - (3e)	b^5 ♀ 100xq58.58.55(6e)	... ...	... ...
	b^6 ♀ 100xq58.58.57(5e)	b^7 ♀ 100xq58.58.57(5e)	c^1 ♂ 100xq58.58.55(5-6e)	c^2 ♂ 100xq58.58.56(6e)	... ...	... ...
	c^3 ♂ 100yq58.58.55(5e)	c^4 ♀ 75 q58.55. 1(3e)	c^5 ♀ 60 q58.58. 1(3e)	c^6 ♀ 100xq58.57.55(5e)	... ...	... ...
	c^7 ♀ 100xq58.57.55(6e)	c^8 ♀ 100xq58.58.55(5e)	... ...	... ...	... ...	... ...
5H60	a^1 ♂ 00 p - - - (alb.)	a^2 ♂ 00 p - - - (alb.)	a^3 ♂ 00 p - - - (alb.)	a^4 ♂ 00 p - - - (alb.)	4H63a^3 ♂ 00 p - - - (alb.)	4H57b^3 ♀ 00 p - - - (alb.)
	a^5 ♀ 00 p - - - (alb.)	a^6 ♀ 00 p - - - (alb.)	a^7 ♀ 00 p - - - (alb.)	a^8 ♀ 00 p - - - (alb.)	... ...	... ...
5H62	a^1 ♂ 100xq58.60.55(5-6e)	a^2 ♂ 100yq60.60. 2(5e)	a^3 ♂ 100xq58.60.55(5ew)	a^4 ♀ 100xq58.57.55(6e)	4H47c^1 ♂ 100xq58.57.55(6e)	4H49c^3 ♀ 100xq57.57.55(6e)
	a^5 ♀ 100xq58.60.55(5-6e)	a^6 ♀ 100xq58.57.55(5-6e)	a^7 - p - - - -	b^1 ♀ 100xq58.58.55(6e)	... ...	... ...
	b^2 ♀ 100xq60.60.54(5-6e)	... ...	... ...	... ...	... ...	... ...
5H63	a^1 ♂ 100xq58.58.55(6e)	a^2 ♀ 100xq58.58.55(6ew)	a^3 ♀ 100xq58.60.55(6ew)	b^1 ♂ 100xq58.55.60(6e)	4H46c^1 ♂ 100xq58.58.55(6e)	4H51c^2 ♀ 100xq58.58.55(6e)
	b^2 ♂ 100xq58.57.60(5ew)	b^3 ♀ 100xq58.57.55(6e)	b^4 ♀ 100xq58.58.55(6ew)	... ...	... ...	... ...
5H64	a^1 ♀ 100xq58.58.55(6e)	... ...	... ...	... ...	4H51b^4 ♂ 100xq58.58.55(6e)	4H46c^5 ♀ 100xq57.57.55(5e)
5H69	a^1 ♂ 100xq58.60.60(6e)	a^2 ♀ 100xq58.60.60(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq58.57.55(6e)	4H65b^4 ♂ 80 q58.55. 3(4e)	4H49b^2 ♀ - q - - - (6e)

MATING	OFFSPRING				PARENTS	
					Father	Mother
5H30	c^2 ♀ 100xq58.58.55(6e)	c^3 ♀ 100xq58.56.55(6e)	c^4 ♀ 100xq58.58.55(6e)	c^5 ♀ 100xq58.58.55(6e)	... ...	... ...
	c^6 ♀ 00 p – – –(alb.)	c^7 – q – – – –	... ...	... ...	... ...	... ...
5H31	a^1 ♂ 100xq58.58.57(6e)	a^2 ♂ 100xp23.23.18(6f)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq58.57.55(6e)	4H30a^3 ♂ 100xq58.57.55(6e)	4H50a^4 ♀ 100xq58.56.56(6e)
	a^5 ♀ 100xq58.58.55(6e)	a^6 ♀ 100xq58.57.55(6e)	a^7 ♀ 100xp23.23.18(6f)	a^8 – q – – – –	... ...	... ...
	a^9 – q – – – –	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ 100xp26.23.15(6f)	b^3 ♂ 100xq58.58.55.5(6e)	... ...	... ...
	b^4 ♂ 100yq57.57.56(5–6e)	b^5 ♂ 100xq58.58.55(6e)	b^6 ♂ 100xq58.58.55(5e)	b^7 ♀ 100xq58.57.56(6e)	... ...	... ...
	b^8 ♀ 100xp22.22.18(6f)	b^9 ♀ 100xp15.15.18(6f)	b^{10} – q – – – –	c^1 ♂ 100xq58.58.57(6e)	... ...	... ...
	c^2 ♂ 100xq58.58.57(6e)	c^3 ♂ 100xq58.58.55(6e)	c^4 ♂ 100xq58.58.55(6e)	c^5 ♂ 100xp26.26.23(6f)	... ...	... ...
	c^6 ♀ 100xq58.58.55(6e)	c^7 ♀ 100xq58.58.57(6e)	c^8 ♀ – q – – –(6e)	c^9 ♀ – p – – –(6f)	... ...	... ...
	c^{10} ♀ – p – – –(6f)	... ...	... ...	... ...	... ...	... ...
5H32	a^1 ♂ – q – – –(6e)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♀ – q – – –(6e)	a^4 ♀ 100xq58.57.55(6e)	4H51a^1 ♂ 100xq58.57.55(6e)	4H30a^4 ♀ 100xq58.57.55(6e)
	a^5 ♀ 90 q58.58. 1(4e)	a^6 ♀ 00 p – – –(alb.)	b^1 ♂ 100xq58.57.55(5e)	b^2 ♀ 00 p – – –(alb.)	... ...	... ...
	b^3 ♀ 100xq58.58.55(6e)	b^4 ♀ 100xq58.57.55(6e)	b^5 – q – – – –	c^1 ♂ 100xq58.57.55(6e)	... ...	... ...
	c^2 ♀ 100xq58.58.56(6e)	c^3 ♀ 100xq58.57.55(6e)	c^4 ♀ 00 p – – –(alb.)	c^5 – q – – – –	... ...	... ...
	c^6 – q – – – –	... ...	... ...	... ...	... ...	... ...
5H33	a^1 ♂ 100xq57.57.55(6e)	a^2 ♂ 100xq58.57.56(6e)	a^3 ♀ 100xq55.55.54(6e)	a^4 ♀ 100xq58.57.55(6e)	4H50b^1 ♂ 100xq58.58.55(6e)	4H30a^3 ♀ 100xq58.57.55(6e)
	b^1 ♂ 100xp23.15.18(6f)	b^2 ♂ 100xq58.57.55(6e)	b^3 ♂ 100xq58.58.55(6e)	b^4 ♂ 100xq58.57.55(6e)	... ...	... ...
	b^5 ♀ 100xq58.57.56(6e)	b^6 ♀ 100xq58.57.55(6e)	b^7 ♀ 100xq58.57.55(5e)	... ...	... ...	... ...
5H34	a^1 ♂ 100xp23.23.15(6f)	a^2 ♀ 100xq58.57.55(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq58.58.56(6e)	4H51 { b^2 ♂ 100xq58.58.55(6e) b^3 ♂ 100xq58.57.55(6e) }	4H30a^5 ♀ 100xq58.57.56(6e)
	a^5 ♀ 100xq58.58.57(6e)	a^6 ♀ 100xp23.23.18(6f)	b^1 ♂ 100xq58.57.55(6e)	c^1 ♂ 100xq58.58.55(6e)	... ...	... ...
	c^2 ♂ 100xq58.57.55(6e)	c^3 ♀ 100xq58.58.55(6e)	c^4 ♀ 100xp23.23.17(6f)	c^5 – p – – – –	... ...	... ...
	c^6 – p – – – –	... ...	... ...	... ...	... ...	... ...
5H35	a^1 ♂ – p – – –(5b)	a^2 ♀ 100zp39.35.28(5b)	a^3 – p – – – –	a^4 – p – – – –	4H14b^1 ♂ 75 p39.39.28(5b)	4H15b^4 ♀ 100zp35.35.28(5b)
	a^5 – p – – – –	... ...	... ...	... ...	... ...	... ...
5H36	a^1 ♂ 100zp37.37. 3(5b)	a^2 ♂ 100zp35.35.30(5b)	a^3 ♀ 100xp22.22.18(6f)	a^4 ♀ 100zp35.35.28(5b)	4H14b^2 ♂ 100zp39.35.28(5b)	4H22b^3 ♀ 75 p39.39. 3(5b)
	a^5 ♂ 100zp26.35. 6(5b)	a^6 – p – – – –	a^7 – p – – – –	b^1 ♂ 100xp23.23.18(6f)	... ...	... ...
	b^2 ♂ 100xp23.23.18(6f)	b^3 ♀ 100xp23.23.18(6f)	b^4 ♀ 100xp22.22.18(6f)	b^5 ♀ 100zp39.35.28(5b)	... ...	... ...
	b^6 ♀ 75 p39.39. 3(5b)	... ...	... ...	... ...	... ...	... ...
5H37	a^1 ♂ 100zp35.35.28(5–6b)	a^2 ♀ – p – – –(5b)	a^3 ♀ 100zp39.35.30(5b)	b^1 ♂ 100xp23.23.22(6f)	4H16b^3 ♂ 100 p39.39.28(5b)	4H15b ♀ – p – – –(5b)
	b^2 ♂ 100zp35.35. 6(5b)	b^3 ♀ 100xp13.13.18(6f)	c^1 ♂ 100yp18.18.18(5b)	c^2 ♂ 100xp39.39.30(5b)	... ...	... ...
	c^3 ♂ 85 p36.36. 3(5b)	c^4 ♀ – p – – –(5b)	... ...	... ...	... ...	... ...
5H40	a^1 ♂ 75 q46.46. 2(5–6c)	a^2 ♀ 100xq58.58.56(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100zq48.46.14(5c)	4H15b^2 ♂ – p – – –(5b)	4H47a^5 ♀ 00 p – – –(alb.)
	a^5 ♀ – q – – –(5c)	a^6 ♀ 00 p – – –(alb.)	b^1 ♂ 100xq58.58.55(6e)	b^2 ♂ 00 p – – –(alb.)	... ...	... ...
	b^3 ♀ 100xq58.58.55(5e)	b^4 ♀ 00 p – – –(alb.)	c^1 ♂ 80 q48.46.11(5c)	c^2 ♀ 00 p – – –(alb.)	... ...	... ...
	c^3 ♀ 00 p – – –(alb.)	... ...	... ...	... ...	... ...	... ...
5H44	a^1 ♂ 95 q58.55. 1(4e)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	A2b^1 ♂ 00 p – – –(alb. A1)	4H17b^4 ♀ 100xp23.23.18(6f)
	a^5 ♀ 100xq58.58.57(6e)	a^6 ♀ 100xq58.58.55(6e)	a^7 ♀ 00 p – – –(alb.)	a^8 ♀ 00 p – – –(alb.)	... ...	... ...
	b^1 ♂ 00 p – – –(alb.)	b^2 ♂ 00 p – – –(alb.)	b^3 ♀ 00 p – – –(alb.)	b^4 ♀ 100xq58.57.55(6e)	... ...	... ...
	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ 100xq58.58.55(6e)	c^3 ♂ 00 p – – –(alb.)	c^4 ♀ 100xq58.57.55(6e)	... ...	... ...
	c^5 ♀ 00 p – – –(alb.)	c^6 ♀ 00 p – – –(alb.)	... ...	... ...	... ...	... ...
5H45	a^1 ♂ 100xq58.57.55(6e)	a^2 ♀ 100xq58.57.55(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq57.57.56(6e)	A2b^1 ♂ 00 p – – –(alb. A1)	4H48a^6 ♀ – p – – –(6f)
	a^5 ♀ 100xq58.58.55(6e)	... ...	... ...	... ...	... ...	... ...
5H46	a^1 ♂ 75 q48.46.11(5c)	a^2 ♂ 75 q48.46.11(5c)	a^3 ♂ 100zq48.46.11(5c)	a^4 ♂ 100zq48.46.28(5c)	4H48a^1 ♂ 00 p – – –(alb.)	4H69a^5 ♀ 80 p39.39.28(5b)
	a^5 ♂ – q – – –(5d)	a^6 ♀ 100xq58.58.55(6e)	a^7 ♀ 100xq58.58.55(6e)	b^1 ♂ 100xq58.58.55(6e)	... ...	... ...
	b^2 ♂ 80 q48.46. 2(5c)	b^3 ♂ 75 q48.46. 2(5c)	b^4 ♂ 100zq48.46. 4(5c)	b^5 ♂ 75 q48.46. 4(5c)	... ...	... ...
	b^6 ♂ 75 q49.46. 4(5c)	b^7 ♂ 60 q61.58. 1(3–4e)	b^8 ♀ 100xq58.58.55(6e)	c^1 ♂ 100xq58.57.55(6e)	... ...	... ...
	c^2 ♂ 70 q61.55. 1(3e)	c^3 ♀ 100xq55.55.55(6e)	c^4 ♀ – q – – –(2–3e)	c^5 – q – – – –	... ...	... ...
5H47	a^1 ♂ 00 p – – –(alb.)	a^2 ♀ 00 p – – –(alb.)	a^3 ♀ 100xq58.58.57(6e)	a^4 ♀ 85 q58.57. 2(4e)	4H46a^1 ♂ 00 p – – –(alb.)	4H16c^2 ♀ 100zp40.40.31(5b)
	a^5 ♀ – q – – –(5c)	a^6 – q – – – –	a^7 – q – – – –	a^8 – p – – – –	... ...	... ...
	a^9 – p – – – –	b^1 ♂ 00 p – – –(alb.)	b^2 ♂ 00 p – – –(alb.)	b^3 ♂ 00 p – – –(alb.)	... ...	... ...
	b^4 ♂ 100zq48.46.11(5c)	b^5 ♀ 00 p – – –(alb.)	b^6 ♀ 70 q48.46. 1(4c)	b^7 – q – – – –	... ...	... ...
	b^8 – p – – – –	c^1 ♂ 00 p – – –(alb.)	c^2 ♂ 00 p – – –(alb.)	c^3 ♂ 00 p – – –(alb.)	... ...	... ...
	c^4 ♂ 00 p – – –(alb.)	c^5 ♀ 00 p – – –(alb.)	c^6 ♀ 100xq58.58.57(6e)	c^7 – q – – – –	... ...	... ...
5H50	a^1 ♀ 90 q48.46. 2(5c)	a^2 ♀ 100 q49.46.12(5c)	a^3 ♀ 75 q49.46. 7(5c)	a^4 ♀ 65 q48.46. 1(4c)	4H16b^1 ♂ 100zp39.39.30(5b)	4H55a^6 ♀ 00 p – – –(alb.)
	b^1 ♂ 75 q49.46.11(5c)	b^2 ♂ 75 q49.46.11(5c)	b^3 ♀ 90 q49.48. 1(4c)	b^4 ♀ 65 q49.48. 1(3–4c)	... ...	... ...
	b^5 ♂ – q – – –(5c)	c^1 ♂ 80 q49.46. 7(5c)	c^2 ♀ – q – – –(3–4c)	c^3 – q – – – –	... ...	... ...
5H51	a^1 ♂ 80 p35.35. 3(5b)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♀ 80 q49.46.14(5c)	4H36b^3 ♂ 00 p – – –(alb.)	4H16c^3 ♀ 75 p39.39. 4(5b)
	a^5 ♀ 100zp35.35. 3(5b)	a^6 ♀ 00 p – – –(alb.)	a^7 ♀ 00 p – – –(alb.)	a^8 – p – – – –	... ...	... ...
	a^9 – p – – – –	b^1 – q – – – –	b^2 ♀ 00 p – – –(alb.)	b^3 ♀ 00 p – – –(alb.)	... ...	... ...
	b^4 ♀ 00 p – – –(alb.)	... ...	... ...	... ...	... ...	... ...
5H52	a^1 ♀ 50 q48.46. 4(2–3c)	a^2 ♀ 55 q49. 1. 1(2c)	a^3 ♂ – q – – –(–c)	b^1 ♂ 50 q48.46. 1(3c)	4H57b^2 ♂ 00 p – – –(alb.)	4H69a^6 ♀ 75 p39.39.28(5b)
	b^2 ♀ 100xq49.48.47(6c)	b^3 ♀ 100yq49.48. 1(5c)	b^4 ♀ 30 q48. 1. 2(2c)	b^5 ♀ 50 q49.46. 1(3c)	... ...	... ...
	b^6 ♀ 55 q49.46. 1(3c)	c^1 ♂ 100yq48.46. 2(5c)	c^2 ♂ 50 q48. 1. 1(3c)	c^3 ♀ 100zq48.46.12(5c)	... ...	... ...
5H53	a^1 ♂ 100xq57.57.56(6e)	a^2 ♂ 100xp15.15.15(6f)	a^3 ♂ 100xp22.17.17(6f)	a^4 ♂ 75 p36.35. 2(5b)	4H22c^1 ♂ 75 p39.35. 2(5b)	4H33c^5 ♀ 00 p – – –(alb.)
	a^5 ♀ – p – – –(5b)	a^6 ♀ 100xp15.15.15(6f)	a^7 ♀ 100xq58.58.55(6e)	a^8 ♀ 100zq48.46.43(5c)	... ...	... ...
5H54	a^1 ♂ 100xp15.15.15(6f)	a^2 ♂ 75 q61. 1. 1(2e)	b^1 ♂ 100xq58.58.56(5e)	b^2 ♂ 100xq58.57.56(5e)	4H58b^1 ♂ 00 p – – –(alb.)	4H69b^3 ♀ 100yp22.15.18(5f)
	b^3 ♀ 60 q58. 1. 1(3e)	b^4 ♀ 100xp10.10.10(6f)	b^5 ♀ 100xp15.15.15(6f)	c^1 ♀ 45 q61.61. 1(3e)	... ...	... ...
	c^2 – p – – – –	... ...	... ...	... ...	... ...	... ...
5H55	a^1 ♂ 90 q57.56. 2(4e)	a^2 ♂ 60 q57. 1. 1(3e)	a^3 ♂ 75 q58. 1. 1(3e)	a^4 ♂ 100yq58.57.55(4e)	4H45a^1 ♂ 100xp23.23.17(6f)	4H63a^7 ♀ 00 p – – –(alb.)
	a^5 ♀ 80 q58.58. 2(4e)	... ...	... ...	... ...	... ...	... ...
5H56	a^1 ♂ 100xq58.58.55(6e)	b^1 ♂ – q – – –(6e)	b^2 ♂ 90 q57.57.55(4e)	b^3 ♂ 80 q57.55. 1(4e)	4H65b^6 ♂ 00 p – – –(alb.)	4H41b^2 ♀ 100xp26.26.17(6f)
	b^4 ♀ 100xq58.58.55(6e)	b^5 ♀ 100xq58.57.55(6e)	c^1 ♂ 80 q58.55. 2(4e)	c^2 ♀ 100xq58.58.55(6e)	... ...	... ...
	c^3 ♀ 95 q58.55. 1(5e)		... ...	... ...	... ...	... ...
5H57	a^1 ♂ 100xq57.57.56(5e)	a^2 ♂ – q – – –(3–4e)	a^3 ♀ 100xq58.57.55(6e)	a^4 ♀ 100xq58.58.55(5e)	4H66a^1 ♂ 100xq57.57.55(5e)	4H52b^2 ♀ 100yq58.58.56(5e)
	a^5 ♀ – q – – –(5e)	b^1 ♂ 100yq57.57.56(5e)	b^2 ♂ 100xq58.57.55(5e)	b^3 ♂ 100xq57.57.56(5e)	... ...	... ...
	b^4 ♂ 75 q58.57. 4(4e)	b^5 ♀ 100xq58.58.56(5e)	b^6 – q – – – –	c^1 ♂ – q – – –(6e)	... ...	... ...
	c^2 ♂ 100xq58.57.55(6e)	c^3 ♀ 100xq58.58.56(6e)	c^4 ♀ 100xq58.58.55(6e)	c^5 – q – – – –	... ...	... ...
5H58	a^1 ♂ – q – – –(6e)	a^2 ♂ 60 q58. 1. 1(3e)	a^3 ♀ 00 p – – –(alb.)	a^4 ♀ 100yq58.58.55(5e)	4H66a^3 ♂ 100yq58.57.55(5e)	4H49a^3 ♀ 100xq58.58.55(5e)
	a^5 ♀ 100xq58.58.55(5e)	... ...	... ...	... ...	... ...	... ...
5H59	a^1 ♂ 100xq58.57.56(6e)	a^2 ♂ 100xq58.57.55(5e)	a^3 ♂ 40 q58.58. 1(2e)	a^4 ♀ – q – – –(6e)	4H52c^1 ♂ 100yq58.58.56(5e)	4H51a^4 ♀ 100xq58.57.56(6e)
	a^5 ♀ 100xq58.58.55(6e)	a^6 ♀ 75 q58.55. 1(4e)	a^7 ♀ 75 q58.57. 1(4e)	b^1 ♂ 100xq57.57.56(6e)	... ...	... ...
	b^2 ♂ 100xq58.58.56(6e)	b^3 ♂ 65 q58. 1. 1(4e)	b^4 ♂ – q – – –(3e)	b^5 ♀ 100xq58.58.55(6e)	... ...	... ...
	b^6 ♀ 100xq58.58.57(5e)	b^7 ♀ 100xq58.58.57(5e)	c^1 ♂ 100xq58.58.55(5–6e)	c^2 ♂ 100xq58.58.56(6e)	... ...	... ...
	c^3 ♂ 100yq58.58.55(5e)	c^4 ♀ 75 q58.55. 1(3e)	c^5 ♀ 60 q58.58. 1(3e)	c^6 ♀ 100xq58.57.55(5e)	... ...	... ...
	c^7 ♀ 100xq58.57.55(6e)	c^8 ♀ 100xq58.58.55(5e)	... ...	... ...	... ...	... ...
5H60	a^1 ♂ 00 p – – –(alb.)	a^2 ♂ 00 p – – –(alb.)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	4H63a^3 ♂ 00 p – – –(alb.)	4H57b^3 ♀ 00 p – – –(alb.)
	a^5 ♀ 00 p – – –(alb.)	a^6 ♀ 00 p – – –(alb.)	a^7 ♀ 00 p – – –(alb.)	a^8 ♀ 00 p – – –(alb.)	... ...	... ...
5H62	a^1 ♂ 100xq58.60.55(5–6e)	a^2 ♂ 100yq60.60. 2(5e)	a^3 ♂ 100xq58.60.55(5ew)	a^4 ♀ 100xq58.57.55(6e)	4H47c^4 ♂ 100xq58.57.55(6e)	4H49c^4 ♀ 100xq57.57.55(6e)
	a^5 ♀ 100xq58.60.55(5–6e)	a^6 ♀ 100xq58.57.55(5–6e)	a^7 – p – – – –	b^1 ♀ 100xq58.58.55(6e)	... ...	... ...
	b^2 ♀ 100xq60.60.54(5–6e)	... ...	... ...	... ...	... ...	... ...
5H63	a^1 ♂ 100xq58.58.55(6e)	a^2 ♀ 100xq58.58.55(6ew)	a^3 ♀ 100xq58.60.55(6ew)	b^1 ♂ 100xq58.55.60(6e)	4H46c^1 ♂ 100xq58.58.55(6e)	4H51c^2 ♀ 100xq58.58.55(6e)
	b^2 ♂ 100xq58.57.60(5ew)	b^3 ♀ 100xq58.57.55(6e)	b^4 ♀ 100xq58.58.55(6ew)	... ...	... ...	... ...
5H64	a^1 ♀ 100xq58.58.55(6e)		... ...	... ...	4H51b^4 ♂ 100xq58.58.55(6e)	4H46c^5 ♀ 100xq57.57.55(5e)
5H69	a^1 ♂ 100xq58.60.60(6e)	a^2 ♀ 100xq58.60.60(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq58.57.55(6e)	4H65b^4 ♂ 80 q58.55. 3(4e)	4H49b^2 ♀ – q – – –(6e)

MATING	OFFSPRING				PARENTS	
					Father	Mother
5H72	a^1 ♂ 45 q61. 1. 1(2e)	a^2 ♂ 65 q42.50. 1(4g)	a^3 ♀ 70 q58.57. 1(4e)	a^4 ♀ 70 q58.57. 1(4e)	4H60c^2 ♂ 65 q58.55. 1(3–4e)	4H56b^4 ♀ 80 q58.55. 3(4e)
	a^5 ♀ 50 q61.60. 1(3e)	a^6 ♀ 95 q41.26. 1(4–5g)	a^7 ♀ 00 p – – –(alb.)	b^1 ♂ 85 q58.60. 1(4e)	... ...	... ...
	b^2 ♂ 65 q61.60. 1(3–4e)	b^3 ♂ 00 p – – –(alb.)	b^4 ♂ 00 p – – –(alb.)	b^5 ♀ 85 q58.55. 1(4e)	... ...	... ...
	b^6 ♀ 90 q58.55. 3(4e)	b^7 ♀ 85 q58.55. 1(4e)	b^8 ♀ 60 q58.60. 1(3–4e)	b^9 ♀ 90 q42.50. 1(4g)	... ...	... ...
5H73	a^1 ♂ 90 q58.60. 2(4e)	a^2 ♂ 50 q58.58. 1(3e)	a^3 ♀ 85 q61.60. 1(4e)	a^4 ♀ 80 q58.57. 1(4e)	4H59c^3 ♂ 60 q58. 1. 1(4e)	4H56b^5 ♀ 90 q58.55. 1(4e)
	a^5 ♀ 00 p – – –(alb.)	b^1 ♂ 00 p – – –(alb.)	... ...	... ...	... ...	... ...
5H76	a^1 ♂ 100zq 49.48.43(5c)	a^2 ♂ 100zq 49.46. 2(5c)	a^3 ♀ 100zq 48.46.43(5–6c)	a^4 ♀ 100xq60.60.55(5e)	4H48c^4 ♂ 00 p – – –(alb.)	4H75b^3 ♀ 80 p40.36.28(5b)
	a^5 ♀ 100xp23.23.15(6f)	b^1 ♂ 100xq58.60.60(6e)	b^2 ♂ 100yq58.60.55(5e)	b^3 ♂ 100xq58.57.55(5e)	... ...	... ...
	b^4 ♀ 100xq58.57.60(5e)	b^5 ♀ 100zp35.35. 3(5b)	c^1 ♂ 100xq58.60.60(5–6e)	c^2 ♀ 100xq60.60.60(5–6e)	... ...	... ...
	c^3 ♀ 100xq58.60.55(5e)	... ...	... ...	... ...	... ...	... ...
5H77	a^1 ♂ 100xp35.35.34(5b)	a^2 ♂ 100zp36.33.28(5b)	a^3 ♂ 100zq 48.46.14(5c)	a^4 ♀ 100zp36.36. 2(5b)	4H48c^4 ♂ 00 p – – –(alb.)	4H75b^4 ♀ 75 p39.35.28(5b)
	b^1 ♀ 100zp36.36. 3(5b)	b^2 ♀ 100zq 48.46.43(5c)	b^3 ♀ 100zq 48.46.43(5c)	c^1 ♂ 100zq 49.46.14(5c)	... ...	... ...
	c^2 ♂ 100zq 49.46.43(5c)	... ...	... ...	... ...	... ...	... ...
5H78	a^1 ♂ 75 p39.35. 1(5b)	a^2 ♂ 100zq 48.46.11(5c)	a^3 ♀ 75 p35.35. 4(5b)	a^4 ♀ 100zq 49.46.43(5c)	4H48c^4 ♂ 00 p – – –(alb.)	4H75b^5 ♀ 75 p39.39. 4(5b)
	a^5 ♀ 100zq 49.46.43(5c)	b^1 ♂ 80 p36.36. 4(5b)	b^2 ♂ 75 p34.34. 3(5b)	b^3 ♂ 100zq 48.46.43(5c)	... ...	... ...
	b^4 ♀ 75 p36.36. 4(5b)	b^5 ♀ 80 p36.36. 3(5b)	b^6 ♀ 100zq 49.46.43(5c)	b^7 ♀ 100zq 49.46.43(5c)	... ...	... ...
5H81	a^1 ♂ 95 p36.33. 3(5b)	a^2 ♂ 50 p39. 1. 1(2–3b)	a^3 ♂ 35 p37.37. 1(2b)	a^4 ♀ 30 p39. 1. 1(2b)	4H69c^2 ♂ 100xp39.39.31(6b)	4H75a^5 ♀ 100zp39.35.30(5b)
	a^5 ♀ 40 p39. 1. 1(2b)	b^1 ♂ 95 p37.35.28(5b)	b^2 ♂ 35 p37. 1. 1(2b)	b^3 ♂ 25 p37. 1. 1(2b)	... ...	... ...
	b^4 ♀ 75 p35.35. 3(5b)	b^5 ♀ 80 p39.35. 3(5b)	b^6 ♀ 80 p39.35. 3(5b)	c^1 ♂ 100zp 36.35.28(5b)	... ...	... ...
	c^2 ♂ 100zp 36.36. 6(5b)	c^3 ♂ 100zp 39.36. 4(5b)	c^4 ♂ 95 p39.35.30(5b)	c^5 ♀ 100zp 37.36. 3(5b)	... ...	... ...
5H82	a^1 ♂ 100zp 39.35. 3(5b)	a^2 ♂ 100zp 36.36. 9(5b)	a^3 ♂ 100zp 35.35. 4(5b)	a^4 ♂ 100zp 39.39.15(5–6b)	4H69c^3 ♂ 100zp39.33.30(5b)	4H76a^6 ♀ 100zp35.35.28(5b)
	a^5 ♀ 100xp39.39.31(5–6b)	a^6 ♀ 100xp39.39 31(5–6b)	a^7 ♀ 100xp39.39.35(5b)	a^8 ♀ 100zp 39.39.28(5b)	... ...	... ...
	b^1 ♂ 100zp 39.35.28(5b)	b^2 ♀ 100zp 26.34.28(5b)	... ...	... ...	... ...	... ...
5H84	a^1 ♂ 100xp26.26. 8(6f)	a^2 ♂ 100xp26.26. 8(6f)	a^3 ♂ 100xp19.23.18(6f)	a^4 ♀ 100xp26.23.23(6f)	4H51c^1 ♂ 100xp26.23.15(6f)	4H39b^3 ♀ 100xp23.23.18(6f)
	a^5 ♀ 100xp26.23.18(6f)	a^6 ♀ 100xp23.23.10(6f)	b^1 ♂ 100xp26.26.17(6f)	b^2 ♂ 100xp23.23.15(6f)	... ...	... ...
	b^3 ♂ 100xp23.23.15(6f)	b^4 ♀ 100xp23.23.18(6f)	b^5 ♀ 100xp23.23.15(6f)	c^1 ♂ 100xp23.23.15(6f)	... ...	... ...
	c^2 ♂ 100xp23.23.15(6f)	c^3 ♂ 100xp22.22.18(6f)	c^4 ♀ 100xp23.23.15(6f)	c^5 ♀ 100xp23.23.15(6f)	... ...	... ...
5H85	a^1 ♂ 100xp23.23.18(6f)	a^2 ♀ 100xp23.23.15(6f)	a^3 ♀ 100xp22.22.23(6f)	a^4 ♀ 100xp22.22.18(6f)	4H38c^1 ♂ 100xp22.22.15(6f)	4H45c^2 ♀ 100xp15.15.15(6f)
5H87	a^1 ♂ 100xp26.23.17(6f)	a^2 ♂ 100xp23.23.15(6f)	a^3 ♀ 95 p22.23. 1(5f)	a^4 ♀ 85 p22.22. 1(4f)	4H48b^2 ♂ 100xp22.22.18(6f)	4H51c^3 ♀ 100xp23.23.18(6f)
	b^1 ♂ 100xp23.23.15(6f)	b^2 ♂ 100xp23.23.15(5f)	b^3 ♂ 85 p26.23. 1(4–5f)	b^4 ♀ 100xp26.23.15(6f)	... ...	... ...
	b^5 ♀ 100xp23.15.15(6f)	b^6 ♀ 100xp26.23.15(6f)	b^7 ♀ 80 p26. 1. 1(4f)	c^1 ♀ 100xp23.23.17(6f)	... ...	... ...
	c^2 ♀ 100xp23.23.15(6f)	c^3 ♀ 100xp23.23.10(6f)	c^4 ♀ 100xp23.23.15(6f)	c^5 ♀ 100xp26.26.23(6f)	... ...	... ...
6H1	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 100xq58.58.55(6e)	a^3 ♂ 80 q48.46.11(5c)	a^4 ♀ 100xq58.58.55(6e)	5H40a^1 ♂ 75 q46.46. 2(5–6c)	5H50a^1 ♀ 90 q48.46. 2(5c)
	a^5 ♀ 100xq58.58.55(6e)	a^6 ♀ 100zq 48.46. 7(5c)	a^7 ♀ 100zq 46.46.14(5c)	a^8 ♀ 00 p – – –(alb.)	... ...	... ...
	b^1 ♂ 100zq 48.46.43(6d)	b^2 ♂ 60 q48.46. 4(4c)	b^3 ♂ 75 p35.35. 2(5b)	b^4 ♂ 100zp 36.36. 4(5b)	... ...	... ...
	b^5 ♂ 00 p – – –(alb.)	b^6 ♂ 00 p – – –(alb.)	b^7 – q – – – –	b^8 – p – – – –	... ...	... ...
	c^1 – q – – – –	c^2 – q – – – –	c^3 – q – – – –	c^4 – q – – – –	... ...	... ...
	c^5 – p – – – –	c^6 – p – – – –	c^7 – p – – – –	... ...	... ...	... ...
6H2	a^1 ♂ 100zq 48.46.11(5c)	a^2 ♂ 100zq 52.46.14(5c)	a^3 ♂ 100zq 49.48. 7(5c)	a^4 ♀ 100xp23.22.17(5fw)	5H46a^2 ♂ 75 q48.46.11(5c)	5H50a^2 ♀ 100zq49.46.12(5c)
	a^5 ♀ 80 p35.35. 4(5b)	a^6 ♀ 00 p – – –(alb.)	a^7 – q – – – –	b^1 ♂ 100xq58.57.55(6e)	... ...	... ...
	b^2 ♂ 100zq 48.46. 7(5c)	b^3 ♂ 75 q48.46. 4(5c)	b^4 ♂ 100zq 48.46. 4(5c)	b^5 ♂ 00 p – – –(alb.)	... ...	... ...
	b^6 ♀ 100zq 52.46.14(5c)	b^7 ♀ 100zp39.35.28(5b)	b^8 ♀ 100xq58.57.55(5e)	c^1 ♂ 90 q48.46.14(5c)	... ...	... ...
	c^2 ♂ 100zq 48.46.11(5c)	c^3 ♂ 100zq 52.46. 7(5c)	c^4 ♂ 80 p36.35. 3(5b)	c^5 ♀ 80 p35.35. 3(5b)	... ...	... ...

	c^6 ♀ 75 p36.35. 3(5b)	c^7 ♀ 100zq 52.46.14(5c)	c^8 ♀ 00 p– – –(alb.)	… … …	… … … …	… … … …
6H3	a^1 ♂ 100zp 36.36.30(5b)	a^2 ♂ 100xq58.60.55(6e)	a^3 ♂ 100xq58.58.55(6e)	a^4 ♂ 100zq 48.46.14(5c)	5H46a^3 ♂ 100zq 48.46.11(5c)	5H51a^4 ♀ 80 q49.46.14(5c)
	a^5 ♂ 100xp17.17.15(6f)	a^6 ♂ 100zq 48.46.11(5c)	a^7 ♀ 100zq 52.46. 4(5c)	a^8 ♀ 100xq58.58.55(6e)	… … … …	… … … …
	b^1 ♂ 100zq 49.46.43(5c)	b^2 ♀ 100xq57.57.56(6e)	b^3 ♀ 100xq58.57 56(6e)	b^4 ♀ 75 q48.48. 2(5c)	… … … …	… … … …
	b^5 ♀ 100zq 49.46.14(5c)	b^6 ♀ 100zq 48.46.11(5c)	c^1 ♂ 80 q48.46. 4(5c)	c^2 ♂ 100zp39.35. 4(5b)	… … … …	… … … …
	c^3 ♂ 00 p– – –(alb.)	c^4 ♀ 00 p– – –(alb.)	c^5 ♀ 00 p– – –(alb.)	c^6 ♀ 100zq 48.46.43(5c)	… … … …	… … … …
6H4	a^1 ♂ 100zp39.39. 3(5bw)	a^2 ♂ 100xp23.23.15(6f)	a^3 ♂ 75 p36.36. 3(5b)	a^4 ♀ 80 q48.48. 4(5c)	5H46a^4 ♂ 100zq 48.46.28(5c)	5H53a^8 ♀ 100zq 48.46.43(5c)
	a^5 ♀ 100zp39.39.28(5bw)	a^6 ♀ 75 p35.35. 3(5b)	a^7 ♀ – p– – –(5b)	a^8 ♀ 00 p– – –(alb.w)	… … … …	… … … …
	a^9 – q– – – –	a^{10} – p– – – –	b^1 ♂ 00 p– – –(alb.)	b^2 ♂ 00 p– – –(alb.)	… … … …	… … … …
	b^3 ♀ 100zq 48.46.11(5cw)	b^4 ♀ 00 p– – –(alb.)	b^5 ♀ 00 p– – –(alb.)	… … …	… … … …	… … … …
6H5	a^1 ♂ 100xp26.26.23(6f)	a^2 ♂ 100xp36.36.30(6b)	a^3 ♂ 80 p36.35. 3(5b)	a^4 ♂ 100zq 52.46.43(5c)	5H47b^4 ♂ 100zq 48.46.11(5c)	5H52b^2 ♀ 100xq49.48.47(6c)
	a^5 ♂ 00 p– – –(alb.)	a^6 ♀ 100xq49.48.47(6c)	a^7 – q– – – –	b^1 ♂ 100zp39.36. 3(5b)	… … … …	… … … …
	b^2 ♂ 85 p36.34. 3(5b)	b^3 ♂ 100xq49.46.47(6c)	b^4 ♂ 00 p– – –(alb.)	b^5 ♀ 100xq48.46.46(6c)	… … … …	… … … …
	b^6 ♀ 100zq 48.46.43(5c)	b^7 ♀ 00 p– – –(alb.)	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ 85 p34.34. 3(5b)	… … … …	… … … …
	c^3 ♂ 100zq 46.46.43(5c)	c^4 ♀ 80 p36.35. 3(5b)	c^5 ♀ 100zq 48.46.11(5c)	… … …	… … … …	… … … …
6H6	a^1 ♂ 75 q46.46.11(5c)	a^2 – p– – –	… … …	… … …	5H50b^1 ♂ 75 q49.46.11(5c)	5H52b^3 ♀ 100yq49.48. 1(5c)
6H7	a^1 ♂ 100xp26.23.18(6f)	a^2 ♂ 65 q49.46. 3(4c)	a^3 ♂ 00 p– – –(alb.)	b^1 ♂ 100zp35.35. 2(5b)	5H50b^2 ♂ 75 q49.46.11(5c)	5H47b^6 ♀ 70 q48.46. 1(4c)
	b^2 ♂ 100zq 52.46.43(5c)	b^3 ♂ 00 p– – –(alb.)	b^4 ♂ 00 p– – –(alb.)	b^5 ♀ 100xq58.58.55(6e)	… … … …	… … … …
	b^6 ♀ 100xq58.57.55(5e)	b^7 ♀ 00 p– – –(alb.)	c^1 ♂ 100zq 48.46. 4(5c)	c^2 ♂ 100zq 48.46. 7(5c)	… … … …	… … … …
	c^3 ♂ 100zp39.35.30(5b)	c^4 ♂ 00 p– – –(alb.)	c^5 ♀ 45 q49.46. 1(3c)	c^6 ♀ 40 q49.46. 1(3c)	… … … …	… … … …
	c^7 – p– – – –	… … …	… … …	… … …	… … … …	… … … …
6H8	a^1 ♀ 00 p– – –(alb.)	a^2 ♀ 00 p– – –(alb.)	… … …	… … …	5H46b^6 ♂ 75 q49.46. 4(5c)	5H50b^3 ♀ 90 q49.48. 1(4c)
6H10	a^1 ♂ 100zq 48.46. 7(5c)	a^2 ♂ 55 q48.46. 1(3c)	a^3 ♀ 80 q58.55. 1(3e)	a^4 ♀ 25 p23. 1. 1(1f)	5H46b^3 ♂ 75 q48.46. 2(5c)	5H52b^5 ♀ 50 q49.46. 1(3c)
	a^5 ♀ 35 q49. 1. 1(2c)	a^6 ♀ 00 p– – –(alb.)	b^1 ♂ 75 p36.35. 3(5b)	b^2 ♂ 60 p37. 1. 1(3b)	… … … …	… … … …
	b^3 ♀ 100zq49.48.43(5c)	b^4 ♀ 100zq 49.46.43(4c)	b^5 ♀ 100zp36.35. 3(5b)	b^6 ♀ 35 q48. 1. 1(2c)	… … … …	… … … …
	c^1 ♂ 85 q48.46. 3(4c)	c^2 ♀ 100zq 48.46.43(5c)	c^3 ♀ 50 p37.35. 1(2b)	c^4 ♀ 45 p37. 1. 3(2b)	… … … …	… … … …
	c^5 ♀ 00 p– – –(alb.)	c^6 ♀ 00 p– – –(alb.)	c^7 ♀ 00 p– – –(alb.)	… … …	… … … …	… … … …
6H11	a^1 ♂ 100zq 48.48.28(5c)	a^2 ♂ 100zq 48.48. 2(5c)	a^3 ♂ 50 q48.35. 3(3c)	a^4 ♂ 00 p– – –(alb.)	5H46b^4 ♂ 100zq 48.46. 4(5c)	5H52b^6 ♀ 55 q49.46. 1(3c)
	a^5 ♀ 55 p37.39. 4(3b)	a^6 ♀ 00 p– – –(alb.)	a^7 – q– – – –	b^1 ♂ 100zq48.46. 7(5c)	… … … …	… … … …
	b^2 ♂ 100xp23.23.18(6f)	b^3 ♂ 85 q48.46.33(4c)	b^4 ♀ 00 p– – –(alb.)	… … …	… … … …	… … … …
6H12	a^1 ♀ 100zq 48.46.14(5c)	a^2 ♀ 00 p– – –(alb.)	b^1 ♂ 100zq49.46.43(5c)	b^2 ♂ 100zq 49.46. 4(5c)	5H46b^5 ♂ 75 q48.46. 4(5c)	5H52b^4 ♀ 30 q48. 1. 2(2c)
	b^3 ♀ 100zq 48.48.11(5c)	b^4 ♀ 100zq 48.46.14(5c)	c^1 ♂ 90 p23. 1.15(4f)	c^2 ♂ 100zp36.36. 3(5b)	… … … …	… … … …
	c^3 ♂ 00 p– – –(alb.)	c^4 ♂ 00 p– – –(alb.)	… … …	… … …	… … … …	… … … …
6H13	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 100zp35.35. 3(5b)	b^1 ♂ 50 q48. 1. 1(2–3c)	b^2 ♂ 00 p– – –(alb.)	5H40c^1 ♂ 80 q48.46.11(5c)	5H50c^2 ♀ – q– – –(3–4c)
	b^3 ♂ 00 p– – –(alb.)	b^4 ♀ 100xp23.17.17(6f)	b^5 ♀ 90 p26.26. 3(5f)	b^6 ♀ 90 q48.35.28(4c)	… … … …	… … … …
	b^7 ♀ 00 p– – –(alb.)	c^1 ♂ 80 q48.46. 3(4c)	c^2 ♂ 100zp36.36. 3(5b)	c^3 ♂ 00 p– – –(alb.)	… … … …	… … … …
	c^4 ♂ 00 p– – –(alb.)	c^5 ♀ 75 q48.46. 7(4c)	c^6 ♀ 70 q49.48. 3(4c)	c^7 ♀ 60 q48.46. 2(3c)	… … … …	… … … …
	c^8 ♀ 00 p– – –(alb.)	c^9 ♀ 00 p– – –(alb.)	… … …	… … …	… … … …	… … … …
6H14	a^1 ♂ 100zp39.35. 3(5b)	a^2 ♂ 00 p– – –(alb.)	a^3 ♀ 100zq 49.48.20(5c)	a^4 ♀ 100zq 48.48.12(5c)	5H50c^1 ♂ 80 q49.46. 7(5c)	5H52c^3 ♀ 100zq48.46.12(5c)
	a^5 ♀ 100xp22.22.17(6f)	a^6 ♀ 00 p– – –(alb.)	a^7 – q– – – –	b^1 ♂ 50 p37. 1. 1(2b)	… … … …	… … … …
	b^2 ♀ 100zp39.39.30(5b)	c^1 ♂ 100xp23.23.15(6f)	c^2 ♂ 00 p– – –(alb.)	c^3 ♂ 00 p– – –(alb.)	… … … …	… … … …
	c^4 ♀ 100zq 52.48.11(5c)	c^5 ♀ 100zq 48.46.14(5c)	c^6 ♀ 60 q48.46. 4(4c)	… … …	… … … …	… … … …
6H15	a^1 ♂ 100xq58.60.55(6e)	a^2 ♂ 100xq58.60.55(6e)	a^3 ♂ 100xq57.57.55(6e)	a^4 ♀ 100xq58.60.55(6e)	5H57c^2 ♂ 100xq58.57.55(6e)	5H59c^7 ♀ 100xq58.57.55(6e)
	a^5 ♀ 100xq58.58.55(6e)	a^6 ♀ 100xq58.58.57(5e)	b^1 ♂ 100xq58.60.55(6e)	b^2 ♂ 100xq58.58.55(6e)	… … … …	… … … …
	b^3 ♂ 100xq58.58.55(5e)	b^4 ♀ 100xq58.58.55(6e)	c^1 ♀ 100xq58.58.55(6e)	c^2 ♀ 100xq58.57.55(6e)	… … … …	… … … …
	c^3 ♀ 100xq58.58.55(6e)	c^4 ♀ 100xq58.55.55(6e)	c^5 ♀ 100xq58.58.57(5–6e)	… … …	… … … …	… … … …
6H16	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 100xq58.57.55(5e)	a^3 ♂ 00 p– – –(alb.)	a^4 ♂ 00 p– – –(alb.)	5H32c^1 ♂ 100xq58.57.55(6e)	5H34c^3 ♀ 100xq58.58.55(6e)
	a^5 ♂ 00 p– – –(alb.)	a^6 ♀ 100xq57.57.55(6e)	a^7 ♀ 100xq58.57.55(6e)	… … …	… … … …	… … … …
6H17	a^1 ♂ 100xq58.55.55(6e)	a^2 ♂ – q– – –(6e)	a^3 ♂ 100xq60.57.55(6e)	a^4 ♀ 100xq58.60.55(6e)	5H34c^1 ♂ 100xq58.58.55(6e)	5H32c^2 ♀ 100xq58.58.56(6e)
	a^5 – q– – – –	b^1 ♂ 100xq58.58.45(6e)	b^2 ♂ 100xq58.57.55(6e)	b^3 ♂ 100yq58.55. 1(5ew)	… … … …	… … … …

MATING	OFFSPRING				PARENTS	
					Father	Mother
5H72	a^1 ♂ 45 q61. 1. 1(2e)	a^2 ♂ 65 q42.50. 1(4g)	a^3 ♀ 70 q58.57. 1(4e)	a^4 ♀ 70 q58.57. 1(4e)	4H60c^2 ♂ 65 q58.55. 1(3–4e)	4H56b^4 ♀ 80 q58.55. 3(4e)
	a^5 ♀ 50 q61.60 1(3e)	a^6 ♀ 95 q41.26. 1(4–5g)	a^7 ♀ 00 p – – –(alb.)	b^1 ♂ 85 q58.60 1(4e)	... ...	... ...
	b^2 ♂ 65 q61.60. 1(3–4e)	b^3 ♂ 00 p – – –(alb.)	b^4 ♂ 00 p – – –(alb.)	b^5 ♀ 85 q58.55. 1(4e)	... ...	... ...
	b^6 ♀ 90 q58.55. 3(4e)	b^7 ♀ 85 q58.55. 1(4e)	b^8 ♀ 60 q58.60. 1(3–4e)	b^9 ♀ 90 q42.50. 1(4g)	... ...	... ...
5H73	a^1 ♂ 90 q58.60. 2(4e)	a^2 ♂ 50 q58.58. 1(3e)	a^3 ♀ 85 q61.60. 1(4e)	a^4 ♀ 80 q58.57. 1(4e)	4H59c^3 ♂ 60 q58. 1. 1(4e)	4H56b^6 ♀ 90 q58.55. 1(4e)
	a^5 ♀ 00 p alb	b^1 ♂ 00 p (alb.)			... ...	... ...
5H76	a^1 ♂ 100zq49.48.43(5c)	a^2 ♂ 100zq 49.46. 2(5c)	a^3 ♀ 100zq 48.46.43(5–6c)	a^4 ♀ 100xq60.60 55(5e)	4H48c^4 ♂ 00 p – – –(alb.)	4H75b^3 ♀ 80 p40.36.28(5b)
	a^5 ♀ 100xp23 23 15(6f)	b^1 ♂ 100xq58.60 60(6e)	b^2 ♂ 100yp58.60 55(5e)	b^3 ♂ 100xq58.57 55 5e	... ...	... ...
	b^4 ♀ 100xq58.57.60 5e	b^5 ♀ 100zp35 35 3(5b)	c^1 ♂ 100xp58 60.60 5–6e	c^2 ♀ 100xp60 60 60 5–6e)	... ...	... ...
	c^3 ♀ 100xq58.60.55(5e)	... ...	... ...	... ...	... ...	... ...
5H77	a^1 ♂ 100zp35 35 31 5b	a^2 ♂ 100zp36 33 28 5b	a^3 ♂ 100zq 48.46.14 5c	a^4 ♀ 100zp36 36 2 5b	4H48c^4 ♂ 00 p – – –(alb.)	4H75b^4 ♀ 75 p39.35 28 5b)
	b^1 ♀ 100zp36.36. 3(5b)	b^2 ♀ 100zq 48.46 43 5c	b^3 ♀ 100zq 48.46.43(5c)	c^1 ♂ 100zq 49 46 14 5c	... ...	... ...
	c^2 ♂ 100zq49.46.43(5c)				... ...	... ...
5H78	a^1 ♂ 75 p39.35. 1(5b)	a^2 ♂ 100zq 48 46 11(5c)	a^3 ♀ 75 p35 35 4(5b)	a^4 ♀ 100zq 49.46.43 5c	4H48c^4 ♂ 00 p – – –(alb.)	4H75b^5 ♀ 75 p39.39. 4(5b)
	a^5 ♀ 100zq49.46.43(5c)	b^1 ♂ 80 p36 36 1 5b	b^2 ♂ 75 p34.34. 3(5b)	b^4 ♂ 100zq 48.46 43 5c	... ...	... ...
	b^3 ♀ 75 p36 36. 4 5b	b^5 ♀ 80 p36 36 3 5b	b^6 ♀ 100zq 49.46.43(5c)	b^7 ♀ 100zq 49.46 43 5c	... ...	... ...
5H81	a^1 ♂ 95 p36 33 3 5b	a^2 ♂ 50 p39 1. 1 2 3b	a^3 ♂ 85 p37.37 1(2b)	a^4 ♀ 30 p39. 1. 1 2b	4H69c^2 ♂ 100xp39 39 31 6b	4H75a ♀ 100zp39 35 30(5b)
	a^5 ♀ 40 p39 1 1 2b	b^1 ♂ 95 p37 35 28 5b	b^2 ♂ 75 p37 1 1(2b)	b^3 ♂ 25 p37 1 1 2b	... .	. . .
	b^4 ♀ ... p35 35 3 5b	b^5 ♀ 80 p39 35 3 5b	b^6 ♀ 80 p39 35 3(5b)	c^1 ♂ 100 p36 35 28 5b	... ...	. . .
	c^2 ♂ 100zp36 36 6 5b	c^3 ♂ 100zp39 36 1 5b	c^4 ♂ 95 p34 35 30 5b	c^5 ♀ 100zp37 36 3 5b	... .	... ...
5H82	a^1 ♂ 100zp 39 35 3 5b	a^2 ♂ 100zp36 36 9 5b	a^3 ♂ 100zp35 35 4 5b	a^4 ♂ 100zp 39 39 15 5 6b	4H69c^3 ♂ 100zp39 33.30 5b	4H76a^6 ♀ 100zp35.35.28(5b)
	a^5 ♀ 100xp34 34 31 5 6b	a^6 ♀ 100xp39 39 31 5 6b	a^7 ♀ 100xp34 39 35 5b	a^8 ♀ 100zp39 39.28 5b	.. . .	. .
	b^1 ♂ 100zp39 35 28 5b	b^2 ♀ 100zp26 34 28 5b				
5H84	a^1 ♂ 100xp26 26 8 6f	a^2 ♂ 100xp26 26 8 6f	a^3 ♂ 100xp19 23.18 6f	a^4 ♀ 100xp26 23 23 6f	4H51c^1 ♂ 100xp26 23 15 6f	4H39b^3 ♀ 100xp23.23.18(6f)
	a^5 ♀ 100xp26 23 18 6f	a^6 ♀ 100xp23 23 10 6f	b^1 ♂ 100xp 26 26 17 6f	b^2 ♂ 100xp23 23 15 6f	... ...	... .
	b^3 ♂ 100xp23 23 15 6f	b^4 ♀ 100xp23 23 18 6f	b^5 ♀ 100xp23 23.15 6f	c^1 ♂ 100xp23 23 17 6f	.. ..	. .
	c^2 ♂ 100xp23 23 15 6f	c^3 ♂ 100xp22 22 18 6f	c^4 ♀ 100xp23 23 15 6f	c^5 ♀ 100xp23 23 15 6f	... .	. ...
5H85	a^1 ♂ 100xp 23 23 18 6f	a^2 ♀ 100xp23 23 15 6f	a^3 ♀ 100xp22.22 23 6f	a^4 ♀ 100xp22 22 18 6f	4H38c^1 ♂ 100xp22 22 15 6f	4H45c^2 ♀ 100xp15 15.15(6f)
5H87	a^1 ♂ 100xp 26 23 17 6f	a^2 ♂ 100xp23 23 15 6f	a^3 ♀ 95 p22 23 1 5f	a^4 ♀ 85 p22 22 1 4f	4H48b^2 ♂ 100xp22 22 18 6f	4H51c^3 ♀ 100xp23 23 18 6f)
	b^1 ♂ 100xp23 23 15 6f	b^2 ♂ 100xp23 23 15 5f	b^3 ♂ 85 p26 23 1 4 5f	b^4 ♀ 100xp26 23 15 6f	. . .	
	b^5 ♀ 100xp23 15 15 6f	b^6 ♀ 100xp26 23 15 6f	b^7 ♀ 80 p26 1 1 4f	c^1 ♀ 100xp23 23 17 6f	.. ..	.
	c^2 ♀ 100xp23 23 15 6f	c^3 ♀ 100xp23 23 10 6f	c^4 ♀ 100xp 23 23 15 6f	c^5 ♀ 100xp26 26 23 6f	.. . .	.
6H1	a^1 ♂ 100xp58 58 55 6e	a^2 ♂ 100xq58 58 55(6e)	a^3 ♂ 80 q48.46.11(5c)	a^4 ♀ 100xq58.58.55(6e)	5H49a^1 ♂ 75 q46.46. 2(5–6c)	5H50a^1 ♀ 90 q48.46. 2(5c)
	a^5 ♀ 100xq58.58.55(6e)	a^6 ♀ 100zq 48.46 7 5c	a^7 ♀ 100zq 46 46 14 5c	a^8 ♀ 00 p – – –(alb.)	... ...	.. ..
	b^1 ♂ 100zq48.46.43(6d)	b^2 ♂ 60 q48.46. 4(4c)	b^3 ♂ 75 p35.35. 2 5b	b^4 ♂ 100zp36.36. 4(5b)	.. .	. .
	b^5 ♂ 00 p – – –(alb.)	b^6 ♂ 00 p – – –(alb.)	b^7 q –	b^8 p	... . . .	. ..
	c^1 – q – – – –	c^2 – q – – –	c^3 q – –	c^4 q	... . .	. ...
	c^5 – p – – – –	c^6 – p	c^7 – p – – – –	. . .	... ...	... ...
6H2	a^1 ♂ 100zq48.46.11(5c)	a^2 ♂ 100zq 52 46 14(5c)	a^3 ♂ 100zq 49.48. 7(5c)	a^4 ♀ 100xp23 22 17 5(fe)	5H46a^2 ♂ 75 q48.46.11(5c)	5H50a^2 ♀ 100zq49.46.12(5c)
	a^5 ♀ 80 p35.35. 4(5b)	a^6 ♀ 00 p – – (alb.)	a^7 – q – – – –	b^1 ♂ 100xq58 57 55 6e	. . . .	. ...
	b^2 ♂ 100zq48.46. 7(5c)	b^3 ♂ 75 q48 46 4 5c	b^4 ♂ 100zq 48.46. 4(5c)	b^5 ♂ 00 p alb.)	.. ..	. ...
	b^6 ♀ 100zq52.46.14(5c)	b^7 ♀ 100zp39 35 28 5b	b^8 ♀ 100xq58 57.55 5e	c^1 ♂ 90 q48 46 14 5e	... .	. . ..
	c^2 ♂ 100zq48 46.11(5c)	c^3 ♂ 100zq52 46 7(5c)	c^4 ♂ 80 p36.35. 3 5b)	c^5 ♀ 80 p35 35. 3(5b)	.. . .	. ...
	c^6 ♀ 75 p36.35. 3(5b)	c^7 ♀ 100zq 52.46.14(5c)	c^8 ♀ 00 p alb	... .		
6H3	a^1 ♂ 100zp36.36.30(5b)	a^2 ♂ 100xq58 60.55(6e)	a^3 ♂ 100xq58 58 55(6e)	a^4 ♂ 100zq 48.46 14 5c	5H46a ♂ 100zq 48 46 11 5c	5H51a^1 ♀ 80 q49 46 14 5c
	a^5 ♂ 100xp17 17 15 6f	a^6 ♂ 100zq 48 46 11 5c	a^7 ♀ 100zq 52.46 4 5c	a^8 ♀ 100xq58.58 55 6e	.	. ..
	b^1 ♂ 100zq 49 46 43 5c	b^2 ♀ 100xq57 57 56 6e	b^3 ♀ 100xq58 57 56 6e	b^4 ♀ 75 q48.48. 2 5c	... .	. .
	b^5 ♀ 100zq 49 46 11 5c	b^6 ♀ 100zq 48.46 11 5c	c^1 ♂ 80 q48.46. 4(5c)	c^2 ♂ 100zp39.35. 4(5b)	. .	..
	c^3 ♂ 00 p alb	c^4 ♀ 00 p – alb.	c^5 ♀ 00 p – – –(alb.)	c^6 ♀ 100zq 48.46.43(5c)	... ...	.. . .
6H4	a^1 ♂ 100zp39 39 3 5bw	a^2 ♂ 100xp23 23 15 6f	a^3 ♂ 75 p36.36. 3(5b)	a^4 ♀ 80 q48.48. 4(5c)	5H46a^4 ♂ 100zq 48.46 28 5c	5H53a^8 ♀ 100zq 48 46.43 5c)
	a^5 ♀ 100zp 39 39 28 5bw	a^6 ♀ 75 p35 35 3 5b	a^7 ♀ – p – – –(5b)	a^8 ♀ 00 p – – –(alb.w)	. .	. . . . .
	a^9 q	a^{10} – p – – – –	b^1 ♂ 00 p – – –(alb.)	b^2 ♂ 00 p – – –(alb.)	..	. ..
	b^3 ♀ 100zq 48 46 11 5c	b^4 ♀ 00 p – – –(alb.)	b^5 ♀ 00 p – – –(alb.)	... ...		
6H5	a^1 ♂ 100xp26 26 23 6f	a^2 ♂ 100xp36.36.30(6b)	a^3 ♂ 80 p36.35. 3(5b)	a^4 ♂ 100zq 52.46.43(5c)	5H47b^1 ♂ 100zq 48 46 11 5c	5H52b^2 ♀ 100xq49 48 47(6e)
	a^5 ♂ 00 p alb	a^6 ♀ 100xq49.48.47(6e)	a^7 – q – – – –	b^1 ♂ 100zp39 36 3 5b	.. ..	...
	b^2 ♂ 85 p36 34 3 5b	b^3 ♂ 100xq49 46 47 6e	b^4 ♂ 00 p – – –(alb.)	b^5 ♀ 100xq48.46.46(6e)	. . .	...
	b^6 ♀ 100zq 48 46 13 5c	b^7 ♀ 00 p – – –(alb.)	c^1 ♂ 100xq58.58.55(6e)	c^2 ♂ 85 p34.34. 3(5b)	. .. . .	.. .. .
	c^3 ♂ 100zq 46 46 43 5	c^4 ♀ 80 p36.35. 3(5b)	c^5 ♀ 100zq 48.46.11(5c)	... .. .		
6H6	a^1 ♂ 75 q46 46 11 5c	a^2 – p – – – –	...	...	5H50b^1 ♂ 75 q49.46.11(5c)	5H52b^3 ♀ 100yq49.48. 1(5c)
6H7	a^1 ♂ 100xp26 23 18 6f	a^2 ♂ 65 q49.46 3 4c	a^3 ♂ 00 p – – –(alb.)	b^1 ♂ 100zp35.35. 2 5b	5H50b^2 ♂ 75 q49.46.11(5c)	5H47b^6 ♀ 70 q48.46. 1(4c)
	b^2 ♂ 100zq 52 46 13 5c	b^3 ♂ 00 p – – –(alb.)	b^4 ♂ 00 p – – –(alb.)	b^5 ♀ 100xq58 58 55 6e	.	... ...
	b^6 ♀ 100xq58 57 55 5e	b^7 ♀ 00 p – – –(alb.)	c^1 ♂ 100zq 48.46. 4(5c)	c^2 ♂ 100zq 48.46 7 5c	..	..
	c^3 ♂ 100zp39 35 30 5b	c^4 ♂ 00 p – – –(alb.)	c^5 ♀ 45 q49.46. 1(3c)	c^6 ♀ 40 q49.46. 1(3c)	. . .	. .. .
	c^7 – p –	. ... ..	... .	... .	...	...
6H8	a^1 ♀ 00 p – alb	a^2 ♀ 00 p alb			5H46b^6 ♂ 75 q49.46. 4(5c)	5H50b^3 ♀ 90 q49.48. 1(4c)
6H10	a^1 ♂ 100zq 48 46 7 5c	a^2 ♂ 65 q48 46 1 3c	a^3 ♀ 80 q58.55. 1(3e)	a^4 ♀ 25 p23 1 1 1f	5H46b^3 ♂ 75 q48.46. 2(5c)	5H52b^5 ♀ 50 q49.46. 1(3c)
	a^5 ♀ 35 q49. 1. 1 2c	a^6 ♀ 00 p alb	b^1 ♂ 75 p36.35. 3(5b)	b^2 ♂ 60 p37. 1. 1 3b)	.. . .	. .. ..
	b^3 ♀ 100zq49.48 43 5c	b^4 ♀ 100zq 49.46 43(4c)	b^5 ♀ 100zp36.35. 3(5b)	b^6 ♀ 35 q48. 1 1 2c	.. .	.
	c^1 ♂ 85 q48.46. 3 4c	c^2 ♀ 100zq 48.46.43(5c)	c^3 ♀ 50 p37.35. 1(2b)	c^4 ♀ 45 p37 1 3 2b)	. . .	. . ...
	c^5 ♀ 00 p – – –(alb.)	c^6 ♀ 00 p – – –(alb.)	c^7 ♀ 00 p – – –(alb.)	... .	... ...	... ...
6H11	a^1 ♂ 100zq 48.48.28(5c)	a^2 ♂ 100zq 48.48. 2(5c)	a^3 ♂ 50 q48.35. 3(3c)	a^4 ♂ 00 p – – –(alb.)	5H46b^4 ♂ 100zq 48.46. 4(5c)	5H52b^6 ♀ 55 q49.46. 1(3c)
	a^5 ♀ 55 p37.39. 4(3b)	a^6 ♀ 00 p – – –(alb.)	a^7 – q – – – –	b^1 ♂ 100zq48.46. 7(5c)	. ...	... .
	b^2 ♂ 100xp23.23.18(6f)	b^3 ♂ 85 q48.46.33(4c)	b^4 ♀ 00 p – – –(alb.)	. . .	... ...	... ...
6H12	a^1 ♀ 100zq 48.46.14(5c)	a^2 ♀ 00 p – – –(alb.)	b^1 ♂ 100zq49.46.43(5c)	b^2 ♂ 100zq 49 46. 4(5c)	5H46b^5 ♂ 75 q48.46. 4(5c)	5H52b^4 ♀ 80 q48. 1. 2(2c)
	b^3 ♀ 100zq 48.48.11(5c)	b^4 ♀ 100zq 48.46.14(5c)	c^1 ♂ 90 p23. 1.15(4f)	c^2 ♂ 100zp36.36. 3 5b)	. ...	... .
	c^3 ♂ 00 p – – –(alb.)	c^4 ♂ 00 p – – –(alb.)	... ...	... .	... ...	... ...
6H13	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 100zp35.35. 3(5b)	b^1 ♂ 50 q48. 1 1(2–3c)	b^2 ♂ 00 p – – –(alb.)	5H40c^1 ♂ 80 q48.46.11(5c)	5H50c^2 ♀ – q – – –(3–4c)
	b^3 ♂ 00 p – – –(alb.)	b^4 ♀ 100xp23.17 17 6f)	b^5 ♀ 90 p26.26. 3(5f)	b^6 ♀ 90 q48.35.28(4c)	... ...	... .
	b^7 ♀ 00 p – – –(alb.)	c^1 ♂ 80 q48.46. 3(4c)	c^2 ♂ 100zp36.36. 3(5b)	c^3 ♂ 00 p – – –(alb.)	... ...	. ...
	c^4 ♂ 00 p – – –(alb.)	c^5 ♀ 75 q48.46. 7(4c)	c^6 ♀ 70 q49.48. 3(4c)	c^7 ♀ 60 q48.46. 2(3c)	... .	.. ..
	c^8 ♀ 00 p – – –(alb.)	c^9 ♀ 00 p – – –(alb.)	... ...	... .	... ...	... ...
6H14	a^1 ♂ 100zp39.35. 3(5b)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 100zq 49.48.20(5c)	a^4 ♀ 100zq 48.48.12(5c)	5H50c^1 ♂ 80 q49.46. 7(5c)	5H52c^3 ♀ 100zq48.46.12(5c)
	a^5 ♀ 100xp22.22.17(6f)	a^6 ♀ 00 p – – –(alb.)	a^7 – q – – – –	b^1 ♂ 50 p37. 1. 1(2b)	... ...	. . ..
	b^2 ♀ 100zp39.39.30(5b)	c^1 ♂ 100xp23.23.15(6f)	c^2 ♂ 00 p – – –(alb.)	c^3 ♂ 00 p – – –(alb.)	... ...	... .
	c^4 ♀ 100zq 52.48.11(5c)	c^5 ♀ 100zq 48.46.14(5c)	c^6 ♀ 60 q48.46. 4(4c)	... .	... ...	... ...
6H15	a^1 ♂ 100xq58.60.55(6e)	a^2 ♂ 100xq58.60.55(6e)	a^3 ♂ 100xq57.57.55(6e)	a^4 ♀ 100xq58.60.55(6e)	5H57c^3 ♂ 100xq58.57.55(6e)	5H59c^7 ♀ 100xq58.57.55(6e)
	a^5 ♀ 100xq58.58 55(6e)	a^6 ♀ 100xq58.58.57(5e)	b^1 ♂ 100xq58.60 55 6e)	b^2 ♂ 100xq58.58.55(6e)	... .	... .
	b^3 ♂ 100xq58.58.55(5e)	b^4 ♀ 100xq58.58.55(6e)	c^1 ♀ 100xq58.58.55(6e)	c^2 ♀ 100xq58.57.55(6e)	... .	. . . .
	c^3 ♀ 100xq58.58.55(6e)	c^4 ♀ 100xq58.55 55(6e)	c^5 ♀ 100xq58.58.57(5–6e)	... ...	... ...	... ...
6H16	a^1 ♂ 100xq58.58.55(6e)	a^2 ♂ 100xq58.57.55(5e)	a^3 ♂ 00 p – – –(alb.)	a^4 ♂ 00 p – – –(alb.)	5H32c^1 ♂ 100xq58 57 55(6e)	5H34c^3 ♀ 100xq58.58.55(6e)
	a^5 ♂ 00 p – – –(alb.)	a^6 ♀ 100xq57.57.55(6e)	a^7 ♀ 100xq58.57.55(6e)	... ...	... ...	... ...
6H17	a^1 ♂ 100xq58.55 55(6e)	a^2 ♂ – q – – –(6e)	a^3 ♂ 100xq60.57.55(6e)	a^4 ♀ 100xq58 60.55(6e)	5H34c^1 ♂ 100xq58.58.55(6e)	5H32c^2 ♀ 100xq58.58.56(6e)
	a^5 q	b^1 ♂ 100xq58.58.45(6e)	b^2 ♂ 100xq58.57.55(6e)	b^3 ♂ 100yq58.55. 1(5ew)	... ...	... ...

MATING	OFFSPRING				PARENTS			
					Father		Mother	
6H17	b^4 ♀ 100xq58.58.55(6e)	b^5 ♀ 100xq58.57.55(6e)	b^6 ♀ 100xq58.55.55(6e)	b^7 ♀ 100xq58.58.55(5e)	...	...	...	...
	c^1 ♂ 100xq58.60.55(6e)	c^2 ♂ 100xq60.57.55(5–6e)	c^3 ♂ 100yq58.55.45(5e)	c^4 ♂ 100yq58.58.55(5e)	...	...	...	...
	c^5 ♀ 100xq58.55.55(6e)	c^6 ♀ 100xq58.58.55(5–6e)	c^7 ♀ 100xq58.60.55(5e)	...	...	...	...	...
6H18	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xp22.22.15(6f)	a^3 ♀ 100xq58.57.55(6e)	a^4 ♀ 100xq58.58.55(6e)	5H29c^1	♂ 100xq58.58.55(6e)	5H30c^3	♀ 100xq58.56.55(6e)
	a^5 ♀ 100xq58.58.55(6e)	a^6 ♀ 100xq58.57.55(5–6e)	a^7 ♀ 100xp23.23.15(6f)	a^8 ♀ 100xp23.23.15(6f)	...	...	...	...
	b^1 ♂ 100xq58.57.55(6e)	b^2 ♂ 100xq58.60.55(6e)	b^3 ♀ 100xq58.60.55(6e)	b^4 ♀ 100xq58.58.55(6e)	...	...	...	...
	b^5 ♀ 100xq58.58.55(6e)	b^6 ♀ 100xq58.58.55(6e)	b^7 ♀ 100xp17.15.15(6f)	c^1 ♂ 100xq58.58.55(6e)	...	...	...	...
	c^2 ♂ 100xq58.60.55(5–6e)	c^3 ♂ 100xp23.23.15(6f)	c^4 ♀ 100xq60.60.55(6e)	c^5 ♀ 100xq58.58.55(6e)	...	...	...	...
	c^6 ♀ 100xq58.57.55(6e)	c^7 ♀ 100xq58.58.55(6e)	c^8 ♀ 100xq58.57.55(6e)	c^9 – p – – – –	...	...	...	...
6H19	a^1 ♂ 100xq58.58.55(6e)	a^2 ♀ 100xq58.57.55(6e)	a^3 ♀ 100xp26.26.23(6f)	b^1 ♂ 100xq57.57.55(6e)	5H31c^4	♂ 100xq58.58.55(6e)	5H29c^3	♀ 100xq58.58.56(6e)
	b^2 ♂ 100xq58.58.55(6e)	b^3 ♀ 100xq58.58.55(6e)	b^4 ♀ 100xq58.60.55(6e)	c^1 ♂ 100xp22.23.15(6f)	...	...	...	...
	c^2 ♀ 100xq58.60.55(6e)	c^3 ♀ 100xq58.58.55(6e)	c^4 ♀ 100xq57.57.55(6e)	c^5 ♀ 100xq57.55.55(6e)	...	...	...	...
	c^6 ♀ 100xq57.55.55(6e)	c^7 ♀ 100xq58.57.55(6e)	c^8 – q – – – –	c^9 – q – – – –	...	...	...	...
	c^{10} – p – – – –	...	...	...	...	...	...	...
6H20	a^1 ♂ 85 q58.55. 4(4e)	a^2 ♂ – q – – –(4e)	a^3 ♂ 90 q56.55. 2(3e)	a^4 ♂ – q – – –(4e)	5H27c^1	♂ 85 q58.44. 1(4e)	5H26c^4	♀ 90 q58.55. 2(4e)
	a^5 ♀ 90 q58.55. 2(4–5e)	a^6 – q – – – –	a^7 – q – – – –	...	...	...	...	...
6H21	a^1 ♀ 95 q58.58. 2(4e)	a^2 ♀ 85 q58.60. 1(4e)	...	...	5H26c^1	♂ 90 q58.57. 1(5e)	5H27c^6	♀ 75 q58.56. 1(4e)
6H22	a^1 ♂ 100xq56.58.55(6e)	a^2 ♂ – q – – –(5e)	a^3 ♀ 100xq57.55.55(6e)	a^4 ♀ 100xq57.57.55(6e)	5H33b^2	♂ 100xq58.57.55(6e)	5H58a^4	♀ 100yq58.58.55(5e)
	a^5 ♀ 80 q58.55. 1(4e)	a^6 ♀ 00 p – – –(alb.)	b^1 ♂ 100xq57.57.55(6e)	b^2 ♂ 100xq58.57.55(6e)	...	...	...	...
	b^3 ♂ 100yq56.55. 1(5e)	b^4 ♀ 100xq58.57.55(6e)	b^5 ♀ 100xq58.57.55(6e)	b^6 ♀ 100xq58.58.55(6e)	...	...	...	...
	b^7 ♀ 00 p – – –(alb.)	c^1 ♀ 100xq58.58.55(6e)	c^2 ♀ 100xq58.58.55(6e)	c^3 ♀ 100xq58.58.55(6e)	...	...	...	...
	c^4 ♀ 95 q58.57. 2(4–5e)	...	...	...	...	...	...	...
6H23	a^1 ♂ 100xq58.58.55(6e)	a^2 ♀ 100xq58.58.55(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq58.55.55(5e)	5H58a^2	♂ 60 q58. 1. 1(3e)	5H33b^5	♀ 100xq58.57.56(6e)
	a^5 ♀ 100yq58.58.55(5ew)	b^1 ♂ 100xq58.60.55(6e)	b^2 ♂ 100xq57.57.55(6e)	b^3 ♀ 100xq58.60.55(6e)	...	...	...	...
	b^4 ♀ 100xq58.55.55(5–6e)	b^5 ♀ 100xq58.60.55(5e)	...	...	...	...	...	...
6H24	a^1 ♀ 100zp37.35.30(5b)	a^2 ♀ 100zq 52.46.14(5c)	b^1 ♂ 100zp36.34. 3(5b)	b^2 ♂ 100zq48.46.43(5c)	5H33b^3	♂ 100xq58.58.55(6e)	5H35a^2	♀ 100zp39.35.28(5b)
	b^3 ♂ 100zq 48.46.43(5c)	b^4 ♂ 100zq 48.46. 4(5c)	b^5 ♂ 100zq 48.46. 7(5c)	b^6 ♀ 100zp36.36. 4(5b)	...	...	...	...
	b^7 ♀ 100zq 48.46.11(5c)	...	...	...	...	...	...	...
6H25	a^1 ♂ 100xq58.60.55(5–6e)	a^2 ♂ 100xq58.57.56(5e)	a^3 ♂ 100zq 52.46.12(5c)	a^4 ♂ 100zq 49.46.14(5c)	5H31b^6	♂ 100xq58.58.55(5e)	5H37a^2	♀ – p – – –(5b)
	a^5 ♀ 95 q58.60. 4(5e)	a^6 ♀ 100zq 48.46. 2(5c)	a^7 ♀ 100zq 48.46.43(5c)	a^8 – q – – – –	...	...	...	...
	a^9 – q – – – –	b^1 ♂ 100zq 44.47.11(5c)	b^2 ♂ 100zq 48.46. 7(5c)	...	...	...	...	...
6H26	a^1 ♂ 100zp36.35. 3(5b)	a^2 ♂ 100zq 52.46.14(5c)	a^3 ♂ 100zq 48.46. 7(5c)	a^4 ♂ 55 q49.46. 3(3c)	5H29c^4	♂ 100xq58.58.55(6e)	5H37a^3	♀ 100zp39.35.30(5b)
	a^5 ♀ 75 p34.36. 3(5b)	a^6 ♀ 100zq 48.46.43(5c)	b^1 ♀ 100zp39.35. 3(5b)	b^2 ♀ 50 p39.39. 1(2–3b)	...	...	...	...
	b^3 ♀ 100zq 48.46. 7(5c)	b^4 ♀ 100zq 48.46.11(5c)	b^5 ♀ 100zq 48.46. 4(5c)	c^1 ♀ 60 q48.47. 1(3c)	...	...	...	...
	c^2 ♀ 100zp26.37. 4(5b)	c^3 – q – – – –	...	...	...	...	...	...
6H27	a^1 ♂ 100xp22.22.18(6f)	a^2 ♂ 100xp22.15.18(6f)	a^3 ♀ 100xp22.22.17(6f)	a^4 ♀ 100xp22.15.17(6f)	5H36b^2	♂ 100xp23.23.18(6f)	5H37b^3	♀ 100xp13.13.18(6f)
	a^5 ♀ 100xp22.22.17(6f)	b^1 ♂ 100xp26.26.17(6f)	b^2 ♂ 100xp22.15.15(6f)	b^3 ♀ 100xp23.23.18(6f)	...	...	...	...
	b^4 ♀ 100xp23.23.17(6f)	...	...	...	...	...	...	...
6H28	a^1 ♂ 100xp23.23.10(6f)	a^2 ♂ 100xp26.15.15(6f)	a^3 ♀ 100xp26.26.23(6f)	a^4 ♀ 100xp23.23.15(6f)	5H33b^1	♂ 100xp23.15.18(6f)	5H31b^8	♀ 100xp22.22.18(6f)
	b^1 ♂ 100xp23.23.18(6f)	b^2 ♂ 100xp26.26.15(6f)	b^3 ♂ 100xp26.26.23(5–6f)	b^4 ♀ 100xp23.23.15(6f)	...	...	...	...
	b^5 ♀ 100xp26.26.23(6f)	b^6 ♀ – p – – –(6f)	c^1 ♂ 100xp26.23.15(6f)	c^2 ♂ 100xp26.23.15(6f)	...	...	...	...
	c^3 ♀ 100xp23.23.15(6f)	c^4 ♀ 100xp23.23.15(6f)	c^5 ♀ 100xp23.23.15(5f)	c^6 100x 23.23.15 –	...	...	...	...
6H29	a^1 ♂ 100xp22.23.18(6f)	a^2 ♂ 100xp19.19. 6(6f)	a^3 ♂ 100xp23.23.15(6f)	a^4 ♀ 100xp26.26.15(6f)	5H30c^1	♂ 100xp15.15.15(6f)	5H31a^7	♀ 100xp23.23.18(6f)
	a^5 ♀ 100xp26.23.18(6f)	a^6 – p – – – –	b^1 ♂ 100xp23.15.10(6f)	b^2 ♀ 100xp26.23.15(6f)	...	...	...	...
	b^3 – p – – – –	b^4 – p – – – –	b^5 – p – – – –	b^6 – p – – – –	...	...	...	...

6H30	a^{1} ♀ 100xp22.22.23(6f)	a^{2} ♀ 100xp50.26.23(6f)	b^{1} ♀ 100xp26.26.23(6f)	b^{2} ♀ 100xp26.22.17(6f)	5H31a^{2} ♂ 100xp23.23.18(6f)	5H30a^{3} ♀ 100xp23.23.18(6f)
	b^{3} ♀ 100xp22.15.15(6f)	c^{1} ♂ 100xp24.24.24(6f)	c^{2} ♂ 100xp26.26.23(6f)	c^{3} ♂ 100xp26.26.23(6f)	… … … …	… … … …
	c^{4} ♀ 100xp50.50.26(6f)	c^{5} ♀ 100xp22.15.15(6f)	c^{6} ♀ 100xp26.23.15(6f)	… … …	… … … …	… … … …
6H31	a^{1} ♂ 50 q44. 1. 1(3ch)	a^{2} ♂ 55 q44.54. 4(3ch)	a^{3} ♂ 55 q55. 1. 3(3ch)	a^{4} ♀ 45 q58.58. 1(3ch)	5H27c^{2} ♂ 35 q44. 1. 1(3ch)	5H27b^{5} ♀ 80 q57.57. 1(4ch)
	a^{5} ♀ 60 q58.44. 4(3ch)	b^{1} ♂ 55 q44.54. 1(3ch)	b^{2} ♂ 60 q44.43. 2(3ch)	b^{3} ♂ 55 q45.54. 1(3ch)	… … … …	… … … …
	b^{4} ♂ 60 q50.26. 2(4g–ch)	b^{5} ♀ 85 q26.26. 4(4g–ch)	… … …	… … …	… … … …	… … … …
6H32	a^{1} ♂ 55 q44.44. 1(3ch)	a^{2} ♀ 40 q58.44. 1(2ch)	… … …	… … …	5H27c^{3} ♂ 40 q52. 1. 1(3ch)	5H27b^{6} ♀ – q– – –(4ch)
6H33	a^{1} ♂ – p– – –(6f)	a^{2} ♂ 100zp44.44. 1(5b)	a^{3} ♀ 100xp15.15.18(6f)	a^{4} ♀ 100xp22.22.17(6f)	5H53a^{4} ♂ 75 p36.35. 2(5b)	5H51a^{5} ♀ 100yp35.35. 3(5b)
	a^{5} ♀ 75 p36.35. 4(5b)	… … …	… … …	… … …	… … … …	… … … …
6H34	a^{1} ♂ 100xp17.17.17(6f)	a^{2} ♂ 100xp22.17.17(6f)	a^{3} ♂ 00 p– – –(alb.)	a^{4} ♂ 00 p– – –(alb.)	5H51a^{1} ♂ 85 p35.35. 3(5b)	5H53a^{6} ♀ 100xp15.15.15(6f)
	a^{5} ♀ 100xp15.15.15(6f)	a^{6} ♀ 75 p36.35. 3(5b)	b^{1} ♂ 100zp36.36.30(5b)	b^{2} ♀ 100xp22.22.15(6f)	… … … …	… … … …
	b^{3} ♀ 100zp36.35.30(5b)	b^{4} ♀ 100zp36.33. 3(5b)	b^{5} ♀ 100zp35.34. 3(5b)	b^{6} ♀ 00 p– – –(alb.)	… … … …	… … … …
	b^{7} – p– – – –	… … …	… … …	… … …	… … … …	… … … …
6H35	a^{1} ♂ 100xp22.10.10(6f)	a^{2} ♂ 00 p– – –(alb.)	a^{3} ♀ 100xp15.15.10(6f)	a^{4} ♀ 100xp15.15.15(6f)	5H53a^{2} ♂ 100xp15.15.15(6f)	5H54b^{4} ♀ 100xp10.10.10(6f)
	a^{5} ♀ 100xp15.15.10(6f)	a^{6} ♀ 00 p– – –(alb.)	a^{7} – p– – – –	… … …	… … … …	… … … …
6H36	a^{1} ♂ 100xp22.22.15(6f)	a^{2} ♂ 100xp15.15.15(6f)	a^{3} ♂ 100xp17.17.15(6f)	a^{4} ♂ 100xp15.15.15(6f)	5H53a^{3} ♂ 100xp22.17.17(6f)	5H54b^{5} ♀ 100xp15.15.15(6f)
	a^{5} ♂ 100xp22.22.15(5f)	a^{6} ♀ 100xp15.15.15(6f)	a^{7} ♀ 100xp23.23.15(5f)	b^{1} ♂ – p– – –(6f)	… … … …	… … … …
	b^{2} ♂ 100xp15.15.15(6f)	b^{3} ♂ 100xp23.23.18(5f)	b^{4} ♀ 100xp15.15.10(6f)	b^{5} ♀ 100xp23.23.15(6f)	… … … …	… … … …
	b^{6} ♀ 100xp23.23.17(6f)	b^{7} ♀ 100xp17.17.15(6f)	b^{8} ♀ 100xp15.15.15(5f)	b^{9} ♀ 00 p– – –(alb.)	… … … …	… … … …
	c^{1} ♂ 100xp26.26.17(6f)	c^{2} ♂ 100xp23.23.15(6f)	c^{3} ♀ 100xp23.23.15(6f)	c^{4} ♀ 100xp15.15.15(6f)	… … … …	… … … …
	c^{5} ♀ 100xp15.15.15(6f)	c^{6} ♀ 100xp15.15.10(6f)	c^{7} ♀ 00 p– – –(alb.)	… … …	… … … …	… … … …
K1	a^{1} – q– – –(6e)	a^{2} – q– – –(5d)	a^{3} – q– – –(5d)	… … …	1H143b^{2} ♂ 00 p– – –(alb.)	1H120b^{4} ♀ 100zq49.46.12(5c)
K2	a^{1} – q– – –(5c)	a^{2} – q– – –(5c)	a^{3} – q– – –(5d)	a^{4} – q– – –(4c)	1H143b^{3} ♂ 100zq49.46.14(5c)	1H101b^{5} ♀ 00 p– – –(alb.)
	a^{5} 00 p– – –(alb.)	… … …	… … …	… … …	… … … …	… … … …
K3	a^{1} – q– – –(5c)	a^{2} – q– – –(5c)	a^{3} – q– – –(5c)	… … …	1H109b^{3} ♂ 00 p– – –(alb.)	1H120b^{5} ♀ 100zq49.48.43(5c)
K4	a^{1} – q– – –(5c)	a^{2} 00 p– – –(alb.)	a^{3} 00 p– – –(alb.)	a^{4} 00 p– – –(alb.)	1H148b^{2} ♂ 00 p– – –(alb.)	1H111b^{6} ♀ 100xq58.57.55(6e)
K5	a^{1} – q– – –(5c)	a^{2} – q– – –(5c)	a^{3} – q– – –(5c)	a^{4} 00 p– – –(alb. w)	1H111b^{2} ♂ 60 q58.55. 1(3e)	1H148b^{4} ♀ 00 p– – –(alb.)
	a^{5} 00 p– – –(alb.)	a^{6} 00 p– – –(alb.)	… … …	… … …	… … … …	… … … …
K6	a^{1} – q– – –(5c)	a^{2} – q– – –(5c)	a^{3} 00 p – –(alb. w)	a^{4} 00 p– – –(alb.)	1H111b^{4} ♂ 00 p– – –(alb.)	1H101b^{2} ♀ 100xq49.48.46(6c)
	a^{5} 00 p– – –(alb.)	a^{6} 00 p– – –(alb.)	a^{7} 00 p– – –(alb.)	… … …	… … … …	… … … …
K10	a^{1} – q– – –(4b)	a^{2} – p– – –(4bw)	a^{3} – p– – –(3bw)	… … …	1H101b^{4} ♂ 00 p– – –(alb.)	1H36a^{4} ♀ 30 r37. 1. 1(2c)
K11	a^{1} – q– – –(6e)	a^{2} – p– – –(6b)	a^{3} – p– – –(6f)	a^{4} 00 p– – –(alb.)	1H101b^{1} ♂ 00 p– – –(alb.)	1H32a^{4} ♀ 100xq58.57.55(6e)
K12	a^{1} – q– – –(3c)	a^{2} – q– – –(3c)	a^{3} – q– – –(3bc)	a^{4} 00 p– – –(alb.)	1H120b^{3} ♂ 00 p– – –(alb.)	1H32a^{3} ♀ 100zq49.46. 2(4c)
	a^{5} 00 p– – –(alb.)	… … …	… … …	… … …	… … … …	… … … …
K15	a^{1} – p– – –(6b)	a^{2} – p– – –(3b)	a^{3} 00 p– – –(alb.)	a^{4} 00 p– – –(alb.)	1H112a^{4} ♂ 00 p– – –(alb.)	1H117b^{1} ♀ 100xq41.41.50(5g)
	a^{5} 00 p– – –(alb.)	… … …	… … …	… … …	… … … …	… … … …
K16	a^{1} – q– – –(3e)	a^{2} – q– – –(3e)	a^{3} – q– – –(2e)	a^{4} – q– – –(4c)	1H118b^{6} ♂ 00 p– – –(alb.)	1H124b^{1} ♀ 65 q58. 1. 1(3-4e)
	a^{5} – q– – –(3c)	… … …	… … …	… … …	… … … …	… … … …
K19	a^{1} – q – –(4c)	a^{2} – q– – (3c)	a^{3} 00 p – –(alb.)	… … …	1H119a^{3} ♂ 55 q49. 1. 1(3c)	1H149a^{4} ♀ 00 p– – –(alb.)
K21	a^{1} – q– – –(6e)	a^{2} – q– – –(6d)	a^{3} – q– – –(4e)	a^{4} – q– – –(4e)	1H114a^{2} ♂ 10 q58. 1. 1(1e)	1H152a^{3} ♀ 00 p– – –(alb.)
	a^{5} – q– – –(4e)	… … …	… … …	… … …	… … … …	… … … …
K23	a^{1} – q– – (5c)	a^{2} – q– – –(5c)	a^{3} – q– – –(5c)	a^{4} – q– – –(4c)	1H119a^{5} ♂ 00 p– – –(alb.)	1H87a^{1} ♀ 100xq58.58.57(6e)
	a^{5} 00 p– – –(alb.)	a^{6} 00 p– – –(alb.)	a^{7} 00 p– – –(alb.)	a^{8} 00 p– – –(alb.)	… … … …	… … … …
K24	a^{1} – q– – –(1e)	a^{2} – q– – –(2c)	a^{3} – q– – –(2c)	a^{4} 00 p– – –(alb.)	1H100b^{3} ♂ 00 p– – –(alb.)	1H65b^{2} ♀ 100xq58.58.56(6e)
	a^{5} 00 p– – –(alb.)	a^{6} 00 p– – –(alb.)	… … …	… … …	… … … …	… … … …
K27	a^{1} – q– – –(6e)	a^{2} – q– – –(6e)	a^{3} 00 p– – –(alb.)	a^{4} 00 p– – –(alb.)	1H118a^{2} ♂ 55 q49. 1. 2(3c)	1H88b^{2} ♀ 00 p– – –(alb.)
	a^{5} 00 p– – –(alb.)	… … …	… … …	… … …	… … … …	… … … …
K28	a^{1} – q– – –(5c)	a^{2} – q– – –(4c)	a^{3} – q– – –(4c)	a^{4} – q– – –(6g)	1H122a^{5} ♂ 25 q46. 1. 1(2c)	1H16a^{6} ♀ 00 p– – –(alb.)
	a^{5} 00 p– – –(alb.)	a^{6} 00 p– – –(alb.)	a^{7} 00 p– – –(alb.)	… … …	… … … …	… … … …

MATING	OFFSPRING				PARENTS			
					Father		Mother	
6H17	b^4 ♀ 100xq58.58.55(6e)	b^5 ♀ 100xq58.57.55(6e)	b^6 ♀ 100xq58.55.55(6e)	b^7 ♀ 100xq58.58.55(5e)	...	...	...	...
	c^1 ♂ 100xq58.60.55(6e)	c^2 ♂ 100xq60.57.55(5–6e)	c^3 ♂ 100yq58.55.45(5e)	c^4 ♂ 100yq58.58.55(5e)	...	...	...	...
	c^5 ♀ 100xq58.55.55(6e)	c^6 ♀ 100xq58.58.55(5–6e)	c^7 ♀ 100xq58.60.55(5e)	...	...	...	...	...
6H18	a^1 ♂ 100xq58.57.55(6e)	a^2 ♂ 100xp22.22.15(6f)	a^3 ♀ 100xq58.57.55(6e)	a^4 ♀ 100xq58.58.55(6e)	5H29c^1	♂ 100xq58.58.55(6e)	5H30c^3	♀ 100xq58.56.55(6e)
	a^5 ♀ 100xq58.58.55(6e)	a^6 ♀ 100xq58.57.55(5–6e)	a^7 ♀ 100xp23.23.15(6f)	a^8 ♀ 100xp23.23.15(6f)	...	...	...	...
	b^1 ♂ 100xq58.57.55(6e)	b^2 ♂ 100xq58.60.55(6e)	b^3 ♀ 100xq58.60.55(6e)	b^4 ♀ 100xq58.58.55(6e)	...	...	...	...
	b^5 ♀ 100xq58.58.55(6e)	b^6 ♀ 100xq58.58.55(6e)	b^7 ♀ 100xp17.15.15(6f)	c^1 ♂ 100xq58.58.55(6e)	...	...	...	...
	c^2 ♂ 100xq58.60.55(5–6e)	c^3 ♂ 100xp23.23.15(6f)	c^4 ♀ 100xq60.60.55(6e)	c^5 ♀ 100xq58.58.55(6e)	...	...	...	...
	c^6 ♀ 100xq58.57.55(6e)	c^7 ♀ 100xq58.58.55(6e)	c^8 ♀ 100xq58.57.55(6e)	c^9 – p – –	...	...	...	...
6H19	a^1 ♂ 100xq58.58.55(6e)	a^2 ♀ 100xq58.57.55(6e)	a^3 ♀ 100xp26.26.23(6f)	b^1 ♂ 100xq57.57.55(6e)	5H31c^4	♂ 100xq58.58.55(6e)	5H29c^3	♀ 100xq58.58.56(6e)
	b^2 ♂ 100xq58.58.55(6e)	b^3 ♀ 100xq58.58.55(6e)	b^4 ♀ 100xq58.60.55(6e)	c^1 ♂ 100xp22.23.15(6f)	...	...	...	...
	c^2 ♀ 100xq58.60.55(6e)	c^3 ♀ 100xq58.58.55(6e)	c^4 ♀ 100xq57.57.55(6e)	c^5 ♀ 100xq57.55.55(6e)	...	...	...	...
	c^6 ♀ 100xq57.55.55(6e)	c^7 ♀ 100xq58.57.55(6e)	c^8 q – – –	c^9 – q –	...	...	...	...
	c^{10} p – –	...	...	...	...	...	...	...
6H20	a^1 ♂ 85 q58.55. 4(4e)	a^3 ♂ – q – – –(4e)	a^2 ♂ 90 q56.55. 2(3e)	a^4 ♂ – q – –(4e)	5H27c^1	♂ 85 q58.44. 1(4e)	5H26c^4	♀ 90 q58.55. 2(4e)
	a^5 ♀ 90 q58.55. 2(4–5e)	a^6 – q – – – –	a^7 – q	...	...	...	...	...
6H21	a^1 ♀ 95 q58.58. 2(4e)	a^2 ♀ 85 q58.60. 1(4e)	...	...	5H26c^1	♂ 90 q58.57. 1(5e)	5H27c^6	♀ 75 q58.56. 1(4e)
6H22	a^1 ♂ 100xq56.58.55(6e)	a^2 ♂ – q – – –(5e)	a^3 ♀ 100xq57.55.55(6e)	a^4 ♀ 100xq57.57.55(6e)	5H33b^2	♂ 100xq58.57.55(6e)	5H58a^4	♀ 100yq58.58.55(5e)
	a^5 ♀ 80 q58.55. 1(4e)	a^6 ♀ 00 p – – –(alb.)	b^1 ♂ 100xq57.57.55(6e)	b^2 ♂ 100xq58.57.55(6e)	...	...	...	...
	b^3 ♂ 100yq56.55. 1(5e)	b^4 ♀ 100xq58.57.55(6e)	b^5 ♀ 100xq58.57.55(6e)	b^6 ♀ 100xq58.58.55(6e)	...	...	...	...
	b^7 ♀ 00 p – – –(alb.)	c^1 ♀ 100xq58.58.55(6e)	c^2 ♀ 100xq58.58.55(6e)	c^3 ♀ 100xq58.58.55(6e)	...	...	...	...
	c^4 ♀ 95 q58.57. 2(4–5e)	...	...	...	...	...	...	...
6H23	a^1 ♂ 100xq58.58.55(6e)	a^2 ♀ 100xq58.58.55(6e)	a^3 ♀ 100xq58.58.55(6e)	a^4 ♀ 100xq58.55.55(5e)	5H58a^2	♂ 60 q58. 1. 1(3e)	5H33b^5	♀ 100xq58.57.56(6e)
	a^5 ♀ 100yq58.58.55(5e)	b^1 ♂ 100xq58.60.55(6e)	b^2 ♂ 100xq57.57.55(6e)	b^3 ♀ 100xq58.60.55(6e)	...	...	...	...
	b^4 ♀ 100xq58.55.55(5–6e)	b^5 ♀ 100xq58.60.55(5e)	...	...	...	...	...	...
6H24	a^1 ♀ 100zp37.35.30(5b)	a^2 ♀ 100zq52.46.14(5c)	b^1 ♂ 100zp36.34. 3(5b)	b^2 ♂ 100zq48.46.43(5c)	5H33b^4	♂ 100xq58.58.55(6e)	5H35a^2	♀ 100zp39.35.28(5b)
	b^3 ♂ 100zq48.46.43(5c)	b^4 ♂ 100zq48.46. 4(5c)	b^5 ♂ 100zq48.46. 7(5c)	b^6 ♀ 100zp36.36. 4(5b)	...	...	...	...
	b^7 ♀ 100zq48.46.11(5c)	...	...	...	...	...	...	...
6H25	a^1 ♂ 100xq58.60.55(5–6e)	a^2 ♂ 100xq58.57.56(5e)	a^3 ♂ 100zq52.46.12(5c)	a^4 ♂ 100zq49.46.14(5c)	5H31b^6	♂ 100xq58.58.55(5e)	5H37a^2	♀ – p – – –(5b)
	a^5 ♀ 95 q58.60. 4(5e)	a^6 ♀ 100zq48.46. 2(5c)	a^7 ♀ 100zq48.46.43(5c)	a^8 – q – – – –	...	...	...	...
	a^9 – q –	b^1 ♂ 100zq44.47.11(5c)	b^2 ♂ 100zq48.46. 7(5c)		...	...	...	...
6H26	a^1 ♂ 100zp36.35. 3(5b)	a^2 ♂ 100zq52.46.14(5c)	a^3 ♂ 100zq48.46. 7(5c)	a^4 ♂ 55 q49.46. 3(3c)	5H29c^4	♂ 100xq58.58.55(6e)	5H37a^3	♀ 100zp39.35.30(5b)
	a^5 ♀ 75 p34.36. 3(5b)	a^6 ♀ 100zq48.46.43(5c)	b^1 ♀ 100zp39.35. 3(5b)	b^2 ♀ 50 p39.39. 1(2–3b)	...	...	...	...
	b^3 ♀ 100zq48.46. 7(5c)	b^4 ♀ 100zq48.46.11(5c)	b^6 ♀ 100zq48.46. 4(5c)	c^1 ♀ 60 q48.47. 1(3e)	...	...	...	...
	c^2 ♀ 100zp36.37. 4(5b)	c^3 q – – –	...	...	...	...	...	...
6H27	a^1 ♂ 100xp22.22.18(6f)	a^2 ♂ 100xp22.15.18(6f)	a^3 ♀ 100xp22.22.17(6f)	a^4 ♀ 100xp22.15.17(6f)	5H36b^2	♂ 100xp23.23.18(6f)	5H37b^4	♀ 100xp13.13.18(6f)
	a^5 ♀ 100xp22.22.17(6f)	b^1 ♂ 100xp26.26.17(6f)	b^2 ♂ 100xp22.15.15(6f)	b^3 ♀ 100xp23.23.18(6f)	...	...	...	...
	b^4 ♀ 100xp23.23.17(6f)	...	...	...	...	...	...	...
6H28	a^1 ♂ 100xp23.23.10(6f)	a^2 ♂ 100xp26.15.15(6f)	a^3 ♀ 100xp26.26.23(6f)	a^4 ♀ 100xp23.23.15(6f)	5H33b^1	♂ 100xp23.15.18(6f)	5H31b^8	♀ 100xp22.22.18(6f)
	b^1 ♂ 100xp23.23.18(6f)	b^2 ♂ 100xp26.26.15(6f)	b^3 ♂ 100xp26.26.23(5–6f)	b^4 ♀ 100xp23.23.15(6f)	...	...	...	...
	b^5 ♀ 100xp26.26.23(6f)	b^6 ♀ – p – – –(6f)	c^1 ♂ 100xp26.23.15(6f)	c^2 ♂ 100xp26.23.15(6f)	...	...	...	...
	c^3 ♀ 100xp23.23.15(6f)	c^4 ♀ 100xp23.23.15(6f)	c^5 ♀ 100xp23.23.15(5f)	c^6 100x 23.23.15	...	...	...	...
6H29	a^1 ♂ 100xp22.23.18(6f)	a^2 ♂ 100xp19.19. 6(6f)	a^3 ♂ 100xp23.23.15(6f)	a^4 ♀ 100xp26.26.15(6f)	5H30a^1	♂ 100xp15.15.15(6f)	5H31a^7	♀ 100xp23.23.18(6f)
	a^5 ♀ 100xp26.23.18(6f)	a^6 – p – – –	b^1 ♂ 100xp23.15.10(6f)	b^2 ♀ 100xp26.23.15(6f)	...	...	...	...
	b^3 p –	b^4 – p – – – –	b^5 – p – –	b^6 – p – – – –	...	...	...	...
6H30	a^1 ♀ 100xp22.22.23(6f)	a^2 ♀ 100xp50.26.23(6f)	b^1 ♀ 100xp26.26.23(6f)	b^2 ♀ 100xp26.22.17(6f)	5H31a^2	♂ 100xp23.23.18(6f)	5H30a^3	♀ 100xp23.23.18(6f)
	b^3 ♀ 100xp22.15.15(6f)	c^1 ♂ 100xp24.24.24(6f)	c^2 ♂ 100xp26.26.23(6f)	c^3 ♂ 100xp26.26.23(6f)	...	...	...	...
	c^4 ♀ 100xp50.50.26(6f)	c^5 ♀ 100xp22.15.15(6f)	c^6 ♀ 100xp26.23.15(6f)	...	...	...	...	...
6H31	a^1 ♂ 50 q44. 1. 1(3ch)	a^2 ♂ 55 q44.54. 4(3ch)	a^3 ♂ 55 q55. 1. 3(3ch)	a^4 ♀ 45 q58.58. 1(3ch)	5H27c^2	♂ 35 q44. 1. 1(3ch)	5H27b^5	♀ 80 q57.57. 1(4ch)
	a^5 ♀ 60 q58.44. 4(3ch)	b^1 ♂ 55 q44.54. 1(3ch)	b^2 ♂ 60 q44.43. 2(3ch)	b^3 ♂ 55 q45.54. 1(3ch)	...	...	...	...
	b^4 ♂ 60 q50.26. 2(4g–ch)	b^5 ♀ 85 q26.26. 4(4g–ch)	...	...	...	...	...	...
6H32	a^1 ♂ 55 q44.44. 1(3ch)	a^2 ♀ 40 q58.44. 1(2ch)	...	...	5H27c^3	♂ 40 q52. 1. 1(3ch)	5H27b^6	♀ – q – – –(4ch)
6H33	a^1 ♂ – p – – –(6f)	a^2 ♂ 100zp44.44. 1(5b)	a^3 ♀ 100xp15.15.18(6f)	a^4 ♀ 100xp22.22.17(6f)	5H53a^4	♂ 75 p36.35. 2(5b)	5H51a^6	♀ 100yp35.35. 3(5b)
	a^5 ♀ 75 p36.35. 4(5b)	...	...	...	...	...	...	...
6H34	a^1 ♂ 100xp17.17.17(6f)	a^2 ♂ 100xp22.17.17(6f)	a^3 ♂ 00 p – (alb.)	a^4 ♂ 00 p – – –(alb.)	5H51a^1	♂ 85 p35.35. 3(5b)	5H53a^6	♀ 100xp15.15.15(6f)
	a^5 ♀ 100xp15.15.15(6f)	a^6 ♀ 75 p36.35. 3(5b)	b^1 ♂ 100zp36.36.30(5b)	b^2 ♀ 100xp22.22.15(6f)	...	...	...	...
	b^3 ♀ 100zp36.35.30(5b)	b^4 ♀ 100zp36.33. 3(5b)	b^5 ♀ 100zp35.34. 3(5b)	b^6 ♀ 00 p – – –(alb.)	...	...	...	...
	b^7 – p – – –	...	...	...	...	...	...	...
6H35	a^1 ♂ 100xp22.10.10(6f)	a^2 ♂ 00 p – – –(alb.)	a^3 ♀ 100xp15.15.10(6f)	a^4 ♀ 100xp15.15.15(6f)	5H53a^2	♂ 100xp15.15.15(6f)	5H54b^4	♀ 100xp10.10.10(6f)
	a^5 ♀ 100xp15.15.10(6f)	a^6 ♀ 00 p – – –(alb.)	a^7 – p – –	...	...	...	...	...
6H36	a^1 ♂ 100xp22.22.15(6f)	a^2 ♂ 100xp15.15.15(6f)	a^3 ♂ 100xp17.17.15(6f)	a^4 ♂ 100xp15.15.15(6f)	5H53a^3	♂ 100xp22.17.17(6f)	5H54b^5	♀ 100xp15.15.15(6f)
	a^5 ♂ 100xp22.22.15(5f)	a^6 ♀ 100xp15.15.15(6f)	a^7 ♀ 100xp23.23.15(5f)	b^1 ♂ – p – – –(6f)	...	...	...	...
	b^2 ♂ 100xp15.15.15(6f)	b^3 ♂ 100xp23.23.18(5f)	b^4 ♀ 100xp15.15.10(6f)	b^5 ♀ 100xp23.23.15(6f)	...	...	...	...
	b^6 ♀ 100xp23.23.17(6f)	b^7 ♀ 100xp17.17.15(6f)	b^8 ♀ 100xp15.15.15(5f)	b^9 ♀ 00 p – – –(alb.)	...	...	...	...
	c^1 ♂ 100xp26.26.17(6f)	c^2 ♂ 100xp23.23.15(6f)	c^3 ♀ 100xp23.23.15(6f)	c^4 ♀ 100xp15.15.15(6f)	...	...	...	...
	c^5 ♀ 100xp15.15.15(6f)	c^6 ♀ 100xp15.15.10(6f)	c^7 ♀ 00 p – – –(alb.)	...	...	...	...	...
K1	a^1 – q – – –(6e)	a^2 – q – – –(5d)	a^3 – q – – –(5d)	...	1H143b^2	♂ 00 p – – –(alb.)	1H120b^4	♀ 100zq49.46.12(5c)
K2	a^1 – q – – –(5c)	a^2 – q – – –(5c)	a^3 – q – – –(5d)	a^4 – q – – –(4c)	1H143b^3	♂ 100zq49.46.14(5c)	1H101b^5	♀ 00 p – – –(alb.)
	a^5 00 p – – –(alb.)	...	...	...	...	...	...	...
K3	a^1 – q – – –(5c)	a^2 – q – – –(5c)	a^3 – q – – –(5c)	...	1H109b^3	♂ 00 p – – –(alb.)	1H120b^5	♀ 100zq49.48.43(5c)
K4	a^1 – q – – –(5c)	a^2 00 p – – –(alb.)	a^3 00 p – – –(alb.)	a^4 00 p – – –(alb.)	1H148b^2	♂ 00 p – – –(alb.)	1H111b^6	♀ 100xq58.57.55(6e)
K5	a^1 – q – – –(5c)	a^2 – q – – –(5c)	a^3 – q – – –(5c)	a^4 00 p – – –(alb. w)	1H111b^2	♂ 60 q58.55. 1(3e)	1H148b^4	♀ 00 p – – –(alb.)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	...	...	...	...	...	...
K6	a^1 – q – – –(5c)	a^2 – q – – –(5c)	a^3 00 p – – –(alb. w)	a^4 00 p – – –(alb.)	1H111b^4	♂ 00 p – – –(alb.)	1H101b^2	♀ 100xq49.48.46(6c)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	a^7 00 p – – –(alb.)	...	...	...	...	...
K10	a^1 – q – – –(4b)	a^2 – p – – –(4bc)	a^3 – p – – –(3bw)	...	1H101b^4	♂ 00 p – – –(alb.)	1H36a^4	♀ 30 r37. 1. 1(2c)
K11	a^1 – q – – –(6e)	a^2 – p – – –(6b)	a^3 – p – – –(6f)	a^4 00 p – – –(alb.)	1H101b^1	♂ 00 p – – –(alb.)	1H32a^4	♀ 100xq58.57.55(6e)
K12	a^1 – q – – –(3c)	a^2 – q – – –(3c)	a^3 – q – – –(3bc)	a^4 00 p – – –(alb.)	1H120b^3	♂ 00 p – – –(alb.)	1H32a^3	♀ 100zq49.46. 2(4c)
	a^5 00 p – – –(alb.)	...	...	...	...	...	...	...
K15	a^1 – p – – –(6b)	a^2 – p – – –(3b)	a^3 00 p – – –(alb.)	a^4 00 p – – –(alb.)	1H112a^4	♂ 00 p – – –(alb.)	1H117b^1	♀ 100xq41.41.50(5g)
	a^5 00 p – – –(alb.)	...	...	...	...	...	...	...
K16	a^1 – q – – –(3c)	a^2 – q – – –(3c)	a^3 – q – – –(2e)	a^4 – q – – –(4c)	1H118b^6	♂ 00 p – – –(alb.)	1H124b^1	♀ 65 q58. 1. 1(3-4e)
	a^5 – q – – –(3c)	...	...	...	...	...	...	...
K19	a^1 – q – – –(4c)	a^2 – q – – –(3c)	a^3 00 p – – –(alb.)	...	1H119a^3	♂ 55 q49. 1. 1(3c)	1H149a^4	♀ 00 p – – –(alb.)
K21	a^1 – q – – –(6e)	a^2 – q – – –(6d)	a^3 – q – – –(4c)	a^4 – q – – –(4c)	1H114a^2	♂ 10 q58. 1. 1(1e)	1H152a^3	♀ 00 p – – –(alb.)
	a^5 – q – – –(4c)	...	...	...	...	...	...	...
K23	a^1 – q – – –(5c)	a^2 – q – – –(5c)	a^3 – q – – –(5c)	a^4 – q – – –(4c)	1H119a^5	♂ 00 p – – –(alb.)	1H87a^1	♀ 100xq58.58.57(6e)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	a^7 00 p – – –(alb.)	a^8 00 p – – –(alb.)	...	...	...	...
K24	a^1 – q – – –(1e)	a^2 – q – – –(2c)	a^3 – q – – –(2c)	a^4 00 p – – –(alb.)	1H100b^3	♂ 00 p – – –(alb.)	1H65b^2	♀ 100xq58.58.56(6e)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	...	...	...	...	...	...
K27	a^1 – q – – –(6e)	a^2 – q – – –(6e)	a^3 00 p – – –(alb.)	a^4 00 p – – –(alb.)	1H118a^2	♂ 55 q49. 1. 2(3c)	1H88b^2	♀ 00 p – – –(alb.)
	a^5 00 p – – –(alb.)	...	...	...	...	...	...	...
K28	a^1 – q – – –(5c)	a^2 – q – – –(4c)	a^3 – q – – –(4c)	a^4 – q – – –(6g)	1H122a^5	♂ 25 q46. 1. 1(2c)	1H16a^6	♀ 00 p – – –(alb.)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	a^7 00 p – – –(alb.)	...	...	...	...	...

MATING	OFFSPRING				PARENTS	
					Father	Mother
K29	a^1 – q – – –(3e)	a^2 – q – – –(3c)	a^3 – q – – –(2d)	a^4 00 p – – –(alb.)	1H121a^5 ♂ 55 q48.46. 2(3c)	1H16a^7 ♀ 00 p – – –(alb.)
	a^5 00 p – – –(alb.)	a^6 00 p – – –(alb.)	... ...	... ...	... ...	... ...
K30	a^1 00 p – – –(alb.)	... ...	... ...	... ...	1H100a^2 ♂ 30 q46. 1. 1(2c)	1H152a^5 ♀ 00 p – – –(alb.)
K50	a^1 – q – – – –	a^2 – p – – – –	a^3 – p – – – –	a^4 – p – – – –	2H96a^1 ♂ 100xq58.57.55(6e)	2H26a^2 ♀ 00 p – – –(alb.)
K53	a^1 – q – – – –	a^2 – q – – – –	a^3 ♂ 00 p – –(alb.)	a^4 – p – – – –	2H156a^3 ♂ 100xq58.57.55(6e)	2H52a^5 ♀ 00 p – – –(alb.)
	a^5 – p – – – –	a^6 – p – – – –	b^1 ♂ – q – – (5e)	b^2 – q – – – –	... ...	... ...
	b^3 – q – – – –	b^4 ♂ 00 p – – –(alb.)	b^5 ♂ 00 p – – –(alb.)	b^6 – p – – – –	... ...	... ...
	b^7 – p – – – –	c^1 – q – – – –	c^2 – q – – – –	c^3 – q – – – –	... ...	... ...
	c^4 – q – – – –	c^5 – q – – – –	c^6 – p – – – –	... ...	... ...	... ...
K54	a^1 – q – – – –	a^2 – q – – – –	a^3 – q – – – –	a^4 – q – – – –	2H49a^2 ♂ 100xq58.57.55(6e)	2H63b ♀ 00 p – – –(alb.)
	a^5 – p – – – –	a^6 – p – – – –	b^1 ♂ – q – – –(3e)	b^2 – q – – – –	... ...	... ...
	b^3 – q – – – –	b^4 – q – – – –	b^5 p – – – –	b^6 – p – – – –	... ...	... ...
	b^7 – p – – – –	b^8 – p – – – –	c^1 – q – – – –	c^2 – q – – – –	... ...	... ...
K55	a^1 – q – – – –	a^2 – q – – – –	a^3 – q – – – –	a^4 ♂ 00 p – – –(alb.)	2H102a^1 ♂ 100xq58.57.55(6e)	2H96a^7 ♀ 00 p – – –(alb.)
	a^5 – p – – – –	a^6 – p – – – –	b^1 – q – – – –	b^2 – q – – – –	... ...	... ...
	b^3 – q – – – –	b^4 – q – – – –	b^5 – q – – – –	b^6 – p – – – –	... ...	... ...
	b^7 – p – – – –	b^8 – p – – – –	c^1 – q – – – –	c^2 – q – – – –	... ...	... ...
	c^3 – p – – – –	... ...	... ...	... ...	... ...	... ...
K56	a^1 – q – – – –	a^2 – q – – – –	a^3 – p – – – –	b^1 – q – – – –	2H108a^2 ♂ 100xq58.56.56(6ew)	2H113a ♀ 00 p – – –(alb.)
	b^2 – p – –	c^1 – p – – – –	c^2 – p – – – –	c^3 – p – – – –	... ...	... ...
	c^4 – p – – – –	... ...	... ...	... ...	... ...	... ...
K57	b^1 – q – – – –	b^2 – q – – – –	b^3 – p – – – –	b^4 – p – – – –	2H117a^1 ♂ – q – – –(6e)	2H116a ♀ 00 p – – –(alb.)
K59	a^1 ♂ – q – – –(6e)	a^2 ♀ – q – – –(c)	a^3 – q – – – –	a^4 – q – – – –	1H130b^1 ♂ 30 p37. 1. 1(2a-b)	2H147a ♀ 00 p – – –(alb.)
	a^5 – q – – – –	a^6 – q – – – –	a^7 – q – – – –	b^1 – q – – – –	... ...	... ...
	b^2 – q – – – –	b^3 – q – – – –	b^4 – q – – – –	b^5 – q – – – –	... ...	... ...
	b^6 – q – – – –	b^7 – q – – – –	b^8 – q – – – –	b^9 – q – – – –	... ...	... ...
	c^1 – q – – – –	c^2 – q – – – –	c^3 – q – – – –	c^4 – q – – – –	... ...	... ...
	c^5 – q – – – –	c^6 – q – – – –	c^7 – q – – – –	c^8 – q – – – –	... ...	... ...
	c^9 – q – – – –	c^{10} – q – – – –	... ...	... ...	... ...	... ...

EXPLANATION OF RECORDS.

In the nomenclature adopted by Weldon, $0H80$ (say) represents the eightieth cross of the original races, i.e. Japanese waltzers and ordinary white mice. The resulting offspring of the first litter of this cross are a^1, a^2, a^3, etc., of the second litter of the same cross b^1, b^2, b^3, etc., and of the third litter c^1, c^2, c^3, c^4, etc. The matings of hybrids are denoted by $1H90$ for the first generation of hybrids, $2H90$ for the second generation and so on. Thus a mouse $6H17b^2$ denotes that it was the second mouse of the second litter resulting from the 17 mating in the 6th generation of hybrids. The characters of any mouse are given by a scheme of the following kind:

$$4H64 \mid c^2 ♂ 75q58.55.1(4e) \mid$$

This signifies that the mouse $4H64c^2$ was a male; 75 denotes that the pigmented areas covered ·75 of the total skin as determined by our scale of pigmented areas (see Plate I). *q* denotes that the eyes are black in this case, *p* would have marked that they were pink, the ordinary albino colour, *r* and *d.r* that they were red or dark red corresponding to blood and very dark blood colour; as a rule mice with such eyes have only incompletely albinotic eyes, i.e. there is more or less pigment found on dissection. The next three figures in the present case, 58, 55 and 1, mark the skin colour of the mouse on the mouse skin pigment scale of the Biometric Laboratory. This scale was formed by matching glass mosaics to the actual colours of the mouse skins. This scale contains 61 tints. Tints 1—9 are the "belly whites," leading up to 10—18, the palest blue greys; 19—27 are pale blue greys; 28—33 are yellow; 34—39, fawn yellow; 40—41 are chocolate; 43—45, chinchilla; 46—54, wild colour of various intensities, and 55—61, black of various intensities.

Of the three tints obtained from this scale the first, 58 in our case, marks the colour at the centre of the median plane of the skin, or if this should fall on a leucotic patch in the case of the piebald, the tint of the nearest dark patch. The second number gives the colour of the skin halfway between the centre of the back and the centre of the belly, 55 in our case; and the third number is that of the colour of the belly. Very often the first two numbers are the same, but we found in a good many cases that the tint shaded off from centre of the back to the belly, and our second number is a rough measure of the lightening of the pigment as we proceed from back towards belly. The final symbols given in brackets are Weldon's appreciation of the area of pigmentation given by the numbers 1 to 6 and his appreciation of colour as given by the letters *a—g*, *ch*. In the case of mice alive at his death and not then classed, these colour appreciations and pigment extents were provided as far as possible in accordance with his recorded judgments of other mice.

In the figures on p. 44, reproduced from *Biometrika*, Vol. III. p. 3, Fig. 6 represents a normal waltzing mouse.

Fig. 1 is Weldon's Class 1, more white and less extent of colour patches than a normal waltzer.

Fig. 2 is Weldon's Class 2, having about the same amount of colour patches as a normal waltzer.

Fig. 3 is Weldon's Class 3, having somewhat greater extent of colour patches than Class 2.

Fig. 4 is Weldon's Class 4.

Fig. 5 is Weldon's Class 5 with only a whitish belly.

Weldon's Class 6 is a whole colour mouse with nearly the same colour on the belly as on the back. Read with a planimeter Weldon's Classes are represented very roughly by:

1	2	3	4	5	6
14	53	68	77	92	100

We ourselves found considerable difficulty with "whole coloured" mice, and finally arranged them in three classes as follows:

$100x$ are mice with the skin pigmented all over, i.e. the same colour on belly as on the back; $100y$ are those with a line of paler belly pigment, but not sufficient to take the skin out of the

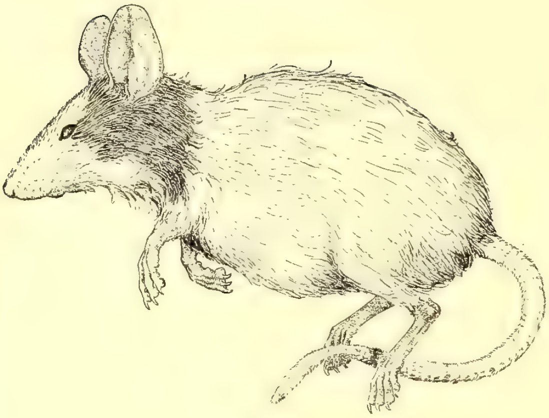


Fig. 1.

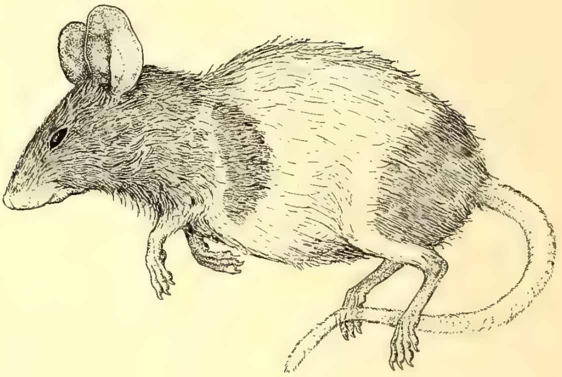


Fig. 2.

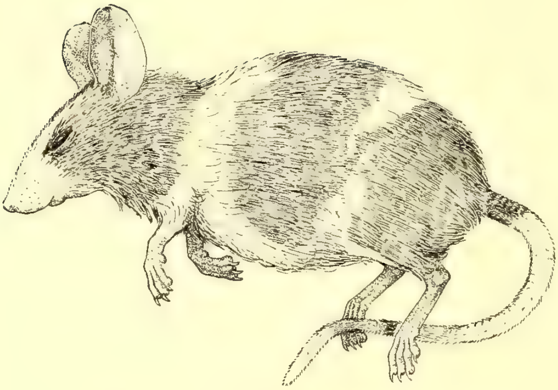


Fig. 3.

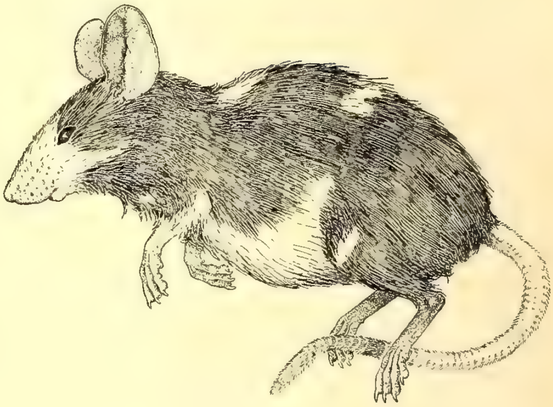


Fig. 4.

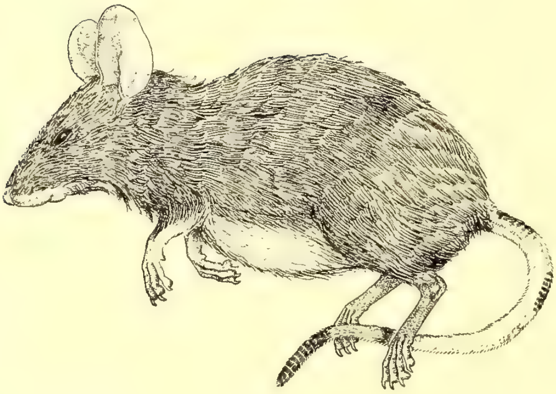


Fig. 5.

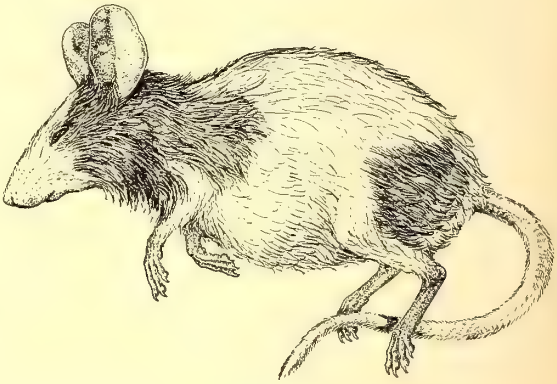


Fig. 6.

100 percentage; $100z$ have a skin pigmented all over, but a strip of paler belly colour which, if white, would make the skin fall in the 80 to 95 percentage classes. This means that our $100x$ and $100y$ ought to be Weldon's 6. As a rule our $100z$ are also his 6, but sometimes he denotes them by 5–6, and on some occasions he has called them 5. We have carefully considered these cases, but believe that our judgment, where not modified, must stand, as the belly pigment in these cases is often almost as intense as the back—e.g. 55 as against 58—and even when it is 9 against 19, this is not a pure white (1 only) but a shade of grey. We think our $100x$ for a pigmented belly with lighter hairs is more definite than terming the whole belly white, in which case our first number giving the extent of piebaldism would have to be modified, but the measurement would be exceedingly difficult as the pigment is there and only gets more plentiful towards the back. As a rule Darbishire and Weldon judged the amount of pigment on the living mouse and the nature of the belly pigment was not so open to full observation as when determined from the skin.

Weldon's colour classification was as follows:

a = yellow
b = fawn-yellow
c = pale wild colour
d = dark wild colour
e = black
f = pale blue grey ("lilac")
g = chocolate
ch = chinchilla

Roughly Weldon's a = our 28—33; his b = our 34—39; his c and d = our 46—54; his e = our 55—61; his f = our 10—27; his g = our 40—42; his ch = our 43—45. Naturally there is a good deal of personal equation in colour-determination, perhaps less with a colour scale than with mere verbal classification. Thus we should *a priori* have supposed c would range from about 46 to 49 or 50 and d from 50 to even 55 or 56. But we find c used for as dark a tint as 55 and d for as light a tint as 48. In the later records Weldon rarely, if ever, uses d, only c, so that its practical use is in the first or second hybrid generations, as in Mr Darbishire's paper. There d may be as low as 45, a tint we should call chinchilla. Our experience would make us doubt whether a rigid line can possibly be drawn between the various colour classes, with the possible but not certain exception of the yellows. Even here such judgments as b–c and b–f may occur. The differences of d and e, and f and g, may be so slight that intermediate classes may be formed as d–e, f–g.

A w following the Weldon colour class in the brackets at the end of the description of the individual mouse marks that it was a waltzer. Intermediates in colour or in piebald class are marked by a hyphen, e.g. a–b or 3–4.

Of the original stocks, the Japanese Waltzers were either bought or bred in the Laboratory. They were of pure stock, for they bred true to the parents when crossed only among themselves. There were very slight differences of the fawn colour in the body, and all eyes were pink. A waltzer is distinguished by the letter W, and as a rule by its laboratory number. The

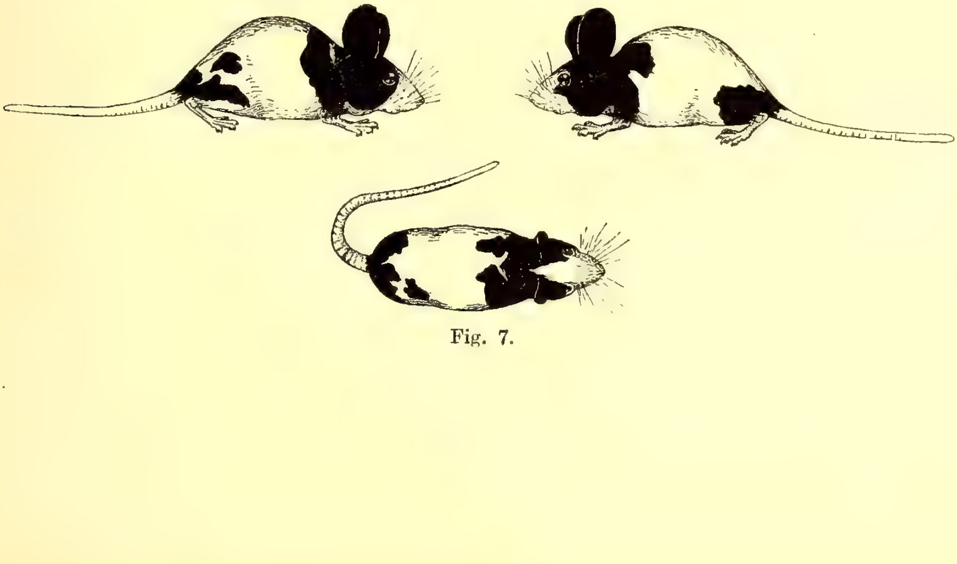


Fig. 7.

skins of these Waltzers were not preserved, but in the case of most of the animals a rough appreciation of the extent of the colour patches was conveyed by the use of the letters α and β and their combination $\alpha\beta$. Figure 7 on p. 45 represents an average waltzer $=(\alpha\beta)$, all waltzers with appreciably less colour are noted by α, all waltzers with appreciably more colour by β.

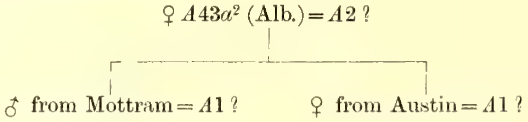
With regard to the albino parents we have distinguished ten classes, which are indicated in the following manner, the great difficulty is to be certain of the ancestry of albinos bought from dealers. In the Index Bt = Bought, Lb = Laboratory-bred.

Albino Classes.

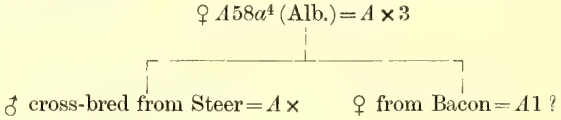
(1)	Parents and grandparents all albino, ancestry beyond unknown	$A2$
(2)	Both parents albino, but one or more grandparents *known* not to be albino	$A\times 3$
(3)	One parent albino, and the other not albino	$A\times 2$
(4)	Neither parent albino	$A\times 1$
(5)	Nothing known of ancestry	$A1\,?$
(6)	Said to be pure-bred albino	$A1$
(7)	Both parents albino, no definite knowledge of grandparents	$A2\,?$
(8)	Back to grandparents all albino, but beyond *known* to be cross-bred	$A2\times$
(9)	Both parents and two grandparents known to be albino, no definite knowledge of the other two	$A2\frac{1}{2}$
(10)	Back to grandparents all albino, 1 or 2 grandparents pure bred albinos, no definite knowledge of others' ancestry	$A1\frac{1}{2}$

In Weldon's later breeding work on first crossss, i.e. the high numbered $0H$ and $1H$, he paid much attention to the ancestry of his albinos, and the following pedigrees illustrate the ancestry in typical cases. The reader must note that the offspring stand at the *top* of the pedigrees, and we descend to parents and ancestry.

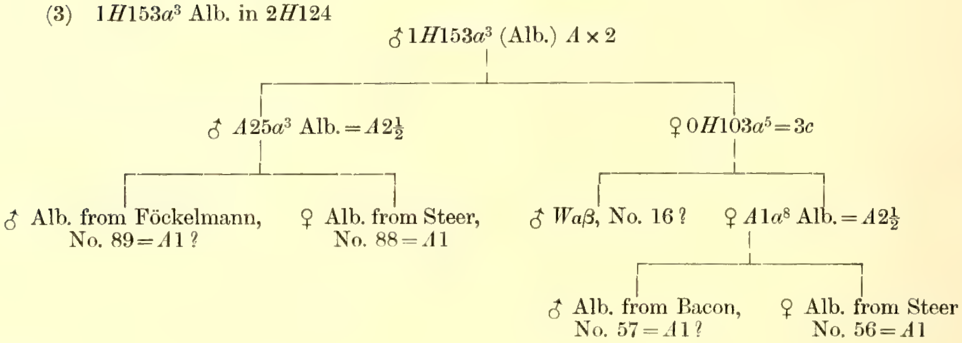
(1) $A43a^2$ Alb. in $1H143$, $2H152$


♀ $A43a^2$ (Alb.) = $A2\,?$
♂ from Mottram = $A1\,?$
♀ from Austin = $A1\,?$

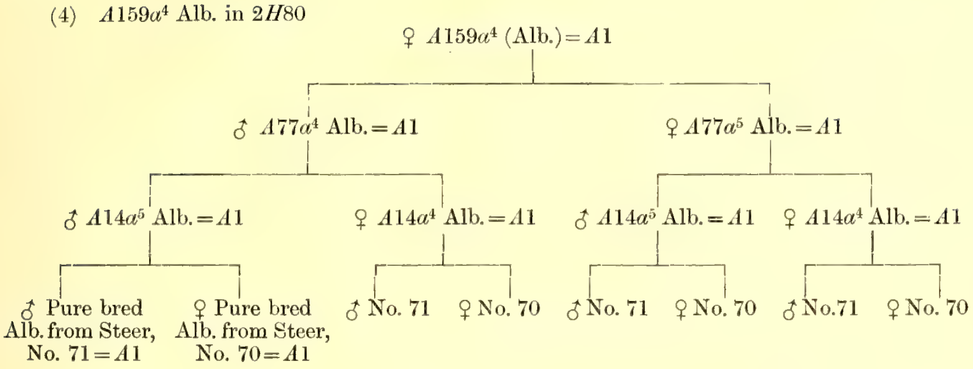
(2) $A58a^4$ Alb. in $2H155$


♀ $A58a^4$ (Alb.) = $A\times 3$
♂ cross-bred from Steer = $A\times$
♀ from Bacon = $A1\,?$

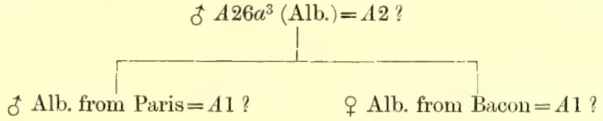
(3) $1H153a^3$ Alb. in $2H124$


♂ $1H153a^3$ (Alb.) $A\times 2$
♂ $A25a^3$ Alb. = $A2\frac{1}{2}$
♀ $0H103a^5 = 3c$
♂ Alb. from Föckelmann, No. 89 = $A1\,?$
♀ Alb. from Steer, No. 88 = $A1$
♂ $W\alpha\beta$, No. 16 ?
♀ $A1a^8$ Alb. = $A2\frac{1}{2}$
♂ Alb. from Bacon, No. 57 = $A1\,?$
♀ Alb. from Steer No. 56 = $A1$

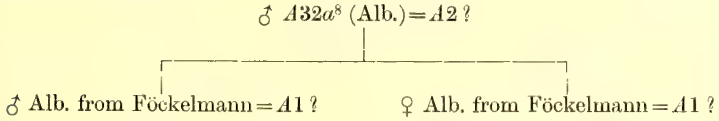
(4) $A159a^4$ Alb. in $2H80$


♀ $A159a^4$ (Alb.) = $A1$
♂ $A77a^4$ Alb. = $A1$
♀ $A77a^5$ Alb. = $A1$
♂ $A14a^5$ Alb. = $A1$
♀ $A14a^4$ Alb. = $A1$
♂ $A14a^5$ Alb. = $A1$
♀ $A14a^4$ Alb. = $A1$
♂ Pure bred Alb. from Steer, No. 71 = $A1$
♀ Pure bred Alb. from Steer, No. 70 = $A1$
♂ No. 71
♀ No. 70
♂ No. 71
♀ No. 70
♂ No. 71
♀ No. 70

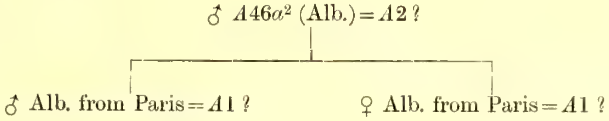
(5) $A26a^3$ Alb. in $1H146$


♂ $A26a^3$ (Alb.) = $A2$?
♂ Alb. from Paris = $A1$?
♀ Alb. from Bacon = $A1$?

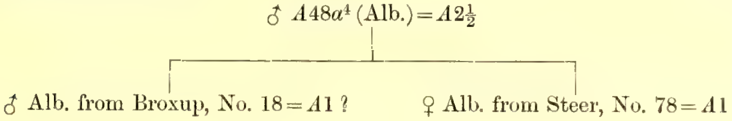
(6) $A32a^8$ Alb. in $1H156$


♂ $A32a^8$ (Alb.) = $A2$?
♂ Alb. from Föckelmann = $A1$?
♀ Alb. from Föckelmann = $A1$?

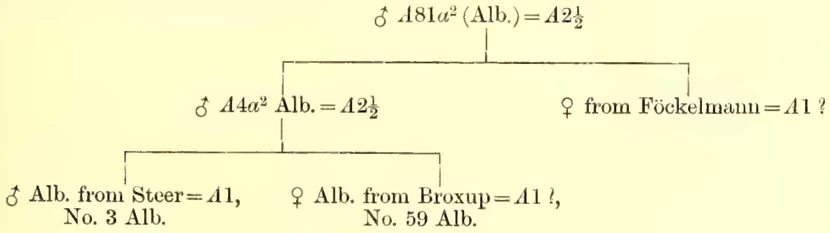
(7) $A46a^2$ Alb. in $1H149$


♂ $A46a^2$ (Alb.) = $A2$?
♂ Alb. from Paris = $A1$?
♀ Alb. from Paris = $A1$?

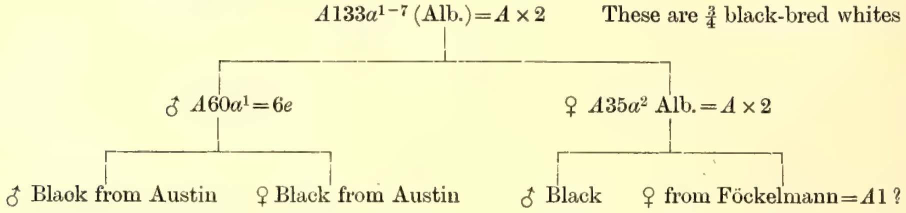
(8) $A48a^4$ Alb. in $1H157$


♂ $A48a^4$ (Alb.) = $A2\frac{1}{2}$
♂ Alb. from Broxup, No. 18 = $A1$?
♀ Alb. from Steer, No. 78 = $A1$

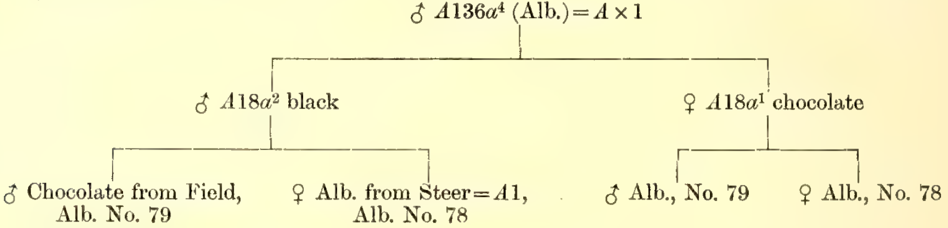
(9) $A81a^2$ Alb. in $1H158$, $1H270$


♂ $A81a^2$ (Alb.) = $A2\frac{1}{2}$
♂ $A4a^2$ Alb. = $A2\frac{1}{2}$
♀ from Föckelmann = $A1$?
♂ Alb. from Steer = $A1$, No. 3 Alb.
♀ Alb. from Broxup = $A1$?, No. 59 Alb.

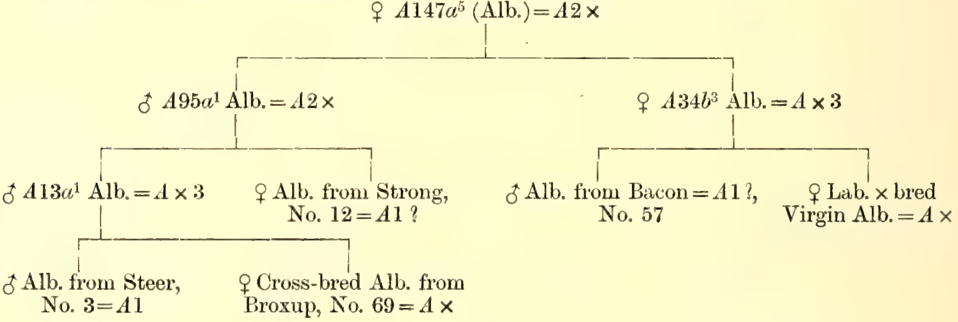
(10) ♀ $A133a^1$ Alb. in $1H164$; ♀ $A133a^2$ Alb. in $1H165$; ♀ $A133a^4$ Alb. in $1H167$; ♂ $A133a^6$ Alb. in $1H168$; ♂ $A133a^7$ Alb. in $1H169$.


$A133a^{1-7}$ (Alb.) $= A \times 2$
These are $\frac{3}{4}$ black-bred whites
♂ $A60a^1 = 6e$
♀ $A35a^2$ Alb. $= A \times 2$
♂ Black from Austin
♀ Black from Austin
♂ Black
♀ from Föckelmann $= A1$?

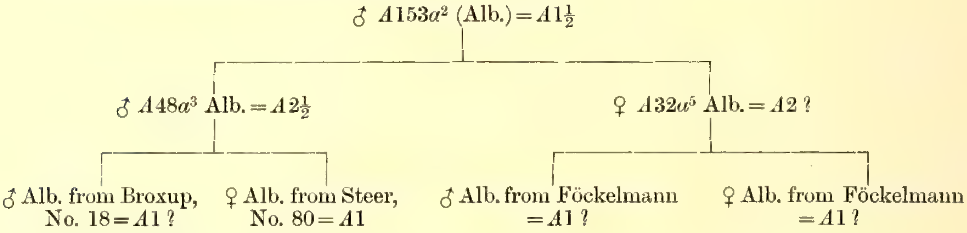
(11) $A136a^4$ Alb. in $1H171$


♂ $A136a^4$ (Alb.) $= A \times 1$
♂ $A18a^2$ black
♀ $A18a^1$ chocolate
♂ Chocolate from Field, Alb. No. 79
♀ Alb. from Steer $= A1$, Alb. No. 78
♂ Alb., No. 79
♀ Alb., No. 78

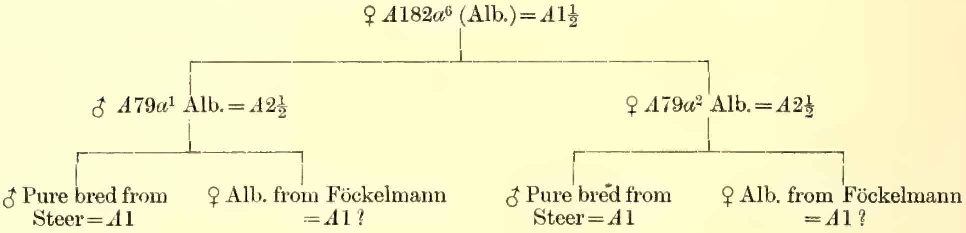
(12) $A147a^5$ Alb. in $1H187$


♀ $A147a^5$ (Alb.) $= A2\times$
♂ $A95a^1$ Alb. $= A2\times$
♀ $A34b^3$ Alb. $= A \times 3$
♂ $A13a^1$ Alb. $= A \times 3$
♀ Alb. from Strong, No. $12 = A1$?
♂ Alb. from Bacon $= A1$?, No. 57
♀ Lab. × bred Virgin Alb. $= A\times$
♂ Alb. from Steer, No. $3 = A1$
♀ Cross-bred Alb. from Broxup, No. $69 = A\times$

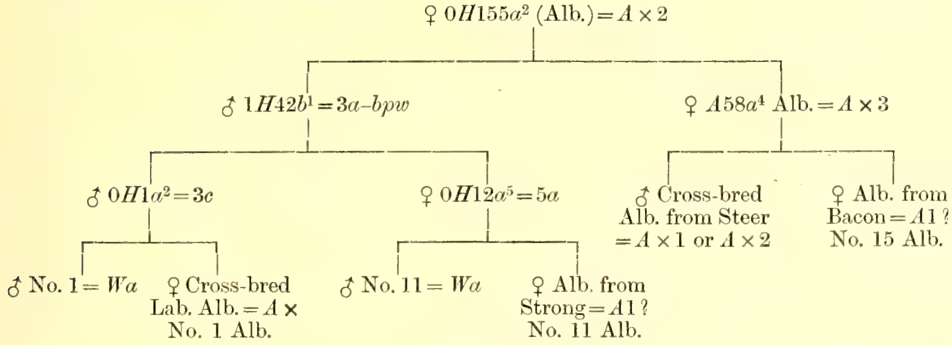
(13) $A153a^2$ Alb. in $1H183$, $1H273$


♂ $A153a^2$ (Alb.) $= A1\frac{1}{2}$
♂ $A48a^3$ Alb. $= A2\frac{1}{2}$
♀ $A32a^5$ Alb. $= A2$?
♂ Alb. from Broxup, No. $18 = A1$?
♀ Alb. from Steer, No. $80 = A1$
♂ Alb. from Föckelmann $= A1$?
♀ Alb. from Föckelmann $= A1$?

(14) $A182a^6$ Alb. in $0H164$, $1H274$


♀ $A182a^6$ (Alb.) $= A1\frac{1}{2}$
♂ $A79a^1$ Alb. $= A2\frac{1}{2}$
♀ $A79a^2$ Alb. $= A2\frac{1}{2}$
♂ Pure bred from Steer $= A1$
♀ Alb. from Föckelmann $= A1$?
♂ Pure bred from Steer $= A1$
♀ Alb. from Föckelmann $= A1$?

(15) $0H155a^2$ Alb.; $0H156a^1$ Alb.


♀ $0H155a^2$ (Alb.) = $A \times 2$
♂ $1H42b^1 = 3a$–bpw
♀ $A58a^4$ Alb. = $A \times 3$
♂ $0H1a^2 = 3c$
♀ $0H12a^5 = 5a$
♂ Cross-bred Alb. from Steer = $A \times 1$ or $A \times 2$
♀ Alb. from Bacon = $A1$? No. 15 Alb.
♂ No. 1 = *Wa*
♀ Cross-bred Lab. Alb. = $A \times$ No. 1 Alb.
♂ No. 11 = *Wa*
♀ Alb. from Strong = $A1$? No. 11 Alb.

$0H156a^1$ is the same except that the mother is $A58a^5$ instead of a^4.

In the following Notes on Individual Mice, we have entered chiefly Weldon's remarks, but occasionally we show reason for differing from the account given of individual mice in Darbishire's memoir. We have only done so where on due consideration it seemed necessary, and chiefly because we found a wrong transcription from the records, or a characterisation not in accordance with the skins in our possession. From $0H$ to $2H$ matings we reproduce largely the results as given by Darbishire, although Weldon himself increased the number of these matings and had preserved for his own work all the skins possible. From $3H$ to $6H$ the matings are taken entirely from the card indices of Weldon, which were completed after his death when the stock in existence was killed and the skins preserved.

The notes we think throw considerable light on the difficulties of a large series of experiments of this kind; they mark the extent to which the mothers destroyed their offspring; the reasons why gaps occur in the matings, or in the skins preserved*; the occurrence of deformities or pathological states; the possibility of the young mating at a very early stage, and the difficulty of preventing at least some escapes. We note also how hard at times is colour classification by mere verbal categories. The following terms are abbreviated:

outbred = *OB*; cross-bred = *XB*; inbred = *IB*; pure-bred = *PB*.

NOTES ON INDIVIDUAL MICE.

$0H$ *Matings.*

$0H8$ or 7 A wild coloured mouse from Cross 7 got loose but was caught; it was put back with the young of Cross 8. Anyone of the litter called $8a$ therefore may be $7a$.—$0H8$ or $7a^2$ A huge mouse, looks like a rat.—$0H8$ or $7a^6$ Died young.—$0H9a^1$ Mangy.—$0H17$, $0H19$ Young all eaten.—$0H35a^3$, $0H35a^4$, $0H35a^5$ Mangy.—$0H45$ In Book of Crosses it is distinctly stated under Cross 45 that the father was the ♂ from Cross 10. Under Cross 6 the ♂ from Cross 10 is said to be No. 6. No. 6 in picture is $W\beta$ but under Cross 45 in *Biometrika*, Vol. III. the father is stated to be *Wa*.—$0H45a^3$ This mouse waltzed, but not in a very close circle.—$0H53$, $0H55$ Young eaten.—$0H61a^7$ Skin not worth keeping.—$0H62$ Mother inbred 5 years.—$0H72$ Offspring died young.—$0H84a^7$ The colour of coat is almost cream.—$0H84a^8$ Escaped.—$0H85a^6$ Mange on face.—$0H87$ Offspring born dead.—$0H90$ Three offspring eaten.—$0H91a^1$ Died young.—

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* Occasionally if two skins in one mating were identical only one was preserved, but usually all were kept, if possible.

0*H*99 Offspring born dead.—$0H102a^6$ Given away.—$0H113a^1$ Killed by relatives.—$0H113a^3$ Skin not kept.—$0H113a^7$ Killed by his doe.—$0H113a^8$ Escaped.—$0H114a^1$ Ill and killed.—$0H117a^3$ Mange on face.—$0H120a^8$ Unhealthy and very small.—$0H124a^3$ Escaped.—0*H*128 The two 3*cw* offspring were killed and mangled beyond recognition.—0*H*130, 0*H*131, 0*H*132 The three waltzer fathers are brothers.— $0H138a^2$ Very small.—$0H139a^5$ Escaped.—0*H*142 Mother *OB*, *XB*.—$0H142a^5$ Skin destroyed.—0*H*143 Mother extracted recessive Albino from Prof. Cuénot.—0*H*144 Mother refused to suckle young, so all died; mother *IB*, *PB*.—0*H*145 These 5's are catalogued as 6's. The colour is present but light on belly.—0*H*147 Several offspring died young; mother *IB*, *XB*.—0*H*150 Several offspring died young.—$0H152a^5$, $0H152a^6$, $0H152a^7$ Died young.—0*H*153 Several offspring died young. Two 4*c*'s are *w*.—$0H156a^4$ The lightest of *c*'s.—0*H*159 Mother *OB*, *PB*.—$0H159a^1$, $0H159a^2$, $0H159a^3$, $0H159a^4$ Escaped.—0*H*160 Mother *OB*, *PB*.—0*H*161, 0*H*162, 0*H*163 Mothers *IB*, *PB*.—$0H163a^5$ Died young.—0*H*165, 0*H*166 Mothers *OB*, *PB*.—$0H166a^1$ Escaped and killed.—0*H*167 Mother *OB*, *PB*.—0*H*168 Mother ate young.—0*H*169 Father *IB*, *PB*.—$0H169a^1$ Escaped.—0*H*170 Mother black inbred.—0*H*171 Mother black inbred.—0*H*172 Mother black inbred, several offspring died young.—0*H*175, 0*H*176 Mothers $\frac{7}{8}$ black bred.—0*H*179, 0*H*184 Mothers *OB*, *XB*.—0*H*185 Mother *IB*, *PB*.

1*H Matings.*

1*H*1 All 4 young died at birth.—$1H3a^1$, $1H3a^2$ Died young.—$1H3a^4$, $1H3a^5$ Very pale "wild."—$1H4a^4$ Escaped.—$1H5a^4$ Mangy.—$1H6a^3$ Escaped and died.—$1H7a^3$ Very light "wild."—$1H9a^4$ *Very* light *c*.—$1H18a^1$ Died young, also 2 albino offspring of 1*H*18 mating.—$1H18a^4$ No head movements.—$1H19a^3$, $1H28b^1$ Escaped.—$1H29a^2$ Is misprinted *e* for *c* in *Biometrika* and so in Catalogue.—$1H29a^3$ Skin destroyed.—1*H*34*a*, 1*H*35*a* Both young born in the same cage, no good therefore and killed.—$1H36a^4$ The colour of the coat is intermediate between fawn and *c*.—$1H36a^5$ The coat colour is nearer *c* than that of the last.—$1H37a^2$, $1H37a^4$ The coat colour is intermediate between fawn and *c*.—$1H42b^4$ The coat colour is curious. There is a "blaze" of *c* on the face and top of head.—$1H43a^2$, $1H43a^3$ Escaped.—$1H43a^4$ Too rotten to skin.—1*H*46 All young killed.—$1H49a^2$ Described as chocolate; it is intermediate between chocolate and a very pale homogeneous *c*.—$1H49b^5$ Mangy.—$1H51a^1$, $1H51a^2$ *Very* pale *c*.—1*H*55, 1*H*56 Note that the does in these two crosses are sisters, and that the one in 1*H*55 is crossed with an outbred pure-bred albino, and that the one in 1*H*56 is crossed with an inbred pure-bred albino.—$1H55a^1$ Colour only very slightly darker than fawn, though in the direction of *c*.—$1H56a^1$ Mange.—$1H57a^5$, $1H57a^6$ Died young.—$1H58a^1$ Colour between fawn and *c* with, perhaps, a soft chocolate tinge.—$1H59a^1$, $1H59a^5$, $1H59a^6$, $1H59a^7$ Back legs paralysed of these albino young.—$1H59a^4$ The colour is rather a soft *c*.—$1H60a^5$ Mange and a damaged foot.—$1H60a^6$ Died young.—$1H63a^4$ The *d* is very soft.—$1H64a^1$ Skin not worth keeping.—$1H64a^2$ Rather tending to chocolate.—$1H66a^2$ The colour is not quite *f*: it has a suspicion of yellow in it.—$1H66a^4$ The yellow is very dull, not unlike that of a normal waltzer, but merging very slightly to lilac.—$1H66a^6$ The colour is almost completely intermediate between that of a waltzer and lilac.—1*H*67 One ♂ and one ♀ offspring. Both mangy and killed.—1*H*68 Note that in 1*H*68, the only hybrid incest pair, there are 3 waltzers out of 5 young. One ♂ and one ♀ offspring mangy.—$1H70a^2$ Waltzer's colour.—$1H70a^3$ General form of waltzer. Mangled beyond recognition.—1*H*71 Albino offspring died young, the rest eaten.—1*H*72 Young eaten.—$1H80a^6$ The colour is a very pale chocolate.—1*H*81, 1*H*82, 1*H*83*a* Young born together: they are described as 81*a* and were killed.—1*H*84, 1*H*85*a* Born together. They are described as 1*H*84 and were chloroformed. 1*H*84 comes to an end. 1*H*85 goes on as 1*H*85*b*.—1*H*88, 1*H*89 Two or one family: born in cage before isolation; the young are described as 1*H*88*a* and were killed. One ♀ is left with ♂ as 1*H*88*b*. 1*H*89 comes to an end.—1*H*90, 1*H*91 Born together. 1*H*90 continued as 1*H*90*b*. 1*H*91 comes to an end. Young recorded and killed.

Note by W. F. R. WELDON:

The last two crosses had three objects (1st) to see whether whites (produced by crossing whites with first crosses) when crossed with first crosses—i.e. 1st generation hybrids—give the Mendelian numbers in their offspring, i.e. 50 °/₀ albinos and 50 °/₀ hybrids, (2nd) to see whether albinos produced in this way behave the same when crossed with waltzers as do ordinary pure-bred albinos, and (3rd) whether albinos whose ancestry has been "watered down" in this way for many generations will breed true when paired. With regard to the "watering down" of the albino ancestry of an albino, if hybrids at the first generation are paired with albinos, and the resulting albinos are paired back with hybrids of the first generation, the following table shows what happens:

The fractions in brackets after Hyb. or Alb. indicate the amount of albino ancestry in the animal.

Hyb. $(\frac{1}{2})$ × *Alb.* ——— $\frac{3}{2}=1\frac{1}{2}$
Alb. $(\frac{3}{4})$ × *Hyb.* ——— $\frac{5}{4}=1\frac{1}{4}$
Alb. $(\frac{5}{8})$ × *Hyb.* ——— $\frac{9}{8}=1\frac{1}{8}$
Alb. $(\frac{9}{16})$ × *Hyb.* ——— $\frac{17}{16}=1\frac{1}{16}$
Alb. $(\frac{17}{32})$ × *Hyb.* ——— $\frac{33}{32}=1\frac{1}{32}$
Alb. $(\frac{33}{64})$ × *Hyb.* ——— $\frac{65}{64}=1\frac{1}{64}$
Alb. $(\frac{65}{128})$ × *Hyb.* ——— $\frac{129}{128}=1\frac{1}{128}$
Alb. $(\frac{129}{256})$ × *Hyb.* ....

From which it will be seen that to find the albino ancestry of an albino in a given generation the denominator of the fraction indicating the albino ancestry of the previous generation must be doubled to produce that of the desired generation: and to find the numerator of the generation in question that of the previous one must be doubled and 1 subtracted.

But it is evident, that continually breeding back with a first cross is bad: because the ancestry of the desired albino can never be reduced below a half. The first step (that taken in 1*H*90 to 91) is all right because the ♀ in 1*H*91 is Alb. $(\frac{3}{4})$ and the young produced (i.e. 1*H*91$a^{1,2}$, etc.) will be alb. $(\frac{5}{8})$, i.e. an $\frac{1}{8}$ less albino ancestry $\frac{5}{8}=\frac{160}{256}$; then the degree in which the "watering down" diminishes is shown in the following series:

$\frac{160}{256}$, $\frac{144}{256}$, $\frac{136}{256}$, $\frac{132}{256}$, $\frac{130}{256}$, $\frac{129}{256}$,

or 16, 8, 4, 2, 1, —

or ancestry of ♀ in 1*H*91, i.e. *H*6a^2 = ·750
" " 1*H*91a^1, etc. = ·625
" " Albino in next generation = ·562
" " " " " = ·531
" " " " " = ·515
" " " " " = ·507
" " " " " = ·503

that is to say starting with Alb. $(\frac{3}{4})$ (e.g. *H*6a^2 in 1*H*91) we can in the 6th generation produce Alb. (·503).

1*H*94 Parents doubtful, at any rate in colour of father and the 4 does *H*84*a* were pregnant when sexed. Offspring destroyed.—1*H*95 The parent females *H*62*a* etc. were pregnant when sexed.—1*H*95a^3 Colour a mixture, between fawn, chocolate and *c*.—1*H*95a^4 Colour pale chocolate.—1*H*95a^6 Killed by relations.—1*H*96a^2 Colour a very pale fawn.—1*H*97a^1 *Pale* fawn, with *little* white on belly.—1*H*98a^4 Much paler than a^3.—1*H*98a^5 Extremely like 1*H*98a^4, but with less white below.—1*H*98a^6 A very blue fawn colour.—1*H*99a^1 *Dark* fawn. The belly lighter but hardly any real white.—1*H*99a^2 Pale *c*. Skin not kept.—1*H*99a^4 Died young.—1*H*100a^3 Skin not kept.—1*H*100a^5 Rather dark fawn.—1*H*100a^6 Pale fawn.—1*H*101a^1 Colour a pale blue-grey, like a mixture of fawn and lilac.—$1H101a^4$ Colour not black.—$1H101a^5$ Died young.—$1H101a^7$ No record except skin.—$1H101b^6$ Mixture of dark *b* and very pale chocolate with a look of fawn.—$1H102a^3$ Colour exactly intermediate between *b* and *c*.—$1H102b^2$ The palest lilac imaginable: if not placed beside an albino, it is taken for an albino.—$1H102b^3$ A pale but obvious *f*.—$1H102b^4$ A darker *f* than $1H102b^3$.—$1H103a^1$ Colour dark fawn.—$1H103a^2$ Colour yellow fawn. Skin rotten and not kept.—$1H103a^3$ Colour very dark fawn or very pale "wild."—$1H103b^3$ The appearance, behaviour and complaints of waltzers are present in this mouse.—$1H104a^5$ Died young.—$1H105a^3$ Colour a brownish lavender.—$1H105b^1$ Escaped.—$1H105b^2$ Died young.—$1H107a^1$ *Very* pale "wild."—$1H108$, $1H109a$ All offspring died young.—$1H109a^1$ Very like pure race.—$1H109a^2$ Colour greyer than in $1H109a^1$.—$1H109b^5$ Died young.—$1H111a^1$ A uniform pale grey almost intermediate between *a* and *e*.—$1H111a^2$ Colour a grey fawn. Died young.—$1H111a^4$ Colour paler than a^3 but the belly less white.—$1H111a^6$ Died young.—$1H112a^3$ Colour called *f* but it is really too brown for lilac.—$1H112a^4$ Exhibits waltzing tendencies.—$1H114a^4$ Skin not worth keeping.—$1H115a^2$, $1H115a^4$ Died young.—$1H115b^1$ Some parts of ears are clothed with yellow hair.—$1H117b^1$ A sharp patch of white between fore legs, another much smaller on belly.—$1H118b^2$, $1H118b^7$ Escaped.—$1H121a^1$ Colour very pale.—$1H121a^3$ Colour *very* dark.—$1H121b^4$ A very dark *d*.—$1H121b^5$ A measly specimen. Escaped.—$1H122a^6$, $1H122a^7$ Skins not kept.—$1H124a^1$, $1H124a^4$, $1H124a^5$ Skins not kept. Head destroyed in $1H124a^1$.—$1H124b^4$ Rather of a dark *b*.—$1H124b^5$ Died young.—$1H125a^2$, $1H125a^4$, $1H125a^5$, $1H125a^6$, $1H125a^7$ Skins not kept.—$1H125b$ All offspring died.—$1H126a^3$ Died young.—$1H126b^2$ Waltzing and neurotic tendency very well pronounced.—$1H126b^3$ Called erroneously *e* in *Biometrika*, Vol. III.—$1H127b^4$ Darker than $1H127b^2$.—$1H127b^5$ Mange.—$1H128b^1$ A dark *b*.—$1H129a^1$ Died before being painted.—$1H129a^2$ Skin not kept.—$1H129b$ Catalogue, *Biometrika*, Vol. III. and skins differ wholly from Children of Hybrids record book.—$1H129b^7$ Died young. Only mentioned in mating book.—$1H130a^2$ Very pale *b*.—$1H133b^1$, $1H133b^2$ Died young.—$1H133b^3$ Very pale.—$1H133b^4$, $1H133b^5$ Very pale, died young.—$1H135a^3$, $1H135a^4$ Died young.—$1H136b^3$ Colour intermediate between *b* and chocolate.—$1H137a^1$ Colour intermediate between chocolate and *c*.—$1H137b^2$ Died young.—$1H137b^3$, $1H138a^2$, $1H138a^3$ Died young.—$1H139b^1$ Called 6*e*. But by no means a pure black. Several hairs on chest, between front legs, have *brown* pigment; and a silvery appearance given by a very few scattered white (?) hairs on sides, and a very few on back. Claws white. Hair on hands and feet white: most caudal hairs white. Skin of tail pink with a few dark patches.—$1H140a^1$ Died young.—$1H140a^2$ Died Young. Distinctly a 5*c*. Called 6*c* in *Biometrika*, Vol. III. but 5*c* in Mating book.—$1H140a^6$ Died young.—$1H142b^3$ A *very small* admixture of long whitish hairs on sides and belly. Digits pink with some white hairs. Tail with patches of pink skin, and on these white hairs. Vibrissae black.—$1H146b^5$ Killed before sexing.—$1H149a^6$ Died young.—$1H149b^4$, $1H150b^3$ Escaped.—$1H152b$ Young eaten.—$1H152c^6$ Skin colour same as $1H152c^5$.—$1H153a^5$, $1H154a^2$, $1H155a^5$, $1H155a^6$ Escaped.—$1H156a^1$ The hair of the belly rather a dark brown than black. On haunches and sides of belly a *very* few white hairs. One or two white vibrissae. Nails pink. Digits and tail with white hairs.—$1H158a^1$ Killed accidentally.—$1H160a$ Offspring died young.—$1H161a^6$ Eaten.—$1H162a^1$ Not very healthy, skin destroyed.—$1H163a^1$, $1H163a^3$ Skins not kept.—$1H164$ Mother black bred.—$1H164a^2$ *c* very pale.—$1H165$, $1H166$ Mothers black bred.—$1H167$ Mother again black bred.—$1H168$, $1H169$ Fathers black bred.—$1H170$ Mother black bred.—$1H170a^5$ Very pale. (We should certainly have called it a pale blue grey verging on chocolate. K. P. and F. J. W.).—$1H171$, $1H172$ Fathers black bred.—$1H172a^4$ Escaped.—$1H172a^5$ Killed accidentally.—$1H173a^5$ Really a very dark chocolate, a very wonderful colour.—$1H177$ All offspring eaten.—$1H178a^8$ Colour same as $1H178a^7$.—$1H179a^5$, $1H179a^6$ Coats same as $1H179a^3$.—$1H182a^6$ Yellow underneath.—$1H187a$ One stillborn offspring only.—$1H196a^9$ Skin identical with $1H196a^3$.—$1H201a^1$ Very pale.—$1H204a^2$ Skin like $1H204a^1$.—$1H204a^4$ Skin like $1H204a^3$.—$1H205a^2$ Skin like $1H205a^1$.—$1H205a^6$ Skin like $1H205a^3$.—$1H206a^7$ Skin like $1H206a^6$.—$1H206$, $1H207$ Mothers *IB*, *PB*.—$1H208$, $1H209$ Fathers *IB*, *PB*.—$1H211a^3$ (We consider this mouse a chocolate. K. P. and F. J. W.).—$1H217a^4$ Colour almost chocolate.—$1H220a^3$, $1H220a^4$, $1H220a^5$ Escaped.—$1H222a^6$ Darker than $1H222a^5$.—$1H222a^8$ Same colour as $1H222a^7$.—$1H227a^2$ Not found in Catalogue.—$1H237$ Both parents from extracted albinos.—$1H237a^4$ Paler, more *f* than $1H237a^3$.—$1H237a^7$ Little browner than $1H237a^6$.—$1H238$ Both parents from extracted albinos.—$1H238a^4$ Skin like $1H238a^3$.—$1H239a^6$ Escaped.—$1H239$, $1H240$, $1H241$, $1H242$ Both parents from extracted albinos.—$1H242a^4$, $1H242a^5$ Skins like $1H242a^3$.—$1H243$, $1H244$ Both parents from extracted albinos.—$1H244a^3$ Colour like $1H244a^2$.—$1H245$ Both parents from extracted albinos.—$1H246$ Mother from Cuénot's extracted ♀ albino.—$1H246a^4$ Skin like $1H246a^3$.—$1H248$ Both parents from extracted albinos, the mother from Cuénot's extracted ♀ albino.—$1H248a^5$, $1H248a^7$ Colour like $1H248a^3$.—$1H249$ Both parents from extracted albinos, the father from Cuénot's extracted ♀ albino.—$1H249a^6$ Colour like $1H249a^5$.—$1H250$ Both parents from extracted albinos, the father from Cuénot's extracted ♀ albino.—$1H250a^2$ Skin not kept.—$1H252$ Both parents from extracted albinos.—$1H252a^2$ The waltzing movements of this mouse were extraordinary; it was incapable of locomotion.—$1H253$ Both parents are hybrids from the extracted $G'G'$ $1H42b^1$.—$1H254$ Mother is hybrid from the extracted $G'G'$, $1H42b^1$.—$1H255$, $1H256$, $1H257$, $1H258$ Both parents are hybrids from the extracted $G'G'$ $1H42b^1$.—$1H262a^1$ The *b* is curious: rather like a very pale *g*.—$1H263$ Both parents from extracted albinos.—$1H266$ One 5*a* offspring is a *w*.—$1H270a^1$ Black generally: but a few of the longer hairs on the sides and thighs white or yellow. A little white hair on the toes. The nails pink. The tail mottled brown and pink, the hairs on it white. A mixture of white hairs round the anus.—$1H270a^4$ Escaped.—$1H273a^2$ Considerable mixture of long brownish and whitish hairs over whole body. All digits and most of dorsum of R. pes with white hairs. Some brown (and white ?) vibrissae. Tail hairs mixed.—$1H278$, $1H280$ Fathers are hybrid from the extracted $G'G'$ $1H42b^1$.—$1H282$ Young eaten.—$1H292$ One 5*c* offspring is *w*.—$1H297$ One 5*c* offspring is *w*.—$1H301$ Two of the 2*c* offspring are *w*.—$1H308$ One 2*c* offspring is *w*.—$1H310$ One 5*c* offspring is *w*.—$1H312$ One 2*c* offspring is *w*.—$1H313a^2$ No record except skin. 6*e* taken from skin.—$1H324a^1$ No record except skin. 2*e* taken from skin.—$1H329$ One 5*c* offspring and one 5*b* are *w*.

2*H Matings.*

$2H12a^2$ Darker than a^1.—$2H12a^3$ The colour simulates pale *g*.—$2H22a^3$ Skin like a^2.—$2H22a^5$ Skin like a^1.—$2H27a^4$ (Skin we think must be described as an *a* like the Catalogue and not *b*, *c* like *Biometrika*. K. P. and F. J. W.).—$2H28a^5$ Colour like a^2.—$2H32a^3$, $2H32a^4$, $2H32a^5$ Skins like a^2.—$2H33a^5$ Hardly any hair.—$2H38a^1$ Colour intermediate between chocolate and *c*.—$2H38a^2$ The *b* is dark tending to chocolate.—$2H38a^4$, $2H38a^5$ The *b* is not very yellow.—$2H40a^1$ Homogeneous grey tending to chocolate.—$2H40a^3$ Tending to dark *b*.—$2H45a^2$, $2H45a^3$ Colour like a^1.—$2H49a^1$ Much blacker than any hybrid in present stock. No white or brown hairs on back: a very few scattered brown and white hairs on sides; more on belly: and grey ring round perineum. Tail hairs white. Dorsum of digits white. Claws pink. Vibrissae all black.—$2H49a^2$ A distinctly greater mixture of yellow or brown hairs along sides of body than in $2H49a^1$, and a group of very few white hairs in middle of belly. Dorsum of feet and hands with much pink skin and many white hairs. About $\frac{1}{2}$ the tail pink: the tail hairs white. Brown hairs round base of ears, as is usual in "black" mice. Vibrissae black.—$2H79a^{2,\,3,\text{ or }4}$ One of these 4*c*'s given as 5*c* in *Biometrika*.—$2H79a^2$ Colour like a^1.—$2H79a^3$, $2H79a^4$ Skins like a^2.—$2H79a^7$ Skin like a^1.—$2H82a^3$ Skin like a^2.—$2H92a^1$ or a^2 One of these 3*c*'s is a waltzer.—$2H93b^*$ Supplied from skins as no records are given.—$2H96a^1$ Fairly black. A few (very few) yellowish hairs on sides and haunches. Perineal patch grey; a good many wild-coloured hairs mixed with the black on belly, and a very small spot of a few nearly white hairs about umbilicus. Digits pink, with white hairs. Whiskers black. Hairs of tail some black (?), some white.—$2H101a^2$, a^3, a^4 or a^5 One of these 6*c*'s is a waltzer.—$2H102a^1$ Very good pure black dorsally: perineum, sides, and upper part of limbs with mixture of wild-coloured hairs.

A few white hairs on belly. Brown hairs round bases of ears well marked. Fingers and toes pink, with white hairs. Tail with dark skin, the hairs wild colour and white. Vibrissae black.—$2H104a^1$ or a^2 One of these is a waltzer.—$2H108a^1$ A very good black, with very few white and brown hairs on sides and belly. The perineal patch wild-coloured. Hardly any brown hairs at base of ears. Digits pink, with white hairs. Tail skin pink and dark. Hairs of tail some white, some brown or black. Vibrissae black.—$2H108a^2$ Colour almost like $2H108a^1$, but with more brown hairs on belly.—$2H109$ One $4c$ offspring is a waltzer.—$2H114a^1$ A very good deep black, with a very few whitish hairs dorsally and few scattered brownish hairs on sides, and more, but not many, on belly. Hardly any brown hair round base of ears. Perineal patch grey. Digits white. Skin of tail pigmented, but many hairs white. Two or three white vibrissae on L. side, three on R. side, the rest black.—$2H117a^1$ A brownish black above, with considerable mixture of whitish or white hairs on limbs and a few on sides of trunk. Belly and perineal patch grey from mixture of white and brown hairs. Digits and dorsum of hands and feet with much white hair. Tail brown, with much white hair. Hair at base of ears pale brown. Some white vibrissae.—$2H124a^1$ A very black mouse, with very small admixture of brownish hairs on sides and belly. Perineal patch very small, brown. Hardly any brown hairs round base of ears. Toes and fingers pink with white hairs. Vibrissae black.—$2H143$ One $6e$ offspring is a waltzer.—$2H147a^1$ or a^3 One of these has pink eyes.—$2H151a^1$, a^3 or a^4 One of these has pink eyes.—$2H155a^1$ Really very little white. A small patch of pure white on belly: the fingers and toes pink with white hairs. The skin of the tail nearly all dark, but with many white hairs. The rest a very good black with the long hairs darker than usual. Pale brown hair round base of ears. Vibrissae black.—$2H155a^2$ A good black with very few lighter hairs. Fingers and toes pink with white hairs: the tail skin pink for about $\frac{1}{4}$ its length, with white hairs. Brown hairs round base of ears. Whiskers black.—$2H155a^3$ A fair black above, and at sides, with a very few white hairs. Perineal hairs mixed with white. Considerable mixture of white hairs on belly. Hairs round base of ears pale brown. Digits pink, and much white hair on hands and feet. Most of tail pink, the hairs white. Vibrissae black.—$2H156a^2$ A poor black, with many long, whitish hairs at sides. Perineal patch considerable, yellowish white. Belly with not much more white than sides. Much white hair on fore-limbs, and on hands and feet. Digits pink. Hair round base of ears very pale. Some white vibrissae.—$2H156a^3$ Almost exactly like $2H156a^2$, but the whitish hairs on sides and belly a little more plentiful.—$2H156a^4$ Almost like $2H156a^2$, but without any white whiskers, and with the hair round the base of the ears still paler.—$2H300a^2$ A very good black; extremely few brown or white hairs scattered on belly and perineal area. Skin of tail dark above, also below except near base. Vibrissae black. Most hairs on ears black, the rest deep brown. Claws pink. Dorsum of manus and pes black; white hairs on phalanges III, II, and parts of I in each digit.—$2H300b^3$ Very good black. Very little brownish hair on belly and perineum. Ears and hair behind them brown. Phalanges of feet pink, with white hairs; those of hand darker, with few or no white hairs. Tail skin dark, with some white hairs. Whiskers black.—$2H300c^3$ A good black: a little scattered (white? or) yellowish hair on sides and belly. Hands and feet pink. Tail pigmented with pale hairs. Many pale whiskers. Perineal patch small. Ears brown.—$2H300c^4$ A rustier black than $2H300c^3$; the hands with more white hair: a little unpigmented skin round base of tail.—$2H300c^5$ Generally like $2H300c^3$, but the pale hairs whiter, and the distal $\frac{1}{4}$ of the tail unpigmented.—$2H301a^1$ Much scattered pale brown (*b* or *c*?) hair on sides: belly with distinct brown tinge. Ears and hair behind brown. Tail dark with brown hairs. Distal phalanges of foot pink, with white hairs; those of manus pink with few or no white but some brown hairs.—$2H301a^2$ A very uniform brown-black, darker than *d*.—$2H301a^4$ A yellowish lilac.—$2H301a^5$ Little scattered pale (*a* and white) hair. Perineum pale *a*. Ears pale brown; hair behind them nearly white. Nostrils pink. All digits more or less pink, with pale or white hairs. Tail, skin darker above, paler below, with white hairs. Many vibrissae white.—$2H301a^6$ General colour black, with a little scattered white hair; but about 3 very small white spots on R. haunch. Whiskers black. Otherwise like $2H301a^5$.—

$2H302b^2$ A good black: the scattered hairs distinctly yellow. Digits pale with yellowish hairs. Perineal patch yellowish. Tail pigmented with yellowish hairs. Many pale whiskers.—$2H302b^3$ Like $302b^2$ but with less coloured hair. The toes and tail darker.—$2H302b^4$ Small white blaze on forehead: white snout: white patch on belly: grizzly line on haunches.—$2H302b^5$ An extremely good black: even the digits with much pigment. Ears still brown.—$2H302b^6$ Body nearly or quite as black as $302b^5$, but the digits pink.—$2H302c^4$ Fair black. Long hairs on sides, and quarters yellowish white. Digits pink. Ears brown. Whiskers black. Tail pigmented with whitish hairs.—$2H302c^5$ Fair black. Long hairs on sides yellower than in $302c^4$. Digits pinker. Perineal patch larger and yellowish white. Some pale whiskers. Tail pigmented with whitish hairs.—$2H302c^6$ Rather more scattered pale hairs than $2H302c^5$. One or two isolated pale hairs on forehead. The digits pinker than in $302c^5$; the whiskers conspicuously mixed black and pale.—$2H303a^1$ A good black above: scattered pale hairs at sides and below: perineum pale grey. Ears brown: hair behind them pale. Most digits pink with white hairs. Skin of tail dark with white hairs. Some white vibrissae.—$2H303a^2$ Fair black. Feet, hands and tail pink, with white hairs. A white patch on one side of snout. All (?) whiskers white. A white patch across belly, running as narrow band up each side.—$2H303a^3$ A large white belly patch, with narrow white band obliquely across back. Hands, feet and tail pink.—$2H303a^4$ Body a good black, with very few pale hairs on belly and perineum. Tail dark, with some pale hairs below. Digits pink: those of R. pes with white hairs only: those of L. with fewer: those of manus with very few or none. Ears brown, the hair behind them brown. Whiskers black.—$2H303b^3$ Minute white patch on forehead: large white belly patch; faint white bar across haunches.—$2H303b^4$ Small white belly patch.—$2H303b^5$ A fair black. Hands and feet very pink, with white hair. The scattered lateral hairs more yellowish.—$2H304$ One black eyed offspring born dead, a mating previous to a.—$2H304a^3$ A fair black. Scattered yellowish hairs on sides. Perineal patch yellowish. Tail pigmented with pale hairs. Digits of manus slightly pigmented, of pes pink. Hair on and behind ears brown. Whiskers black.—$2H304a^6$ Good black. Few scattered yellowish hairs. Perineal patch yellowish. Digits pink. A few pale whiskers. Tail feebly pigmented with pale or nearly white hairs.—$2H304b^2$ Tail coloured.—$2H304b^3$ Tail ringed pink. —$2H305a^1$, $2H305a^3$, $2H305a^4$, $2H305a^5$ A yellowish lilac.—$2H307a^1$ Killed by ♀.—$2H308a^1$ A fairly typical fawn, the belly white.—$2H308a^2$ A pale and rather "blue" fawn. Belly coloured except for a small angular patch of pure white.—$2H308a^3$ A rather dark fawn. Belly white.—$2H308a^4$ A rather "blue" fawn: belly coloured with sharply defined white patch.—$2H308a^5$ Paler than a^4. The white of the belly comes on to the sides at the waist.—$2H308a^6$ Almost exactly intermediate between fawn and lilac. Belly white.—$2H309a^1$ A typical fawn. Skin not kept.—$2H309a^2$ A typical fawn.—$2H309a^3$, $2H309a^4$ Rather grey fawn.—$2H309c^1$, c^2, c^3, c^4 Died before sexing.—$2H309c^5$, c^6, c^7, c^8 Died and half eaten.—$2H312a^4$ Some scattered pale hairs at sides: belly with many pale hairs: perineum pale. Ears and hair behind them brown. Tail skin dark, with white hair, especially at tip. Digits pink with white hair. Some white vibrissae.—$2H312a^5$ A fair black: tail pinkish with white hairs, and a narrow linear streak of white on belly. Digits pink with white hairs.—$2H312b^1$ Escaped.—$2H312b^3$ Fairly uniform, but very rusty black. Perineum grey. Tail dark with white hairs. Hands and feet pink with white hairs. Ears and hair behind them brown.—$2H312b^6$ A good black. Ears dark brown, hair behind them lighter. Very little paleness on belly and perineum. Digits pink, with some white hairs.—$2H317$ Data marked * supplied from skins.

3*H* *Matings.*

$3H1a^2$ A dark chocolate.—$3H1a^3$ A yellowish lilac.—$3H1a^4$ A yellowish lilac: colour very uniform.—$3H1a^5$ A yellowish lilac: indistinguishable from $3H1a^4$.—$3H1a^8$ Some scattered light hairs on belly. Hairs behind ears nearly white. White hairs on digits. Whiskers black. Tail with dark skin.—$3H1b^3$ A rusty black, with scattered yellow (a ?) hairs. Belly and perineum with so much yellow hair that the mouse can hardly be put in Class 6. Ears brown: hair behind them very pale. Feet pink: terminal phalanges of hand with white or pale yellow hairs. Tail skin dark, hairs pale. Whiskers black.—$3H1b^4$ Body a good black above (darker than in $3H1b^3$), belly and perineum with much yellowish scattered hair. Ears and hair behind them brown. All digits with white or pale hairs. Tail pigmented except at tip, with pale hairs. A few pale whiskers.—$3H1b^7$ Almost exactly like $3H1b^3$. Escaped.—$3H2a^3$ A rusty black: a narrow pure white patch on belly; much scattered pale hair among black, at sides and below. Perineal patch with much white hair. Hair on and behind ears fawn coloured. Digits and dorsum of hands and feet with white hairs. Tail with patches of pink skin and some white hairs. Some pale whiskers.—$3H2a^4$ No definite white patch but much scattered pale (fawn?) hair on sides and belly. Hands and feet with much pink skin and white (?) hair. *Tail* almost all dark skinned, with pale hair. Hair on ears pale fawn: behind ears nearly white. Vibrissae black.—$3H2a^7$ Much scattered white hair. Much pink skin and white hair on hands, feet and tail. Hair behind ears whiter than in a^4. Whiskers black.—$3H2b^4$ Much fawn hair scattered on sides, more on belly. Perineum yellowish white. Tail skin feebly pigmented: hair light. Digits pink, almost all hair on them white or pale. Snout and chin with pale hairs. Hair on inside of ears and behind them pale fawn: on outside dark or black. A few fawn whiskers.—$3H2b^5$ A good uniform lilac, but very pale (nearly white) at root of tail.—$3H3a^3$ Poor black. Much scattered pale hair on sides: a very small spot (3—4 sq. mm.) pure white on belly. Digits pink, with white hairs; some white hairs on dorsum of hands and feet. Skin of tail dark near base, the rest pink. Caudal hairs all pale. Hair behind ears nearly or quite white: hair inside ears fawn. Some pale vibrissae.—$3H3a^4$ Black: much scattered pale (white?) hair on sides and belly. Perineum nearly white. Only last 2 phalanges of digits pink. Hair behind ears dark; within ears very pale. Some pale whiskers. Tail dark skinned with pale hair.—$3H3a^5$ Black, with little mixture of pale hairs. Digits pink, with white hairs. A pink portion, with white hair, near extremity of tail. Some pale whiskers.—$3H3a^8$ Very pale "pearl grey" rather than "lilac."—$3H3b^3$ More white than $3b^2$.—$3H3b^4$ A good black above: a fair quantity of scattered fawn hair on belly, on fore-limbs and throat. Perineal patch pale fawn. Tail skin pigmented, hair fawn. Digits pink, hair white or fawn. Hair within and behind ears pale. Many pale whiskers.—$3H3b^5$ A very good black. Few scattered fawn hairs on sides, belly and throat. Tail pigmented except at tip: its hairs white or pale. Digits pink, hairs white or pale. Perineum with very little pale hair. Inside and behind ears, hair fawn: darker outside. Whiskers black.—$3H3b^6$ A good black above: dark slaty grey, with small median white patch, below. Digits and whole R. foot, pink with white hairs. Tail nearly all pink. Many pale whiskers.—$3H3b^7$ Like $3b^5$, but with less pale hair, and with no pink on tail.—$3H3b^9$ A very good black: very little pale hair on belly and throat. Digits pink, with some pale hair. Tail pigmented. Whiskers black.—$3H3c^8$ Died before sexing.—$3H4a^5$ Good black. Very few scattered pale hairs. Terminal phalanges pink, with white hairs. Perineal patch very dark. Tail pale-skinned, but not pink, with light hairs. Hair within and behind ears pale. Vibrissae black.—$3H4a^6$ Very like $4a^5$, but feet whiter, and tip of tail pink.—$3H4b^1$ A very small patch pure white in middle of belly. Digits pink. Terminal 12—15 mm. of tail pink.—$3H4b^2$ A good black, with scattered pale hairs ventrally. Perineal patch small. Only phalanges II and III pink, except on R. foot, where they are all pink. Tail slightly pigmented, with small pink patches; hair fawn (?). Hair on and behind ears brown. Many pale whiskers.—$3H4b^3$ Almost exactly like $4b^2$, but both hind feet have pink digits.—$3H4c^3$ Small white belly streak.—$3H4c^7$ Dwarf.—$3H5a^1$ A very dark chocolate: eyes quite black.—$3H5a^2$ Good black. Some scattered (fawn?) hair on sides and belly. Very little pale hair behind ears: hair within ears brown. Digits pink, with white hair. Tail feebly pigmented, with about 1 cm. at end pink: the hair fawn (?). Some pale whiskers.—$3H5a^3$ More pink on toes than a^2, and $\frac{1}{3}$ of tail pink.—$3H5a^4$ Less pink on toes than a^2, and none on tail.—$3H5b^3$ A good black: some scattered (fawn?) hair below: perineal patch small. Terminal cm. of tail pink. Hair behind ears very pale. Digits feebly pigmented.—$3H5b^4$ Much as $5b^3$, but digits pink, and tail ringed with pink.—$3H5b^7$ A good black, like $5b^3$, but with the digits pink.—$3H6a^2$ A good black: very little scattered light hair; a small white spot on forehead. Belly with much scattered white hair and a pure white patch. Hair in and behind ears brown. Digits and all L. hand pink with white hairs. More than $\frac{1}{2}$ the tail pink skinned, with white hairs. Whiskers mostly white.—$3H6a^3$ Rusty black: scattered white hairs on sides: perineal patch very pale: belly almost slate-coloured, with linear central streak of white. Tail feebly pigmented, the hair white. Hair in ears brown. Digits white. Some white whiskers.—$3H6a^4$ Back and sides fairly good black. Snout and whiskers white, with white streak on forehead. Hair behind ears nearly white, within ears pale yellow. Digits and parts of dorsum of manus and pes pink, with white hairs. Tail pink, with patches of pale pigment, the hair white. Belly grey (i.e. black with much scattered white hair), with large central white patch.—$3H6b^1$ Fair black above: much scattered white hair at sides and below. Tail pigmented, hair white. Digits pink with white hairs. Perineal patch whitish. Hair on ears yellow, nearly white behind ears. Whiskers black.—$3H6b^2$ A very slaty black: much diffuse white hair, and about a dozen conspicuous white hairs on forehead. Hands and feet with much white hair. Many white whiskers.—$3H6b^3$ Very like $3H6b^2$.—$3H6b^4$ Very like $3H6b^2$.—$3H6b^5$ Median white belly patch.—$3H6c^4$ Mange.—$3H7a^1$ Very good black dorsally: perineum yellowish, belly with fair mixture of light hairs. Tail dark, hair yellowish except at the extreme tip, where it is pure white. Hair behind ears black, inside ears yellow. Skin of ears darker than usual. Digits dark, with extremely little light hair.—$3H7a^3$ Much light hair on sides and belly. Perineal patch almost white. Hair behind ears almost white: within ears pale yellow. Much white on digits and on dorsum of L. foot. Tail with patches of dark and of pink skin, early all the hair white. Whiskers black.—$3H7a^4$ Almost identical with $7a^3$, but with less white on the ears.—$3H7a^5$ Almost like $7a^3$, but with more pink at tip of tail.—$3H7b^1$ A rusty black dorsally, the belly dark slate grey. All digits and dorsum of hands and feet pink, with much pale hair. Tail feebly pigmented, the hair pale. Hair behind ears pale, inside ears yellow. Many pale whiskers. Perineal patch yellow-white.—$3H7b^2$ Body a good black; slightly grey below, from a little scattered pale hair. Perineum yellow-white. Feet and the digits of the hands white. Tail feebly pigmented, the hairs white. Very few pale hairs behind ears. A few pale whiskers.—$3H7b^5$ A rusty black above: belly grey, with faint linear white streak.—$3H8a^3$ A very rusty black. Hind feet and digits of hands white; $\frac{2}{3}$ of tail pink, with white hair on all the tail. Belly with much diffuse white hair, and a small white patch. Pale patch behind ear well marked. Hair within ears yellow. Some light whiskers.—$3H9a^2$ Generally a good black, with scattered whitish hairs at sides, and a line of thickly grouped whitish hairs, making a very pale band, along middle of belly. Perineal patch pale yellowish white. Pale tufts behind ears well marked: hair inside ears yellow: some white whiskers. Skin of tail dark except at tip, where it is bright pink: the tail hair white. Fingers white. The R. toes and part of dorsum of R. foot white; only terminal phalanges of L. foot white.—$3H9a^3$ No white patch, but much scattered pale hair on sides and shoulders. The pale hair yellowish. Perineal patch yellow. Digits white. Extreme tip of tail white. Pale patch behind ear less conspicuous than in a^2. Some whiskers pale.—$3H10c^2$ Eye-colour not noted (but *not* pink).—$3H11a^1$ Poor black, with much pale or white hair on sides, fore-limbs, and inside of thighs: rather less on belly. Digits white. Tail feebly pigmented below, darker above. Pale patch behind ears very small: hair inside ears yellow. Nearly all whiskers pale.—$3H11a^2$ Nearly completely white below (belly darkened with age).—$3H11a^4$ Poor black, but with little scattered pale hair. Perineal patch very pale. Hair behind ears nearly white: hair within ears yellow. Terminal phalanges of digits white. Tail with pigmented skin but with white hair. Some whiskers white.—$3H11a^5$ Almost like a^4, but with more white on toes.—$3H11a^6$ A pale ring on tail. A patch of densely clustered white hairs on belly. Otherwise like a^4.—$3H11a^7$ Exactly like a^5. Later; developed much brown hair on shoulders.—$3H12b$ Nine offspring born, seven eaten.—$3H14a^4$ A "chinchilla" mixture of black and white.—$3H14a^5$ "Chinchilla" grey like a^4.—$3H14b^1$ The black with a good many scattered white hairs. Cf. the chinchilla of $14a$.—$3H14b^3$ The black with scattered white hairs.—$3H14c^1$

"Grey" = "chinchilla" mixture of dark grey or black hairs with white ones.—$3H25a^2$ Much white on head: fawn patch round each ear: fawn patch on L. side, and large patch on hind quarters.—$3H25a^3$ Head and much of shoulders fawn: white blaze on face. Large fawn patch on hind quarters: no lateral patch.—$3H25a^4$ Head fawn with white streak on forehead. Fawn continued backwards on left side as large patch. A large fawn patch on hind quarters. Much white on right side.—$3H25a^5$ Head and shoulders fawn: muzzle white: hind quarters fawn: large blotches of fawn on L. side and over back to R. side. Much more colour than the others.—$3H25a^6$ Head and shoulder patch small. Snout white. Hind quarter patch small. —$3H25a^7$ Snout white. Head and shoulder patch larger. A patch on R. side. Hind quarter patch small.—$3H25a^8$ Head with practically no white. Patch on hind quarters divided.—$3H26a^1$ Head patch covers both eyes.—$3H26a^2$ Head patch very small: area round L. eye white.—$3H29a^3$ Escaped.—$3H31$ *Z* 139 albino mother pure-bred for 3 generations.—$3H31a^1$ A fairly uniform, very dark black: the tail with some pink: the belly slightly paler than the back. Digits white haired. Whiskers black. Patch behind ears not very light.—$3H31a^2$ Perfect wild-colour, grey beneath.—$3H31a^3$ Perfect wild-colour, grey beneath, but the tail ringed with pink.—$3H31a^6$ With some pink on tail.—$3H31b^7$ With much pink on tail.—$3H31b^8$ With a little pink on tail.—$3H33$ The *Z* mother pure-bred for 3 generations.—$3H33a^1$ Fair black, with few scattered light hairs. Belly with more light hair. Perineum nearly white. Tail very slightly pigmented with white hair. Digits pink, with white hair. Some white hair on dorsum of pes. Ears pale. Hair behind them very pale. Some white whiskers. Tip of muzzle with some white hairs.—$3H33a^2$ A very fair black: belly darker than in a^1: perineum yellowish. Some white on digits, but less than in a^1. Many white whiskers. Ears as in a^1.—$3H33a^3$, $3H33a^4$ Almost identical with a^2.—$3H33a^5$ Very good black. Perineum nearly white. Digits pink with white hair. Tail pigmented, with little pink patch near extremity. Ears darker than in males: a few white whiskers.—$3H33a^6$ Very like a^5 but no pink on tail and no (?) pale whiskers. Perineum and hair on tail yellowish.—$3H37a^1$ Killed by brother.—$3H39b^3$ A dwarf.—$3H40$ Ancestry black.—$3H40a^2$ Killed by brother.—$3H42$ Ancestry fawn.—$3H42a^1$ Disappeared and record lost.—$3H50a^2$ Dark lilac.—$3H51b^3$ Minute white ventral spot.—$3H53a^1$ Mangy.—$3H53c^4$ Belly pale but not white.—$3H59a^1$ Very dark chocolate. Eyes black.—$3H59b^1$ Ears malformed.—$3H59b^3$ Very small white ventral streak.—$3H61a^2$, $3H61a^3$ Small white belly patch.—$3H64a^2$ Very small white ventral patch.—$3H64a^3$ Very pale lilac.—$3H65b^1$ Small ventral white streak.—$3H65b^2$ Small oblique ventral streak.—$3H65b^3$ Very small linear ventral streak.—$3H65c^3$ Fairly large ventral patch.—$3H65c^4$ Small ventral patch.—$3H65c^5$ Very small ventral patch.—$3H66b^3$, $3H66b^4$, $3H66b^5$ Very dark lilac.—$3H66b^6$, $3H66b^7$ Very brown lilac.—$3H71a^1$, $3H71a^2$, $3H71a^3$, $3H71a^4$ Dwarfs.

4*H* *Matings.*

$4H1a^1$ Extremely good black above, dark slate colour below. Longer hairs at sides, and perineal patch, yellowish. Tail pigmented, hairs yellowish. Digits pink, with pale hairs. Hair on and behind ears brown. Whiskers black.—$4H1a^2$ Like $4H1a^1$, but the long hairs more nearly white and some whiskers white.—$4H1a^3$ Like $4H1a^1$, but more pink on feet.—$4H2a^2$ Good black above. Scattered hairs on sides and belly fawn. Digits pink.—$4H2a^3$ Like $4H2a^2$, but with less scattered fawn hair.—$4H2c^3$ Tip of tail pink.—$4H3b^1$ Fair black: little scattered very pale (? white) hair at sides. Perineal patch nearly white. Hair behind ears very pale yellow. Digits pink, hairs white. Tail pigmented, with white hairs. Many white whiskers.—$4H3b^2$ Better black than $3b^1$; no white whiskers.—$4H3b^3$ Good black. Scattered hairs yellower than in the other two: tail pink at end: toes much pinker than in $3b^1$ or $3b^2$.—$4H3b^5$ A very good black, with few scattered pale hairs. Some whiskers white. Digits pink with white hair. Tail pink except at base and tip.—$4H3b^6$ Like $4H3b^5$, but tail pigmented.—$4H4a^2$ Good black above. Tail pigmented. Digits pink, with white hairs. Some white whiskers. Ear hairs brown. Slaty grey below, with very small linear streak of nearly pure white (say 5 mm. × ½ mm.)

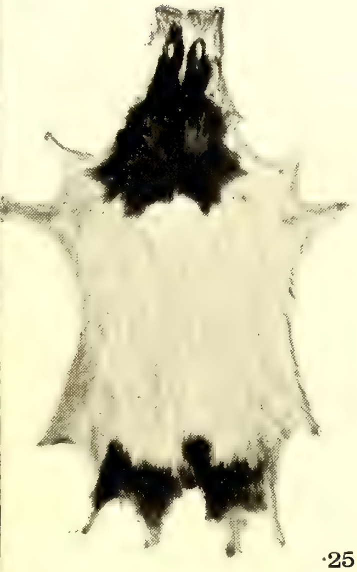
on belly.—$4H4a^3$ Very small white blaze on forehead: white patch on belly.—$4H4a^4$ Minute white streak on forehead: larger but still very small white streak on belly. Most of the whiskers white. Digits pink with white hairs. Tail mottled pink and white.—$4H4b^1$ Narrow white belly streak. Tail pink.—$4H4b^2$ White speck on forehead. Trace of white ventral streak. Tail pink.—$4H6a^2$ Blaze on forehead: large white patch on belly.—$4H6b^1$ Small white blaze on forehead, long patch on belly.—$4H6b^2$ No blaze on forehead: white patch on belly.—$4H6b^3$ Like $4H6b^2$, but a smaller mouse.—$4H6b^4$ White blaze on forehead: white belly patch.—$4H6b^5$ No blaze. White belly patch.—$4H6c^2$ Streak on belly.—$4H6c^3$ Blaze on forehead; streak on belly. —$4H7a^1$ Small white ventral patch, unusually far back.—$4H7a^2$ Very faint streak of scattered white hairs on belly.—$4H8$ Ancestry black.—$4H12a^1$, $4H12a^2$, $4H12a^3$ Dwarfs.—$4H12a^4$ No record.—$4H12a^5$, $4H12a^6$ Dwarfs.—$4H12a^7$ Omitted in Index, referred to in Mating Book, no record.—$4H13b^1$ Small blaze on forehead and *very* thin white band across haunches.—$4H13b^6$ Very small white blaze on forehead.—$4H14a^1$, $4H14a^2$, $4H14a^3$, $4H14a^4$, $4H14a^5$, $4H14a^6$ Belly uniformly white.—$4H15a^2$ Dark lilac.— $4H15a^3$, $4H15a^4$ Very pale lilac.—$4H15b^3$ Very pale lilac--pearl grey. Killed (tumour).—$4H16b^2$ White face.—$4H17a^3$ Bred with one brother before sexing as $5H1$.—$4H20b^3$ Small linear white streak on belly.—$4H20b^4$ Belly white.—$4H23a^1$, $4H23a^2$ Eaten.--$4H23b^2$ Small white belly streak.—$4H23c^1$, $4H23c^2$ *Very* little white on belly.—$4H24a^1$ A very good black above: scattered white hairs below. Tail pigmented. Fingers slightly pigmented, toes pink. Whiskers black. Perineal patch small. Faint trace of white line below appeared later.—$4H24a^2$ Small white line on back part of belly. Tail ringed pink and dark.—$4H24a^3$ Good black above: scattered greyish hairs sides and below. Digits pink. Whiskers black. Tail pigmented.—$4H24b^1$, $4H24b^2$, $4H24b^4$, $4H24b^5$, $4H24c^1$, $4H24c^2$ Dwarfs.—$4H25x$ This the only young of the first brood. Eaten. Next brood called $4H25a$.—$4H25a^1$ A good black. Very little scattered pale hair. Digits largely pigmented. Tail pigmented. Whiskers black.—$4H25a^2$ A good black. Very like $4H25a^1$ but digits pinker. —$4H26a^2$ Much scattered hair (nearly white) at sides. Some white whiskers. Fingers and all feet pink. Tail pigmented.—$4H26a^3$ Face and most of belly pure white.—$4H26a^5$ Few scattered white hairs close together on belly.—$4H27$ Ancestry black. Father uncertain.—$4H27c^1$ Dwarf. —$4H28$ Ancestry black. Father uncertain.—$4H28a^1$, $4H28a^2$ Linear white belly streak.—$4H29$ Father's ancestry fawn, mother's ancestry black.—$4H30a^7$, $4H30a^8$ Escaped while brood was being sexed.—$4H33a^2$ Much scattered pale hair ventrally. Tail partly pink.—$4H36a^4$ Mangy.—$4H38$, $4H39$, $4H40$ Ancestry black.—$4H40a^1$ Very pale. Face nearly white.—$4H41$, $4H45$ Ancestry black.—$4H45b^1$ Very light *f.*—$4H45b^2$, $4H45b^5$ Dark *f.*—$4H46b^4$ The palest "coloured" mouse yet seen.—$4H49c^2$ Killed by brothers.—$4H51a^2$ Very small ventral streak.—$4H51a^5$ Very small linear white ventral streak.—$4H51b^4$ Perhaps a trace of ventral white.—$4H52a^3$ Very small ventral streak.—$4H58b^3$ Nearest intermediate between lilac and chocolate yet seen.—$4H63a^3$ Albino with traces of colour on shoulders.—$4H66a^1$ Trace of white *dorsal* streak.—$4H66a^2$ Merest trace of ventral white streak.—$4H66a^6$ Trace of blaze on forehead.—$4H66b^3$ Small white ventral patch.—$4H69$ Fawn ancestry.—$4H69b^1$ Small white patch.—$4H69b^2$ Belly white.—$4H71$ Ancestry black.—$4H75a^2$ Record lost.—$4H84b^5$ A few white hairs on belly.

$5H$ *Matings.*

$5H1a^5$ Escaped.—$5H3x$ Eaten soon after birth.—$5H8a^1$ Very small white ventral streak.—$5H8a^3$ Very pale lilac.—$5H8a^4$ Small forehead blaze.—$5H8b^5$ Small white ventral patch.—$5H8c^1$ Dark lilac.—$5H8c^2$ Pale lilac.—$5H8c^3$, $5H8c^4$ Dark lilac.—$5H10a^3$ Very small ventral streak and forehead blaze.—$5H10a^5$ Very small forehead blaze. No ventral streak.—$5H11a^5$ Escaped.—$5H12a^3$ Dwarf.—$5H12b^1$ Very small white patch on forehead.--$5H14a^4$ Few white ventral hairs.—$5H15a^2$ Tip of tail pink.—$5H15a^3$ Tail lost.—$5H15b^2$ Small round belly patch.—$5H19b^4$ Few white hairs on forehead.—$5H19b^5$ Very few white hairs on forehead.—$5H29b^2$ Very small white patch.—$5H30b^3$ Extremely pale (nearly white). Ought to have a new colour category.—$5H30b^4$ Normal lilac.—$5H31b^3$, $5H31b^4$ Very small ventral group of white hairs.—$5H31b^6$ Few white hairs on belly.—$5H32b^1$ Poor black, but no white patch ventrally: small white patch on forehead.—$5H32b^4$ Scattered white ventral hairs.—$5H35a^1$ Escaped.—$5H37a^1$ Belly very pale but *not* white.—$5H37b^3$ Pale.—$5H40a^1$ Belly pale but not white.—$5H40a^5$ Escaped.—$5H40b^1$ Trace of white ventral group of hairs.—$5H52a^3$ Escaped.—$5H54a^1$ Very pale.—$5H54b^1$ Very small ventral streak.—$5H62a^1$ A faint streak of white on belly.—$5H82a^4$, $5H82a^5$, $5H82a^6$ A few white hairs on belly.

$6H$ *Matings.*

$6H1a^7$ White blaze on nose.—$6H7b^6$ A small patch of white hairs on belly.—$6H11a^2$ Small white patch on back.—$6H15c^5$, $6H17c^2$, $6H17c^6$, $6H18a^6$, $6H25a^1$ A few white hairs on belly.—$6H27$ Fawn ancestry.—$6H28$ Black ancestry.—$6H28b^3$ A few white hairs on belly.—$6H28c^6$ No record except skin.—$6H29$, $6H30$ Black ancestry.—$6H33a^3$, $6H34a^1$, $6H34a^2$, $6H35a^3$, $6H35a^4$, $6H35a^5$ Very pale.



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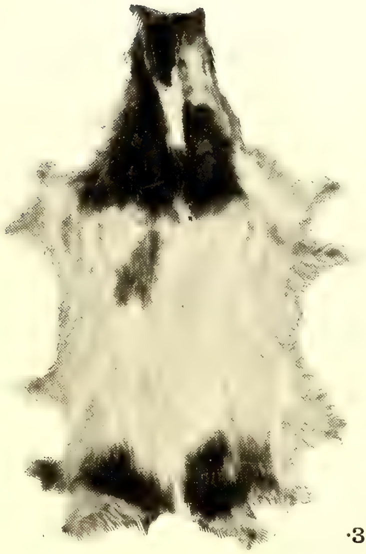

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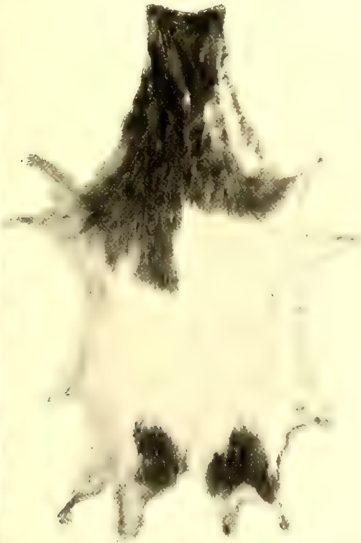

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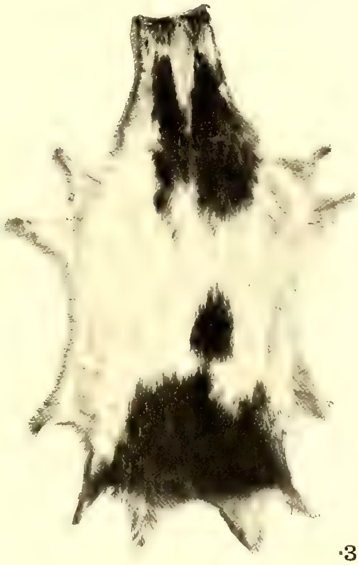

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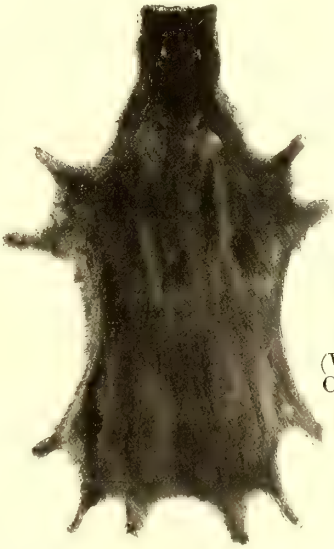

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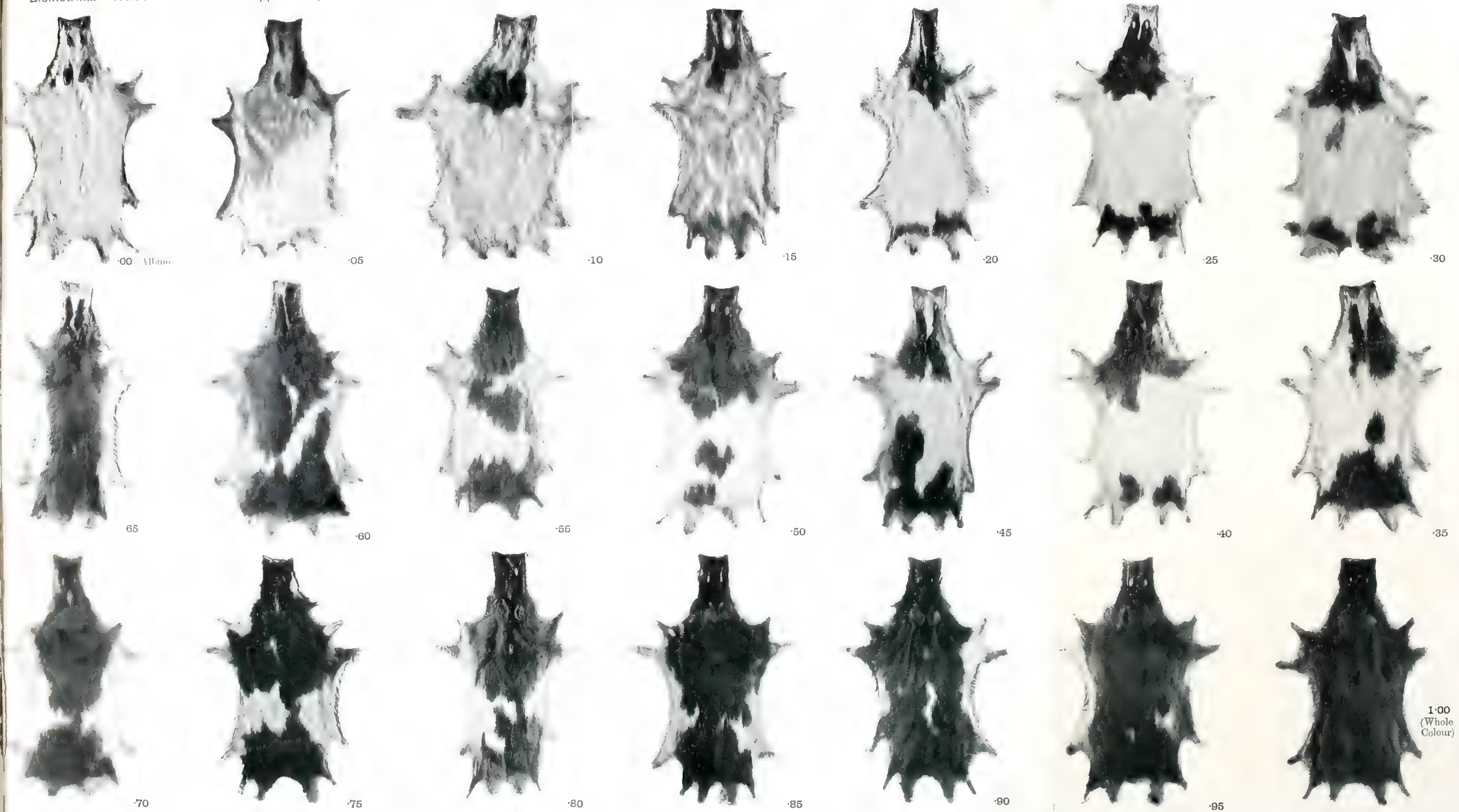
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The actual photographs are somewhat deceptive, as shadow or roughness on the actual skin gives here and there a dark appearance which is not due to the presence of pigment. The series, however, forms an approximate scale sufficient for practical purposes.

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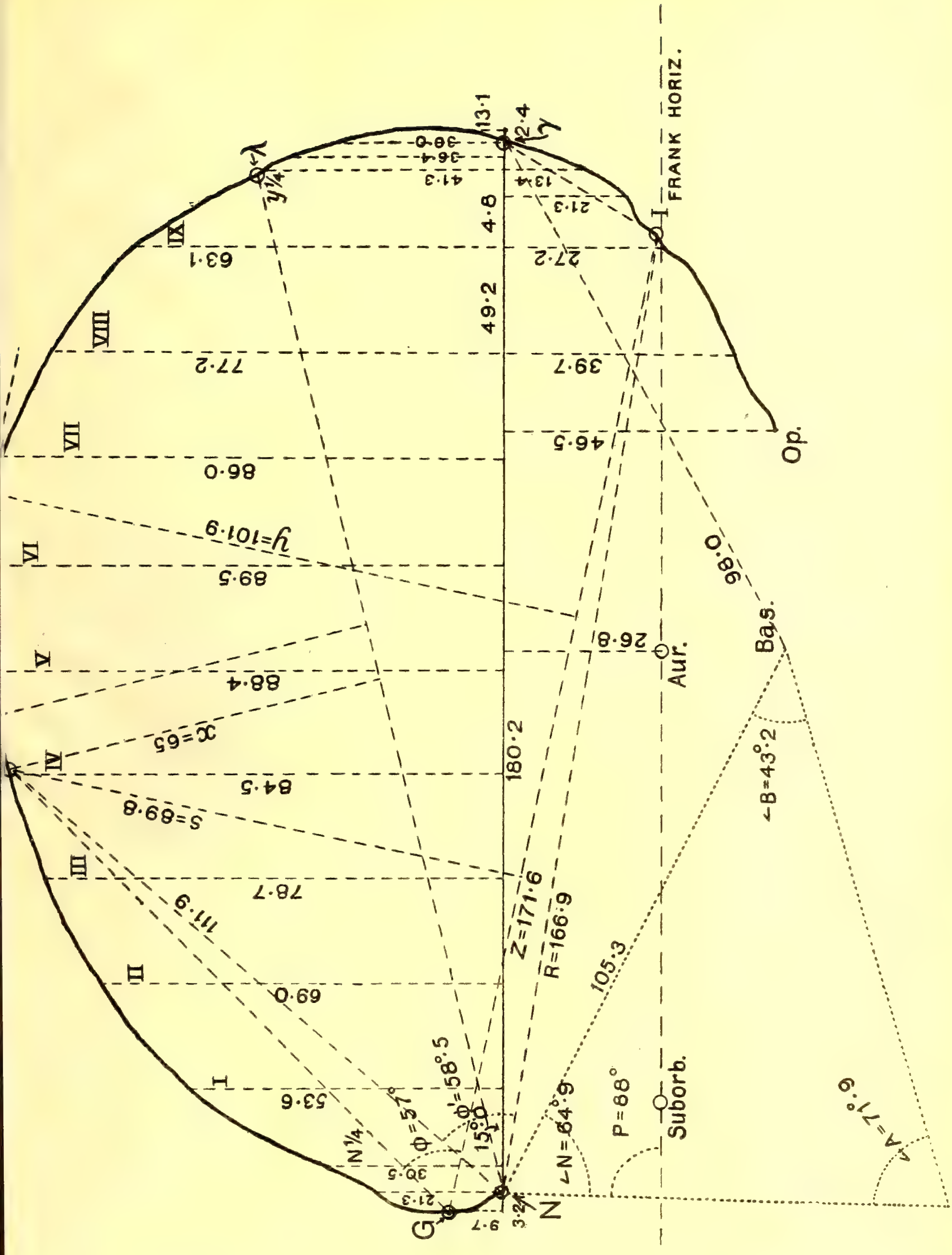
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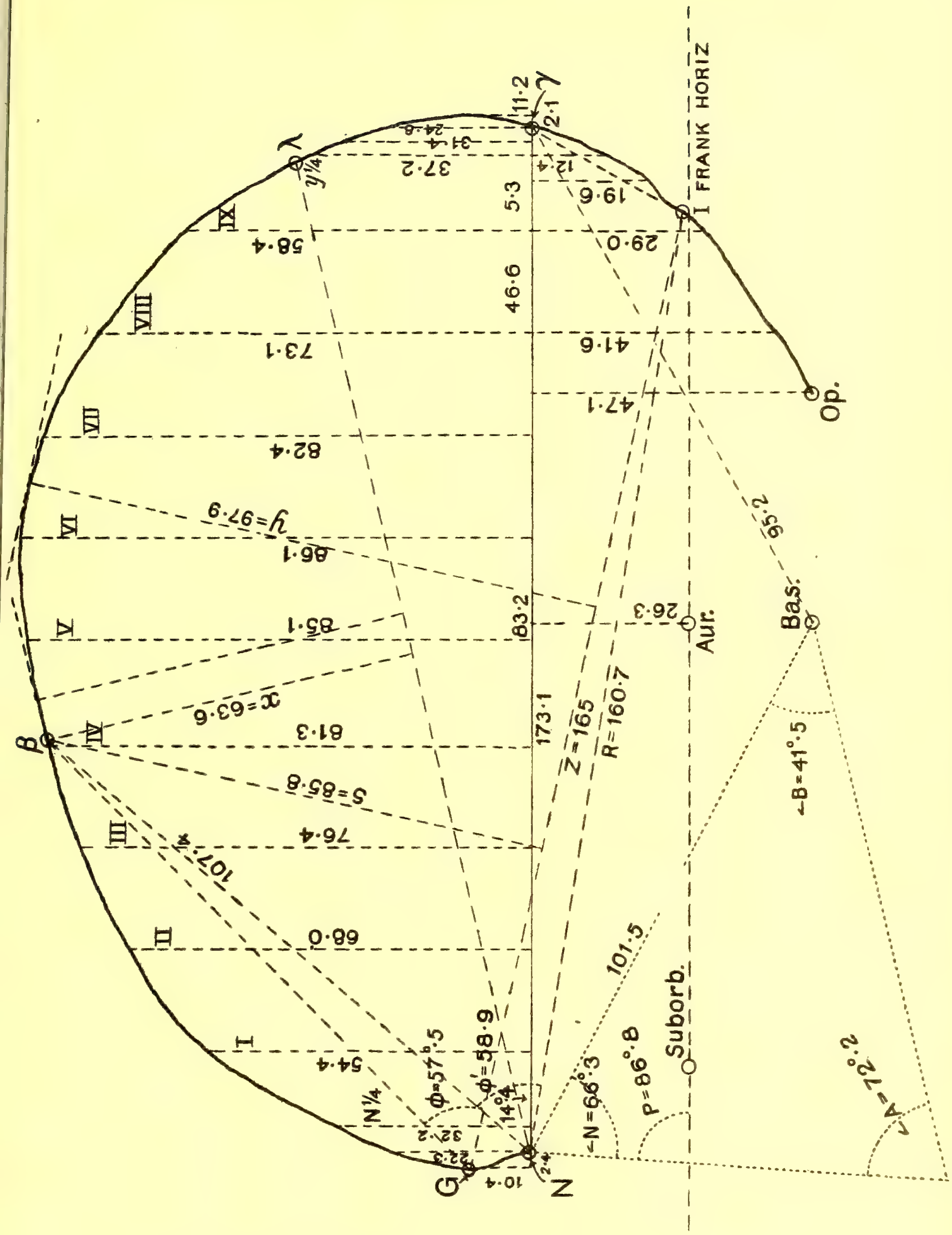
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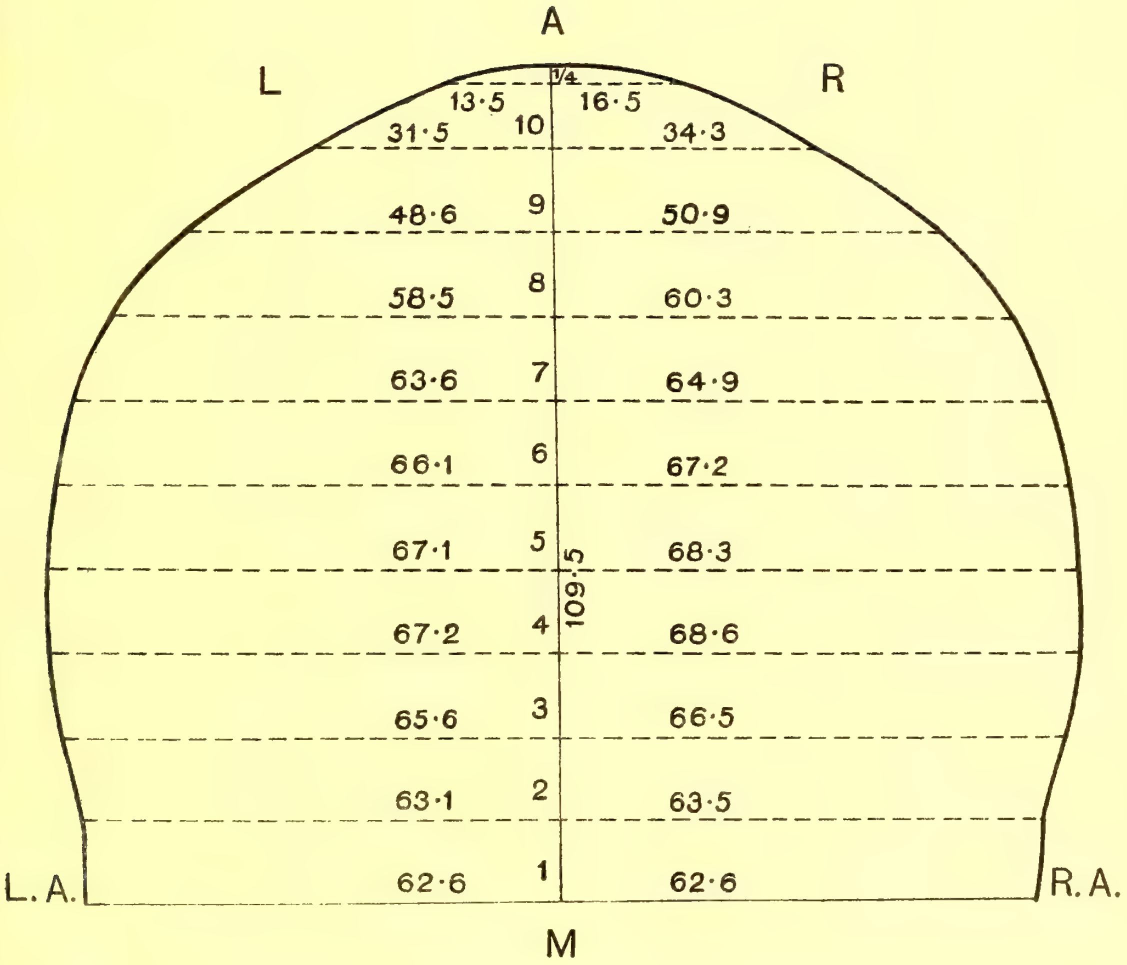
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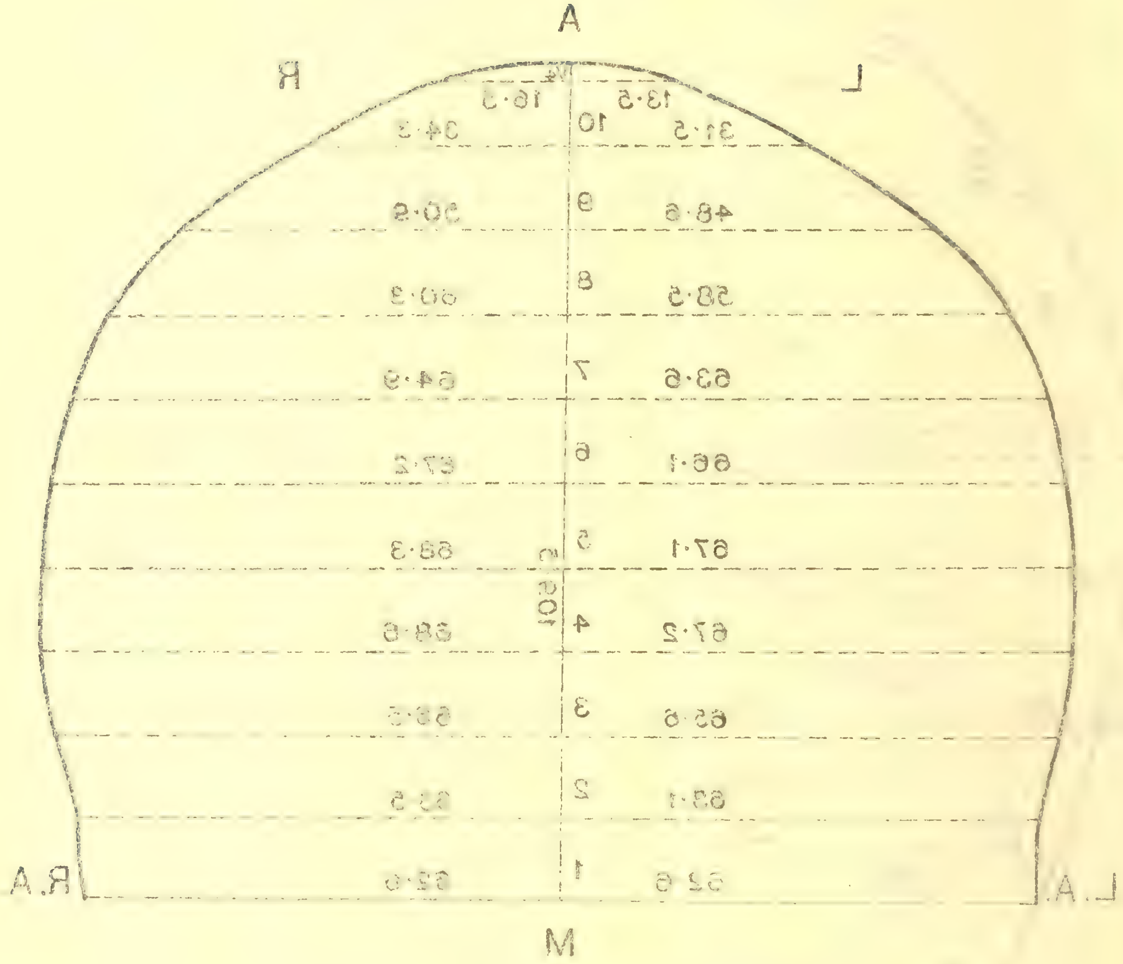
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Fig. III. Moriori Type Vertical Contour. ♂. (33 skulls.)

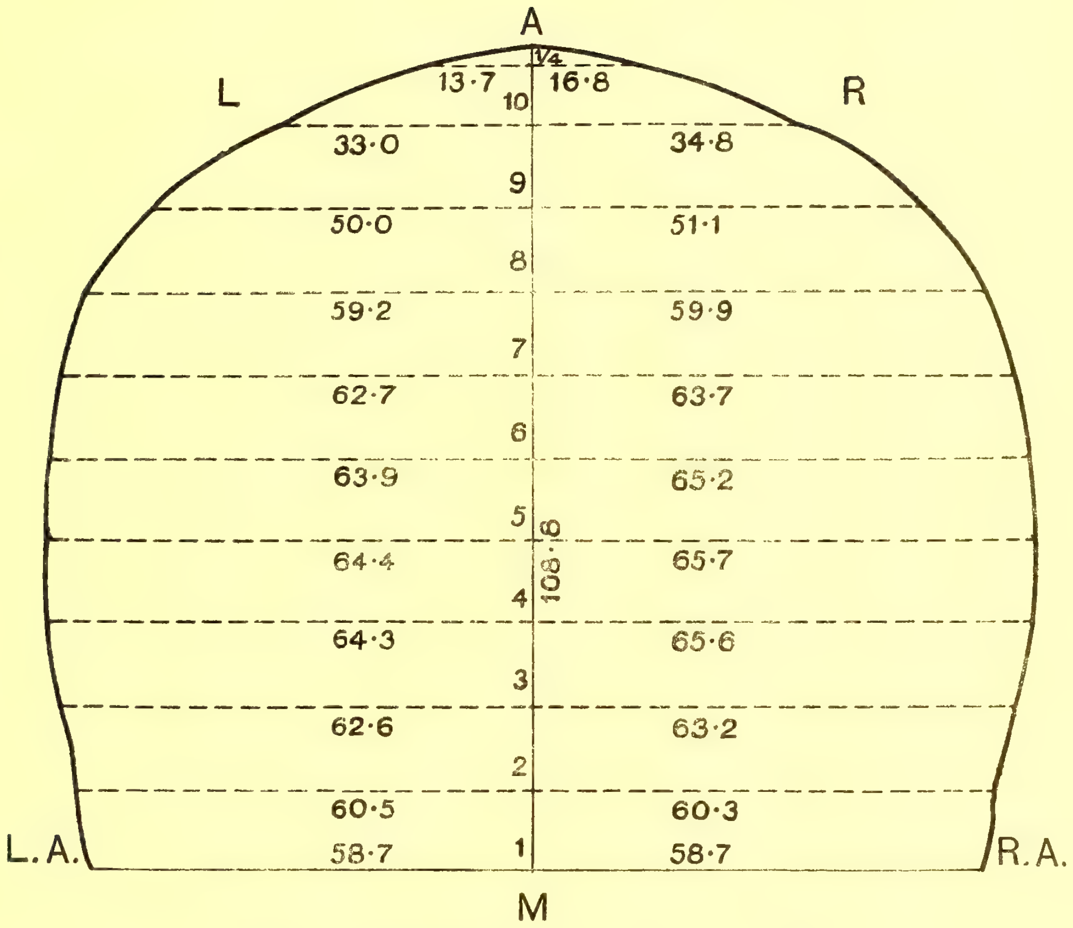
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Fig. IV. Moriori Type Vertical Contour. ♀. (21 skulls.)

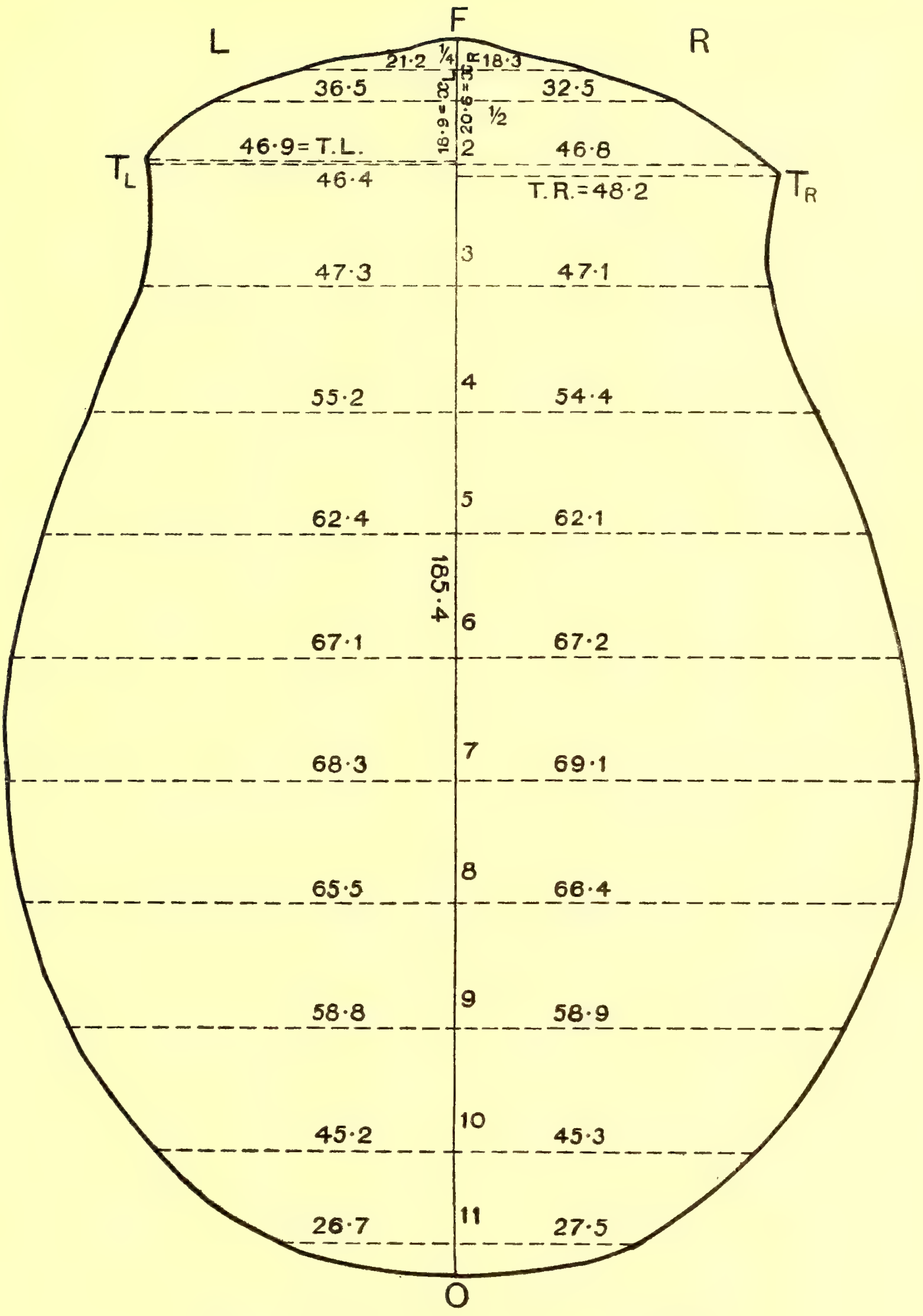
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FIG. V. Moriori Type Horizontal Contour. ♂. (33 skulls.)

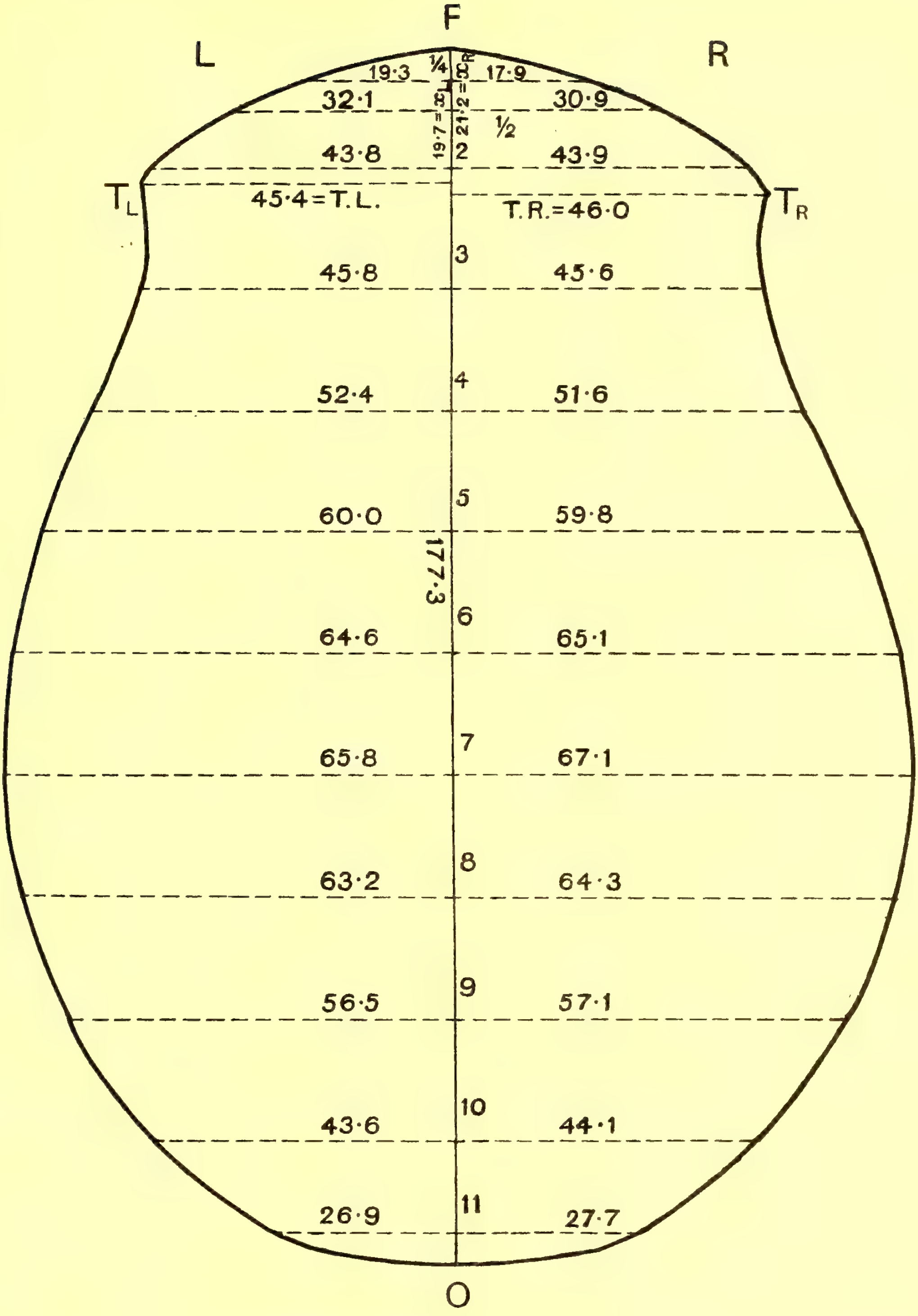
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Fig. VI. Moriori Type Horizontal Contour. ♀. (21 skulls.)

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VOLUME XI MAY, 1916 No. 3

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ON THE GENERAL THEORY OF MULTIPLE CONTINGENCY WITH SPECIAL REFERENCE TO PARTIAL CONTINGENCY.

BY KARL PEARSON, F.R.S.

(1) Let there be l variates or characteristics $A, B, C, \dots L$, each of these variates or characteristics being subdivided into categories $A_1, A_2, \dots A_\alpha, B_1, B_2, \dots B_\beta, C_1, C_2, \dots C_\gamma, \dots L_1, L_2, \dots L_\lambda$, where $\alpha, \beta, \gamma, \dots \lambda$ are arbitrary numbers. Then if N be the total population, and $n_{a_1}, n_{a_2}, \dots n_{a_\alpha}$ the number of individuals in the A-categories; $n_{b_1}, n_{b_2}, \dots n_{b_\beta}$ those in the B-categories and so on, we have relations

$$S_1^\alpha (n_{a_u}) = S_1^\beta (n_{b_v}) = S_1^\gamma (n_{c_w}) = \dots = N.$$

Further, if there be no relationship whatever between the variates or characteristics, we should anticipate that the frequency of the group $A_u, B_v, C_w, \dots L_\psi$ in a sample of M would *on the average* be

$$M \cdot \frac{n_{a_u}}{N} \cdot \frac{n_{b_v}}{N} \cdot \frac{n_{c_w}}{N} \cdots \frac{n_{l_\psi}}{N}.$$

Actually we find in the sample M the number $m_{uvw\dots\psi}$, and the problem arises whether the system represented by $m_{uvw\dots\psi}$ is so improbable that in the selected population M the characteristics $A, B, C, \dots L$ cannot be considered independent, i.e. M is really not a random sample from the supposed population N. Clearly the answer to this problem has already been given. We have to find the value of χ^2:

$$= S_{uvw\dots\psi} \left[\frac{\left(m_{uvw\dots\psi} - M \cdot \frac{n_{a_u}}{N} \cdot \frac{n_{b_v}}{N} \cdots \frac{n_{l_\psi}}{N} \right)^2}{M \cdot \frac{n_{a_u}}{N} \cdot \frac{n_{b_v}}{N} \cdots \frac{n_{l_\psi}}{N}} \right] \quad \dots\dots\dots\dots\dots\dots \text{(i)},$$

and apply the tables for "goodness of fit." Of course in many cases the sampled population is not known and accordingly we can only put for $\frac{n_{a_u}}{N}, \frac{n_{b_v}}{N}, \dots \frac{n_{l_\psi}}{N}$ the values given by the sample itself, i.e. $\frac{m_{a_u}}{M}, \frac{m_{b_v}}{M}, \dots \frac{m_{l_\psi}}{M}$, and test from this substitution the degree of divergence from independence. If we take the mean value of χ^2, i.e. $\phi^2 = \chi^2/M$, ϕ^2 is termed the mean square contingency, and $C_2 = \sqrt{\phi^2/(1+\phi^2)}$

gives a measure of the divergence from independence*. This is a multiple contingency coefficient.

Another case not infrequently arises; the population N has the characteristics $A, B, C, \ldots$ not independent, but related; the cell $uvw \ldots \psi$ contains $n_{uvw \ldots \psi}$ and the question arises how far it is safe to consider the population M as a sample of this population. In this case†

$$\chi^2 = S_{uvw \ldots \psi} \left[\frac{m_{uvw \ldots \psi} - \frac{M}{N} n_{uvw \ldots \psi}}{\frac{M}{N} n_{uvw \ldots \psi}} \right] \quad \text{..........................(ii)}.$$

In both these cases we have the relation

$$S_{uvw \ldots \psi} (m_{uvw \ldots \psi}) = M \quad \text{..............................(iii)},$$

and accordingly the number of cell frequencies is one more than the number of independent variates. Thus in using the tables‡ of "goodness of fit" the n' of the argument is $\alpha\beta\gamma \ldots \lambda$, but the value of P, the probability, has actually been determined from $n' - 1$.

(2) Now there are a number of cases in which not only do the cell-contents of the sample obey the linear relation (iii), but also other linear relations are imposed on the cell-contents. In the most general case we can suppose q linear relations between the cell-contents $m_{uvw \ldots \psi}$, and obtain the probability P corresponding to a value of χ^2 limited by these q relations. The theory of sampling, when such conditions are introduced, I term the theory of partial contingency. The reason for this terminology will be clearer as we develop the theory.

As far as we are concerned at present, it is of no importance whether we are dealing with one or other of our two cases, i.e. whether we are questioning the possibility of our material being a sample from a population with independent $A, B, C, \ldots L$ characteristics, i.e. determining a coefficient of mean squared contingency, or are investigating the possibility of its being a probable sample from a population with any associations between these characteristics. We can accordingly write χ^2 in the form

$$\chi^2 = S_{uvw \ldots \psi} \left\{ \frac{(m_{uvw \ldots \psi} - \overline{m}_{uvw \ldots \psi})^2}{\overline{m}_{uvw \ldots \psi}} \right\} \quad \text{.....................(iv)},$$

or for convenience we may even drop the descriptive subscripts and, numbering the cells in some sequence 1, 2, 3, ... s, ... $(\alpha\beta\gamma \ldots \lambda)$, write

$$\chi^2 = S_s \frac{(m_s - \overline{m}_s)^2}{\overline{m}_s} \quad \text{.................................(v)}.$$

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* If the characteristics may be assumed to be continuous variates, certain corrections for units of grouping can be made. There is also a correction due to the necessarily positive value of ϕ^2. These corrections, which have been for some time in use, will be considered elsewhere.

† We must in this case of course actually know the value of $n_{uvw \ldots \psi}$, it cannot be judged from the sample.

‡ For discussion of the deduction of P from χ^2: see *Phil. Mag.* Vol. L. p. 157 (July 1900); and for Tables: see *Tables for Statisticians and Biometricians*, Table XII (Cambridge University Press).

Here $\overline{m}_{uvw\ldots\psi} = \overline{m}_s$ will be either

$$M \cdot \frac{n_{a_u}}{N} \cdot \frac{n_{b_v}}{N} \cdots \frac{n_{l_\psi}}{N} \quad \text{or} \quad \frac{M}{N} n_{uvw\ldots\psi},$$

as the case may be.

Now we shall find it convenient to write

$$X_s = (m_s - \overline{m}_s)/\sqrt{\overline{m}_s} \quad \text{.............................(vi)},$$

and thus

$$\chi^2 = S_s (X_s^2) \quad \text{.................................(vii)}.$$

Further, if our q linear equations be of type

$$h_{t1} m_1 + h_{t2} m_2 + \ldots + h_{ts} m_s + \ldots = H_t \quad \text{..................(viii)},$$

where h and H are constants and t takes every value from 1 to q, we can write our conditions in the form

$$\left.\begin{aligned} &k_{t1} X_1 + k_{t2} X_2 + \ldots + k_{ts} X_s + \ldots = K_t \\ \text{where}\quad &k_{ts} = \frac{h_{ts}\sqrt{\overline{m}_s}}{\sqrt{h^2_{t1}\overline{m}_1 + h^2_{t2}\overline{m}_2 + \ldots + h^2_{ts}\overline{m}_s + \ldots}} \\ \text{and}\quad &K_t = \frac{H_t - h_{t1}\overline{m}_1 - h_{t2}\overline{m}_2 - \ldots - h_{ts}\overline{m}_s - \ldots}{\sqrt{h^2_{t1}\overline{m}_1 + h^2_{t2}\overline{m}_2 + \ldots + h^2_{ts}\overline{m}_s + \ldots}} \end{aligned}\right\} \text{...............(ix)}.$$

We shall speak of the first of (ix) as the *prepared condition*. Clearly it corresponds to a plane in n-dimensional space in which the constants $k_{t1}, k_{t2}, \ldots k_{ts}, \ldots$ are the direction-cosines and K_t the perpendicular from the origin on the plane. It is convenient to use the notation

$$k_{t1} k_{t'1} + k_{t2} k_{t'2} + \ldots + k_{ts} k_{t's} + \ldots = \cos(tt') \quad \text{..............(x)},$$

for (tt') is now the angle between the tth and t'th planes.

Assuming that the frequency surface with which we have to deal may be taken as

$$z = z_0 e^{-\frac{1}{2}\chi^2} \quad \text{..................................(xi)},$$

we may suppose before applying equations of condition (ix) that $K_1, K_2, \ldots K_t, \ldots K_q$ are variates and that we eliminate $X_1, X_2, \ldots X_q$, expressing our χ^2 in terms of $K_1, K_2, \ldots K_q, X_{q+1}, X_{q+2}, \ldots X_n$. We shall have then

$$z = z_0 \text{ expt.} - \tfrac{1}{2} \text{(quadratic function of these } n \text{ new variables)}.$$

We now proceed to put $K_1, K_2, \ldots K_q$ constant, but leave the other $n - q$ quantities to vary; we are therefore seeking the value of χ^2 for certain variates constant. This is the essence of partial contingency, and the analogy in contingency to partial correlation.

(3) As a rule in partial contingency we do not seek to discuss χ^2 when single cells of our multiple contingency solid are constant in frequency, although our theory covers that case. What we require usually is the value of χ^2 when we make the contents of certain marginal total cells constant. For example, let us consider

a population of uniform sex and let there be three characteristics: (i) frequency of age groups, (ii) frequency of occupational categories and (iii) frequency of survival and of death, the latter classified by various special disease classes—the whole, say, representing the returns for one year of a large area or country. We then require to determine, whether the like contingency solid for a sub-district, or for another population entirely, may be considered as significantly different from the above general population, i.e. we require to find the probability of its being a random sample of this general population. Now we may do this in the most universal manner, by assuming that not only survivals and deaths, but that age groups and occupations all have frequencies, which are random samples of the above general population. But this is not very often what we require; we admit that the age distribution is differentiated, we admit that the occupational frequencies are peculiar to the locality, and we ask whether, notwithstanding these differences, the death distribution is to be considered as a random sample.

In other words, we do not only fix the size of our sample M; we fix all one face—that of age groups and occupational categories of our contingency solid—and ask what is the distribution of samples of M taken from this solid, subject to the linear conditions that the totals of age-occupational categories are constant. For example, if A be age and B occupation, we make $n_{a_u b_v}$ constant for all values of u and v, but

$$n_{a_u b_v} = n_{a_u b_v c_1} + n_{a_u b_v c_2} + n_{a_u b_v c_3} + \cdots,$$

where $n_{a_u b_v c_s}$ is the frequency of the uvs cell, c denoting the category C of type of death and survival.

Now clearly in making this investigation we shall be studying the mean square contingency and the resulting probability of a *partial* sample—a sample of survival and death-type in a population of constant age groups and occupational classes. Again, we might treat a population as a sample with only constant age groups or only constant occupational frequencies and again investigate its probability as a sample with regard to deaths and occupations or with regard to deaths and age groups respectively. These would be partial contingencies of the order α or β, while the previous partial contingency was of the order $\alpha + \beta$, α being the number of categories in A (i.e. age groups) and β being the number in B (i.e. mortality and survival classes).

(4) Now the value of χ^2 given above, and of the frequency surface (xi), was discussed by me in the year 1900* from the general normal frequency surface by evaluation of determinants. The demonstration depends on two hypotheses:

(i) The approach of the binomial $M(p+q)^n$ to the normal curve

$$y = \frac{M}{\sqrt{2\pi}\sqrt{npq}} e^{-\frac{x^2}{2npq}}.$$

We know that this is true if n be considerable and neither p nor q very small.

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* *Phil. Mag.* Vol. L. p. 157, 1900. I have recently given a more elementary proof with the probable error of P: see *Phil. Mag.* April, 1916.

The hypothesis is justified therefore if no cell be taken so small that its contents are very small compared with the size of the sample.

(ii) That the sampling takes place out of a population indefinitely greater than the sample. If this be not true, then the distribution of frequency of any given cell for a series of samples follows not a binomial but a hypergeometrical series. The necessary modifications in the formulae are not very substantial and have been discussed elsewhere*.

Supposing the above two conditions to be fulfilled, then in true random samples the mean of a cell frequency will be

$$\overline{m}_s = M \frac{n_s}{N} \quad \text{...(xii)},$$

where M is size of sample, n_s the contents of the sth cell in the sampled population N, and condition (i) amounts to saying that no cell is to be chosen so that n_s/N is indefinitely small.

Further, the frequency of the sth cell will follow the binomial

$$\left\{\frac{n_s}{N} + \left(1 - \frac{n_s}{N}\right)\right\}^M,$$

and thus have a standard deviation given by

$$\sigma^2_{n_s} = M \frac{n_s}{N}\left(1 - \frac{n_s}{N}\right) \quad \text{...............................(xiii)}.$$

Lastly the correlation $r_{ss'}$ between deviations in the sth and s'th cells is given by

$$\sigma_{n_s}\sigma_{n_{s'}}r_{ss'} = -M \frac{n_s}{N}\frac{n_s}{N} \quad \text{.............................(xiv)}.$$

(5) The following deduction of the value of χ^2 is a variant from my *Phil. Mag.* proof. I owe the suggestion of it to Mr H. E. Soper, although I have deviated somewhat from his track. Let an indefinitely large population N consist of the classes $C_0, C_1, \ldots C_l$ in the quantities $n_0, n_1, n_2, \ldots n_l$ respectively. Then $p_s = n_s/N =$ chance of drawing a member of the class C_s, and the standard deviation of the distribution of frequency in samples of M drawn from the population will in this class C_s be as above

$$\sigma_s = \sqrt{Mp_s(1 - p_s)} \quad \text{...........................(xiii)}^{\text{bis}}.$$

Further, the mean of samples for this class will be Mp_s by (xii).

In the next place the correlation between deviations from the means in classes C_s and C_t will be in our present notation

$$\sigma_s\sigma_t r_{ss'} = -Mn_sn_{s'}/N^2 \quad \text{...........................(xiv)}^{\text{bis}},$$

or by (xiii)$^{\text{bis}}$

$$r_{ss'} = -\frac{\sqrt{p_sp_{s'}}}{\sqrt{1-p_s}\sqrt{1-p_{s'}}} \quad \text{............................(xv)}.$$

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* *Phil. Mag.* p. 239, 1899 and *Biometrika*, Vol. v. p. 174.

The distribution of frequency of the different classes in these samples of M will be given by the terms of the multinomial

$$(p_0C_0 + p_1C_1 + p_2C_2 + \dots + p_lC_l)^M \quad\dots\dots\dots\dots\dots\dots\text{(xvi)},$$

the general term being

$$\frac{M!}{u_0!\, u_1!\, u_2!\, \dots\, u_l!}\, p_0^{u_0} p_1^{u_1} p_2^{u_2} \dots p_l^{u_l}\, C_0^{u_0} C_1^{u_1} \dots C_l^{u_l} \quad\dots\dots\dots\text{(xvii)},$$

where C_0, C_1, ... C_l are only logical symbols to denote that this general term is the frequency of the group, where the class C_s occurs u_s times in the sample. Clearly $u_0 + u_1 + \dots + u_l = M$.

But (xvi) may be put into the form of a binomial, it is

$$= \underset{m=0}{\overset{m=M}{S}} \frac{M!}{m!\,(M-m)!} (p_0C_0)^{M-m} (p_1C_1 + p_2C_2 + \dots + p_lC_l)^m \dots\text{(xviii)}.$$

Let $(p_1 + p_2 + \dots + p_l) = \lambda$, then the mth term of the above series may be read as

$$\frac{M!}{m!\,(M-m)!} (p_0C_0)^{M-m} \lambda^m (p_1'C_1 + p_2'C_2 + \dots + p_l'C_l)^m \dots\dots\text{(xix)},$$

where

$$p_1' + p_2' + \dots + p_l' = 1,$$

and

$$\frac{p_1}{p_1'} = \frac{p_2}{p_2'} = \frac{p_3}{p_3'} = \dots = \frac{p_l}{p_l'} = \lambda = 1 - p_0 \quad\dots\dots\dots\dots\dots\dots\text{(xx)}.$$

Now it is clear that the factor

$$(p_1'C_1 + p_2'C_2 + \dots + p_l'C_l)^m$$

gives the frequency distribution of samples of m drawn from a population of indefinitely large size of which the proportions of the classes C_1, C_2, ... C_l are p_1', p_2', ... p_l' and in which no class C_0 occurs. But by (xx) these proportions are the same as in the original population which contains C_0.

Hence if we take samples of M from an indefinitely large population with classes C_0, C_1, C_2, ... C_l, those that contain m of the classes C_1, C_2, ... C_l will be distributed in the same proportions as if we had extracted m from an indefinitely large population consisting only of those l classes in the same proportions.

Now thus far the nature of the class C_0 is at our choice. In the original population N it appears with the total frequency n_0. Let $n_0 + n' = N$, and suppose n_0 is indefinitely greater than n', then p_0 will be indefinitely greater than p_1, p_2, ... p_n. It follows that p_s if s be not zero is very small compared to unity, because

$$p_0 + p_1 + p_2 + \dots + p_n = 1.$$

Hence in such a system from (xiii)$^{\text{bis}}$, if s be not zero,

$$\sigma_s = \sqrt{Mp_s} \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\text{(xxi)},$$

and from (xv) to the same degree of approximation $r_{st} = 0$. That is to say, if p_0 be large in taking samples of size M from an indefinitely large population, there will be no correlation in deviations in the frequency of the classes C_1, ... C_l.

On the other hand r_{s0} is not zero, but equals by (xv)

$$-\sqrt{p_s p_0}/\sqrt{1-p_0} = -\sqrt{p_s}/\sqrt{1-p_0}$$

to the same degree of approximation. Hence by (xx)

$$r_{s0} = -\sqrt{p_s'} \quad \text{.................................(xxii).}$$

But if we form the partial correlation of deviations in classes C_s and C_t for constant frequency of number in C_0, we have

$$\begin{aligned} {}_0r_{st} &= (r_{st} - r_{s0}r_{t0})/\sqrt{(1-r_{s0}^2)(1-r_{t0}^2)} \\ &= (0 - \sqrt{p_s'}\sqrt{p_t'})/\sqrt{(1-p_s')(1-p_t')}, \end{aligned}$$

or

$${}_0r_{st} = -\frac{\sqrt{p_s' p_t'}}{\sqrt{1-p_s'}\sqrt{1-p_t'}} \quad \text{........................(xxiii),}$$

which agrees with (xv), the classes being now reduced by unity. Further, the reduced standard deviation must now be

$$\begin{aligned} {}_0\sigma_s &= \sigma_s\sqrt{1-r_{0s}^2} \\ &= \sigma_s\sqrt{1-p_s'} \\ &= \sqrt{Mp_s(1-p_s')} \\ &= \sqrt{M\lambda p_s'(1-p_s')} \quad \text{..........................(xxiv).} \end{aligned}$$

Now take the mean value ${}_0\bar{x}_s$ of x_s the frequency in class C_s for a constant x_0 or for constant frequency in class C_0; in our case, if the sample is to be m this will be $M - m$, we have

$$\begin{aligned} {}_0\bar{x}_s - \bar{x}_s &= r_{0s}\frac{\sigma_s}{\sigma_0}(x_0 - \bar{x}_0), \\ {}_0\bar{x}_s - Mp_s &= \frac{-\sqrt{p_s'}\sqrt{Mp_s}}{\sqrt{Mp_0(1-p_0)}}(M - m - Mp_0), \\ {}_0\bar{x}_s &= p_s\left\{M - \frac{1}{1-p_0}[M(1-p_0) - m]\right\} \end{aligned}$$

or

$${}_0\bar{x}_s = \frac{mp_s}{1-p_0} = mp_s' \quad \text{.....................................(xxv).}$$

The partial values ${}_0\bar{x}_s$, ${}_0r_{ts}$ of the means and correlations of classes for constant number in class C_0 are given by (xxiii) and (xxv), and are what we might anticipate. But (xxiv) should be $\sqrt{mp_s'(1-p_s')}$. It is accordingly needful to take

$$\lambda = m/M,$$

or

$$p_0 = 1 - m/M \quad \text{................................(xxvi).}$$

These results have been reached on the assumption that p_0 is very large as compared with $p_1, p_2, \ldots p_n$. It follows accordingly that the sample M must be large as compared with m, and further the sum of the classes $C_1, C_2, \ldots C_l$ must be to that of the class C_0 in the total population in the ratio of the partial sample m to the total sample M. Without this condition it is not possible to replace Mp_s by mp_s'. Assuming these conditions to be satisfied, then samples of the size m in classes $C_1, C_2, \ldots C_l$ picked out of very large samples of M will reproduce the same distribution of frequencies in those classes as samples of m picked out of an indefinitely large population with the same relative frequencies in those classes.

But in the case of samples of M, the deviations have their correlations zero for the classes $C_1, C_2, \ldots C_n$, or they will be approximately distributed by the product of their independent probabilities. The standard deviation being $\sqrt{mp_s'}$ and the mean mp_s', we see that the frequency distribution would really follow a Poisson's binomial limit, but as shown by L. Whitaker* this binomial limit is approximately Gaussian with fairly low values of mp_s'; see the Diagrams for $mp_s' = 10$ and $= 30$ in the plate of her memoir. We may accordingly therefore take the distribution of the frequency in the sth class or cell to be given by

$$z_s = \frac{1}{\sqrt{2\pi}\sqrt{\overline{m}_s}} e^{-\frac{1}{2}\frac{(m_s - \overline{m}_s)^2}{\overline{m}_s}} \quad \text{.........................(xxvii)},$$

and the general distribution to be

$$\left.\begin{aligned} z &= \left(\frac{1}{\sqrt{2\pi}}\right)^l \frac{1}{\sqrt{m^l p_1' p_2' \ldots p_l'}} e^{-\frac{1}{2}\chi^2} \\ \text{where} \quad \chi^2 &= S_s \left\{\frac{(m_s - \overline{m}_s)^2}{\overline{m}_s}\right\} \end{aligned}\right\} \quad \text{...............(xxviii)}.$$

Here, if the size of the sample only be fixed, we shall have

$$S(m_s) = m = mS(p_s') = S(\overline{m}_s),$$

or

$$S(m_s - \overline{m}_s) = 0.$$

If we take

$$X_s = \frac{m_s - \overline{m}_s}{\sqrt{\overline{m}_s}},$$

we have:

$$\chi^2 = X_1^2 + X_2^2 + \ldots + X_l^2 \quad \text{.........................(xxix)}$$

subject to the condition:

$$\sqrt{\overline{m}_1}X_1 + \sqrt{\overline{m}_2}X_2 + \ldots + \sqrt{\overline{m}_l}X_l = 0 \quad \text{..................(xxx)}.$$

It is clear that χ^2 equal to a constant gives a sphere in l-fold space, and that (xxx) is a plane passing through its centre, and therefore cutting the sphere in l-fold space in a sphere of the *same radius* in $(l-1)$-fold space. Hence if we desire to find the volume of the frequency surface (xxviii) which lies outside a value of $\chi = \chi_0$ subject to the condition (xxx), all we have to do is to transfer to polar coordinates and integrate the value of z for the $(l-1)$-fold surface beyond the value χ_0*. Accordingly

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* *Biometrika*, Vol. x. p. 36. If $\overline{m}_s = mp_s'$ be the mean, $\beta_1 = 1/\overline{m}_s = \beta_2 - 3$, so that $\beta_1 = \cdot 03$, $\beta_2 = 3\cdot 03$ already for $\overline{m}_s = 33$.

$$P = \frac{\int_{\chi_0}^{\infty} e^{-\frac{1}{2}\chi^2}\chi^{l-2}d\chi}{\int_0^{\infty} e^{-\frac{1}{2}\chi^2}\chi^{l-2}d\chi} \qquad \text{.........................(xxxi)}$$

is the chance of a sample occurring with as great or greater deviation as the χ_0 sample from the general population. This is the expression from which the Tables of "Goodness of Fit" were calculated, the arguments being χ_0^2 and l, i.e. the value of χ^2 for the sample and the total number of categories in the sample. Thus far there is only difference of method of deduction, not of results.

(6) We now propose to replace condition (xxx) by a series of q linear equations of form (viii). These in the case of sampling will, if the size of the sample be fixed, either directly or indirectly involve (xxx).

The type of these equations in their prepared form is

$$k_{t1}X_1 + k_{t2}X_2 + \dots + k_{ts}X_s + \dots = K_t.$$

Each such plane will intersect

$$\chi^2 = X_1^2 + X_2^2 + \dots + X_s^2 + \dots$$

in a sphere of lower order. For example, if there be n variates X, the first plane gives a sphere of the $(n-1)$th order, this will be intersected by the second plane in a sphere of the $(n-2)$th order, so that ultimately we find ourselves reduced to a sphere of the $(n-q)$th order, by the intersection of the qth plane. If K_1, K_2, ... K_q were all zero, the radius of the sphere of the $(n-q)$th order would be the same, i.e. χ^2 as the radius of the sphere of the nth order. But since these quantities are usually not zero we have to determine the radius of this sphere. The centre of this sphere must lie in every one of the q planes of the nth order, and accordingly on the plane of order $n-(q-1)$ in which they intersect. But the centre of the sphere of $n-q$ order is where the perpendicular, K, from the origin meets this plane of the $(n-\overline{q-1})$th order, and the radius χ' of the sphere of the $(n-q)$th order is given by $\chi'^2 = \chi^2 - K^2$. To determine χ' we must find P.

Now P will be the minimum distance from the origin to the plane of the $(n-\overline{q-1})$th order in which the q planes intersect. In other words to find P we must make

$$D^2 = X_1^2 + X_2^2 + \dots + X_s^2 + \dots + X_u^2$$

a minimum subject to the q conditions of type

$$k_{t1}X_1 + k_{t2}X_2 + \dots + k_{ts}X_s + \dots + k_{tu}X_u = K_t,$$

where

$$k_{t1}^2 + k_{t2}^2 + \dots + k_{ts}^2 + \dots + k_{tu}^2 = 1.$$

Using the method of indeterminate multipliers we find n equations of the form

$$X_s + \lambda_1 k_{1s} + \lambda_2 k_{2s} + \dots + \lambda_q k_{qs} = 0.$$

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* *Phil. Mag.* Vol. L. p. 158.

Multiply by X_s and add the series, and we find

$$-K^2 = \lambda_1 K_1 + \lambda_2 K_2 + \dots + \lambda_q K_q \quad \text{..............(xxxii).}$$

Multiply by k_{1s} and add the series, and we find, by aid of (x),

$$-K_1 = \lambda_1 + \lambda_2 \cos(12) + \lambda_3 \cos(13) + \dots + \lambda_q \cos(1q).$$

Similarly:

$$-K_2 = \lambda_1 \cos(21) + \lambda_2 + \lambda_3 \cos(23) + \dots + \lambda_q \cos(2q),$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$-K_t = \lambda_1 \cos(t1) + \lambda_2 \cos(t2) + \dots + \lambda_t + \dots + \lambda_q \cos(tq),$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$-K_q = \lambda_1 \cos(q1) + \lambda_2 \cos(q2) + \dots + \dots + \dots + \lambda_q.$$

These are q equations to find $\lambda_1, \lambda_2, \dots \lambda_q$, and we can then substitute in (xxxii) to find the required P^2.

Now consider the determinant

$$R = \left| \begin{array}{cccccc} 1, & K_1, & K_2, & K_3, & \dots & K_q \\ K_1, & 1, & \cos(12), & \cos(13), & \dots & \cos(1q) \\ K_2, & \cos(21), & 1, & \cos(23), & \dots & \cos(2q) \\ K_3, & \cos(31), & \cos(32), & 1, & \dots & \cos(3q) \\ \dots & \dots & \dots & \dots & \dots & \dots \\ K_q, & \cos(q1), & \cos(q2), & \cos(q3), & \dots & 1 \end{array} \right| \quad \text{...(xxxiii),}$$

and let us call the first row and the first column the 0 row and 0 column, then clearly

$$-\lambda_t = -R_{0t}/R_{00},$$

where R_{st} is the minor of the sth row and tth column, and

$$-K^2 = \frac{K_1 R_{01} + K_2 R_{02} + \dots + K_t R_{0t} + \dots + K_q R_{0q}}{R_{00}},$$

$$1 - K^2 = R/R_{00},$$

or

$$K^2 = 1 - R/R_{00} \quad \text{..........................(xxxiv).}$$

If we call Δ the minor R_{00}, we have:

$$R_{0t} = -K_1\Delta_{1t} - K_2\Delta_{2t} - \dots - K_t\Delta_{tt} - \dots.$$

Thus

$$K^2 = S\left(K_t{}^2 \frac{\Delta_{tt}}{\Delta}\right) + 2S\left(K_t K_{t'} \frac{\Delta_{tt'}}{\Delta}\right) \text{.................(xxxv).}$$

From this we deduce that the probability of a sample which gives $\chi^2 = \chi_0{}^2$ with q linear conditions must be obtained from

$$P = \frac{\int_{\chi_0{}^2 - K^2}^{\infty} e^{-\frac{1}{2}\chi^2} \chi^{n-q-1} d\chi}{\int_0^{\infty} e^{-\frac{1}{2}\chi^2} \chi^{n-q-1} d\chi} \quad \text{....................(xxxvi).}$$

We must therefore in order to find P enter the Tables of "Goodness of Fit" with $\chi^2 = \chi_0^2 - K^2$ and with $n' = n - q + 1$*.

The reader who is familiar with the theory of the multiple total and partial correlation coefficients will note how closely analogous the formulae (xxxiii) to (xxxv) are to results in that theory. The fundamental determinant of the correlation may be made to agree with Δ, if we merely write $r_{ss'} = \cos(ss')$. In fact both theories really reduce to the discussion of the formulae of spherical trigonometry in multiple space.

(7) A simple illustration of the above formulae may be taken from the case of the distribution of mortality in two districts, where the problem is to ascertain the probability that the difference of mortality observed allowing for the frequency of the age groups could be due to random sampling.

Let the population sampled be represented by

(α)

D_1	D_2	D_3	......	D_s	......	D_u	Δ
L_1	L_2	L_3	......	L_s	......	L_u	Λ
A_1	A_2	A_3	......	A_s	......	A_u	P

where D_s are the dead, L_s the living and A_s the exposed to risk in the sth age group, there being u such groups in the total population P, of whom in the given period Δ die and Λ survive.

Let the districts be represented by

(β)

d_1	d_2	d_3	......	d_s	......	d_u	δ
l_1	l_2	l_3	......	l_s	......	l_u	λ
a_1	a_2	a_3	......	a_s	......	a_u	p

and

(γ)

d_1'	d_2'	d_3'	......	d_s'	......	d_u'	δ'
l_1'	l_2'	l_3'	......	l_s'	......	l_u'	λ'
a_1'	a_2'	a_3'	......	a_s'	......	a_u'	p'

respectively. Then the problem is to ascertain the probability that the last two distributions could both be samples of the same first population.

The general formula has been given by me†; we have to evaluate

$$\chi^2 = S\left\{\frac{\left(\dfrac{f_t}{p} - \dfrac{f_t'}{p'}\right)^2}{\left(\dfrac{1}{p} + \dfrac{1}{p'}\right)\dfrac{F_t}{P}}\right\},$$

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* The tables are constructed for $n' - 1$ independent variables; in our case there are $n - q$ such variables, hence $n' - 1 = n - q$.

† *Biometrika*, Vol. VIII. p. 252.

where f_t must take the value of every frequency in the cells of (β), f_t' the corresponding cell frequency for (γ), and F_t that for (α). We have to seek for P under the argument $n' = 2u$ in our tables, if we have no restrictions on our variates. Actually we have such restrictions, for we are going to seek the *partial* χ^2 when we suppose the age groups in each sample to be *constant*. Now

$$d_s + l_s = a_s \quad \text{and} \quad d_s' + l_s' = a_s'.$$

Thus it follows that

$$\frac{d_s}{p} - \frac{d_s'}{p'} + \frac{l_s}{p} - \frac{l_s'}{p'} = \frac{a_s}{p} - \frac{a_s'}{p'} \quad \text{......................(xxxvii).}$$

We have accordingly u equations of condition or n' as argument will be reduced to $u + 1$. Now we take

$$X_s = \sqrt{\frac{pp'}{p+p'}}\, \frac{\left(\frac{d_s}{p} - \frac{d_s'}{p'}\right)}{\sqrt{D_s/P}},$$

$$Y_s = \sqrt{\frac{pp'}{p+p'}}\, \frac{\left(\frac{l_s}{p} - \frac{l_s'}{p'}\right)}{\sqrt{L_s/P}}.$$

Thus
$$\chi^2 = S_1^u (X_s^2) + S_1^u (Y_s^2),$$

with u conditions of form

$$\sqrt{D_s/P}\, X_s + \sqrt{L_s/P}\, Y_s = \sqrt{\frac{pp'}{p+p'}} \left(\frac{a_s}{p} - \frac{a_s'}{p'}\right),$$

or in the prepared form

$$\sqrt{\frac{D_s}{A_s}}\, X_s + \sqrt{\frac{L_s}{A_s}}\, Y_s = \sqrt{\frac{pp'}{p+p'}}\, \frac{\left(\frac{a_s}{p} - \frac{a_s'}{p'}\right)}{\sqrt{\frac{A_s}{P}}}.$$

Thus
$$K_s = \sqrt{\frac{pp'}{p+p'}} \left(\frac{a_s}{p} - \frac{a_s'}{p'}\right) \Big/ \sqrt{\frac{A_s}{P}}.$$

Further, all the cosines like $\cos(ss')$ are zero, for no equations of condition involve the same variates*. Thus

$$K^2 = K_1^2 + K_2^2 + \ldots + K_u^2$$
$$= \frac{pp'}{p+p'}\, S_1^u \frac{P}{A_s} \left(\frac{a_s}{p} - \frac{a_s'}{p'}\right)^2 \quad \text{..................(xxxviii).}$$

Accordingly

$$\chi_0^2 = \chi^2 - K^2 = \frac{pp'}{p+p'} \left\{ S_1^u \frac{P}{D_s} \left(\frac{d_s}{p} - \frac{d_s'}{p'}\right)^2 + S_1^u \frac{P}{L_s} \left(\frac{l_s}{p} - \frac{l_s'}{p'}\right)^2 - S_1^u \frac{P}{A_s} \left(\frac{a_s}{p} - \frac{a_s'}{p'}\right)^2 \right\}.$$

We shall now substitute from the relation (xxxvii), getting rid of $\frac{l_s}{p} - \frac{l_s'}{p'}$.

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* This will necessarily be true if our equations of condition refer to parallel rows or columns, not if they refer to certain rows *and* columns.

We find

$$\chi_0^2 = \frac{pp'}{p+p'}\left\{S_1^u \frac{PA_s}{D_sL_s}\left(\frac{d_s}{p} - \frac{d_s'}{p'}\right)^2 - \frac{2P}{L_s}\left(\frac{d_s}{p} - \frac{d_s'}{p'}\right)\left(\frac{a_s}{p} - \frac{a_s'}{p'}\right) + \frac{PD_s}{A_sL_s}\left(\frac{a_s}{p} - \frac{a_s'}{p'}\right)^2\right\}$$

$$= \frac{pp'}{p+p'} S_1^u \left\{\frac{PA_s}{D_sL_s}\left[\frac{d_s}{p} - \frac{d_s'}{p'} - \frac{D_s}{A_s}\left(\frac{a_s}{p} - \frac{a_s'}{p'}\right)\right]^2\right\}.$$

Let $\frac{D_s}{A_s} = p_s$, $\frac{L_s}{A_s} = q_s$, then we may write

$$\chi_0^2 = \frac{pp'}{p+p'}, S_1^u \left\{\frac{P}{A_s p_s q_s}\left(\frac{d_s - p_s a_s}{p} - \frac{d_s' - p_s a_s'}{p'}\right)^2\right\}.$$

Now if we know the population sampled, we have only to insert the values of P, A_s, and p_s, q_s to obtain the value of χ_0^2. But if we do not, and this is usually the case, then our problem is: Are the two districts samples of the same unknown general population? For example, we might enquire whether the death distributions in Bradford and Leeds were, correcting for age, significantly different. It might be supposed that it would be correct to give A_s/P, p_s and q_s, the values found for all England and Wales. But it may be doubted whether this would be satisfactory. The populations of Bradford and Leeds might fairly be considered as samples of a general population which is very far indeed from being that of all England. Accordingly it seems much more reasonable to suppose that they are samples of a population whose mortality characters will be best represented by the combination of those of the two districts themselves, or we take

$$p_s = \frac{d_s + d_s'}{a_s + a_s'}, \qquad \frac{A_s}{P} = \frac{a_s + a_s'}{p + p'}.$$

Substituting the first of these relations we find

$$\chi_0^2 = S_1^u \left(\frac{\frac{a_s}{p}\frac{a_s'}{p'}}{\frac{a_s + a_s'}{p + p'}\frac{A_s}{P}}\right)\left(\frac{a_s a_s'\left(\frac{d_s}{a_s} - \frac{d_s'}{a_s'}\right)^2}{(d_s + d_s')\left(1 - \frac{d_s + d}{a_s + a_s'}\right)}\right) \quad \text{......(xxxix).}$$

If we substitute the second relation, (xxxix) becomes

$$\chi_0^2 = S_1^u \left\{\frac{\frac{a_s}{p}\frac{a_s'}{p'}}{\left(\frac{a_s + a_s'}{p + p'}\right)^2}\frac{a_s a_s'\left(\frac{d_s}{a_s} - \frac{d_s'}{a_s'}\right)^2}{(d_s + d_s')\left(1 - \frac{d_s + d_s'}{a_s + a_s'}\right)}\right\} \quad \text{............(xl),}$$

and this, I take it, is the best measure of significance in the difference of the death distributions allowing for age groups in the two districts.

It is clear that the first factor will in many cases differ but little from unity. For we should anticipate that approximately

$$a_s = \frac{p}{P} A_s, \qquad a_s' = \frac{p'}{P} A_s.$$

Hence it would follow that

$$\frac{a_s}{p}\frac{a_s'}{p'} = \left(\frac{A_s}{P}\right)^2 = \left(\frac{a_s + a_s'}{p + p'}\right)^2,$$

or the first factor is approximately unity. The value of χ_0^2 then becomes

$$Q^2 = S_1^u \left\{ \frac{a_s a_s' \left(\frac{d_s}{a_s} - \frac{d_s'}{a_s'}\right)^2}{(d_s + d_s')\left(1 - \frac{d_s + d_s'}{a_s + a_s'}\right)} \right\} \quad \text{..................(xli)},$$

a quantity which I have shown elsewhere* has an important relation to the question of the significant difference of corrected death *rates*.

It is obvious that the present method is of wide generality and I propose to illustrate it later by considering whether the general death distributions of various national groups allowing for age distribution, occupation distribution and character of mortality are or are not significantly different.

Meanwhile (xxxix) provides us with a formula which enables us readily in the case of districts, special diseases or class groups to assert whether mortality experiences corrected for age distribution are or are not significantly different. As far as I can see (xxxix) and its extensions to occupation groups provide the proper means of ascertaining whether the populations at risk in various insurance offices or friendly societies are or are not materially different in character. It should be a guide to the actuary also as to which classes of the population it is most desirable to cater for.

I have to thank my friends Mr H. E. Soper and Mr A. W. Young for suggestions and help at several points. In the following paper numerical illustrations of (xl) are provided.

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* See *Biometrika*, Vol. XI. p. 164.

ON CRITERIA FOR THE EXISTENCE OF DIFFERENTIAL DEATHRATES.

By KARL PEARSON, F.R.S. and J. F. TOCHER, D.Sc.

(1) To determine whether the general deathrates or the deathrates for special diseases in two towns or in two classes of the general population are significantly different is a problem of very great importance. It is now generally recognised that crude deathrates are of little service for this purpose, the age distribution in the two districts or in the two classes of the population may be widely different, and the general deathrate or the special deathrate, i.e. the deathrate for a special disease, usually is a marked function of age*. The deathrate is therefore corrected by reduction to a "standard population." We ask what would be the deathrate in the given district or class if its age distribution were constituted in a given or "standard" manner. This deathrate is spoken of as the "corrected deathrate." Now we usually suppose the deathrate to be subject to "probable error." In other words in a population of size n, if p be the chance of a person dying in the year, q the chance of a person surviving, then the standard deviation of the number of deaths is $\sqrt{npq}$, or the probable error of the corresponding deathrate m is $\frac{\cdot 67449}{\sqrt{n}}\sqrt{pq}$. If there be a second town of deathrate m' and population n', and chances p' and q', we are tempted to compare without very full consideration $m' - m$ with $\cdot 67449\sqrt{\frac{pq}{n} + \frac{p'q'}{n'}}$ to obtain a measure of significant differentiation probably using a table of the probability integral. This method is for several reasons unreliable and fallacious. In the first place if it be applied to the crude deathrates, p and p' can hardly be applied to the individual, they are so markedly a function of age. In the next place for many diseases, or for many ages in a general deathrate, p and p' will be very small; accordingly there will be no approach to Gaussian distribution, but the binomials will approach Poisson's Exponential Limit, in which case the meaning of the standard deviation of a difference requires much further consideration and probabilities will not be given by a table of the probability integral.

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* It is also well known to be a marked function of class. This is customarily disregarded in a comparison of local deathrates. But to test real sanitary efficiency a standardised class population may be as important as a standardised age population.

But the method outlined above has this importance: it suggests that the deathrate obtained is only a "sample" deathrate, and subject to the variations of sampling; thus it forces the problem upon us in a very definite form: Can two populations dying in a known manner during a given period be considered as samples drawn at random from the same material? Here again the age distribution difficulty arises, for by hypothesis we admit the age distributions of our two samples are not the same. Let us fix our attention for a time on a fairly narrow age group, say that d_s deaths occur in an age group of size a_s in a certain population and that the chance of death in the age group is p_s and the chance of survival q_s in the population out of which the sample is supposed to be drawn. Then undoubtedly the standard deviation of samples would be $\sqrt{a_s p_s q_s}$, but as before the distribution would hardly be Gaussian. Now let us suppose the standard population, size A, to consist of the age groups $A_1, A_2, \dots A_s, \dots$, then the "corrected" deathrate M will be given by

$$M = S\left(\frac{d_s}{a_s}\frac{A_s}{A}\right) \quad \text{......................................(i)},$$

and if δ denote a variation due to random sampling, we shall have

$$\delta . M = S\left(\frac{\delta . d_s}{a_s}\frac{A_s}{A}\right).$$

Now speaking generally we do not "draw" our deaths in such a manner that their total remains constant. A shot so to speak fired at one age group is not to be supposed if it misses that group to have a chance of hitting a second age group. There will thus not be any of the usual *negative* correlation between a variation in d_s and one in $d_{s'}$. Of course epidemics which in a given period attacked individuals in certain age groups only might show a *positive* correlation between δd_s and $\delta d_{s'}$. But in general it will be sufficient to suppose the variations of the d_s's independent, and measured by the probability of death in each group. Thus we should have

$$\sigma_M{}^2 = S\left(\frac{\sigma_{d_s}{}^2}{a_s{}^2}\frac{A_s{}^2}{A^2}\right)$$

$$= S\left(\frac{p_s q_s}{a_s}\frac{A_s{}^2}{A^2}\right) \quad \text{..................................(ii)}.$$

Similarly if there be another population with age groups a'_s and corrected deathrate M', then

$$M' = S\left(\frac{d'_s}{a'_s}\frac{A_s}{A}\right),$$

and

$$\sigma_{M'}{}^2 = S\left(\frac{p_s q_s}{a'_s}\frac{A_s{}^2}{A^2}\right).$$

We do not write p'_s and q'_s in this population, because we are supposing both to be samples from one and the same population. Now clearly

$$M' - M = S\left\{\left(\frac{d'_s}{a'_s} - \frac{d_s}{a_s}\right)\frac{A_s}{A}\right\} \text{.............................(iii)},$$

and there will be no reason for supposing any correlation between d'_s and d_s. Thus it will follow that the standard deviation of $M' - M$ is $\sqrt{\sigma_M{}^2 + \sigma_{M'}{}^2}$. Hence:

$$\sigma^2{}_{M'-M} = S\left\{p_s q_s \left(\frac{1}{a_s} + \frac{1}{a'_s}\right)\frac{A_s{}^2}{A^2}\right\} \quad \text{........................(iv)}.$$

Now assuming p_s and q_s for the moment to be known, can we learn more from the relative values of $M' - M$ and $\sigma_{M'-M}$ than we thought possible from the distribution of the deaths in a single age group owing to the latter's non-Gaussian form?

It seems probable that we can for the following reasons. Let z be the sum of u variates $x_1 + x_2 + \ldots + x_u$, these variates following arbitrary laws of frequency and being in no way correlated together, i.e. z is to be found by taking a random selection of each of our u-variates and adding them together, then from

$$z = x_1 + x_2 + \ldots + x_u$$

we can find the moments of z. Obviously we can measure all variates from their means, and accordingly we find:

$$\begin{aligned}
{}_z\mu_2 &= S\,({}_x\mu_2) = u \times {}_x\bar{\mu}_2,\\
{}_z\mu_3 &= S\,({}_x\mu_3) = u \times {}_x\bar{\mu}_3,\\
{}_z\mu_4 &= S\,({}_x\mu_4) + 6S\,({}_x\mu_2 \cdot {}_x\mu'_2),\\
&= u \times {}_x\bar{\mu}_4 + 6\,\frac{u\,(u-1)}{2}\,({}_x\bar{\mu}_2)^2,
\end{aligned}$$

where ${}_x\bar{\mu}_2$, ${}_x\bar{\mu}_3$ and ${}_x\bar{\mu}_4$ denote the mean values of the moment coefficients for the various x-distributions. Accordingly if B_1 and B_2 be the β-coefficients for z:

$$B_1 = \frac{1}{u}\frac{{}_x\bar{\mu}_3{}^2}{{}_x\bar{\mu}_2{}^3} = \frac{1}{u}\tilde{\beta}_1, \quad \text{....................................(v)},$$

$$B_2 = \frac{1}{u}\frac{{}_x\bar{\mu}_4}{{}_x\bar{\mu}_2{}^2} + 3\left(1 - \frac{1}{u}\right),$$

$$B_2 - 3 = \frac{1}{u}(\tilde{\beta}_2 - 3) \quad \text{....................................(vi)}.$$

where $\tilde{\beta}_1$ and $\tilde{\beta}_2$ denote β-coefficients found from the mean moments. Now in the case of the Poisson's Exponential Limit to the Binomial we have*

$$\beta_1 = \beta_2 - 3 = \frac{1}{np} = \frac{1}{m} \quad \text{..........................(vii)},$$

where m is the mean number of deaths in the age groups. Hence, if we dealt with a fairly large population, where the number of deaths in any group were say 5 to 10 and we made 10 age groups, we could reckon on B_1 and $B_2 - 3$ being of the order ·02 to ·01, or the distribution would be closely Gaussian. Thus there need be small hesitation in applying the tables of the probability integral to the investigation of the relationship of $M' - M$ to $\sigma_{M'-M}$.

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* *Biometrika*, Vol. v. p. 353 and Vol. x. p. 39.

(2) Two points, however, arise in this work. We do not know p_s and q_s nor have we yet selected the standard population, i.e. the values of ratios like A_s/A.

With regard to p_s it may be held by some that the value obtaining in the general population should be given to it. This might be reasonable if that population were immensely large as compared with either group under consideration, but very often the group dealt with is quite considerable as compared to the remainder. Thus in Scotland for certain administrative purposes we consider *Scotland* (*Clyde*), and *Scotland* (*excluding Clyde*). In England for similar purposes we find nine official districts* selected, so that if we were comparing the *North-western* district with the *London* district, it would be curiously difficult to demonstrate why if these are samples, we take them plus the remainder to be more representative and fixed than either alone. We doubt very much whether the material we suppose we are sampling is to be considered as the general population. Rather we look upon that general population as the indefinitely large group who might be considered as living and dying under the same environmental conditions, if they continued indefinitely in force.

Again, is it quite correct to take p_s from the general population of the country when the problem is to discover whether the two districts are themselves random samples from some population which may not be the same as that of the general population of the country? Thus Aberdeen and Inverness might both well be random samples of a population which is not that of Scotland as a whole†. Hence as in cases of probable error it will usually be best to calculate p_s from the observed material itself, i.e. if our two groups be really samples of the same population, then probably the best thing we can do is to take

$$p_s = \frac{d_s + d'_s}{a_s + a'_s} \quad \text{and} \quad q_s = 1 - \frac{d_s + d'_s}{a_s + a'_s}.$$

If p_s be small, as it usually is, then it will be sufficient to take

$$\sigma^2_{M'-M} = S\left\{\left(\frac{d_s + d'_s}{a_s a'_s}\right)\frac{A_s^2}{A^2}\right\}.$$

It remains to consider A_s/A. Here again it is not unusual to take A_s/A as given by the general population. Or if the "corrected deathrates" for a series of years are being compared, it is not unusual to reduce the age distributions to that of a certain year, e.g. in the manner of the English Registrar-General for tuberculosis and cancer deathrates to the population of 1901. For the same reasons as in the case just discussed we might, perhaps, find it fitting to take A_s/A as given by the material under discussion, i.e. $= (a_s + a'_s)/(a + a')$. Or, again, we might reduce one district to the population of the other, i.e. take $A_s/A = a_s/a$, or a'_s/a'. It is of interest to see practically what differences such divergent reductions make.

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* By the Board of Trade for example.

† Or again lawyers and the clergy may have or may not have significantly different mortalities, but the mortality of both differs essentially from that of all England.

Another solution of the-standard-population-problem, which seems to us of some importance, arises from the consideration that we ought to select our standard population so that the probability that the two districts or classes under consideration are samples of one and the same population should be a minimum. In other words we ought to select A_s/A $(= X_s)$ so that

$$Q = S(\lambda_s X_s)/\sqrt{S(\nu_s X_s^2)},$$

where

$$\lambda_s = d'_s/a'_s - d_s/a_s,$$

$$\nu_s = p_s q_s (1/a'_s + 1/a_s),$$

is a maximum, subject to the relation $S(X_s) = 1$.

Proceeding by the usual rules for finding a max.-min. we have

$$\delta Q = Q \times S\left(\frac{\lambda_s}{S(\lambda_s X_s)} - \frac{\nu_s X_s}{S(\nu_s X_s^2)}\right)\delta X_s,$$

$$S(\delta X_s) = 0.$$

Therefore if P be an indeterminate multiplier:

$$\frac{\lambda_s}{S(\lambda_s X_s)} - \frac{\nu_s X_s}{S(\nu_s X_s^2)} + P = 0.$$

Multiply by X_s and sum all such equations and we find $P = 0$. Hence:

$$\chi_s = \frac{\lambda_s}{\nu_s} \cdot \frac{S(\nu_s X_s^2)}{S(\lambda_s X_s)}.$$

Sum all such equations and we have

$$1 = S\left(\frac{\lambda_s}{\nu_s}\right)\frac{S(\nu_s X_s^2)}{S(\lambda_s X_s)}.$$

Thus we deduce:

$$X_s = \frac{\lambda_s}{\nu_s}\,\frac{1}{S\left(\dfrac{\lambda_s}{\nu_s}\right)},$$

whence

$$S(\lambda_s X_s) = S\left(\frac{\lambda_s^2}{\nu_s}\right)\bigg/ S\left(\frac{\lambda_s}{\nu_s}\right),$$

$$S(\nu_s X_s^2) = S\left(\frac{\lambda_s^2}{\nu_s}\right)\bigg/\left\{S\left(\frac{\lambda_s}{\nu_s}\right)\right\}^2,$$

and accordingly

$$Q = \sqrt{S\left(\frac{\lambda_s^2}{\nu_s}\right)}.$$

A little consideration shows that this is a maximum value of Q. The argument we then use is that if on the standard population which provides a maximum for Q, there be no significance in the deathrate difference, there cannot be any significance at all in the difference. While on the other hand if on any population whatever used as standard, we do find a significant difference, such a difference really exists.

Returning to our deathrate symbols we have

$$\frac{A_s}{A} = X_s = \frac{\dfrac{d'_s}{a'_s} - \dfrac{d_s}{a_s}}{p_s q_s\left(\dfrac{1}{a'_s} + \dfrac{1}{a_s}\right)} \; \frac{1}{S\left\{\dfrac{\dfrac{d'_s}{a'_s} - \dfrac{d_s}{a_s}}{p_s q_s\left(\dfrac{1}{a'_s} + \dfrac{1}{a_s}\right)}\right\}} \quad \text{............(viii).}$$

Hence the age classes A_s of the standard population must be taken proportional to

$$\frac{a'_s a_s\left(\dfrac{d'_s}{a'_s} - \dfrac{d_s}{a_s}\right)}{(d'_s + d_s)\left(1 - \dfrac{d'_s + d_s}{a'_s + a_s}\right)}, \text{ or very approximately to } \frac{a'_s a_s\left(\dfrac{d'_s}{a'_s} - \dfrac{d_s}{a_s}\right)}{d'_s + d_s}.$$

We note at once that this method would give irrational values for A_s in the case of any material for which the age deathrate was not invariably greater for one district or class. On the other hand Q the ratio of the difference of corrected deathrates to the standard deviation of that difference is always real and given by

$$Q^2 = S\left\{\frac{a'_s a_s\left(\dfrac{d'_s}{a'_s} - \dfrac{d_s}{a_s}\right)^2}{(d'_s + d_s)\left(1 - \dfrac{d'_s + d_s}{a'_s + a_s}\right)}\right\} \quad \text{....................(ix),}$$

$$= S\left\{\frac{a'_s a_s\left(\dfrac{d'_s}{a'_s} - \dfrac{d_s}{a_s}\right)^2}{d'_s + d_s}\right\} \text{ approximately.}$$

If for every value of s, d'_s/a'_s is either greater than d_s/a_s, or, on the other hand, is always less than d_s/a_s, then the odds are about 50 to 1 if Q be as great as 2 that there is significant divergence of the two corrected deathrates. On the other hand if Q has no significant magnitude, the deathrates will not be significantly different. Supposing the condition as to the relative magnitude of d'_s/a'_s and d_s/a_s be not satisfied, then if Q has no significant magnitude for these irrational age classes, it will certainly have no significant magnitude for any other size of age classes, and accordingly we conclude that the difference of the deathrates is not significant. But if Q be of significant magnitude for the irrational age classes, it does not follow that it will be significant for rational age classes, and further discussion is needful. Of course any case in which for the bulk of groups d'_s/a'_s is greater than d_s/a_s, but for one or two groups d_s/a_s is the greater, will have $Q = (M' - M)/\sigma_{M'-M}$ lessened by the inclusion of these cases, for the numerator of Q is decreased and the denominator increased. We should therefore be at liberty to consider only the groups where the deathrate goes one way, for the age groups are actually independent, and it is really a fictitious balancing of the corrected deathrates which arises, when one age class with deaths in excess compensates for another age class with deaths in defect, and so tends to equalise M and M'. This is of course a grave difficulty which must arise when we deal with any corrected death*rate* at all. Two such deathrates might show no significant difference although the aged were dying in one population and the young in the other in excess*. We might work only with the groups for which $d'_s/a'_s - d_s/a_s$ is of the same sign, and this would indicate, by the same test, differentiation or its absence in the manner of dying; but of course we should then be dropping the idea of a "corrected deathrate"; that idea is, however, essentially imperfect and does not really distinguish effectually between differences in the manner of dying.

(3) We now ask whether it is not feasible to interpret the value of Q in Eqn (ix) as a measure of the probability of a differential mortality without regard to the theory of "corrected deathrates."

As before let d_s be the number of deaths in the age group of size a_s in one district or class. We suppose this to be a sample of a population of which p_s is the chance of dying in this age group, then if there be u age groups in each district or class, we have $2u$ deviations, all of which are independent, and given by $d_s - p_s a_s$ and $d'_s - p_s a'_s$. These deviations have respectively standard deviations $\sqrt{a_s p_s q_s}$ and $\sqrt{a'_s p_s q_s}$. Accordingly if every deviation be measured in terms of its standard deviation we shall have a second moment coefficient given by

$$\Sigma^2 = \frac{1}{2u}\left[S\left(\frac{(d_s - p_s a_s)^2}{a_s p_s q_s}\right) + S\left(\frac{(d'_s - p_s a'_s)^2}{a'_s p_s q_s}\right)\right],$$

and Σ^2 ought to be unity.

Now in the above expression p_s, q_s are the unknown values of the death and survival chances in the population from which the two districts or classes are supposed to be sampled, and the best values we probably can take for them are $p_s = 1 - q_s = (d_s + d'_s)/(a_s + a'_s)$. But inserting these values in the above expression, we find with the value of Q given in Eqn (ix):

$$\Sigma^2 = \frac{1}{2u} Q^2, \quad \text{or} \quad \Sigma = Q \times \sqrt{\frac{1}{2u}}.$$

But the standard deviation of a second moment coefficient is $\sqrt{\mu_4 - \mu_2^2}/\sqrt{n}$ and this in the case of a normal distribution ($\mu_4 = 3\mu_2^2$) equals $\sqrt{\frac{2}{n}}\,\mu_2$, so in our case, since $\mu_2 = 1$ and $n = 2u$, the standard deviation of Σ^2 equals $1/\sqrt{u}$. Thus we have to measure $\Sigma^2 - 1$ in terms of $1/\sqrt{u}$, or to ascertain the probability of a ratio of deviation to standard deviation of magnitude greater than $(\frac{1}{2}Q^2 - u)/\sqrt{u}$. We see again therefore Q arising as a constant which naturally determines the mortality resemblance or difference of the two districts. But while in the previous approach to the solution of the problem from the corrected deathrates we found Q alone sufficed to obtain our criterion, we require in the case of this second criterion to make use explicitly as well as implicitly of the number of age classes. We can thus give a physical meaning to Q, $\sqrt{2}Q$ is the ratio of the standard deviation of the $2u$ age classes' deathrates—each supposed to be drawn from the same population and measured in terms of its own standard deviation—to the standard deviation of this standard deviation, i.e. to $\frac{1}{\sqrt{2\times 2u}}$. It by no means follows that this second criterion will give the same result as may be drawn from the first. But this new aspect of Q frees us from many of the difficulties essentially associated with "corrected deathrates" and the indefinite category of a standard population. We must observe, however, that to evaluate the probability of the occurrence of Q we have again to justify the assumption that Σ^2 will follow a normal distribution. We could not justify this for the distribution of deaths in any single age group, nor even for the distribution of factors like $(d_s - p_s a_s)^2/(a_s p_s q_s)$ and $(d'_s - p_s a'_s)^2/(a'_s p_s q_s)$ summed for our two populations on one occasion, but we can do this for the distribution of Σ^2 on the basis of a number of random samples*.

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* Another factor, sometimes overlooked when deathrate after correction is taken as a measure of local health, is emigration. If the sth age class tend to migrate from A to B, it is usually the healthy who migrate; thus the deathrate of the sth class in A will be inflated and that in B reduced. If the migration be chiefly that of males, as to mining districts and colonies, a spurious correlation between deathrate and sex ratio may be created.

(4) We can again approach the problem by considering quantities like

$$\frac{d_s - p_s a_s}{\sqrt{a_s p_s q_s}} \quad\text{and}\quad \frac{d'_s - p_s a'_s}{\sqrt{a'_s p_s q_s}}.$$

The mean of these deviations measured each in terms of its own standard deviation should be zero; and the standard deviation of this mean should be $\frac{1}{\sqrt{2u}}$, since there are $2u$ variates and, each being measured in terms of its own standard deviation, the standard deviation of the series is as before unity. Thus if

$$\bar{m} = \frac{1}{2u} S_1^u \left[\left(\frac{d_s - p_s a_s}{\sqrt{a_s p_s q_s}}\right) + S_1^u \left(\frac{d'_s - p_s a'_s}{\sqrt{a'_s p_s q_s}}\right)\right],$$

$\bar{m}\Big/\left(\frac{1}{\sqrt{2u}}\right) = \sqrt{2u}\times\bar{m}$ may be looked up in the tables of the probability integral, and the probability of the system, as a result of random sampling from a population p_s, q_s, thus again determined. If we use the values of p_s and q_s so often adopted above, the quantity with which to enter the probability table, i.e. the ratio of deviation to standard deviation, is expressed by

$$\sqrt{2u}\times\bar{m} = \frac{1}{\sqrt{2u}} S_1^u \left\{ \frac{\left(\frac{d_s}{a_s} - \frac{d'_s}{a'_s}\right)\left(\sqrt{a'_s} - \sqrt{a_s}\right)}{\sqrt{\frac{(d_s + d'_s)(l_s + l'_s)}{a_s a'_s}}} \right\} \quad\text{...............(x)},$$

where $l_s = a_s - d_s$ and $l'_s = a'_s - d'_s$ are the survivors in the sth age groups.

---

* We can show in a manner similar to that on p. 161 that the distribution of μ_2's approaches the normal when each of the constituent x^2's is drawn from different populations, none of these populations being in themselves accurately normal.

(5) The previous methods are all more or less inadequate; they test whether a certain *single* character of the distribution does or does not present significant difference in the two populations compared. If we want to test the distributions as a whole, we must adopt a modification of the method given in *Biometrika*, Vol. VIII. p. 250 for determining the probability that two systems of frequency are random samples of the same population. It has been shown in a memoir dealing with partial contingency* that the proper test is to determine

$$\chi_0^2 = S_1^u \left\{ \frac{a_s a'_s}{(a_s + a'_s)^2} \frac{(p+p')^2}{pp'} \frac{a_s a'_s \left(\dfrac{d_s}{a_s} - \dfrac{d'_s}{a'_s}\right)^2}{(d_s + d'_s)\left(1 - \dfrac{d_s + d'_s}{a_s + a'_s}\right)} \right\} \quad \text{.........(xi)}$$

entering the Tables of Goodness of Fit with $n' = s + 1$. Here p and p' are the numbers in the sampled populations. Now the factor

$$\frac{a_s a'_s}{(a_s + a'_s)^2} \frac{(p+p')^2}{pp'}$$

is for most practical purposes unity, as we shall illustrate in the sequel. Hence for such purposes

$$\chi_0^2 = S_1^u \left\{ \frac{a_s a'_s \left(\dfrac{d_s}{a_s} - \dfrac{d'_s}{a'_s}\right)^2}{(d_s + d'_s)\left(1 - \dfrac{d_s + d'_s}{a_s + a'_s}\right)} \right\} = Q^2 \quad \text{...............(xi)}^{\text{bis}}$$

of our earlier investigations. Thus to obtain the probability of the two districts being random samples of the same population all we have to do is to look up Q^2 for $n' = u + 1$, u being the number of age groups, in the Tables of Goodness of Fit. It will be seen that Q has a wide meaning, and generally speaking we may treat it as a constant, which can be used in a variety of criteria for testing the existence of differential deathrates. We propose to illustrate this in the following sections, applying in each case the four tests discussed above, namely:

(*a*) The probable error of the "corrected" deathrates' difference reduced to the standard population of maximum difference.

(*b*) The significance of the means of the $2u$ deathrate classes.

(*c*) The significance of the squared standard deviations of the $2u$ deathrate classes.

(*d*) The general χ_0^2 test of partial contingency with its approximate value Q^2.

The following illustrations have been used:

(i) General Deathrates of Liverpool and Birmingham.

(ii) Cancer Deathrates of Edinburgh and Dundee.

(iii)—(viii) Cancer Deathrates for all England and Wales divided into four groups: (*a*) London, (*b*) County Boroughs other than London, (*c*) Urban Districts other than (*a*) and (*b*), (*d*) Rural Districts.

(ix)—(xiv) Diabetes Deathrates precisely as in the preceding Cancer case.

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* *Biometrika*, Vol. XI. p. 157.

Illustrations. (i) We have for *Birmingham and Liverpool* the following data for all Males in 1911:

Age Group	Birmingham		Liverpool	
	Population	Deaths	Population	Deaths
0—	32,552	2003	45,889	3117
5—	58,653	161	78,518	326
15—	48,431	162	62,751	309
25—	47,212	252	58,216	476
35—	37,897	382	47,711	632
45—	25,431	454	32,664	775
55—	15,384	575	20,198	944
65—	7,535	511	10,215	904
75—	1,944	321	2,194	335
85—	173	58	191	62
Totals	275,212	4879	358,547	7880

Our problem is to discover whether there is actual differentiation between these two systems of deaths, and if so, what is the measure of it. We work with ten age groups.

The actual arithmetical work is indicated on the following page. It leads to $Q^2 = 165{\cdot}5031$ or $Q = 12{\cdot}8648$. Hence applying first test $\dfrac{M' - M}{\sigma_{M'-M}} = 12{\cdot}8648$.

Or: the difference of the two deathrates corrected to the population of maximum ratio is no less than 12·86 times the standard deviation of the difference. We conclude that the chance of such a difference arising from random sampling is enormous*, or the two deathrates are most certainly and markedly different.

There is no difficulty in correcting the deathrates to the population of maximum difference as standard, but it is of interest to note what happens, if the standard population has other values.

For example, when we correct to the male population of all England and Wales for 1901, we find using the formula of p. 161

$$\frac{M' - M}{\sigma_{M'-M}} = 10{\cdot}0198.$$

If we use the general male population of England and Wales in 1911 we find

$$\frac{M' - M}{\sigma_{M'-M}} = 10{\cdot}1567.$$

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* The chance is approximately $3{\cdot}3508/10^{38}$. This test is practically valid in this case for the general deathrate of Liverpool males is greater at all ages except 75 onwards than that of Birmingham and the age groups above 75 contribute nothing of importance to the value of Q^2.

Liverpool and Birmingham Male Deaths from all Causes, 1911.

Liverpool: Dashed letters. Birmingham: Undashed letters.

Age Group	(i) a_s	(ii) d_s	(iii) d_s/a_s	(iv) a'_s	(v) d'_s	(vi) d'_s/a'_s	(vii) $\frac{d'_s}{a'_s}-\frac{d_s}{a_s}$	(viii) $10^3\times\left(\frac{d_s}{a_s}-\frac{d'_s}{a'_s}\right)^2$	(ix) $a_s a'_s$	(x) $a_s a'_s\left(\frac{d_s}{a_s}-\frac{d'_s}{a'_s}\right)^2$
0—5	32,552	2003	·061,532,318	45,889	3117	·067,924,775	+ ·006,392,457	·0408,635,065	1,493,778,728	61,041·037
5—15	58,653	161	·002,744,958	78,518	326	·004,151,914	+ ·001,406,956	·0019,795,252	4,605,316,254	9,116·340
15—25	48,431	162	·003,344,965	62,751	309	·004,924,224	+ ·001,579,259	·0024,940,590	3,039,093,681	7,579·679
25—35	47,212	252	·005,337,626	58,216	476	·008,176,446	+ ·002,838,820	·0080,588,990	2,748,493,792	22,149·834
35—45	37,897	382	·010,079,954	47,711	632	·013,246,421	+ ·003,166,467	·0100,265,133	1,808,103,767	18,128·976
45—55	25,431	454	·017,852,228	32,664	775	·023,726,427	+ ·005,874,199	·0345,062,139	830,678,184	28,663·559
55—65	15,384	575	·037,376,495	20,198	944	·046,737,301	+ ·009,360,806	·0876,246,890	310,726,032	27,227·272
65—75	7,535	511	·067,816,855	10,215	904	·088,497,308	+ ·020,680,453	·4276,811,363	76,970,025	32,918·628
75—85	1,944	321	·165,123,457	2,194	335	·152,689,152	− ·012,434,305	·1546,119,408	4,265,136	659·441
85 and over	173	58	·335,260,116	191	62	·324,607,330	− ·010,652,786	·1134,818,496	33,043	3·750

$p = 275{,}212 \quad d = 4879 \qquad p' = 358{,}547 \quad d' = 7880$

Age Group	(xi) $d_s+d'_s$	(xii) $a_s+a'_s$	(xiii) $\frac{d_s+d'_s}{a_s+a'_s}$	(xiv) $(d_s+d'_s)\left(1-\frac{d_s+d'_s}{a_s+a'_s}\right)$	(xv) $Q_s=(\text{x})/(\text{xiv})$	(xvi) $\frac{a_s a'_s}{pp'}$	(xvii) $\frac{a_s+a'_s}{p+p'}$	(xviii) $\left(\frac{a_s+a'_s}{p+p'}\right)^2$	(xix) $\frac{a_s a'_s}{pp'}\left(\frac{p+p'}{a_s+a'_s}\right)^2$	(xx) $\chi_s^2=(\text{xix})\times(\text{xv})$
0—5	5120	78,441	·065,271,988	4,785·8074	12·7546	·015,1381	·123,771	·015,3193	·98817	12·6037
5—15	487	137,171	·003,550,313	485·2710	18·7861	·046,6709	·216,440	·046,8463	·99626	18·7158
15—25	471	111,182	·004,236,297	469·0047	16·1612	·030,7986	·175,433	·030,7766	1·00071	16·1727
25—35	728	105,428	·006,905,186	722·9730	30·6372	·027,8536	·166,353	·027,6733	1·00652	30·8370
35—45	1014	85,608	·011,844,687	1,001·9895	18·0930	·018,3236	·135,080	·018,2466	1·00422	18·1694
45—55	1229	58,095	·021,155,005	1,203·0005	23·8267	·008,4182	·091,667	·008,4029	1·00182	23·8701
55—65	1519	35,582	·042,690,124	1,454·1537	18·7238	·003,1489	·056,144	·003,1522	·99895	18·7047
65—75	1415	17,750	·079,718,310	1,302·1986	25·2793	·000,780,024	·028,007	·000,78442	·99440	25·1376
75—85	656	4,138	·158,530,691	552·0039	1·1946	·000,043,2234	·006,5293	·000,042632	1·01388	1·2112
85 and over	120	364	·329,670,330	80·4396	·0466	·000,003,3486	·000,57435	·000,000,32988	1·01510	·0473

$d + d' = 12{,}759 \quad p + p' = 633{,}759$

$pp' = 98{,}676{,}436{,}964$

$Q^2 = 165{\cdot}5031$

$Q = 12{\cdot}8648$

$\chi_0^2 = 165{\cdot}4695$

This table exhibits in one out of the fourteen cases dealt with in this memoir the work of determining Q^2 and χ_0^2.

These values of course lead to the same conclusion but show that the ratio $(M' - M)/\sigma_{M'-M}$ varies from standard population to standard population, and may be increased more than 20 % when we pass to the population of maximum difference as standard. Such an increase might be of considerable importance in our estimate of differentiation if the ratio $(M' - M)/\sigma_{M'-M}$ lay between 1·8 and 2·2, say.

Our third test is given by the formula on p. 165:

$$(\tfrac{1}{2}Q^2 - u)/\sqrt{u} = 72\cdot7515/\sqrt{10} = 23\cdot01.$$

The probability of a deviation as great or greater than this arising is immense.

Thus we see that the distribution of the squares of the actual deviations is excessively improbable.

Proceeding to our second test we find

$$\bar{m} = -\cdot16625,$$

and accordingly $\bar{m}\Big/\left(\frac{1}{\sqrt{2u}}\right) = \cdot745$, or $\bar{m}$ does not differ significantly from zero. Thus by this test no essential difference would be indicated. This does not show that it does not exist, but only indicates the inadequacy of the test. In fact since

$$d_s - p_s a_s + d'_s - p_s a'_s = 0,$$

and there is no great difference between $\sqrt{a_s p_s q_s}$ and $\sqrt{a'_s p_s q_s}$, $\bar{m}$ tends to be zero, even with considerable differences between d_s and $p_s a_s$ or d'_s and $p_s a'_s$.

We have seen that the value of Q^2 in the Liverpool and Birmingham case is 165·5031. We will now investigate the value of the factors

$$\left(\frac{a_s}{p}\frac{a'_s}{p'}\right)\Big/\left(\frac{a_s + a'_s}{p + p'}\right)^2$$

for the ten age groups. They run

Age	Factor	Age	Factor
0—5	·98817	*45—55*	1·00182
5—15	·99626	*55—65*	·99895
15—25	1·00071	*65—75*	·99440
25—35	1·00652	*75—85*	1·01376
35—45	1·00422	*85* and over	1·01510

It will be seen that the factors differ very little from unity and introducing them into the several terms of Q^2 we find

$$\chi_0^2 = 165\cdot4695,$$

or χ_0^2 only differs by 0·02 % from Q^2. Applying the test for goodness of fit to χ_0^2 for n' equal *eleven* groups* we find

$$P = 2\cdot40/10^{30},$$

---

* See *Tables for Statisticians*, p. xxxiii.

or the odds against Liverpool and Birmingham general mortality experiences being samples of the same population are gigantic. This test which takes the distributions as wholes seems to us absolutely conclusive. The differences in deathrates in Liverpool and Birmingham are not due to difference of age constitutions, but are fundamental*.

(ii) *Cancer Deathrates in Edinburgh and Dundee,* 1891–1900 (*Males*)†.

Age Group	Edinburgh		Dundee	
	Population	Deaths, Cancer	Population	Deaths, Cancer
0—5	149,763	5	89,775	6
5—15	280,655	8	168,510	2
15—25	274,343	18	142,917	7
25—35	214,063	47	98,953	10
35—45	158,133	117	76,132	47
45—55	115,206	295	59,066	114
55—65	69,954	356	37,337	151
65—75	32,966	266	16,958	109
75—85	10,311	93	4,625	30
85 and over	1,045	8	535	3
Totals	1,306,439	1213	694,808	479

Proceeding as before we find $Q^2 = 28 \cdot 4837$. The factors for the age classes to give χ_0^2 from the Q^2 terms for the separate age groups are given below. They differ more from unity, but some being in excess and some in defect, there is no substantial difference between χ_0^2 and Q^2.

We have $$\chi_0^2 = 28 \cdot 0393,$$

or, χ_0^2 is 1·6 % less than Q^2.

Age Group	Factor: $\frac{a_s a'_s}{pp'} \Big/ \left(\frac{a_s + a'_s}{p + p'}\right)^2$	Age Group	Factor: $\frac{a_s a'_s}{pp'} \Big/ \left(\frac{a_s + a'_s}{p + p'}\right)^2$
0—5	1·03386	*45—55*	·98855
5—15	1·03427	*55—65*	1·00711
15—25	·99360	*65—75*	·98966
25—35	·95387	*75—85*	·94318
35—45	·96788	*85* and over	·98812

We now apply the same tests as before: we have

$$(M' - M)/\sigma_{M'-M} = Q = 5 \cdot 3370.$$

Hence $$\tfrac{1}{2}(1 + \alpha) = \cdot 99999,99527,$$

---

[*] This only means that the differentiation cannot be accounted for by age differences, it might well be accounted for by class or occupation differences.

[†] The results are sums of ten years' population found on assumption of arithmetical progression.

or, the chance of a deviation so great as this appearing is only $4{\cdot}73/10^8$, or the "corrected" cancer deathrates for males are significantly different in Dundee and Edinburgh.

Using the General Population of Scotland, 1891–1901, as a standard population, we find

$$(M' - M)/\sigma_{M'-M} = 4{\cdot}7399.$$

Using the General Population of England and Wales (1901, males) as a standard, we find

$$(M' - M)/\sigma_{M'-M} = 4{\cdot}7630.$$

These again both mark significant deviations in the corrected deathrates, but fail to give the maximum of significance.

Now let us apply the test of distribution of squares of differences. We have

$$\begin{aligned}(\tfrac{1}{2}Q^2 - u)\sqrt{u} &= 4{\cdot}24186/\sqrt{10}\\ &= 1{\cdot}3414.\end{aligned}$$

Such a deviation would occur about once in ten trials and is not necessarily significant.

Again applying the test of mean value, we have

$$\bar{m} = -\,{\cdot}1564,$$

and accordingly
$$\bar{m}\bigg/\left(\frac{1}{\sqrt{2u}}\right) = {\cdot}6994,$$

and this is an insignificant ratio of the deviation to its standard deviation.

We now turn to the χ_0^2 test, where we have $\chi_0^2 = 28{\cdot}0393$. The Tables of Goodness of Fit provide for $n' =$ *eleven*:

$$P = {\cdot}00178,$$

or the odds are nearly 500 to 1 against such a deviation on random sampling. We conclude that there is a significant difference between cancer mortality in Dundee and Edinburgh. It is noteworthy that the corrected deathrates criterion which when analysed seems so very unsatisfactory gives here as in the case of the Liverpool and Birmingham General Deathrates far greater significance to the observed differences of mortality.

We have not considered it worth while to investigate for this case the test of the significance of the mean of the $2u$ deathrates. It is we believe inadequate and further is laborious to calculate. It is, we hold, sufficient and more enlightening to calculate χ_0^2, and, what is almost deduced in the same process, the quantity Q^2.

There is a further point which may be illustrated on the Liverpool-Birmingham and Dundee-Edinburgh data. We have supposed in the course of our work that $(d_s + d'_s)/(a_s + a'_s)$ is an extremely reasonable value to give to p_s. If we assume that the distributions of quantities like d_s are given with sufficient accuracy by the normal curve then the probability of the whole result observed is given by

$$\Pi = \text{Product}\left[\frac{1}{\sqrt{a_s p_s q_s}}\, e^{-\frac{1}{2}\frac{(d_s - p_s a_s)^2}{a_s p_s q_s}}\right]_1^u \times \left[\frac{1}{\sqrt{a'_s p_s q_s}}\, e^{-\frac{1}{2}\frac{(d'_s - p_s a'_s)^2}{a'_s p_s q_s}}\right]_1^u V \ldots\text{(xii)},$$

where V is the continued product of the differentials of the d_s's and the d'_s's, and we require to make this a maximum for the variation of all quantities like p_s. In other words we have u equations found by differentiating the above expression for $p_1, p_2, \ldots p_s, \ldots p_n$ (of course $q_s = 1 - p_s$) to determine the best values of these quantities. Taking the logarithmic differential of the above product with regard to p_s and equating it to zero, we find since $\delta q_s = -\delta p_s$:

$$0 = -\frac{1}{p_s} + \frac{1}{q_s} + \frac{d_s - p_s a_s + d'_s - p_s a'_s}{p_s q_s} + \frac{1}{2}\left(\frac{(d_s - p_s a_s)^2}{a_s p_s^2 q_s^2} + \frac{(d'_s - p_s a'_s)^2}{a'_s p_s^2 q_s^2}\right)(p_s - q_s).$$

Hence:

$$p_s = \frac{d_s + d'_s}{a_s + a'_s} + \frac{1}{2}\frac{q_s - p_s}{(a_s + a'_s)\, q_s p_s}\left\{\frac{(d_s - p_s a_s)^2}{a_s} + \frac{(d'_s - p_s a'_s)^2}{a'_s} - 2p_s q_s\right\}\ldots\text{(xiii)}.$$

Now assuming to a first approximation $p_s = \dfrac{d_s + d'_s}{a_s + a'_s}$ we find to a second approximation $\tilde{p}_s$

$$\tilde{p}_s = \frac{d_s + d'_s}{a_s + a'_s} - \frac{1 - 2p_s}{a_s + a'_s}\left(1 - \frac{1}{2}\frac{a_s a'_s\left(\dfrac{d_s}{a_s} - \dfrac{d'_s}{a'_s}\right)^2}{(d_s + d'_s)\left(1 - \dfrac{d_s + d'_s}{a_s + a'_s}\right)}\right).$$

Calling as before $$Q_s^2 = \frac{a_s a'_s\left(\dfrac{d_s}{a_s} - \dfrac{d'_s}{a'_s}\right)^2}{(d_s + d'_s)\left(1 - \dfrac{d_s + d'_s}{a_s + a'_s}\right)},$$

we have $$\tilde{p}_s = \frac{d_s + d'_s}{a_s + a'_s} - \frac{\left(1 - 2\dfrac{d_s + d'_s}{a_s + a'_s}\right)(1 - \tfrac{1}{2}Q_s^2)}{a_s + a'_s}.$$

Hence we may write $$\tilde{p}_s = \frac{d_s + d'_s - \delta_s}{a_s + a'_s};$$

this amounts to altering the deaths by

$$\delta_s = \left(1 - 2\frac{d_s + d'_s}{a_s + a'_s}\right)(1 - \tfrac{1}{2}Q_s^2),$$

which is usually a very small number, or we may write

$$\tilde{p}_s = \frac{d_s + d'_s}{a_s + a'_s}\left\{1 - (1 - \tfrac{1}{2}Q_s^2)\left(\frac{1}{d_s + d'_s} - \frac{2}{a_s + a'_s}\right)\right\},$$

where the factor

$$f_s = 1 - (1 - \tfrac{1}{2}Q_s^2)\left(\frac{1}{d_s + d'_s} - \frac{2}{a_s + a'_s}\right)$$

will only differ slightly from unity.

The following table gives the values of p_s, $\tilde{p}_s$ for the various age groups in the case of (*a*) Liverpool and Birmingham for General Deathrates, (*b*) Edinburgh and Dundee, Cancer Deathrates, (*c*) Rural and Urban Districts, England and Wales, Cancer Deathrates.

Illustrations of Approximate Deathrates for unknown sampled Populations.

Age Group	(*a*) Liverpool and Birmingham, General Deathrates		(*b*) Edinburgh and Dundee, Cancer Deathrates		(*c*) Rural and Urban Districts, England and Wales, Cancer Deathrates	
	p_s	$\tilde{p}_s$	p_s	$\tilde{p}_s$	p_s	$\tilde{p}_s$
0—5	·065,272	·065,332	·000,046	·000,045	·000,025	·000,025
5—15	·003,550	·003,611	·000,022	·000,021		
15—25	·004,236	·004,299	·000,060	·000,058	·000,033	·000,033
25—35	·006,905	·007,039	·000,182	·000,187	·000,116	·000,116
35—45	·011,845	·011,937	·000,700	·000,698	·000,404	·000,404
45—55	·021,155	·021,335	·002,347	·002,360	·001,500	·001,500
55—65	·042,690	·042,905	·004,725	·004,726	·004,256	·004,256
65—75	·079,718	·080,269	·007,511	·007,532	·007,787	·007,788
75—85	·158,531	·158,498	·008,235	·008,252	·009,653	·009,646
85 and over	·329,670	·328,756	·006,962	·006,405	·008,856	·008,847

It will be seen that the corrective factor contains the inverses of the total number $(d_s + d'_s)$ of deaths and the total numbers of individuals $(a_s + a'_s)$ in the combined age groups s and s'. Hence for big districts as in (*c*) the corrective factor is of small importance even for special diseases. For the general deathrates in two large towns as in (*a*) the difference between p_s and $\tilde{p}_s$ is as a rule less than 1 %. Even in special diseases in towns of moderate size, it is only where the total number of deaths in the combined age groups s and s' is very small that any substantial divergence between p_s and $\tilde{p}_s$ arises, e.g. in (*b*) for the child or extreme old age groups. Thus the value of Q^2 is hardly likely to be modified practically, if we replace $\tilde{p}_s$ by p_s. Accordingly $\frac{d_s + d'_s}{a_s + a'_s}$, besides being easy to calculate, is a reasonable approximation to the better value $\tilde{p}_s$. Of course $\tilde{p}_s$ itself is only an approximation* to the "best value" and this "best value" also depends on the accuracy of replacing the binomial by a normal curve. Thus it is by no means certain that, if we obtained a true best value for the deathrate in the unknown sampled population, it would be markedly nearer to $\tilde{p}_s$ than to p_s. We content ourselves by remarking that neither for practical nor theoretical reasons does there seem likelihood of a great gain resulting from taking any other value than $(d_s + d'_s)/(a_s + a'_s)$ for p_s.

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* We should have to solve a cubic for each age group to find the accurate "best value," on the above hypothesis of normality. Doubts may also be raised as to the legitimacy of the theory which makes Π in (xii) a maximum. They are discussed in another paper.

(iii)—(viii) *Cancer Deathrates for all England and Wales.*

The object of the present illustrations is to ascertain whether there are significant differences in the cancer deathrates associated with urban and rural conditions. We divide the data into four groups: (*a*) London, (*b*) County Boroughs other than London, (*c*) Urban Districts other than County Boroughs, (*d*) Rural Districts. We compare pair and pair these four groups in order to ascertain the degree of their significant differences.

The following data are taken from the Registrar-General's 76*th Annual Report* and are for the year 1913*:

Populations and Cancer Deaths in Age Groups, 1913.

Populations

Age Group	(*a*) London	(*b*) County Boroughs	(*c*) Urban Districts	(*d*) Rural Districts
0—15	648,361	1,796,847	1,976,078	1,242,634
15—25	383,601	1,006,039	1,126,179	716,323
25—35	362,967	938,541	1,017,991	580,305
35—45	293,427	766,070	836,306	496,185
45—55	213,635	536,949	586,962	395,443
55—65	131,423	331,209	369,840	276,814
65—75	68,124	166,717	200,982	180,029
75—85	18,830	43,125	60,407	65,453
85 and over	2,239	4,493	7,361	9,125
		Cancer Deaths		
0—15	25	31	47	34
15—25	24	39	34	27
25—35	51	128	121	65
35—45	130	416	353	185
45—55	472	1075	971	503
55—65	723	1700	1675	1077
65—75	655	1312	1607	1360
75—85	209	450	576	639
85 and over	29	29	73	73
Populations	2,122,607	5,589,990	6,182,106	3,962,311
Total deaths	2,318	5,180	5,457	3,963
Crude death-rates per 100,000	109·21	92·67	88·37	100·02

How far are these differences in the crude deathrates really significant, when allowance is made for age groups? Above all: what is the *numerical measure* of this significance in the six cases? The following table gives the deathrates per 100,000 at each age in the four groups:

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* See pp. 4, 217, 234, 253, and 271.

Male Cancer Statistics, England and Wales, 1913.

Age Group	(*a*) London	(*b*) County Boroughs	(*c*) Urban Districts	(*d*) Rural Districts
0—15	3·86	1·73	2·38	2·74
15—25	6·26	3·88	3·02	3·77
25—35	14·05	13·64	11·89	11·20
35—45	44·30	54·30	42·21	37·28
45—55	220·94	200·21	165·43	127·20
55—65	550·13	513·27	452·90	389·07
65—75	961·48	786·96	799·57	755·43
75—85	1109·93	1043·48	953·53	976·27
85 and over	1295·22	645·45	991·71	800·00
Corrected deathrates*	111·48	101·01	91·97	82·24
Crude deathrates	109·21	92·67	88·27	100·02

In the first place we take the "corrected deathrates" reduced to the male population of 1913. We have:

Pair	$M' - M$	$\sigma_{M'-M}$	$(M'-M)/\sigma_{M'-M}$	Q
London and County Boroughs ...	10·47	2·666	3·93	6·96
London and Urban Districts	19·51	2·522	7·74	8·95
London and Rural Districts	29·24	2·536	11·53	13·09
County Boroughs and Urban Districts	9·04	1·848	4·89	7·39
County Boroughs and Rural Districts	18·77	1·953	9·61	12·25
Urban Districts and Rural Districts...	9·73	1·829	5·32	6·76

To begin with it will be seen that there is a very considerable difference in the values of $(M' - M)/\sigma_{M'-M}$ and Q, or the reduction to the general population of males gives nothing like the same intensity of significance to the differences between the means as the reduction to the standard populations of maximum difference. We are unable to determine what is the standard population of *real* maximum difference in any case†, and this very fact seems to discredit the use of the deathrates corrected to an arbitrary standard population as a means of adequately testing differences in mortality. For, although in this case *all* the differences of the deathrates corrected to the general male population are significant, in the next case—for example diabetes—they may not be, while the differences of the deathrates corrected to the standard population of maximum difference may be—as in the case of diabetes practically, they are—of significance.

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* The "corrected deathrates" are here reduced to the assumed male population of 1913 of all England and Wales. They differ therefore somewhat from the Registrar-General's "corrected deathrates"—110·0, 98·9, 90·2, 80·6 respectively, which are deduced from the general population of England and Wales, 1901.

† Q may be deduced from an unreal population of maximum difference, i.e. one with some age classes negative.

If we consider the order of the significance deduced from the two standard populations we have:

Q	Pair	$(M'-M)/\sigma_{M'-M}$
13·09	London and Rural Districts	11·53
12·25	County Boroughs and Rural Districts	9·61
8·95	London and Urban Districts	7·74
7·39	County Boroughs and Urban Districts	4·89
6·96	London and County Boroughs ...	3·93
6·76	Urban Districts and Rural Districts	5·32

The Q order shows that London and the County Boroughs are markedly differentiated from the Rural and Urban Districts—in the higher degree from the former. Thus the city districts tend most to a high cancer rate. London is differentiated from the County Boroughs, and the Urban from the Rural Districts, but to a less extent than in the previous cases. The $(M'-M)/\sigma_{M'-M}$ test confirms this order except in the case of Urban and Rural Districts which are now more highly differentiated than County Boroughs and Urban Districts. Thus we note that a particular standard population may not only influence the significances of the differentiated deathrates, but also the order of these significances. This result will be confirmed in the case of diabetes.

The following are the values of Q^2 and $\chi_0{}^2$:

Pair	Q^2	$\chi_0{}^2$
London and County Boroughs ...	48·4567	49·2414
London and Urban Districts	80·0869	80·6986
London and Rural Districts	171·3498	164·2686
County Boroughs and Urban Districts	54·5901	54·3279
County Boroughs and Rural Districts	149·9172	151·0230
Urban Districts and Rural Districts ...	45·6922	46·5109

It will be seen that the only considerable difference between Q^2 and $\chi_0{}^2$ arises in the case of the London and Rural Districts pair, and more than half this difference is due to the very heavy deathrate in London from cancer of persons between 65 and 75.

Before we discuss the inferences to be drawn from these results, we will place on record the ratio of $\bar{m}$ to its standard deviation $\frac{1}{\sqrt{2u}}$. The easiest way to calculate this ratio is from the components

$$Q_s{}^2 = \frac{a_s a'_s \left(\dfrac{d_s}{a_s} - \dfrac{d'_s}{a'_s}\right)^2}{(d_s + d'_s)\left(1 - \dfrac{d_s + d'_s}{a_s + a'_s}\right)}.$$

We have
$$\bar{m} = \frac{1}{2u} S_1{}^u \left\{ \frac{\sqrt{a'_s} - \sqrt{a_s}}{\sqrt{a_s + a'_s}} \times Q_s \right\},$$
where Q_s must be given the sign of $\frac{d_s}{a_s} - \frac{d'_s}{a'_s}$. The advantage of this method is that the value of Q_s has usually been tabled as a stage to the finding of $Q^2 = S_1{}^u (Q_s{}^2)$. In the present case we deduce:

Arithmetic Value of $\bar{m} \Big/ \frac{1}{\sqrt{2u}}$.

London and County Boroughs	1·00
London and Urban Districts	1·94
London and Rural Districts	1·94
County Boroughs and Urban Districts	0·06
County Boroughs and Rural Districts	0·49
Urban Districts and Rural Districts	0·26

We can now calculate the probability that each pair are random samples of the same population by aid of our four tests. We place the pairs of districts in order of their improbability as random samples of the same population, taking $\chi_0{}^2$ as our standard test.

Probability P of the Districts being Samples of the same Population.

Paired Districts compared for Cancer Mortality	$\chi_0{}^2$ Test, or Goodness of Fit Test	Test from Difference of Corrected Deathrates $(M' - M)/\sigma_{M'-M} = Q$	Test from Distribution of Squares of Deviations $(\frac{1}{2}Q^2 - u)/\sqrt{u}$	Test from Mean of Deviations $\bar{m} \Big/ \left(\frac{1}{\sqrt{2u}}\right)$
London and Rural Districts	·9625/10^{30}	1·8031/10^{39}	1·9628/10^{144}	·3121
County Boroughs and Rural Districts ...	·5414/10^{27}	·7929/10^{34}	1·9385/10^{107}	·0262
London and Urban Districts	1·1740/10^{13}	1·7497/10^{19}	·2555/10^{23}	·0262
County Boroughs and Urban Districts ...	1·6355/10^{8}	·6606/10^{13}	·5133/10^{9}	·4761
London and County Boroughs	1·4996/10^{7}	1·6586/10^{12}	·1927/10^{6}	·1587
Urban Districts and Rural Districts ...	·4214/10^{7}	·6327/10^{11}	·1962/10^{5}	·3974

The probability* in the first of these tests is deduced from the Goodness of Fit Tables; in the remaining three tests from the Tables of the Probability Integral. It will be seen that the first three tests give absolutely the same order of significance for the six pairs of differences. The fourth test is irregular and confirms the view already expressed that but little is to be gained from its use. The fact is that each test measures a different feature of the difference of the distributions and the worst test will be that which measures the least important characteristic. There is little doubt that the deviation from zero of the mean of all the deviations measured in terms of their S.D.'s is this characteristic. The second test measures the significance of the "corrected" deathrates for the standard population of maximum significance gives results which are most closely in accord with the χ_0^2, but it suffers from two rather serious defects: (i) it is conceivable that M might be very close to M' and yet the actual distribution of deaths very different, (ii) the standard population which gives the maximum significance to the difference of the corrected deathrates may, as we have indicated (p. 164), be an impossible one. Hence the values of the significance may be very considerably exaggerated. It has been our experience, that when we have taken other standard populations, we have found the Q^2 considerably less than for the population of maximum significance, but not always most markedly less.

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* The very high improbabilities given are only rough approximations, sufficient, however, for our present purposes. Since we enter the Goodness of Fit Tables with $n = ten$ (i.e. $u+1$), we must use the first value of P in Equation (xxix), *Tables for Statisticians*, p. xxxi. The integral I may then be calculated by the first term of the Schlömilch formula (*ibid.* p. xxxiii), as this integral will only affect the fourth figure in the decimals. Table IV (*ibid.* p. 11) has been used to approximate to the extreme tails of the probability integral. Very useful work could be done by extending this Table between 5 and 50 to the first decimal place in the argument.

On the whole we think a test which considers the general distribution of deviations more likely to show definite results, than one which considers only a mean, and from this standpoint we hold that the χ_0^2 and $(\frac{1}{2}Q^2 - u)/\sqrt{u}$ are the better criteria. The main assumptions on which these tests are based are for the latter: that the distribution of squared deviations (each measured in terms of its own S.D.) will, even if each deviation be selected from a non-normal frequency, give a second moment distribution which follows the normal law; and for the former: that the Gaussian curve accurately enough describes the frequency given by a binomial. This assumption would be more closely fulfilled by a general than by a special disease deathrate, but is probably more valid than the previous assumption. Hence we believe that while the three first tests have all a certain value, the first and third are to be preferred and the first is best of all.

Judged by the first, second and third tests we conclude that significant differences can be definitely said to exist between all these cancer mortalities, Urban and Rural Districts showing the least but still a very weighty significance; that London and the County Boroughs other than London have significantly different cancer mortality, while both London and the County Boroughs differ conspicuously from the Urban and Rural Districts.

It is clear that the degree of significance is closely associated with some variate which increases with difference of position in the scale (*a*) London, (*b*) County Boroughs, (*c*) Urban Districts, (*d*) Rural Districts, i.e. with some factor which increases with the city character. The increase during the past fifty years in the cancer deathrate has been associated by some with improved diagnosis. Is the variate correlated with the above order that of better diagnosis? It may, perhaps, be doubted whether the general practitioner is much more competent in cancer diagnosis in London now-a-days than in the Rural Districts. But an examination of the terms of Q^2 or χ_0^2 shows that nearly half of the significant difference arises from the terms Q_s^2 and χ_s^2 corresponding to the 45 to 55 group and nearly a third corresponds to the 55 to 65 group. In fact in the case of London and Rural Districts about three-fourths and in the case of County Boroughs and Rural Districts about four-fifths of the value of Q^2 are contributed by the age groups 45 to 65. This is not the period in which the total number of deaths from cancer is a maximum; it may therefore be the period in which the less skilled medical man is less likely to diagnose cancer. It would be equally valid, however, to assert that cancer finds more susceptibility in town than in country dwellers and that this is particularly the case with men from 45 to 65 years of age. Further suggestions are, of course, emigration of cancerous persons to the towns* and the presence of certain occupations with high cancer deathrates in the towns. In both these cases we must find explanation for the particularly marked contributions for the age groups 45–65, unless we are content with the view that these are the age groups where the cancer deathrates are fairly high, and the population of the group with which the result is weighted fairly large.

It would be in every way desirable if we could, applying "Occam's razor," attribute to one source the rising cancer deathrate and the significant differences between cancer mortality in cities and in rural districts. Both *may* be due to differential diagnostic power or to varying accuracy of certification, but we gain little by merely throwing out suggestions, and omitting to demonstrate them.

(ix)—(xiv) *Diabetes Deathrates for all England and Wales.*

As a last illustration of the present method we take the deaths from Diabetes for the year 1913 from the Registrar-General's *Report*† in the same fundamental groupings; of course the populations at risk in age groups will remain the same.

Diabetes Deaths in Age Groups, 1913.

Age Group	(*a*) London	(*b*) County Boroughs	(*c*) Urban Districts	(*d*) Rural Districts
0—15	8	22	22	26
15—25	12	35	46	38
25—35	20	60	69	38
35—45	21	63	66	44
45—55	49	86	102	51
55—65	59	158	196	106
65—75	47	178	181	137
75—85	16	47	57	50
85 and over	0	5	6	1
Total Deaths	232	654	745	491
Crude deathrates	10·930	11·700	12·051	12·392
Corrected deathrates (Both per 100,000)	11·099	12·649	12·433	10·948

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* It must be remembered of course that institutional deaths are since 1911 distributed to their locus of origin, and therefore immigration for operation no longer tends to swell the London or County Boroughs cancer deathrates. It would be otherwise with immigration for permanent residence in the initial stages of the disease. But there is no evidence at present to show that cancerous persons of ages 45 to 65 do move into the larger towns.

† See pp. 217, 235, 253 and 271.

The "Corrected" Deathrates in the table on p. 180 are based on the Age Groups of the total population for England and Wales in 1913. It will be seen that they modify entirely the order of the Crude Deathrates, which order appeared at first sight to be comparable with the cancer results, as showing a difference between the city and rural or urban areas—in this case indeed to the advantage of the former. Proceeding as in the case of cancer to compare significance of corrected deathrates when reduced to the general population of males (1913) as standard and when reduced to the standard population of maximum difference we find:

Pair	$M'-M$	$\sigma_{M'-M}$	$(M'-M)/\sigma_{M'-M}$	Q
London and County Boroughs ...	− 1·550	·8898	1·74	4·16
London and Urban Districts	− 1·334	·8794	1·52	3·16
London and Rural Districts	+ 0·151	·8829	0·17	3·95
County Boroughs and Urban Districts	+ 0·216	·6727	0·32	2·34
County Boroughs and Rural Districts	+ 1·701	·6998	2·43	5·76
Urban Districts and Rural Districts ...	+ 1·485	·6790	2·19	5·03

Now with the single doubtful result for County Boroughs and Rural Districts none of the values of the ratio $(M'-M)/\sigma_{M'-M}$ can be definitely considered as rendering $M'-M$ significant. On the other hand all the differences of the corrected deathrates reduced to the standard population of maximum difference must, with one possible exception, i.e. County Boroughs and Urban Districts, be considered as markedly significant. This illustration is of great interest. For it is quite easy to select a standard population where for *real* age classes there is a significant difference between the "corrected" deathrates of London and the County Boroughs, but reduced to the general population (males) for 1913 there is no such difference. We see therefore that the reduction to an arbitrary standard population may be absolutely misleading as a means of testing whether two class deathrates are differentiated. Again the order of pairs for significance in the case of diabetes is for Q:

Q	Pair	$(M'-M)/\sigma_{M'-M}$
5·49	County Boroughs and Rural Districts ...	2·43
5·03	Urban Districts and Rural Districts ...	2·19
4·16	London and County Boroughs	1·74
3·95	London and Rural Districts	0·17
3·16	London and Urban Districts	1·52
2·34	County Boroughs and Urban Districts	0·32

It will be seen that the significance of London and Rural Districts is considerably displaced by the general population as standard. Or, we conclude, as in the case of cancer, that relative significance as well as absolute is determined by the special standard population selected and we cannot accept the current view that the "standard population" to which the deathrates are corrected is a matter merely of convenience.

It is clear that there is nothing in the "corrected" deathrates to indicate in any group a substantial difference from the rate 11·883 per 100,000 for all England and Wales*. It will accordingly be of some interest in this case to test whether this result of non-significance which flows from the deathrates corrected to the general population is confirmed when more accurate methods are applied to test the distribution of deviations as well as mean rates. The following are the values of Q^2 and χ_0^2:

Pair	Q^2	χ_0^2
London and County Boroughs	17·3188	18·0702
London and Urban Districts	10·0126	9·9831
London and Rural Districts	15·5939	15·4501
County Boroughs and Urban Districts	5·4693	5·4236
County Boroughs and Rural Districts	30·1470	30·4870
Urban Districts and Rural Districts ...	25·2912	25·8263

It will be seen as in previous cases that there is no substantial difference between Q^2 and χ_0^2.

The following table gives the deathrates from Diabetes per 100,000 exposed to risk in each age group of the four categories:

Male Diabetes Statistics, England and Wales, 1913.

Age Group	(*a*) London	(*b*) County Boroughs	(*c*) Urban Districts	(*d*) Rural Districts
0—15	1·23	1·22	1·11	2·09
15—25	3·13	3·48	4·08	5·30
25—35	5·51	6·39	6·78	6·55
35—45	7·16	8·22	7·89	8·87
45—55	22·94	16·02	17·38	12·90
55—65	44·89	47·70	53·00	38·29
65—75	68·99	106·77	90·06	76·10
75—85	84·97	108·99	94·36	76·39
85 and over	0	112·28	81·51	10·96

The problem before us is again whether the above distributions of deathrates as a whole are or are not significantly different, and not whether the differences in one of their statistical constants (i.e. the "corrected" deathrate) are or are not significant. We shall apply only three tests, i.e. (i) the χ_0^2 test or Goodness of Fit Test, (ii) that of the difference of corrected deathrates for the standard population of maximum difference, and (iii) the Test from Distribution of Squares of Deviations.

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* The deviations from the General Rate are (*a*) $-\cdot 784$, (*b*) $+\cdot 766$, (*c*) $+\cdot 550$ and (*d*) $-\cdot 935$, all of the order of the probable errors of the differences.

Proceeding as in the case of Cancer we have the following table:

Probability P of the two Groups being samples of the same Population.

Paired Districts compared for Diabetes Mortality	(i) χ_0^2 Test or Goodness of Fit Test	(ii) Test from Difference of Corrected Deathrates $(M'-M)/\sigma_{M'-M} = Q$	(iii) Test from Distribution of Squares of Deviations $(\frac{1}{2}Q^2-u)/\sqrt{u}$
Rural Districts and County Boroughs	·0004	·20/10^7	·0215
Rural Districts and Urban Districts ...	·0022	·25/10^6	·1121
London and County Boroughs	·0344	·000,02	·4548
Rural Districts and London	·0801	·000,04	·3442
London and Urban Districts	·3520	·000,7	·0916
Urban Districts and County Boroughs	·7943	·009,7	·0184

As before the order of significance, as we might expect, of the first two tests is the same*, but it is no longer as in the case of cancer the same as in the third test. The third test shows only significance between the members of the two pairs Rural Districts and County Boroughs and Urban Districts and County Boroughs, and neither difference is at all emphatic, while the last is out of accord with the χ_0^2 test. The χ_0^2 test shows distinct significance between Rural Districts on the one side and County Boroughs and Urban Districts on the other. The Rural Districts are not very significantly differentiated from London. Probably London and the County Boroughs have a different diabetes mortality. An examination of the table on p. 182 shows that the chief difference between the Rural Districts and the Urban and County Boroughs mortality is the much lessened deathrate after 50 years of age; while the difference between the Rural Districts and London lies in the much greater deathrate from diabetes under 45 in the Rural Districts. Such differences might be so balanced that there existed no significant difference in the corrected deathrates. No one could judge by the corrected deathrates of 10·948 and 11·099 with a probable error of ·60 that the Diabetes mortalities of Rural Districts and of London are as significantly differentiated as they are.

We see that the second test enormously exaggerates the significances determined by the first test and this is precisely what we might anticipate. The second test is the difference between the corrected deathrates, corrected to a standard population which gives the maximum difference between those rates. But this standard population is, if the individual age group deathrates are not all greater in one district than the other, an algebraical fiction and not a real standard population. But with the single possible exception of London and County Boroughs there is no approach in the case of diabetes mortality to greater deathrates in all the age groups of one district class. This was far more nearly the case in the cancer mortality. Hence the second test was more reasonable in that case.

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* The order of significance in the third test will not now be the same as in the second, although it depends only on Q^2 because $(\frac{1}{2}Q^2-u)$ can be positive or negative. It was the same in the case of cancer because Q^2 was always greater than $2u$. Positive or negative values of $(\frac{1}{2}Q^2-u)$ only signify that the mean of the squares or deviations exceed or fall short of the theoretical value respectively.

We have retained this maximum difference of corrected deathrates to the end of our illustrations, because we think it serves to indicate the danger of any argument as to significant differences in mortality based on "corrected deathrates." In the case of diabetes, our district classes give no such significant differences. But our χ_0^2 test shows that these differences actually exist, as indeed might be suspected, although their numerical valency could not be adequately tested by a mere examination of the age group deathrate table on p. 182. The reason for this failure of the corrected deathrate difference lies largely in the fact already insisted on that the significance of the difference in the corrected deathrates depends largely upon the standard population selected—a point often overlooked. What then is to be the standard population selected? Clearly it should be such as (i) to make the corrected deathrate difference a maximum and (ii) at the same time be a real population, i.e. not one with negative age classes. At present we do not see how to reach the maximum of a certain function of variates, subject to the condition that the variates are to take positive values only. The corrected deathrates for diabetes in certain districts show no significant differentiation when reduced to the general population of England and Wales as standard. They show a marked differentiation when reduced to the standard populations of maximum difference. It is true that these populations are merely algebraic fictions, but how far should we approach this marked differentiation, if we could discover the *real* maximum difference population? We cannot say; and in view of this uncertainty, it seems to us needful to drop for the present any criterion of mortality differentiation depending on the so-called corrected deathrates.

We can only conclude that the proper test for differentiated mortality is the χ_0^2 test used for the first time in the present paper. For this test does not depend on the measure of the divergence between two means—"corrected" it may be,—but on the general difference between two frequency distributions as *wholes* and this appears to us the essential feature of any true measure of differential mortality.

We have to thank our colleagues Mr A. W. Young, Mr I. Horwitz and Mr George Rae for much assistance in a piece of arithmetical work more arduous than may appear on the face of this paper.

ON CERTAIN PROBABLE ERRORS AND CORRELATION COEFFICIENTS OF MULTIPLE FREQUENCY DISTRIBUTIONS WITH SKEW REGRESSION.

BY L. ISSERLIS, D.Sc.

(1) In the systematic investigation of the statistical constants of multiple correlation and of their probable errors, it is important to have to hand the probable errors and the mutual correlations of the more fundamental constants—the means, the standard deviations and the correlation coefficients. For the case in which the frequency distribution follows the normal law this need is supplied in the memoir by Pearson and Filon entitled "On the Probable Errors of Frequency Constants and on the Influence of Random Selection on Variation and Correlation*."

As regards the more general case in which the regression is skew, the probable error of a correlation coefficient was first given by Sheppard (*Phil. Trans.* Vol. 192, A, p. 128).

The probable error of a mean and the correlation between deviations in the value of the mean and that of a standard deviation, or of a correlation coefficient, and the correlation between two standard deviations, are given by Pearson (*Biometrika*, Vol. IX. 1913, pp. 1–10).

For reference we give here the results for the case of normal distributions obtained by Pearson and Filon in the memoir referred to above:

$$\Sigma_{\sigma_1} = \sigma_1/\sqrt{2n} \quad \text{..........} (1),$$

$$\Sigma_{r_{12}} = (1 - r^2_{12})/\sqrt{n} \quad \text{..........} (2),$$

$$R_{\sigma_1\sigma_2} = r^2_{12} \quad \text{..........} (3),$$

$$R_{\sigma_1 r_{12}} = r_{12}/\sqrt{2} \quad \text{..........} (4),$$

$$R_{\sigma_1 r_{23}} = \frac{r_{12}(r_{13} - r_{12}r_{23}) + r_{13}(r_{12} - r_{13}r_{23})}{\sqrt{2}\,.\,(1 - r^2_{23})} \quad \text{..........} (5),$$

$$R_{r_{12}r_{13}} = r_{23} - \frac{r_{12}r_{13}(1 - r^2_{23} - r^2_{13} - r^2_{12} + 2r_{12}r_{13}r_{23})}{2(1 - r^2_{12})(1 - r^2_{13})} \quad \text{..........} (6),$$

$$R_{r_{12}r_{34}} = \frac{\left\{\begin{array}{l} (r_{13} - r_{12}r_{23})(r_{24} - r_{23}r_{34}) + (r_{14} - r_{34}r_{13})(r_{23} - r_{12}r_{13}) \\ + (r_{13} - r_{14}r_{34})(r_{24} - r_{12}r_{14}) + (r_{14} - r_{12}r_{24})(r_{23} - r_{24}r_{34}) \end{array}\right\}}{2(1 - r^2_{12})(1 - r^2_{34})} \quad (7)\dagger.$$

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* *Phil. Trans.* Vol. 191, A (1898), pp. 229–311.

† *Phil. Trans.* Vol. 191, A, Equations (xv)–(xviii), (xxxvi), (xxxvii) and (xl).

In the present paper the corresponding results are obtained for the case of skew regression. The method employed is different and by supposing the regression to be linear and the distribution to be normal, a confirmation is obtained of the above results which in Pearson and Filon's memoir depend on very complicated analysis.

(2) We may begin by discussing the correlation that exists between deviations from their means in the case of two correlation coefficients r_{xy} and r_{zt}.

We have
$$r_{xy} = p_{xy}/\sqrt{(p_{x^2}p_{y^2})} \quad \text{......(8)},$$
$$r_{zt} = p_{zt}/\sqrt{(p_{z^2}p_{t^2})} \quad \text{......(9)},$$
where $p_{x^l y^m z^n t^k}$ is employed to denote the mixed moment of orders l, m, n, k in the variables, taken about the means so that
$$\frac{dr_{xy}}{r_{xy}} = \frac{dp_{xy}}{p_{xy}} - \frac{1}{2}\frac{dp_{x^2}}{p_{x^2}} - \frac{1}{2}\frac{dp_{y^2}}{p_{y^2}} \quad \text{......(10)},$$
and
$$\frac{dr_{zt}}{r_{zt}} = \frac{dp_{zt}}{p_{zt}} - \frac{1}{2}\frac{dp_{z^2}}{p_{z^2}} - \frac{1}{2}\frac{dp_{t^2}}{p_{t^2}} \quad \text{......(11)}.$$

It is clear that we shall require the correlations between any one of p_{xy}, p_{x^2}, p_{y^2} and any one of p_{zt}, p_{z^2} and p_{t^2}. It will suffice to find the correlation between p_{xy} and p_{zt}.

Now
$$Np_{xy} = \underset{x}{S}\,\underset{y}{S}\{n_{xy}(x-\bar{x})(y-\bar{y})\} \quad \text{......(12)},$$
$$\therefore \quad Ndp_{xy} = \underset{x}{S}\,\underset{y}{S}\{dn_{xy}(x-\bar{x})(y-\bar{y})\}$$
$$+ \underset{x}{S}\,\underset{y}{S}\{-n_{xy}(y-\bar{y})\,d\bar{x}\} + \underset{x}{S}\,\underset{y}{S}\{-n_{xy}(x-\bar{x})\,d\bar{y}\} \quad \text{......(13)},$$
or
$$Ndp_{xy} = \underset{x}{S}\,\underset{y}{S}\{dn_{xy}XY\} \quad \text{......(14)},$$
if we denote the total population by N, $x-\bar{x}$ by X, $y-\bar{y}$ by Y and remember that
$$\underset{x}{S}\{(x-\bar{x})\,n_{xy}\} = \underset{y}{S}\{(y-\bar{y})\,n_{xy}\} = 0.$$
Similarly
$$Ndp_{zt} = \underset{z}{S}\,\underset{t}{S}\{dn_{zt}ZT\} \quad \text{......(15)}.$$

The mean value of $dn_{xy}dn_{zt}$ in many samples is the mean value of
$$\underset{h}{S}\,\underset{h'}{S}\{dn_{x_s y_{s'} z_h t_{h'}}\} \times \underset{k}{S}\,\underset{k}{S}\{dn_{x_k y_{k'} z_l t_{l'}}\}$$
$$= -\frac{1}{N}\underset{h}{S}\,\underset{h'}{S}\,\underset{k}{S}\,\underset{k'}{S}\{dn_{x_s y_{s'} z_h t_{h'}}dn_{x_k y_{k'} z_l t_{l'}}\} + n_{x_s y_{s'} z_l t_{l'}}(1 - n_{x_s y_{s'} z_l t_{l'}}/N) \quad \text{...(16)},$$
where in the fourfold summation the term
$$(dn_{x_s y_{s'} z_l t_{l'}})^2$$
is omitted.

But clearly the right-hand member of (16) reduces to
$$n_{x_s y_{s'} z_l t_{l'}} - n_{x_s y_{s'}}n_{z_l t_{l'}}/N,$$
hence the mean value of
$$N^2 dp_{xy}dp_{zt} \quad \text{is} \quad N(p_{xyzt} - p_{xy}p_{zt}) \quad \text{......(17)}.$$

Putting $t = z$ we deduce from (17)

$$\text{Mean value of } dp_{xy}dp_{z^2} = (p_{xyz^2} - p_{xy}p_{z^2})/N \quad \text{...............(18)},$$

and putting $y = x$ in this result,

$$\text{Mean value of } dp_{x^2}dp_{z^2} = (p_{x^2z^2} - p_{x^2}p_{z^2})/N \quad \text{...............(19)}.$$

If we multiply (10) and (11), sum for all samples and divide by the number of samples we deduce

$$\begin{aligned} &N\sigma_{r_{xy}}\sigma_{r_{zt}}R_{r_{xy},r_{zt}}/r_{xy}r_{zt} \\ &\quad = \frac{p_{xyzt} - p_{xy}p_{zt}}{p_{xy}p_{zt}} - \frac{1}{2}\frac{(p_{xyz^2} - p_{xy}p_{z^2})}{p_{xy}p_{z^2}} - \frac{1}{2}\frac{(p_{xyt^2} - p_{xy}p_{t^2})}{p_{xy}p_{t^2}} \\ &\quad - \frac{1}{2}\frac{(p_{x^2zt} - p_{x^2}p_{zt})}{p_{x^2}p_{zt}} - \frac{1}{2}\frac{(p_{y^2zt} - p_{y^2}p_{zt})}{p_{y^2}p_{zt}} + \frac{1}{4}\frac{(p_{x^2z^2} - p_{x^2}p_{z^2})}{p_{x^2}p_{z^2}} \\ &\quad + \frac{1}{4}\frac{(p_{x^2t^2} - p_{x^2}p_{t^2})}{p_{x^2}p_{t^2}} + \frac{1}{4}\frac{(p_{y^2z^2} - p_{y^2}p_{z^2})}{p_{y^2}p_{z^2}} + \frac{1}{4}\frac{(p_{y^2t^2} - p_{y^2}p_{t^2})}{p_{y^2}p_{t^2}} \quad \text{......(20)}. \end{aligned}$$

This result like Sheppard's formula for $\sigma^2_{r_{xy}}$ is much simpler when expressed in *reduced moments*. Let us write

$$\frac{p_{x^ly^mz^nt^k}}{\sigma_x{}^l\sigma_y{}^m\sigma_z{}^n\sigma_t{}^k} = q_{x^ly^mz^nt^k},$$

so that q_{x^2} is unity and $q_{xy} = r_{xy}$. The numerical term in (20) is

$$-1 + \tfrac{1}{2}(4) + \tfrac{1}{4}(-4)$$

or zero, hence

$$\begin{aligned} &N\sigma_{r_{xy}}\sigma_{r_{zt}}R_{r_{xy},r_{zt}}/r_{xy}r_{zt} \\ &\quad = \frac{q_{xyzt}}{r_{xy}r_{zt}} - \frac{1}{2}\left(\frac{q_{xyz^2} + q_{xyt^2}}{r_{xy}} + \frac{q_{x^2zt} + q_{y^2zt}}{r_{zt}}\right) + \frac{1}{4}(q_{x^2z^2} + q_{x^2t^2} + q_{y^2z^2} + q_{y^2t^2}) \\ &\qquad \text{.........(21)}. \end{aligned}$$

In the same notation Sheppard's formula becomes

$$\frac{\sigma^2_{r_{xy}}}{r^2_{xy}} = \frac{1}{N}\left\{\frac{q_{x^2y^2}}{r^2_{xy}} + \tfrac{1}{4}(\beta_2 + \beta'_2) + q_{x^2y^2} - \frac{q_{x^3y} + q_{xy^3}}{r_{xy}}\right\} \quad \text{.........(22)*}.$$

To find the correlation between r_{xy} and r_{xz} we have only to replace t by x in (21), thus

$$\begin{aligned} &N\sigma_{r_{xy}}\sigma_{r_{xz}}R_{r_{xy},r_{xz}}/r_{xy}r_{xz} \\ &\quad = \frac{q_{x^2yz}}{r_{xy}r_{xz}} - \frac{1}{2}\left(\frac{q_{xyz^2} + q_{x^3y}}{r_{xy}} + \frac{q_{x^3z} + q_{xy^2z}}{r_{xz}}\right) + \frac{1}{4}(q_{x^2z^2} + q_{x^4} + q_{x^2y^2} + q_{y^2z^2}) \\ &\qquad \text{.........(23)}. \end{aligned}$$

(3) These correlation coefficients will simplify if the regression be linear and simplify to a considerable extent if at the same time the distribution be normal. For with *linear regression*

$$\begin{aligned} Np_{x^2yz} &= \underset{x}{S}\,\underset{y}{S}\,\underset{z}{S}\,(n_{xyz}x^2yz)\dagger \\ &= \underset{x}{S}\,\underset{y}{S}\,(n_{xy}x^2y \times \bar{z}_{xy}), \end{aligned}$$

where $\bar{z}_{xy}$ is the mean value of z for given values of x and y.

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* For the denominator of left-hand side, cf. *Biometrika*, Vol. IX. p. 4.

† The origin being taken at the mean.

But from the usual regression equation

$$\bar{z}_{xy} = x\left(\frac{r_{xz} - r_{xy}r_{yz}}{1 - r^2_{xy}}\right)\frac{\sigma_z}{\sigma_x} + y\left(\frac{r_{yz} - r_{xz}r_{xy}}{1 - r^2_{xy}}\right)\frac{\sigma_z}{\sigma_y},$$

so that for linear regression

$$p_{x^2yz} = \left(\frac{r_{xz} - r_{xy}r_{yz}}{1 - r^2_{xy}}\right)\frac{\sigma_z}{\sigma_x}\,p_{x^3y} + \left(\frac{r_{yz} - r_{xz}r_{xy}}{1 - r^2_{xy}}\right)\frac{\sigma_z}{\sigma_y}\,p_{x^2y^2} \quad \text{............(24).}$$

Further

$$\begin{aligned} p_{x^3y} &= \frac{1}{N}\underset{x}{S}\,\underset{y}{S}\,(n_{xy}x^3y) \\ &= \frac{1}{N}\underset{x}{S}\,(n_x x^3 \bar{y}_x) \\ &= \frac{1}{N}\underset{x}{S}\left(n_x x^4 \frac{\sigma_y}{\sigma_x} r_{xy}\right), \end{aligned}$$

or

$$p_{x^3y} = p_x\, r_{xy}\frac{\sigma_y}{\sigma_x} \quad \text{.......................................(25),}$$

while

$$q_{x^2y^2} = 1 + r^2_{xy}\sqrt{(\beta_2 - 1)(\beta'_2 - 1)} \text{ approximately} \quad \text{.........(26)*,}$$

so that (23) can be evaluated approximately by the use of simple moment coefficients and correlation coefficients only. *If in addition the distribution be normal,* we know that

$$p_{x^3y} = 3p_{xy}p_{x^2} \text{ and } p_{x^2y^2} = (1 + 2r^2_{xy})/p_{x^2}p_{y^2},$$

so that for normal distributions

$$\frac{p_{x^2yz}}{p_{xy}p_{xz}} = \left(\frac{r_{xz} - r_{xy}r_{yz}}{1 - r^2_{xy}}\right)\frac{3}{r_{xz}} + \frac{r_{yz} - r_{xz}r_{xy}}{1 - r^2_{xy}}\,\frac{1 + 2r^2_{xy}}{r_{xy}r_{xz}},$$

or

$$\frac{q_{x^2yz}}{r_{xy}r_{xz}} = 2 + r_{yz}/r_{xy}r_{xz} \quad \text{.....................................(27).}$$

Similarly

$$\frac{q_{xyz^2}}{r_{xy}} = 1 + \frac{2r_{yz}r_{xz}}{r_{xy}} \quad \text{.......................................(28),}$$

and

$$\frac{q_{xy^2z}}{r_{xz}} = 1 + 2r_{yz}r_{xy}/r_{xz} \quad \text{.....................................(29),}$$

Substituting these values in (23) we obtain, after some reduction and using

$$\sigma_{r_{xy}} = (1 - r^2_{xy})/\sqrt{N},$$

$$\begin{aligned} &(1 - r^2_{xy})(1 - r^2_{xz})\,R_{r_{xy}r_{xz}} \\ &\quad = r_{yz}(1 - r^2_{xy})(1 - r^2_{xz}) - \tfrac{1}{2}r_{xy}r_{xz}(1 - r^2_{xy} - r^2_{yz} - r^2_{zx} + 2r_{xy}r_{yz}r_{zx}) \\ &\qquad\qquad \text{.........(30),} \end{aligned}$$

agreeing with (6) the value obtained by Pearson and Filon for normal distributions.

As regards the more general case dealing with the correlation between deviations of r_{xy} and those of r_{zt} given by equation (22), we have *when the regression is linear*

$$\begin{aligned} p_{xyzt} &= \underset{x}{S}\,\underset{y}{S}\,\underset{z}{S}\,\underset{t}{S}\,\{n_{xyzt}(xyzt)\} \\ &= \underset{x}{S}\,\underset{y}{S}\,\underset{z}{S}\,\{n_{xyz}(xyz\bar{t}_{xyz})\}, \end{aligned}$$

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* *Biometrika,* Vol. IX. p. 4.

where $\bar{t}_{xyz}$ is the mean value of t for given values of x, y and z, so that as is well known

$$\bar{t}_{xyz} = -x\frac{\sigma_t}{\sigma_x}\frac{\Delta_{xt}}{\Delta_{tt}} - y\frac{\sigma_t}{\sigma_y}\frac{\Delta_{yt}}{\Delta_{tt}} - z\frac{\sigma_t}{\sigma_z}\frac{\Delta_{zt}}{\Delta_{tt}} \quad\text{..................(31),}$$

where

$$\Delta = \begin{vmatrix} 1, & r_{xy}, & r_{xz}, & r_{xt} \\ r_{xy}, & 1, & r_{yz}, & r_{yt} \\ r_{xz}, & r_{yz}, & 1, & r_{zt} \\ r_{xt}, & r_{yt}, & r_{zt}, & 1 \end{vmatrix} \quad\text{..............................(32),}$$

and Δ_{pq} is the minor corresponding to r_{pq}. Thus

$$q_{xyzt} = -(\Delta_{xt} q_{x^2yz} + \Delta_{yt} q_{xy^2z} + \Delta_{zt} q_{xyz^2})/\Delta_{tt} \quad\text{...............(33),}$$

so that $R_{r_{xy} \cdot r_{zt}}$ can be evaluated approximately in the case of linear regression without employing any mixed moments beyond the simple product moment occurring in a correlation coefficient.

For *normal distributions* we may use (27), 28) and (29) giving

$$q_{xyzt} = -[\Delta_{xt}(2r_{xy}r_{xz} + r_{yz}) + \Delta_{yt}(2r_{xy}r_{yz} + r_{xz}) + \Delta_{zt}(2r_{xz}r_{yz} + r_{xy})]/\Delta_{tt} \quad\text{...(34).}$$

By well-known properties of first minors of a determinant we have from (32)

$$\Delta_{xt} + r_{xy}\Delta_{yt} + r_{xz}\Delta_{zt} + r_{xt}\Delta_{tt} = 0 \quad\text{........................(35),}$$

$$r_{xy}\Delta_{xt} + \Delta_{yt} + r_{yz}\Delta_{zt} + r_{yt}\Delta_{tt} = 0 \quad\text{........................(36),}$$

$$r_{xz}\Delta_{xt} + r_{yz}\Delta_{yt} + \Delta_{zt} + r_{zt}\Delta_{tt} = 0 \quad\text{........................(37).}$$

Multiply these equations by r_{yz}, r_{zx}, r_{xy} respectively and add,

$$\therefore\ (r_{yz} + 2r_{xz}r_{xy})\,\Delta_{xt} + (r_{xz} + 2r_{yz}r_{xy})\,\Delta_{yt}$$
$$+ (r_{xy} + 2r_{yz}r_{xz})\,\Delta_{zt} + (r_{yz}r_{xt} + r_{zx}r_{yt} + r_{xy}r_{zt})\,\Delta_{tt} = 0 \quad\text{......(38).}$$

Combining this result with (33) we see that for normal distributions

$$q_{xyzt} = r_{xy}r_{zt} + r_{yz}r_{xt} + r_{zx}r_{yt} \quad\text{........................(39)*,}$$

an interesting result likely to prove useful in other applications and probably capable of generalisation Particular cases of (39) are obtained by putting $t = x$ so that $q_{x^2yz} = r_{yz} + 2r_{xy}r_{xz}$ which is (27) and $t = x$, $z = y$ giving $q_{x^2y^2} = 1 + 2r^2_{xy}$ which is well known.

If we now substitute these values in equation (21) we find

$$N\sigma_{r_{xy}}\sigma_{r_{zt}}R_{r_{xy}r_{zt}}$$
$$= 2r_{yz}r_{xt} + 2r_{zx}r_{yt} - 2r_{xz}r_{yz}r_{zt} - 2r_{xt}r_{yt}r_{zt}$$
$$- 2r_{xz}r_{xt}r_{xy} - 2r_{yz}r_{yt}r_{xy} + r_{xy}r_{zt}(r^2_{xz} + r^2_{xt} + r^2_{yz} + r^2_{yt}).$$

The right-hand member can be put in the form

$$\tfrac{1}{2}\{(r_{xt} - r_{xz}r_{zt})(r_{yz} - r_{xy}r_{xz}) + (r_{xt} - r_{xy}r_{yt})(r_{yz} - r_{yt}r_{zt})$$
$$+ (r_{xz} - r_{xy}r_{yz})(r_{yt} - r_{yz}r_{zt}) + (r_{xz} - r_{xt}r_{yz})(r_{yt} - r_{xy}r_{xt})\},$$

---

* This result, which is accurate for normal distributions, is given as approximately true for such distributions by H. E. Soper, *Biometrika*, Vol. IX. p. 100.

and if we remember that for normal distributions

$$\sigma_{r_{xy}} = (1 - r^2_{xy})/\sqrt{N}, \quad \sigma_{r_{zt}} = (1 - r^2_{zt})/\sqrt{N},$$

this result agrees with Pearson and Filon's value quoted above as equation (7).

(4) To find the probable error of a standard deviation.

$$\sigma^2_x = p_{x^2},$$

$$\therefore \quad \frac{d\sigma_x}{\sigma_x} = \frac{dp_{x^2}}{2p_{x^2}} \quad \text{...(40).}$$

Hence
$$\frac{\Sigma^2_{\sigma_x}}{\sigma^2_x} = \frac{p_{x^4} - p^2_{x^2}}{4p^2_{x^2}N} \text{ by (17)}$$

$$= \frac{\beta_2 - 1}{4N},$$

$$\therefore \quad \Sigma_{\sigma_x} = \frac{\sigma_x\sqrt{\beta_2 - 1}}{2\sqrt{N}} \quad \text{...(41).}$$

This result is well known, and for normal distributions, i.e. when $\beta_2 = 3$, becomes $\Sigma_{\sigma_x} = \dfrac{\sigma_x}{\sqrt{2N}}$, agreeing with (1).

To find the correlation between a standard deviation σ_x and a correlation coefficient r_{yz}, we multiply (40) by the equation

$$\frac{dr_{yz}}{r_{yz}} = \frac{dp_{yz}}{p_{yz}} - \frac{dp_{y^2}}{2p_{y^2}} - \frac{dp_{z^2}}{2p_{z^2}},$$

and sum for all samples and divide by their number in the usual way, obtaining

$$2N\sigma_{\sigma_x}\sigma_{r_{yz}}R_{\sigma_x, r_{yz}}/\sigma_x r_{yz}$$

$$= (p_{x^2yz} - p_{x^2}p_{yz})/p_{x^2}p_{yz} - \tfrac{1}{2}(p_{x^2y^2} - p_{x^2}p_{y^2})/p_{x^2}p_{y^2} - \tfrac{1}{2}(p_{x^2z^2} - p_{x^2}p_{z^2})/p_{x^2}p_{z^2}$$

$$= \frac{q_{x^2yz}}{r_{yz}} - \frac{q_{x^2y^2} + q_{x^2z^2}}{2} \quad \text{...(42),}$$

a result which as before can be approximated to in the case of linear regression, and which for normal distributions becomes*

$$R_{\sigma_x, r_{yz}} = \frac{\dfrac{r_{yz} + 2r_{xy}r_{xz}}{r_{yz}} - \tfrac{1}{2}(2 + 2r^2_{xy} + 2r^2_{xz})}{2N \cdot \dfrac{\sigma_x}{\sqrt{2N}} \cdot \dfrac{(1 - r^2_{yz})}{\sqrt{N}} \cdot \dfrac{1}{\sigma_x r_{yz}}}$$

$$= \frac{2r_{xy}r_{xz} - (r^2_{xy} + r^2_{xz})\,r_{yz}}{\sqrt{2}\,.\,(1 - r^2_{yz})} \quad \text{...(43),}$$

agreeing with equation (5).

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* For the case $z = x$, i.e. $R_{\sigma_x, r_{xy}}$, cf. *Biometrika*, Vol. IX. p. 8.

ON THE CORRELATION BETWEEN THE "CORRECTED" CANCER AND DIABETES DEATHRATES.

By C. A. CLAREMONT, B.Sc., Biometric Laboratory, University College.

In a paper due to Maynard published in this journal* it is shown that in the United States in the case of both states and great towns there is a marked correlation between the corrected deathrates from cancer and diabetes. The point is of very great interest, and additional investigations by Pearson and others† and by Greenwood and Frances Wood‡ confirm Maynard's result for the material used by him. On the other hand Greenwood and Wood show that it is not apparently true for Switzerland.

Maynard writes§: "If the increased rates observed in the cases of cancer and diabetes were due to a common cause then it is probable that their rates of growth will be found to be fairly highly correlated. Not being able to obtain rates for a sufficient period of time from the United States reports, the rates given in the Registrar's Report for England and Wales, in five-yearly groups for the 35 years 1871 to 1905, were used and here $\rho = \cdot 8060 \pm \cdot 0893$. This high correlation shows I think a strong probability that there is a common factor influencing the increase of both diseases. That this value is explicable on the assumption that the increased rates are merely apparent and due to more careful diagnosis is in view of the facts already mentioned almost inconceivable."

In view of the development of the variate difference correlation method since the publication of Maynard's memoir, it seemed possible to test this point, and at the suggestion of Professor Pearson I undertook the necessary calculations‖. It is to be noted that it is partly the continuous rise in the deathrates from these two diseases during the last few decades that has suggested a possibly organic relation between them. This increase is effectively shown in the Diagrams of the corrected deathrates I and II for these diseases, and in the combined "spot" diagram for individual years, III, which brings out the contemporary rises.

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* *Biometrika*, Vol. VII. pp. 276–304.

† *Journal of R. Statistical Society*, Vol. 73, p. 534.

‡ *Journal of Hygiene*, Vol. XIV. p. 83, 1914.

§ *loc. cit.* p. 289.

‖ I have heartily to thank Miss B. C. B. Cave for revision of my arithmetic and the correction of several errors.

DIAGRAM I. Cancer. Corrected Deathrate per Million. Males.

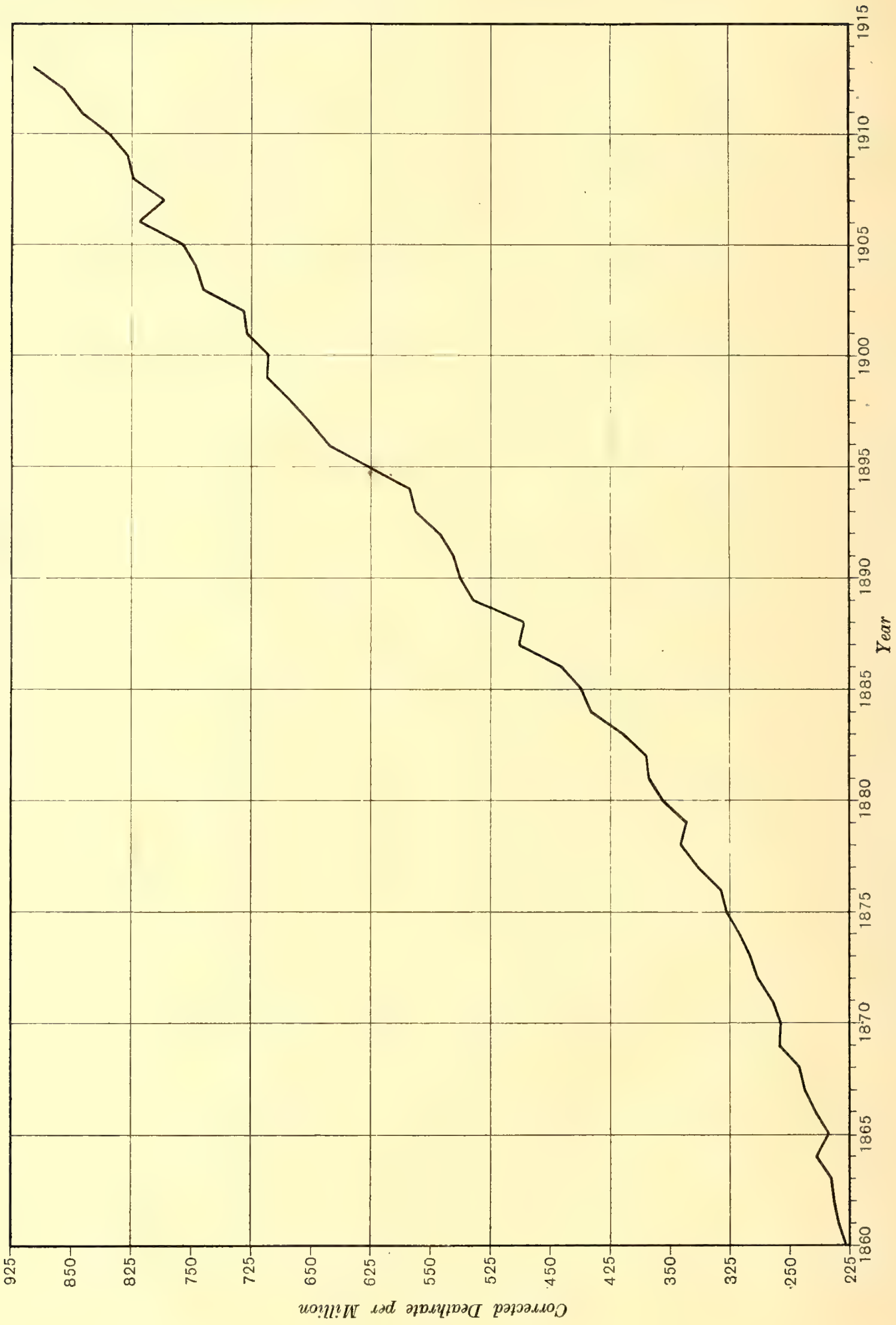


Diagram showing rapid rise of *corrected* cancer deathrate for males between 1860 and 1915.

DIAGRAM II. Diabetes. Corrected Deathrate per Million.

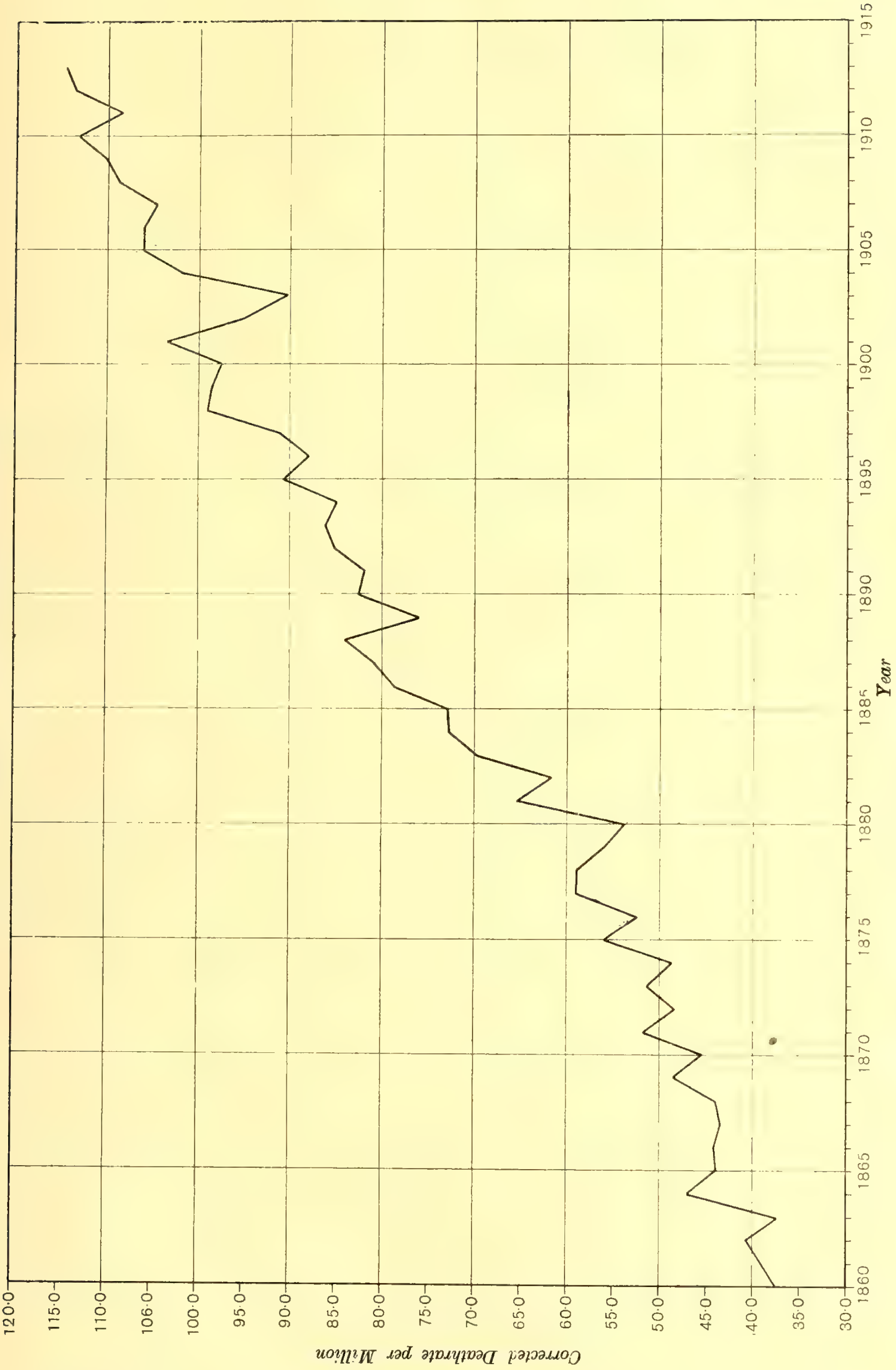
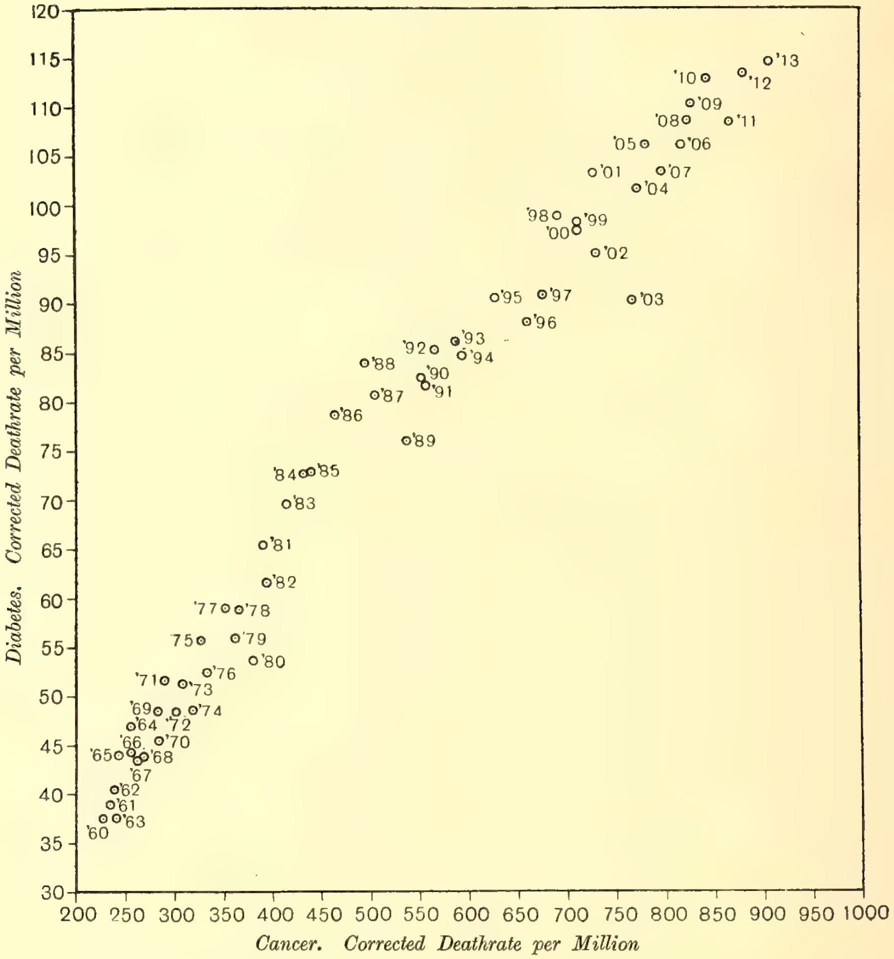


Diagram showing rapid rise of *corrected* diabetes deathrate for males between 1860 and 1915.

Diagram III. Cancer and Diabetes. Corrected Male Deathrates showing scatter of pairs of occurrences in the same years.



The diagram indicates an *apparent* highly correlated change.

It should be noted, however, that Newsholme and King* conclude that the increase of cancer is apparent only and due to improvement in diagnosis and more careful certification of causes of death. Whether there has been a real increase in diabetes is a question that these authors consider to be undecided. It is difficult, however, in the case of England and Wales to believe that the increase in the *corrected* deathrates for both cancer and diabetes that has gone on continuously since 1900 can have anything to do with increased accuracy

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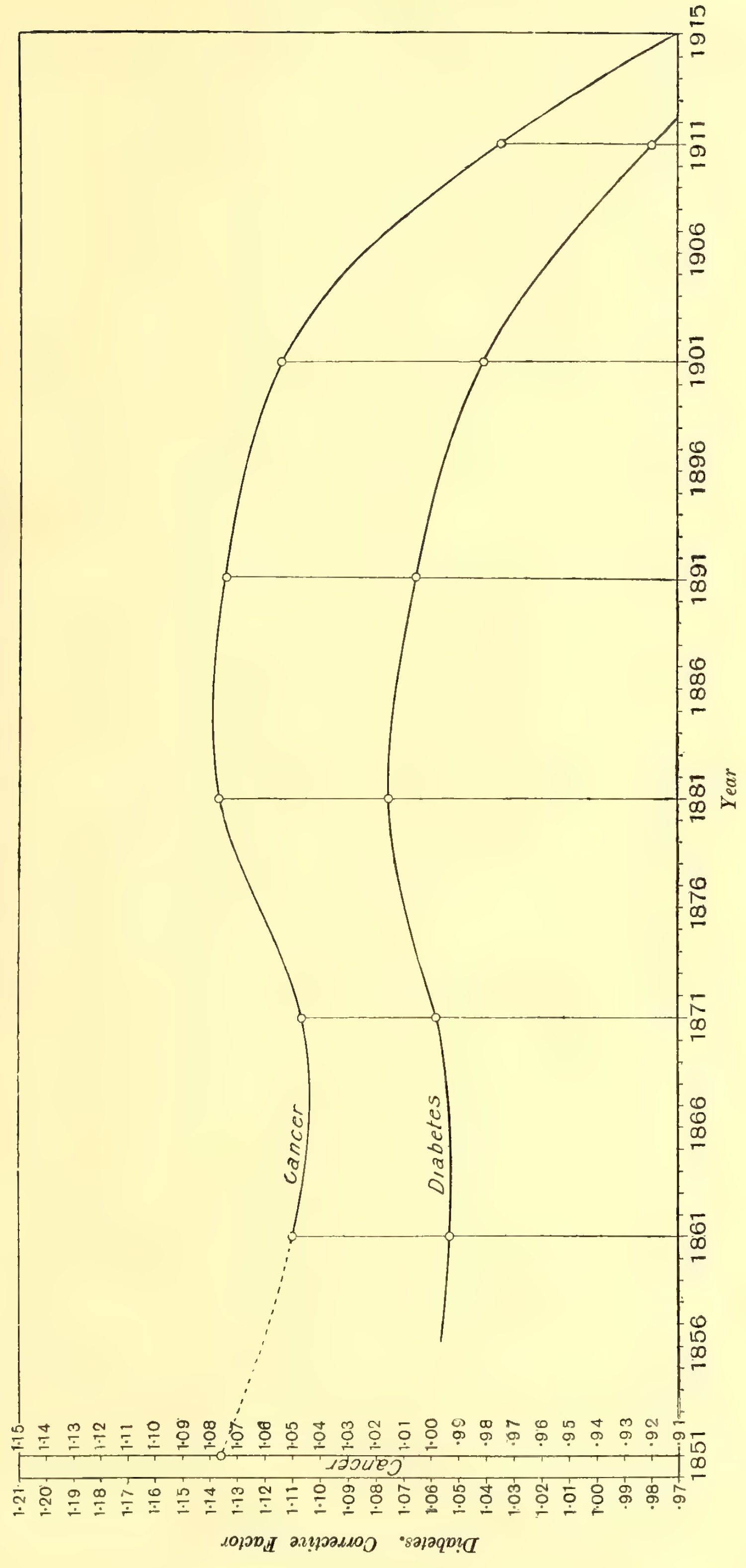
* *R. S. Proc.* Vol. LIV. p. 228 (1893). An examination of our Diagram I shows that the increase of the corrected cancer deathrate from 1873 to 1893 was practically the same as from 1893 to 1913. If improved diagnosis is the source of the increased cancer rate it is certainly remarkable that the improvement should have been so uniform for the space of forty years.

DIAGRAM IV. Diabetes and Cancer. Corrective Factors.



Facsimile of large diagram used for deducing corrective factors for intercensal years from known corrective factors for census years.

of diagnosis of either disease in this recent period, and the general sweep of the plotted curves seems to show that the source of the increase whatever its nature has been continuous since the middle of last century.

In the following investigation I have correlated the deaths from cancer and diabetes for the *male* population only. I had intended to deal also with the female population, but the results for the male were so conclusive, that I did not think it needful to go further. But the idea of dealing with the female deathrates led me to select as standard population the total male and female population of England and Wales as given by the 1901 census. Of course the choice of the standard population to obtain a corrected deathrate is arbitrary, but if female deathrates are to be compared with male, we must take a common standard for both, and this is why the combined male and female age groups of 1901 were taken as the standard population classes.

DIAGRAM V. Associated Inflexion Points in Corrective Factors and °/₀ Males under 45.

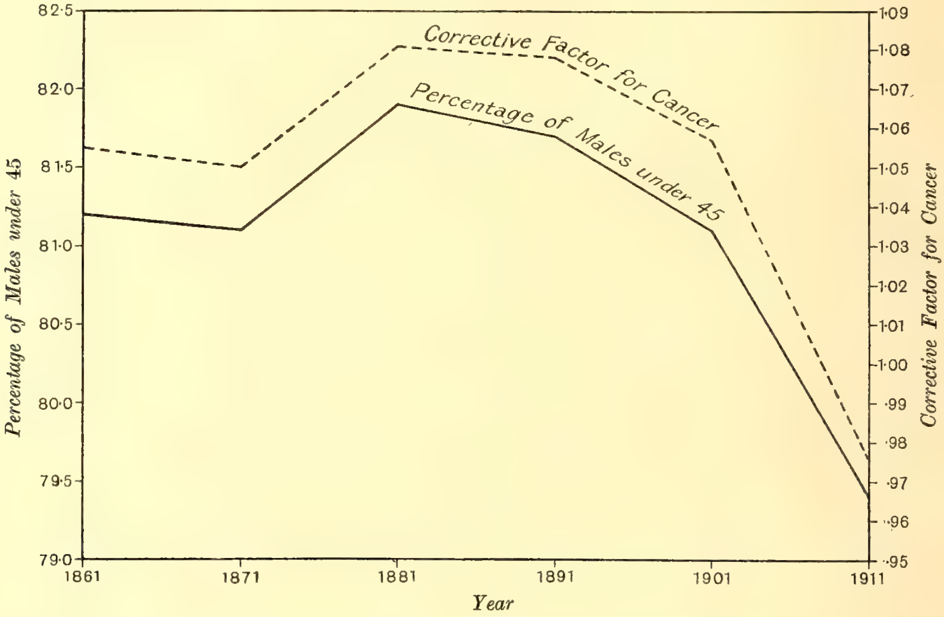


Diagram illustrating that the inflexion point in the corrective factor diagram (IV) corresponds to actual changes in the age distribution of the population.

A word must be said as to the manner of correcting the crude deathrate for age groups. Actually we only know the age groups accurately for the census years; the age groups for intervening years are more or less guesses not beyond suspicion. Accordingly it was considered best to obtain the corrective factors for the crude deathrates in each census year, then to plot these to the year, draw a continuous curve through the plotted points by aid of a spline and read off from this curve the corrective factors for non-census years. This was done on a large diagram of which IV is a small reproduction.

A noteworthy feature of these corrective factor curves is the inflexion which occurs between 1861 and 1871. There are two tests of its reality. First the diabetes curve confirms the cancer curve and secondly as far as the data permit an approximative factor was obtained for the cancer corrective factor curve for the census of 1851; it is seen to substantiate the general sweep of the corrective factor curve and confirms the inflexion between 1861 and 1871. Diagram V shows that the inflexion also occurs in the age distribution of the population as exhibited in parentage of males under 45.

In dealing with the years 1875 to 1880, the few deaths separately recorded under Melanosis, Fungus Haematodes and Sweep's Cancer—now entitled simply cancer—were included under the total deaths from cancer.

The following table showing the work for the year 1911 indicates the manner in which the census year corrective factors were found. The crude deathrates per million of the male population in this year were: Diabetes 110·96 and Cancer 893·57.

TABLE I. *Corrective Factors for Census Year* 1911.

Age Groups	Cancer Deaths (Males)	Diabetes Deaths (Males)	Census Population (Males), 1911	Deathrate per million, Cancer $=r_1$	Death-rate per million, Diabetes $=r_2$	Standard Population, 1901, per million A	Corrected Deathrates	
							Cancer $=\frac{r_1 A}{10^6}$	Diabetes $=\frac{r_2 A}{10^6}$
Under *5*	64	11	1,936,113	33·1	5·7	114,262	3·78	·65
5—10	51	22	1,847,295	27·6	11·9	107,209	2·96	1·27
10—15	39	36	1,747,631	22·3	20·6	102,735	2·29	2·12
15—20	60	67	1,654,895	36·3	40·5	99,796	3·62	4·04
20—25	95	71	1,502,652	63·2	47·3	95,946	6·06	4·54
25—35	317	157	2,831,655	111·9	55·4	161,580	18·08	8·95
35—45	978	183	2,336,508	418·6	78·3	122,848	51·42	9·62
45—55	2901	252	1,694,333	1712·2	148·7	89,222	152·77	13·27
55—65	4627	461	1,085,156	4263·9	424·8	59,741	254·73	25·38
65—75	4602	512	602,764	7634·8	849·4	33,080	252·56	28·10
75—85	1687	155	183,869	9175·0	843·0	12,090	110·93	10·19
85—	168	9	22,737	7388·8	395·8	1,491	11·02	·59
All ages	15,589	1936	17,445,608	893·57	110·96	1,000,000	870·22	108·72

Accordingly for 1911 we have corrective factors for:

$$\text{Cancer} = {}_1F_{1911} = 870\cdot22/893\cdot57 = \cdot97386.$$

$$\text{Diabetes} = {}_2F_{1911} = 108\cdot72/110\cdot96 = \cdot97981.$$

The accompanying Table II gives under each year the total male population, the total deaths from cancer and from diabetes, the crude deathrates, the corrective factors and the corrected deathrates. From the latter the first six differences, Δ_1 to Δ_6, were found and these were correlated for the two diseases. The following correlations were found:

Correlation of Differences of Corrected Deathrates.

Corrected Deathrates	+ ·958 ± ·008,
First Differences	+ ·058 ± ·113,
Second ,, ...	+ ·043 ± ·130,
Third ,, ...	+ ·047 ± ·143,
Fourth ,, ...	+ ·051 ± ·154,
Fifth ,, ...	+ 050 ± ·164,
Sixth ,, ...	+ ·045 ± ·173.

It will be seen that with the first differences we get an enormous drop in the correlation of the cancer and diabetes deathrates, i.e. from + ·958 to + ·058, and that very rapidly the correlation becomes steady* and insignificant. Thus with the removal of the time factor there appears to be no organic relationship between the prevalence of diabetes and cancer. A year in which there is an excess of diabetes deaths is not a year which will probably have an excess of cancer deaths.

Now this result although absolutely conclusive as far as it goes, must not be stretched beyond its exact limitation. There is nothing to show that an increase of cancer at any given epoch will be accompanied by an increase of diabetes. It might be argued that it is conceivable that an increase of diabetes will be *followed* at an interval by an increase of cancer. I have not directly tested this, but it is to some extent indirectly tested by the method of variate differences. The sixth difference correlation correlates functions of the deathrates for seven years in the case of two diseases, and would be likely to indicate if there were any such related succession. At the same time it must be remarked that previous investigators have all dealt with the *contemporaneous* deathrates of cancer and

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* This steadiness can be illustrated also by the method discussed in *Biometrika*, Vol. x. p. 272 and illustrated on p. 346.

Values of $\sigma^2_{\Delta^m x}/\sigma^2_{\Delta^{m-1} x}$ *and their approach to* $4-\frac{2}{m}$.

m	Theoretical Ratio $=4-\frac{2}{m}$	Cancer	Diabetes	Mean
1	2·000	0·655	0·830	0·742
2	3·000	2·589	2·969	2·779
3	3·333	3·431	3·386	3·409
4	3·500	3·446	3·549	3·497
5	3·600	3·719	3·627	3·673
6	3·667	3·653	3·669	3·661

It will be seen that there is a rapid approach to the theoretical values of the ratio.

TABLE II. *Cancer and Diabetes corrected Deathrates and Corrective Factors England and Wales* 1860–1913.

Year	Total Male Population	Deaths from Cancer	Crude Cancer Deathrate	Cancer Corrective Factor	Corrected Cancer Deathrate	Deaths from Diabetes	Crude Diabetes Deathrate	Diabetes Corrective Factor	Corrected Diabetes Deathrate
1860	9,704,394	2100	216·4	1·0544	228·2	346	35·7	1·0542	37·6
1861	9,801,152	2180	222·4	1·0523	234·0	363	37·0	1·0541	39·0
1862	9,923,272	2256	227·3	1·0498	238·6	382	38·5	1·0537	40·6
1863	10,046,909	2311	230·0	1·0480	241·0	359	35·7	1·0536	37·6
1864	10,172,089	2459	242.7	1·0465	252·9	454	44·6	1·0538	47·0
1865	10,298,826	2389	232·0	1·0452	242·5	430	41·8	1·0541	44·0
1866	10,427,146	2532	242·8	1·0444	253·6	437	41·9	1·0545	44·2
1867	10,557,066	2650	251·0	1·0439	262·0	434	41·1	1·0551	43·4
1868	10,688,600	2743	256·6	1·0440	267·9	445	41·6	1·0559	43·9
1869	10,821,775	2933	271·0	1·0446	283·1	497	45·9	1·0569	48·5
1870	10,956,608	2971	271·2	1·0460	283·7	470	42·9	1·0580	45·4
1871	11,092,620	3060	275·9	1·0478	289·1	541	48·8	1·0593	51·7
1872	11,242,495	3228	287·1	1·0506	301·6	513	45·6	1·0605	48·4
1873	11,394,394	3337	292·9	1·0540	308·7	550	48·3	1·0623	51·3
1874	11,548,346	3470	300·5	1·0575	317·8	528	45·7	1·0642	48·6
1875	11,704,378	3614	308·8	1·0615	327·8	614	52·5	1·0662	55·9
1876	11,862,519	3708	312·6	1·0650	332·9	582	49·1	1·0683	52·4
1877	12,022,796	3950	328·5	1·0686	351·0	663	55·1	1·0703	59·0
1878	12,185,238	4164	341·7	1·0719	366·3	669	54·9	1·0722	58·9
1879	12,349,875	4149	336·0	1·0747	361·1	645	52·2	1·0738	56·0
1880	12,516,737	4423	353·4	1·0770	380·6	624	49·9	1·0752	53·7
1881	12,673,435	4611	363·8	1·0787	392·4	769	60·7	1·0762	65·3
1882	12,808,460	4685	365·8	1·0795	394·9	733	57·2	1·0765	61·6
1883	12,944,923	4967	383·7	1·0800	414·4	838	64·7	1·0764	69·6
1884	13,082,837	5346	408·6	1·0804	441·5	886	67·7	1·0759	72·8
1885	13,222,216	5495	415·6	1·0803	449·0	896	67·8	1·0752	72·9
1886	13,363,079	5754	430·6	1·0798	465·0	978	73·2	1·0740	78·6
1887	13,505,441	6262	463·7	1·0794	500·5	1019	75·5	1·0724	80·9
1888	13,649,314	6284	460·4	1·0787	496·6	1070	78·4	1·0714	84·0
1889	13,794,721	6891	499·5	1·0777	538·3	980	71·0	1·0697	76·0
1890	13,941,671	7137	511·9	1·0766	551·1	1076	77·2	1·0679	82·4
1891	14,092,535	7294	517·6	1·0754	556·6	1082	76·8	1·0662	81·9
1892	14,252,190	7547	529·5	1·0740	568·7	1142	80·1	1·0642	85·2
1893	14,413,657	7908	548·6	1·0728	588·5	1169	81·1	1·0622	86·1
1894	14,576,948	8077	554·1	1·0715	593·7	1166	80·0	1·0603	84·8
1895	14,742,091	8628	585·3	1·0698	626·2	1262	85·6	1·0584	90·6
1896	14,909,104	9216	618·1	1·0682	660·3	1243	83·4	1·0560	88·1
1897	15,078,010	9573	634·9	1·0662	676·9	1303	86·4	1·0536	91·0
1898	15,248,823	9932	651·3	1·0639	692·9	1437	94·2	1·0510	99·0
1899	15,421,578	10337	670·3	1·0614	711·5	1448	93·9	1·0480	98·4
1900	15,596,283	10475	671·6	1·0583	710·8	1455	93·3	1·0445	97·5
1901	15,769,412	10891	690·6	1·0550	728·6	1565	99·2	1·0408	103·3
1902	15,933,658	11098	696·5	1·0505	731·7	1462	91·8	1·0368	95·1
1903	16,099,612	11799	732·9	1·0453	766·1	1409	87·5	1·0322	90·3
1904	16,267,291	12086	743·0	1·0390	772·0	1612	99·1	1·0270	101·8
1905	16,436,707	12470	758·7	1·0318	782·8	1708	103·9	1·0214	106·1
1906	16,607,890	13257	798·2	1·0238	817·2	1736	104·5	1·0153	106·1
1907	16,780,848	13199	786·6	1·0150	798·4	1738	103·6	1·0070	104·5
1908	16,955,609	13901	819·8	1·0056	824·4	1842	108·6	1·0022	108·8
1909	17,132,182	14263	832·5	0·9953	828·6	1898	110·8	·9948	110·2
1910	17,310,586	14843	857·5	0·9846	844·3	1982	114·5	·9873	113·0
1911	17,490,847	15589	893·6	0·9739	870·2	1936	111·0	·9798	108·7
1912	17,672,985	16188	916·0	0·9620	881·2	2064	116·8	·9716	113·5
1913	17,857,014	16918	947·4	0·9580	907·6	2122	118·8	·9637	114·5

diabetes. Of course such an investigation as the present does not provide any measure of whether persons contracting diabetes are more liable to die of cancer, for the deaths would be registered as deaths from cancer, yet this not improbably is the vital problem*. The result reached, however, is consistent with a continuous increase of both cancer and diabetes not essentially related organically with each other. In this case Maynard's results for the United States would mean that in that huge population the towns and states were in heterogeneous stages of historical development, whether as to medical training, accuracy of record or cultural conditions, while the non-appearance of the high correlation in the Swiss data would merely signify that the units chosen there were homogeneous in such characters, and this considering the size of Switzerland is not improbable. In other words some American districts present with regard to cancer and diabetes the English conditions of 1870 and others the English conditions of 1910. The time correlation of the English deathrates may thus correspond to the geographical correlation of the American deathrates. If so, since, when the time factor is removed, the English diabetes and cancer deathrates show no organic correlation, we cannot use the English material to support the view that the American relationship is organic in character. The correlations that Maynard has indicated between both cancer and diabetes deathrates and the spread of insanity, suicides and the newspaper press, seem indeed to indicate not an organic relationship of the two diseases, but as Maynard himself has suggested a wider range of both with advancing cultural conditions.

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* The enquiry as to whether cancer patients are suffering or have suffered from diabetes should always be made and the answer recorded.

A CONTRIBUTION TO THE PROBLEM OF HOMOTYPOSIS

DATA FROM THE LEGUME *CERCIS CANADENSIS*

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I. Introductory Remarks.

In 1905, I began the collection of series of data for a comprehensive study of the problem of intra-individual or "homotypic"* correlation for fertility and fecundity characters in plants. Such characters were chosen not merely because of particular interest in that subject in general but because of the conviction that the intra-class and inter-class† correlation methods applied to the problem of fertility and fecundity might yield results of considerable physiological interest. Since that time, circumstances have prevented my carrying out the work along the lines originally laid down. The constants given here are drawn from notes of the work then done.

II. Materials.

The data presented are exclusively those for pod length l, number of ovules formed o, number of seeds developing s, and number of ovules failing to develop f, in the small arborescent legume, *Cercis Canadensis*. Three collections are involved: one from Meramec Highlands, near St Louis, Mo., made in the autumn of 1905; one from the vicinity of Lawrence, Kansas, taken in 1905; one from the neighbourhood of Sharpsburg, Athens County, Ohio, gathered in 1908. These materials have already been considered in dealing with other problems quite distinct from the present one; the reader seeking more comprehensive information with regard to the materials used should consult the papers cited below‡.

For the Meramec Highlands collections two sets of symmetrical homotypic correlation tables were prepared.

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* K. Pearson, and others: *Phil. Trans.* B, Vol. cxcvii. pp. 286–288, 1901.

† *Biometrika*, Vol. ix. pp. 446–472, 1913.

‡ *Bot. Gaz.* Vol. l. pp. 117–127, 1910; *Bull. Torr. Bot. Club*, Vol. xli. pp. 243–256, 1914

I used the first 26 pods taken from 112 trees. The desirability of working with a larger series of data from individual trees then appealed to me and a table comprising the first 100 pods from the 60 trees from which that number could be secured was undertaken. For both of these series all the possible direct and cross homotypic correlations were ascertained for the fertility characters.

The Kansas series comprised only 22 individual trees, from each of which 100 pods were counted. The Ohio series included 150 pods each from 26 trees. For these two series all possible homotypic correlations for the fertility characters have been determined.

III. METHODS.

The three direct intra-individual or homotypic correlations are:

(1) Ovules of first pod and ovules of second pod, Tables I, II*.

(2) Seed developing in first pod and seed developing in second pod, Tables III, IV.

(3) Ovules failing in first pod and ovules failing in second pod, Tables V, VI.

The cross-homotypic relationships are:

(4) Ovules of first pod and seeds developing in second pod, Tables VII, VIII.

(5) Ovules of first pod and ovules failing in second pod.

(6) Seeds developing in first pod and ovules failing in second pod.

Tables for two of the cross correlations are unnecessary. Thus for the relationship between the number of ovules in the first pod and the number of ovules failing in the second pod, $r_{o_1 f_2}$, the slope of the regression line is clearly

$$r_{o_1 f_2}\frac{\sigma_{f_2}}{\sigma_{o_1}} = r_{o_1 o_2}\frac{\sigma_{o_2}}{\sigma_{o_1}} - r_{o_1 s_2}\frac{\sigma_{s_2}}{\sigma_{o_1}},$$

or in terms of correlation

$$r_{o_1 f_2} = r_{o_1 o_2}\frac{\sigma_{o_2}}{\sigma_{f_2}} - r_{o_1 s_2}\frac{\sigma_{s_2}}{\sigma_{f_2}}. \qquad \text{(i)}$$

Again, for number of seeds in the first pod and number of ovules failing to develop in the second pod the slope of the regression straight line is

$$r_{s_1 f_2}\frac{\sigma_{f_2}}{\sigma_{s_1}} = r_{s_1 o_2}\frac{\sigma_{o_2}}{\sigma_{s_1}} - r_{s_1 s_2}\frac{\sigma_{s_2}}{\sigma_{s_1}},$$

whence

$$r_{s_1 f_2} = r_{s_1 o_2}\frac{\sigma_{o_2}}{\sigma_{f_2}} - r_{s_1 s_2}\frac{\sigma_{s_2}}{\sigma_{f_2}}. \qquad \text{(ii)}$$

In calculating the correlations for length of pod in the Meramec Highlands series and for the fertility of characters of the Ohio and Kansas series I have had recourse to the direct and cross intra-class correlation formulae (v)–(xii) of

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* The numbers refer to the tables for the Meramec Highlands series.

Biometrika, Vol. IX. pp. 450–452, 1913. In present notation the product summations are for the several correlations:

For $r_{o_1o_2}$, $\{S[\Sigma(o')]^2 - S[\Sigma(o'^2)]\}/N$, ...(iii)

,, $r_{s_1s_2}$, $\{S[\Sigma(s')]^2 - S[\Sigma(s'^2)]\}/N$, ...(iv)

,, $r_{f_1f_2}$, $\{S[\Sigma(f')]^2 - S[\Sigma(f'^2)]\}/N$,..........................(v)

,, $r_{o_1s_2}$, $\{S[\Sigma(o')\Sigma(s')] - S[\Sigma(o's')]\}/N$, ...(vi)

,, $r_{o_1f_2}$, $\{S[\Sigma(o')\Sigma(f')] - S[\Sigma(o'f')]\}/N$,..................(vii

,, $r_{s_1f_2}$, $\{S[\Sigma(s')\Sigma(f')] - S[\Sigma(s'f')]\}/N$,..................(viii)

where N is the population of individuals resulting from the $n(n-1)$-fold weighting, Σ denotes a summation for the individual pods of a class and S a summation for the classes (individual trees). For data see Table IX.

In the first three of these the negative sign term of the formula is merely the second moment of the unweighted population and in the fourth it is the product moment of the correlation surface r_{os}—the "organic" relationship between ovules and seeds of the same pod—from which the three second moments, $\Sigma(o'^2)$, $\Sigma(s'^2)$, $\Sigma(f'^2)$, may be calculated. The 5th and 6th cross correlations may be determined from the other correlations by (i) and (ii), or since $f = o - s$ the minus terms in (vii) and (viii) are given by

$$S[\Sigma(o'f')] = S[\Sigma(o'^2)] - S[\Sigma(o's')],$$

$$S[\Sigma(s'f')] = S[\Sigma(s'o')] - S[\Sigma(s'^2)],$$

which may be most easily calculated from the published tables for r_{os}, since all classes are equally large.

IV. PRESENTATION OF DATA.

The constants for both direct and cross homotypic correlations are shown for all series in Table A.

The reader, in examining these constants, will remember that the two Meramec Highlands series were taken from the same habitat and in the same year. They are known to differ only in (i) the smaller number of pods from each individual in the first series, (ii) the fact that the first series contains pods from 50 individuals which do not occur in the second lot. The Ohio and Kansas collections on the other hand represent materials of the same species from localities several hundreds of miles distant. Unfortunately, both of these contain too few individuals to be fully trustworthy.

There can be no reasonable question of the statistical trustworthiness of all the direct homotypic correlations. The lowest is that for ovules failing per pod and this is in all cases six or more times its probable error. The coefficients $r_{o_1o_2}$, $r_{s_1s_2}$ are from 25 to 45 times as large as their probable errors. For the most part the cross correlations may also be regarded as statistically trustworthy.

TABLE A.

Homotypic Correlations for Cercis.*

Combination	Meramec Highlands, Mo., 112 Trees	Meramec Highlands, Mo., 60 Trees	Sharpsburg, Ohio, 26 Trees	Lawrence, Kansas, 22 Trees
Ovules of First Pod and Ovules of Second Pod	·3524 ± ·0109	·3527 ± ·0076	·2768 ± ·0100	·3999 ± ·0121
Seeds of First Pod and Seeds of Second Pod	·2677 ± ·0116	·2306 ± ·0082	·1798 ± ·0104	·2019 ± ·0138
Ovules Failing in First Pod and Ovules Failing in Second Pod	·0979 ± ·0123	·0684 ± ·0087	·1756 ± ·0104	·0858 ± ·0143
Ovules of First Pod and Seeds of Second Pod	·2758 ± ·0115	·2622 ± ·0081	·1173 ± ·0106	·2574 ± ·0134
Ovules of First Pod and Ovules Failing in Second Pod ...	·0178 ± ·0125	·0301 ± ·0087	·0762 ± ·0107	·1195 ± ·0142
Seeds of First Pod and Ovules Failing in Second Pod ...	– ·0570 ± ·0125	– ·0261 ± ·0087	– ·1095 ± ·0107	·0344 ± ·0144

The regression equations for the two Meramec Highlands series appear in Table B. The regression lines for the direct relationships $r_{o_1o_2}$, $r_{s_1s_2}$ are given in Diagrams I and II, and for the cross relationships $r_{o_1s_2}$, $r_{s_1o_2}$ in Diagrams III and IV.

TABLE B.

Regression Equations for Meramec Highlands Collection.

2912 Pods	6000 Pods
$o_2 = 3{\cdot}0571 + {\cdot}3524 o_1$	$o_2 = 2{\cdot}9925 + {\cdot}3527 o_1$
$s_2 = 2{\cdot}9563 + {\cdot}2677 s_1$	$s_2 = 3{\cdot}0018 + {\cdot}2306 s_1$
$f_2 = {\cdot}6171 + {\cdot}0979 f_1$	$f_2 = {\cdot}6722 + {\cdot}0684 f_1$
$s_2 = 2{\cdot}4485 + {\cdot}3365 o_1$	$s_2 = 2{\cdot}3982 + {\cdot}3252 o_1$
$f_2 = {\cdot}6089 + {\cdot}0159 o_1$	$f_2 = {\cdot}5943 + {\cdot}0275 o_1$
$f_2 = {\cdot}8671 - {\cdot}0453 s_1$	$f_2 = {\cdot}7965 - {\cdot}0192 s_1$

Inspection indicates that the regression of the ovules of the second pod (Diagrams I and II, upper lines) on the ovules of the first and of the seeds of the second pod (Diagrams III and IV, steeper lines) on the ovules of the first are linear. It is not so clear in the case of ovules or seeds of the second pod on seeds

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* All probable errors are calculated on the basis of the actual, not the weighted, number of pods.

Diagram I. Direct Homotyposis. *Cercis Canadensis* (Meramec Highlands, 1st Series). Ovules on Ovules and Seeds on Seeds.

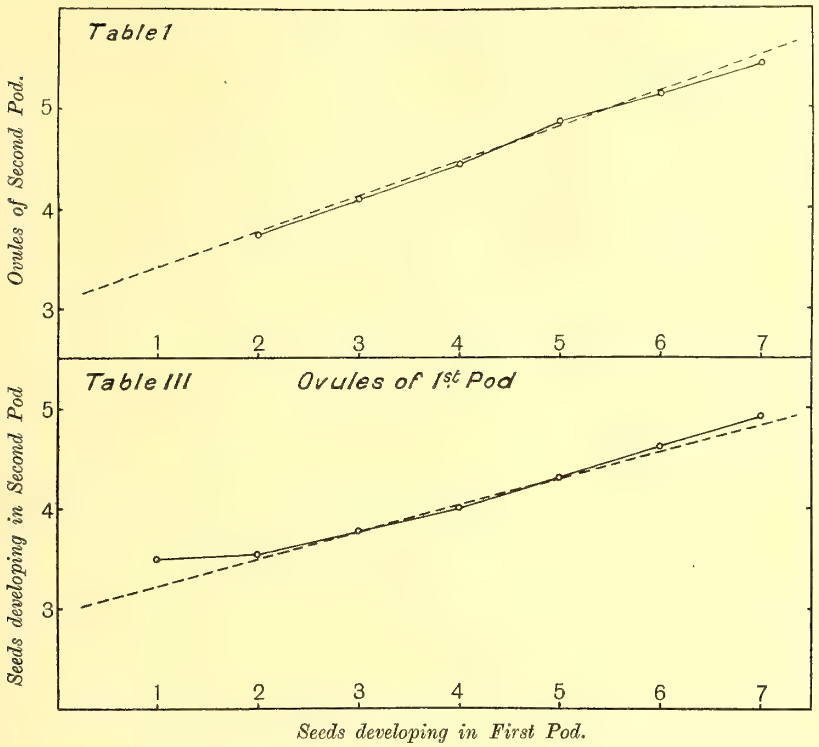


Diagram II. Direct Homotyposis. *Cercis Canadensis* (Meramec Highlands, 2nd Series). Ovules on Ovules and Seeds on Seeds.

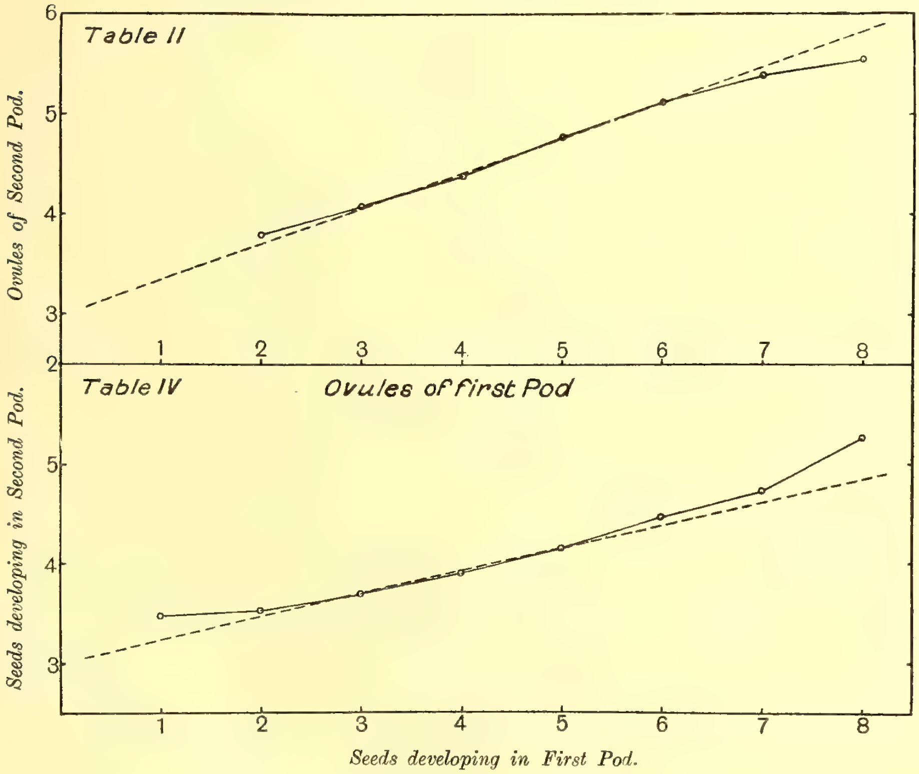


Diagram III. Cross Homotyposis. *Cercis Canadensis* (Meramec Highlands, 1st Series). Ovules on Seeds and Seeds on Ovules.

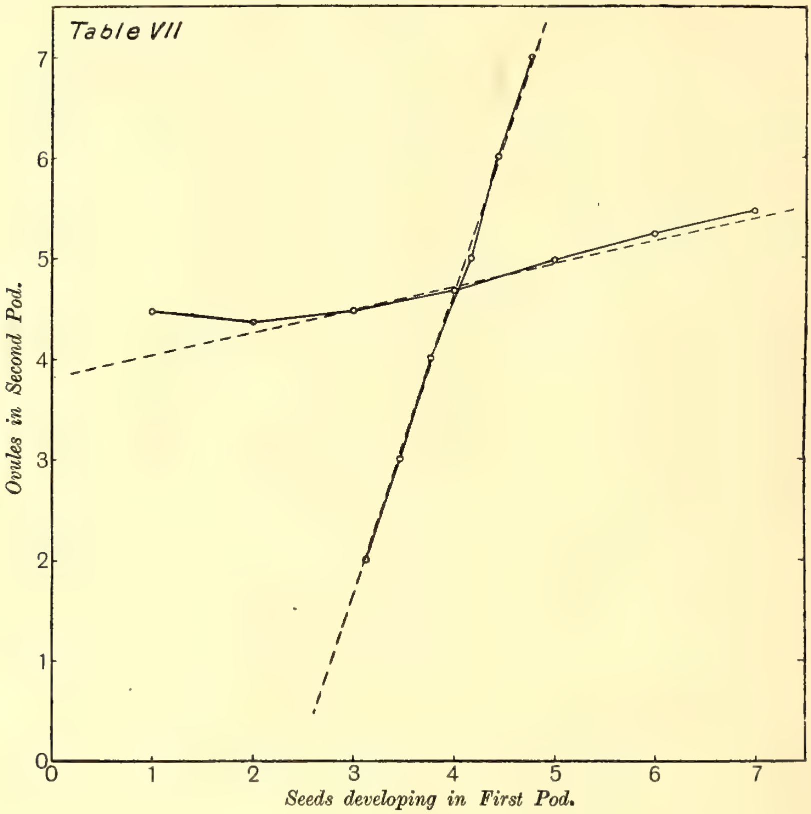
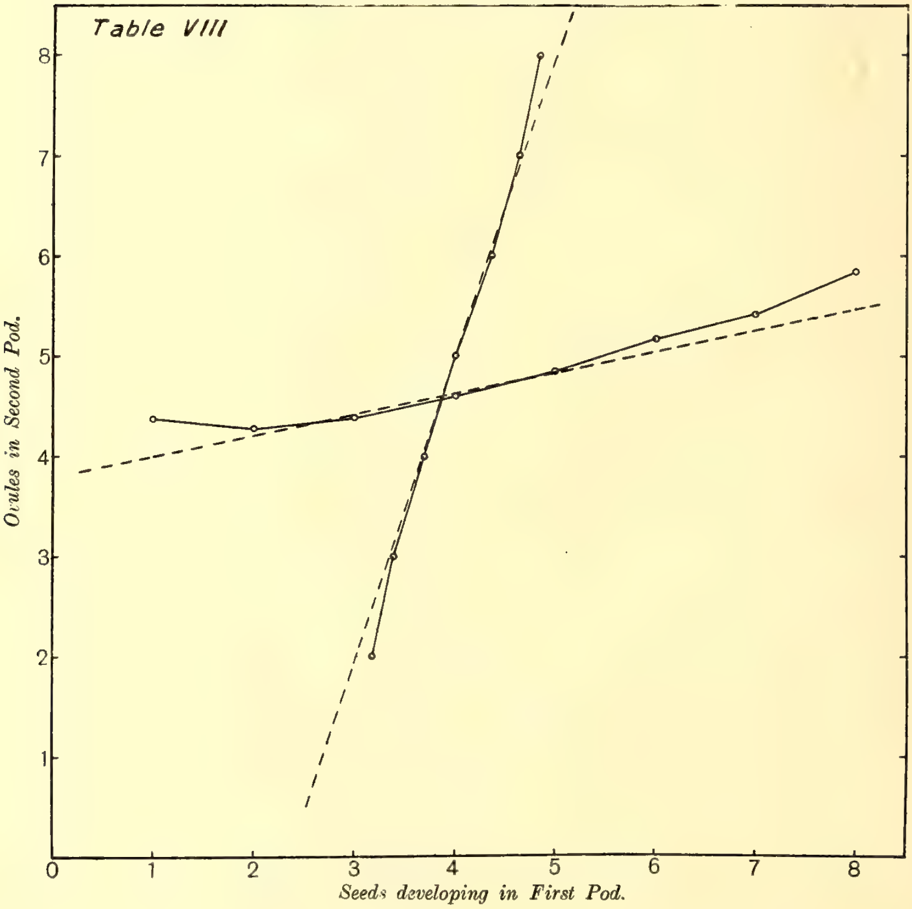


Diagram IV. Cross Homotyposis. *Cercis Canadensis* (Meramec Highlands, 2nd Series). Ovules on Seeds and Seeds on Ovules.



of the first, although even here (see Diagrams I and II, lower lines, III and IV, less steep lines) the deviation from linearity is not great*.

Returning now to the constants and considering them in detail, we note that for all localities the direct homotypic correlations are highest for the number of ovules per pod and lowest for the number of ovules failing†. The averages‡ are

Ovules and Ovules	·3430
Seeds and Seeds	·2164
Ovules failing and Ovules failing	·1198

The cross relationships are highest for ovules formed and seeds developing per pod; much lower for ovules formed and ovules failing to develop; lowest for seeds developing and seeds failing to develop—three of the four values for $r_{s_1f_2}$ are negative in sign.

The averages are:

Ovules and Seeds	+ ·2168
Ovules and Ovules failing	+ ·0712
Seeds and Ovules failing	− ·0440

From these data it appears that the individuality of the trees is more marked for number of ovules per pod than for either number of seeds matured or number of ovules failing per pod.

Number of ovules per pod may be regarded in comparison with number of seeds developing and number of ovules failing to develop as an independent variable§.

The number of seeds per pod is absolutely limited by (i) the number of ovules formed in the pod, and (ii) the number of these which are prevented from developing by peculiarities inherent in ovum or sperm, by accidents of fertilization, inadequacy of food supply, or other unknown causes. Or conversely, the number of ovules failing per pod is dependent upon the number of ovules formed and the number of seeds developing. Thus the number of ovules per pod determines within certain limits the number of seeds developing, or the number of ovules failing, per pod; but in the nature of things it cannot be influenced by either of these.

Now since the three characters under consideration are correlated, i.e. since r_{os}, r_{of}, r_{sf} have been found to have sensible values, it is clear that some homotypic relationship would arise for seeds because of the correlation for ovules, or for ovules failing because of the correlation for ovules and seeds. Thus a statistically significant value of an intra-individual correlation for seeds per pod does not necessarily mean that there are any specific biological factors influencing the number of seeds developing or the number of ovules failing in all of the pods of the same individual in a similar manner. The observed values of correlations for (semi)dependent characters may be merely the resultant of independent variables with which they are correlated.

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* In the case of the relationship for ovules and ovules in the 60 trees from Meramec Highlands I have found the raw value of η to be ·35336, exceeding r by only ·00064.

† In the Ohio series $r_{o_1o_2}$ is very low; $r_{s_1s_2}$ and $r_{f_1f_2}$ are not sensibly different.

‡ The constants for the 112 trees from Meramec Highlands are alone used in obtaining the general averages.

§ This does not mean that in any given pod all three of these characters are not dependent upon the same ultimate causes, but merely that in a proximate sense number of ovules per pod is physiologically independent while number of seeds and number of ovules failing are physiologically dependent.

To remove the influence of $r_{o_1o_2}$ upon $r_{s_1s_2}$ we have recourse to the partial correlation coefficient for two variables, i.e. o_1 and o_2, constant. In our notation this is

$$_{o_1o_2}r_{s_1s_2} = \frac{r_{s_1s_2}(1-r_{o_1o_2}{}^2) - r_{o_1s_1}r_{o_1s_2} - r_{o_2s_1}r_{o_2s_2} + r_{o_1o_2}(r_{o_1s_1}r_{o_2s_2} + r_{o_1s_2}r_{o_2s_1})}{\sqrt{(1-r_{o_1o_2}{}^2 - r_{o_2s_1}{}^2 - r_{o_1s_1}{}^2 + 2r_{o_1o_2}r_{o_1s_1}r_{o_2s_1})}\sqrt{(1-r_{o_1o_2}{}^2 - r_{o_2s_2}{}^2 - r_{o_1s_2}{}^2 + 2r_{o_1o_2}r_{o_1s_2}r_{o_2s_2})}}.$$

But since we are dealing with symmetrical tables we may write*

$$_{o'o'}r_{s's'} = \frac{r_{s's'}(1-r_{o'o'}{}^2) - 2r_{os}r_{o's'} + r_{o'o'}(r_{os}{}^2 + r_{o's'}{}^2)}{1 - r_{o'o'}{}^2 - r_{o's'}{}^2 - r_{os}{}^2 + 2r_{o'o'}r_{os}r_{o's'}},$$

where the correlations without dashes are those for the same pod and those with dashes are for the relationships between different pods.

In applying this formula to the actual data, the values of r_{os} for the individual pods are essential. These have been deduced from Tables X, XI below and Tables VI, VII of a former paper†.

Meramec Highlands.
6000 pods, $r_{os} = \cdot 6855 \pm \cdot 0046$,
2912 pods, $r_{os} = \cdot 6936 \pm \cdot 0065$,
Sharpsburg, Ohio, $r_{os} = \cdot 4553 \pm \cdot 0086$,
Lawrence, Kansas, $r_{os} = \cdot 6032 \pm \cdot 0091$.

The partial correlation coefficients for seeds per pod for constant numbers of ovules per pod are:

Series	$_{o'o'}r_{s's'}$	$r_{s's'}$	$\frac{_{o'o'}r_{s's'}}{r_{s's'}}$
Meramec Highlands			
112 Trees ...	·1063	·2677	·397
60 Trees ...	·0700	·2306	·304
Sharpsburg, Ohio ...	·1645	·1798	·915
Lawrence, Kansas	·0582	·2019	·288

These give a mean value of ·0998 as compared with the mean relationship uncorrected for the influence of ovules per pod, $r_{s's'} = \cdot 2164$.

Apparently, there are specific physiological‡ factors which tend to differentiate the trees with respect to seed production, but the bulk of the interdependence for seeds seems to be merely the resultant of $r_{o'o'}$ and r_{os}*.

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* *Biometrika*, Vol. VII. p. 328. For original see K. Pearson, *Phil. Trans.* A, Vol. CC. p. 31, 1902.

† *Bull. Torr. Bot. Club*, Vol. XLI. pp. 243–256, 1914.

‡ Under the term physiological as used here are included (*a*) the ecological factors which determine whether an ovule shall receive a sperm, (*b*) the nutritional factors which determine the availability of food materials, and other environmental prerequisites for development, and (*c*) the innate vigour or other physiological characters of the individual which determine whether a fertilized ovule shall develop into a seed.

Relative net individuality of the trees with respect to capacity for seed production is highest in the Ohio series. This is due to the fact that both $r_{o_1o_2}$ and r_{os} are lower for this series than for any other.

The frequency distribution of length of pod measured to the nearest millimetre in 50 pods each from 60 trees is shown in Table XII. The summed length for the individual trees, i.e. mean length × 50, is given in Table XIII. By the direct intra-class formulae I find

$$r_{l_1l_2} = \cdot 4784 \pm \cdot 0095,$$

a homotypic value distinctly higher than those found for the fertility characters.

V. DISCUSSION OF RESULTS.

The only homotypic constants available for fertility characters in plants are those furnished for several species of Leguminosae by Pearson and his associates†. They give the direct correlations as shown in Table C.

TABLE C.

Homotypic Correlations for Fertility Characters.

Species of Legume	Ovules	Abortive Ovules	Ripe Seeds
Cytisus Scoparius	—	—	·4155
Lotus Corniculatus	—	—	·2354
,, ,,	—	—	·1884
Lathyrus odoratus	·2182	·2679	·0830
,, ,,	·3658	·1759	·2091
Lathyrus Sylvestris	·1695	·1376	·2184
,, ,,	—	—	·1877
Vicia Faba ...	·1724	·1493	·1243
Vicia Hirsuta ...	·2315	·1827	·2077

In addition to these values I have found in a short series of only 12 trees of the arborescent legume ***Robinia Pseud-acacia*** worked out as an illustration of method‡ the values $r_{o_1o_2} = \cdot 452$, $r_{s_1s_2} = \cdot 449$, $r_{o_1s_2} = \cdot 383$. For a short series (23 plants) of *Cytisus Scoparius*§ I have found $r_{s_1s_2} = \cdot 198$.

The average homotypic values for ovules in *Cercis* are distinctly higher than the comparable values for other species hitherto adequately investigated. The mean values for seeds are about the same as those found by English investigators. The mean correlation for ovules failing is lower.

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* In *Sanguinaria* (*Biometrika*, Vol. VII. p. 328) the correlation for the number of seeds on the two placentae of the same fruit seems to be chiefly due to physiological factors. In this case an organic correlation is superimposed upon a homotypic. The point may profitably be discussed comparatively when other data now in hand are completely analyzed.

† K. Pearson and others, *Phil. Trans.* A, Vol. CXCVII. pp. 364–379, 1901.

‡ *Biometrika*, Vol. IX. pp. 456–458, 463, 1913.

§ *Amer. Nat.* Vol. XLV. pp. 566–571, 1911. Possibly this material is of closely selected ancestry.

The homotypic correlation for seeds matured per pod is statistically largely a resultant of the homotypic correlation for ovules and the "organic" correlation for ovules and seeds of the same pod. Thus homotypic correlation for seeds per pod does not entirely disappear when correction for $r_{o_1 o_2}$ and r_{os} is made by the application of partial correlation formulae. There are, therefore, (proximately) independent ecological and physiological factors tending to differentiate individuals with respect to capacity for seed production.

The general rule for fertility characters in Leguminosae seems to be for the maximum value of the homotypic correlation to be that for number of ovules per pod, much lower values are found for seeds matured or abortive ovules per pod. I believe that the evidence also points in the direction of a lower correlation for ovules than for the more truly vegetative characters of the plant. Thus Pearson's average values for $r_{o_1 o_2}$ are far lower than his average for leaves and other similar organs. The result given above for length of pod $r_{l_1 l_2}$ is practically ·500, and distinctly higher than $r_{o_1 o_2} = \cdot 352$ from the same habitat.

TABLE I.

Ovules per Pod.

Ovules per Pod.	2	3	4	5	6	7	Totals
2	18	111	197	46	3	0	375
3	111	836	2382	942	239	15	4525
4	197	2382	11770	8642	2599	160	25750
5	46	942	8642	12134	5613	598	27975
6	3	239	2599	5613	3742	529	12725
7	0	15	160	598	529	148	1450
Totals	375	4525	25750	27975	12725	1450	72800

TABLE II.

Ovules per Pod.

Ovules per Pod.	2	3	4	5	6	7	8	Totals
2	98	1025	1775	419	49	0	0	3366
3	1025	10608	25134	11257	2300	111	5	50490
4	1775	25184	101272	74059	18242	1296	31	221859
5	419	11257	74059	92240	37849	4012	142	219978
6	49	2300	18242	37849	25458	3833	181	87912
7	0	111	1296	4012	3833	712	35	9999
8	0	5	31	142	181	35	2	396
Totals	3366	50490	221859	219978	87912	9999	396	594000

TABLE III.

Seeds per Pod.

Seeds per Pod.	1	2	3	4	5	6	7	Totals
1	16	95	179	207	94	9	0	600
2	95	710	1596	1691	693	152	13	4950
3	179	1596	5106	6260	2975	747	62	16925
4	207	1691	6260	10164	6106	1821	151	26400
5	94	693	2975	6106	4986	2109	212	17175
6	9	152	747	1821	2109	1096	166	6100
7	0	13	62	151	212	166	46	650
Totals	600	4950	16925	26400	17175	6100	650	72800

TABLE IV.

Seeds per Pod.

Seeds per Pod.	1	2	3	4	5	6	7	8	Totals
1	394	1467	2965	2861	1412	283	23	0	9405
2	1467	7224	17224	16294	7546	1520	104	2	51381
3	2965	17224	47594	52375	27161	6032	388	8	153747
4	2861	16294	52375	71576	45505	12278	1041	30	201960
5	1412	7546	27161	45505	36836	13121	1304	72	132957
6	283	1520	6032	12278	13121	6334	853	70	40491
7	23	104	388	1041	1304	853	134	14	3861
8	0	2	8	30	72	70	14	2	198
Totals	9405	51381	153747	201960	132957	40491	3861	198	594000

TABLE V.

Ovules failing per Pod.

Ovules Failing per Pod.	0	1	2	3	4	5	Totals
0	19140	12531	3620	756	91	12	36150
1	12531	9536	3182	692	97	12	26050
2	3620	3182	1206	313	53	1	8375
3	756	692	313	134	30	0	1925
4	91	97	53	30	4	0	275
5	12	12	1	0	0	0	25
Totals	36150	26050	8375	1925	275	25	72800

TABLE VI.

Ovules Failing per Pod.

Ovules Failing per Pod.	0	1	2	3	4	5	Totals
0	143232	101090	32493	6712	1408	185	285120
1	101090	78862	27035	6332	1227	185	214731
2	32493	27035	10220	2735	596	82	73161
3	6712	6332	2735	884	233	33	16929
4	1408	1227	596	233	92	8	3564
5	185	185	82	33	8	2	495
Totals	285120	214731	73161	16929	3564	495	594000

TABLE VII.

Seeds per Pod.

Ovules per Pod.	1	2	3	4	5	6	7	Totals
2	7	86	154	110	18	0	0	375
3	57	606	1726	1568	471	92	5	4525
4	261	2213	7443	10099	4660	1003	71	25750
5	206	1553	5511	10150	7592	2697	266	27975
6	67	460	1931	4104	3953	1975	235	12725
7	2	32	160	369	481	333	73	1450
Totals	600	4950	16925	26400	17175	6100	650	72800

TABLE VIII.

Seeds per Pod.

Ovules per Pod.	1	2	3	4	5	6	7	8	Totals
2	98	658	1407	953	234	16	0	0	3366
3	1158	7868	19077	15761	5710	882	34	0	50490
4	4206	23467	67408	78277	40285	7727	485	4	221859
5	3066	15160	49891	76785	56232	17265	1525	54	219978
6	800	3923	14687	27312	27097	12491	1492	110	87912
7	72	294	1238	2793	3260	2005	309	28	9999
8	5	11	39	79	139	105	16	2	396
Totals	9405	51381	153747	201960	132957	40491	3861	198	594000

TABLE IX.

Tree	Kansas Series		Ohio Series	
	Total Ovules	Total Seeds	Total Ovules	Total Seeds
	$\Sigma(o')$	$\Sigma(s')$	$\Sigma(o')$	$\Sigma(s')$
1	522	461	740	592
2	471	377	690	578
3	420	362	861	731
4	473	412	791	605
5	400	351	813	576
6	480	413	831	504
7	456	368	826	477
8	591	469	809	519
9	454	346	753	329
10	574	491	843	546
11	429	380	815	490
12	487	382	879	633
13	482	407	762	606
14	552	465	787	453
15	502	418	769	566
16	473	411	863	632
17	396	338	746	641
18	564	408	1039	731
19	614	521	707	510
20	495	437	962	731
21	536	435	886	696
22	444	403	788	649
23	—	—	806	692
24	—	—	800	632
25	—	—	925	651
26	—	—	932	612

TABLE X.

Seeds per Pod.

Ovules per Pod.	1	2	3	4	5	6	7	Totals
2	1	14	—	—	—	—	—	15
3	11	50	120	—	—	—	—	181
4	8	89	380	553	—	—	—	1030
5	3	38	147	421	510	—	—	1119
6	1	7	29	80	169	223	—	509
7	—	—	1	2	8	21	26	58
Totals	24	198	677	1056	687	244	26	2912

TABLE XI.

Seeds per Pod.

Ovules per Pod.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	Totals
2	3	31	—	—	—	—	—	—	34
3	34	176	300	—	—	—	—	—	510
4	37	226	885	1093	—	—	—	—	2241
5	17	71	310	784	1040	—	—	—	2222
6	4	14	54	153	288	375	—	—	888
7	—	1	4	9	15	33	39	—	101
8	—	—	—	1	—	1	—	2	4
Totals	95	519	1553	2040	1343	409	39	2	6000

TABLE XII.

Length of Individual Pods.

L	*f*	*L*	*f*	*L*	*f*	*L*	*f*	*L*	*f*	*L*	*f*
45	2	55	13	65	141	75	213	85	53	95	—
46	1	56	12	66	83	76	152	86	28	96	2
47	1	57	19	67	104	77	122	87	26	97	3
48	—	58	19	68	128	78	117	88	18	98	2
49	1	59	13	69	56	79	53	89	3	99	1
50	1	60	54	70	222	80	187	90	21	100	3
51	2	61	39	71	113	81	82	91	7	101	3
52	3	62	54	72	162	82	86	92	11	102	1
53	4	63	51	73	154	83	64	93	5	103	—
54	5	64	63	74	158	84	56	94	1	104	2

TABLE XIII.

Total Lengths of 50 *Pods for* 60 *Individual Cercis Trees.*

3345	3361	3587	3594	3499	3598	3739	3418	3848	3202
3679	3794	3352	3707	3615	3463	3945	3388	4026	3642
3930	4546	3654	3658	3823	3484	3806	3977	3244	3459
3548	3748	3343	3686	3619	3978	3725	3544	3223	4086
3693	3725	3168	3873	3889	3279	3502	3053	3765	4031
3883	3850	3770	3921	3497	3791	3749	3603	4117	3590

STATION FOR EXPERIMENTAL EVOLUTION,
COLD SPRING HARBOR, LONG ISLAND, U.S.A.

ON THE PROBABLE ERROR OF A COEFFICIENT OF CONTINGENCY WITHOUT APPROXIMATION.

BY ANDREW W. YOUNG, M.A. AND KARL PEARSON, F.R.S.

(1) *Introductory.*

There have been two memoirs dealing with the probable error of a coefficient of contingency, namely that by Blakeman and Pearson in 1906* and that by Pearson in 1914†. In the former paper the authors started from the expression for the mean square contingency

$$\phi^2 = S\left(\frac{n^2_{ss'}}{n_{s.}n_{.s'}}\right) - 1,$$

and varied $n_{ss'}$, $n_{s.}$ and $n_{.s'}$ but neglected the squares and products of these variations. The result was lengthy, and the arithmetical work laborious. In 1914 Pearson gave reasons for considering $n_{s.}$ and $n_{.s'}$ as constant during the sampling and got a much simpler value for σ_{ϕ^2}. The result in actual numerical cases did not differ widely from the much more elaborate formula of the earlier memoir. Recent work in other directions has, however, shown that caution must be used in neglecting the square and product terms of the variations due to random sampling, and the object of the present paper is to consider the variation of ϕ^2 on the hypothesis of Pearson's 1914 note but without approximation.

Let a population of size M be grouped into c divisions—for example, the cells of a contingency table—and let the contents of the sth division be m_s. Let a sample of size N be taken at random from the population and let n_s be the contents of the sth division according to the same grouping.

We shall here consider the variation of the quantity ϕ^2 defined by

$$1 + \phi^2 = S\left(\frac{n_s{}^2}{N\lambda_s}\right) \quad \text{..................................(i)},$$

where λ_s is a number connected with the sth division and is for the present restricted only by the condition

$$S(\lambda_s) = N \quad \text{..(ii)},$$

—a condition which enables us to write

$$\phi^2 = S\left\{\frac{(n_s - \lambda_s)^2}{N\lambda_s}\right\} \quad \text{..................................(iii)}$$

as equivalent to (i).

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* *Biometrika,* Vol. v. p. 191.

† *Biometrika,* Vol. x. p. 570.

These undetermined numbers λ_s are thus in general of the nature of weights and may be chosen in a variety of ways. The most important particular case is that of the population being grouped in a contingency table with, say, two variates. The sth division will be, say, the cell (u, v) and if λ_s be taken to be $N\frac{m_u m_v}{M^2}$, m_u and m_v being as usual the marginal totals of the uth row and vth column of the population M, ϕ^2 will be the mean square contingency*. Other cases will be discussed later.

The object of the present paper is to investigate the variation of the quantity ϕ^2 as determined from the samples of the population. We take the numbers λ to be a property of the whole population and accordingly to have no variation as long as the size N of the samples is constant. It is true that in most cases in practice there will be only one sample and that the values of the numbers λ will have to be deduced from that sample and will therefore deviate from the values which would be used if the sampled population were known. But what we are seeking is the variability of the samples on the understanding that the distribution of the whole population is *definite* although in practice we know only the approximation to that distribution which is given by our sample. If we had wanted the variability of the *calculated* values of ϕ^2 deduced from a large number of random samples, then we should have taken into account the variation of the λ's as well as of the n's. In this lies the difference between the discussions in the two earlier papers of 1906 and 1914.

This investigation follows that of the second paper, but we shall here give the full expressions without approximation, i.e. without neglecting the square of δn_s as was done in 1914. It will appear from the numerical examples worked out later that this squared term makes a fairly great difference and, even if this were not so, it is always preferable to have such formulae in full in order to decide the legitimacy of neglecting any terms. This is especially the case in statistical theory where neglect of the later terms of a Taylor expansion often leads to false results.

(2) *Mean Value of* ϕ^2.

Let $\bar{\phi}^2$ be the mean value of ϕ^2 and let $\bar{n}_s$ be the mean value of n_s, i.e. the value which would be given by taking a very large number of samples. Then we can write

$$\frac{\bar{n}_s}{N} = \frac{m_s}{M}.$$

Also if we define $\delta\phi^2$ and δn_s by the equations

$$\phi^2 = \bar{\phi}^2 + \delta\phi^2,$$
$$n_s = \bar{n}_s + \delta n_s,$$

we have

$$1 + \bar{\phi}^2 + \delta\phi^2 = S\left(\frac{\bar{n}_s^2}{N\lambda_s}\right) + 2S\left(\frac{\bar{n}_s \delta n_s}{N\lambda_s}\right) + S\left(\frac{(\delta n_s)^2}{N\lambda_s}\right) \text{............(iv).}$$

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* *Drapers' Research Memoirs, Biometric Series*, I. On the Theory of Contingency, etc., Cambridge University Press, 1904; *Biometrika*, Vol. v. p. 191, 1906; *Biometrika*, Vol. x. p. 570, 1914.

Sum all such equations for a large number of samples and divide by the number of samples. Then, since Mean $\delta n_s = 0$ and Mean $\delta\phi^2 = 0$, we have

$$1 + \bar{\phi}^2 = S\left(\frac{\bar{n}_s^2}{N\lambda_s}\right) + \text{Mean}\, S\left(\frac{(\delta n_s)^2}{N\lambda_s}\right) \quad \text{.....................(v).}$$

The expression Mean $(\delta n_s)^2$ is typical of several which we have to use in what follows and it will be useful to state or prove all the needful formulae before proceeding further.

(3) *Formulae regarding Products of Deviations.*

The deviations δn_s arrange themselves according to a hypergeometrical series and the moment coefficients of this series are known to be*

$$\left.\begin{aligned} \mu_2 &= \chi_1 Npq \\ \mu_3 &= \chi_1\chi_2 Npq\,(p-q) \\ \mu_4 &= \chi_1 Npq\,(3\chi_3 Npq + \chi_4) \\ \text{where} \qquad \chi_1 &= 1 - \frac{N-1}{M-1} \\ \chi_2 &= 1 - \frac{2\,(N-1)}{M-2} \\ \chi_3 &= \left(1 - \frac{2}{N}\right)\left\{1 - \frac{N-1}{M-2}\left(\frac{N-10}{N-2} + \frac{9}{M-3}\right)\right\} \\ \chi_4 &= 1 - 6\,\frac{N-1}{M-2}\left(1 - \frac{N-2}{M-3}\right) \end{aligned}\right\} \quad \text{.........(vi),}$$

and in the present case

$$\left.\begin{aligned} p &= \frac{m_s}{M} = \frac{\bar{n}_s}{N} \\ q &= 1 - \frac{m_s}{M} = 1 - \frac{\bar{n}_s}{N} \end{aligned}\right\} \quad \text{..............................(vii).}$$

When M is very large as compared with N, as in the majority of cases in practice, we may write $\chi_1 = \chi_2 = \chi_3 = \chi_4 = 1$.

We can now make use of these formulae to derive the following:

(*a*) Mean $(\delta n_s)^2$. This is μ_2 in the notation of (vi) and

$$= \chi_1 \bar{n}_s \left(1 - \frac{\bar{n}_s}{N}\right) \quad \text{....................................}(a).$$

(*b*) Mean $\delta n_s \delta n_{s'}$ where s and s' differ. Suppose first that δn_s remains constant and investigate the mean of $\delta n_{s'}$ for this constant value of δn_s. Now the distribution of $n_{s'}$ for n_s constant is clearly given by the sub-hypergeometrical series with $N' = N - \bar{n}_s - \delta n_s$ as total population and

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* Pearson, *Phil. Mag.* 1899, p. 239; *Biometrika*, Vol. v. p. 174.

$$p' = \frac{\bar{n}_{s'}}{N - \bar{n}_s},$$

$$q' = 1 - \frac{\bar{n}_{s'}}{N - \bar{n}_s},$$

so that the Mean $\delta n_{s'}$ for δn_s constant is

$$(N - \bar{n}_s - \delta n_s) \frac{\bar{n}_{s'}}{N - \bar{n}_s} - \bar{n}_{s'} = -\frac{\delta n_s \bar{n}_{s'}}{N - \bar{n}_s} *.$$

Hence

$$\begin{aligned} \text{Mean } \delta n_s \delta n_{s'} &= -\text{ Mean } \delta n_s \cdot \frac{\delta n_s \bar{n}_{s'}}{N - \bar{n}_s} \\ &= -\text{ Mean } \delta n_s^2 \cdot \frac{\bar{n}_{s'}}{N - \bar{n}_s} \\ &= -\chi_1 \bar{n}_s \left(1 - \frac{\bar{n}_s}{N}\right) \frac{\bar{n}_{s'}}{N - \bar{n}_s} \text{ from } (a) \\ &= -\chi_1 \frac{\bar{n}_s \bar{n}_{s'}}{N} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (b). \end{aligned}$$

(*c*) Mean $(\delta n_s)^3$. Directly from (vi) and (vii)

$$\text{Mean } \delta n_s^3 = \chi_1 \chi_2 \bar{n}_s \left(1 - \frac{\bar{n}_s}{N}\right)\left(1 - \frac{2\bar{n}_s}{N}\right) \quad \ldots\ldots\ldots\ldots\ldots (c).$$

(*d*) Mean $(\delta n_s)^2 \delta n_{s'}$. Using the process of double summation as in (*b*) we have

$$\begin{aligned} \text{Mean } (\delta n_s)^2 \delta n_{s'} &= -\text{ Mean } (\delta n_s)^2 \cdot \frac{\delta n_s \bar{n}_{s'}}{N - \bar{n}_s} \\ &= -\frac{\bar{n}_{s'}}{N - \bar{n}_s} \cdot \chi_1 \chi_2 \bar{n}_s \left(1 - \frac{\bar{n}_s}{N}\right)\left(1 - \frac{2\bar{n}_s}{N}\right) \text{ from } (c) \\ &= -\chi_1 \chi_2 \frac{\bar{n}_s \bar{n}_{s'}}{N}\left(1 - \frac{2\bar{n}_s}{N}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (d). \end{aligned}$$

(*e*) Mean $(\delta n_s)^4$. As in (*c*) we have immediately

$$\text{Mean } (\delta n_s)^4 = \chi_1 \bar{n}_s \left(1 - \frac{\bar{n}_s}{N}\right)\left(3\chi_3 \bar{n}_s \left(1 - \frac{\bar{n}_s}{N}\right) + \chi_4\right) \quad \ldots\ldots\ldots\ldots (e).$$

(*f*) Mean $(\delta n_s)^2 (\delta n_{s'})^2$. We again use the double summation as in (*b*), but in this case the algebra is much more troublesome. From the constants of the sub-hypergeometrical as given in (*b*) we have

$$\chi_1' N' p' q' = \left(1 - \frac{N - \bar{n}_s - \delta n_s - 1}{M - m_s - 1}\right)(N - \bar{n}_s - \delta n_s) \frac{\bar{n}_{s'}}{N - \bar{n}_s}\left(1 - \frac{\bar{n}_{s'}}{N - \bar{n}_s}\right).$$

---

* Since $S(\delta n_s)$ must be zero, we can regard the Mean $\delta n_{s'}$ for a given δn_s as being the result of a distribution of a deviate $-\delta n_s$ distributed over all the divisions except the *s*th. The portion due to the *s*′th is then $\frac{n_{s'}}{N - n_s} \times (-\delta n_s)$, as obtained above.

But this is the Mean $(\delta' n_{s'})^2$, where $\delta' n_{s'}$ is measured from the mean of $n_{s'}$ in the case where δn_s is fixed and we must reduce to the general mean, i.e. where δn_s is not given, to obtain the mean value of $(\delta n_{s'})^2$ for constant δn_s. This is done by adding the square of the difference between these means of $n_{s'}$, namely $\dfrac{\bar{n}_{s'} \delta n_s}{N - \bar{n}_s}$.

We thus obtain that the Mean $(\delta n_{s'})^2$ for constant δn_s

$$= \left(1 - \frac{N - \bar{n}_s - \delta n_s - 1}{M - m_s - 1}\right) (N - \bar{n}_s - \delta n_s) \frac{\bar{n}_{s'}}{N - \bar{n}_s} \left(1 - \frac{\bar{n}_{s'}}{N - \bar{n}_s}\right) + \frac{\bar{n}_{s'}^2 (\delta n_s)^2}{(N - \bar{n}_s)^2}$$

$$= \left\{\frac{N}{M}\left(1 - \frac{N}{M}\right) - \left(1 - \frac{2N}{M}\right) \frac{N \delta n_s}{M (N - \bar{n}_s)} - \frac{N^2 (\delta n_s)^2}{M^2 (N - \bar{n}_s)^2}\right\} \frac{M^2 \bar{n}_{s'} (N - \bar{n}_s - \bar{n}_{s'})}{N^2 \left(\frac{M}{N} (N - \bar{n}_s) - 1\right)} + \frac{\bar{n}_{s'}^2 (\delta n_s)^2}{(N - \bar{n}_s)^2},$$

when we substitute $M\bar{n}_s/N$ for m_s.

Thus we have to evaluate

$$\text{Mean } (\delta n_s)^2 (\delta n_{s'})^2 = \left\{\frac{N}{M}\left(1 - \frac{N}{M}\right) \text{Mean } (\delta n_s)^2 - \left(1 - \frac{2N}{M}\right) \frac{N}{M} \frac{\text{Mean } (\delta n_s)^3}{N - \bar{n}_s} - \frac{N^2}{M^2} \frac{\text{Mean } (\delta n_s)^4}{(N - \bar{n}_s)^2}\right\} \frac{M^2 \bar{n}_{s'} (N - \bar{n}_s - \bar{n}_{s'})}{N^2 \left(\frac{M}{N} (N - \bar{n}_s) - 1\right)} + \frac{\bar{n}_{s'}^2}{(N - \bar{n}_s)^2} \text{Mean } (\delta n_s)^4.$$

Substituting from (a), (c) and (e), we find

$$\text{Mean } (\delta n_s)^2 (\delta n_{s'})^2 = \chi_1 \frac{\bar{n}_s \bar{n}_{s'}}{N} \left\{\frac{N}{M}\left(1 - \frac{N}{M}\right) \frac{M^2 (N - \bar{n}_s)(N - \bar{n}_s - \bar{n}_{s'})}{N^2 \left(\frac{M}{N} (N - \bar{n}_s) - 1\right)} - \chi_2 \left(1 - \frac{2N}{M}\right) \frac{M (N - 2\bar{n}_s)(N - \bar{n}_s - \bar{n}_{s'})}{N^2 \left(\frac{M}{N} (N - \bar{n}_s) - 1\right)} - 3\chi_3 \frac{\bar{n}_s (N - \bar{n}_s - \bar{n}_{s'})}{N \left(\frac{M}{N} (N - \bar{n}_s) - 1\right)} - \chi_4 \frac{N - \bar{n}_s - \bar{n}_{s'}}{(N - \bar{n}_s) \left(\frac{M}{N} (N - \bar{n}_s) - 1\right)} + 3\chi_3 \frac{\bar{n}_s \bar{n}_{s'}}{N} + \chi_4 \frac{\bar{n}_{s'}}{N - \bar{n}_s}\right\}.$$

This expression must be symmetrical in s and s' and this will be the case only if the quantities $\dfrac{M}{N}(N - \bar{n}_s) - 1$ and $N - \bar{n}_s$ in the denominator cancel with factors in the numerator. By taking the two terms in χ_4 together we get rid of the $N - \bar{n}_s$ factor and after a laborious expansion in substituting the values of the χ's we reduce the whole expression to the comparatively simple form

$$\chi_1 \frac{\bar{n}_s \bar{n}_{s'}}{N} \left\{\chi_3 N \left(1 - \frac{\bar{n}_s + \bar{n}_{s'}}{N} + \frac{3\bar{n}_s \bar{n}_{s'}}{N^2}\right) + \chi_4\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (f).$$

This agrees with the value obtained by Isserlis from the differential equation to the hypergeometric series and thus confirms his result obtained by a totally different procedure*.

---

* *Proc. Roy. Soc.* Vol. XCII. p. 28. Our notations are different.

With these formulae we can now proceed to discuss the mean value of ϕ^2 and its variability.

(4) *Mean Value of ϕ^2 (continued).*

At the end of (2) we had arrived at the equation

$$1 + \bar{\phi}^2 = S\left(\frac{\bar{n}_s^2}{N\lambda_s}\right) + \text{Mean } S\left(\frac{(\delta n_s)^2}{N\lambda_s}\right),$$

and substituting from (*a*) we now obtain

$$1 + \bar{\phi}^2 = S\left(\frac{\bar{n}_s^2}{N\lambda_s}\right) + \chi_1 \frac{1}{N} S\left\{\frac{\bar{n}_s}{\lambda_s}\left(1 - \frac{\bar{n}_s}{N}\right)\right\} \quad \text{...............(viii)}.$$

We can usually put $\chi_1 = 1$ and write

$$1 + \bar{\phi}^2 = \left(1 - \frac{1}{N}\right) S\left(\frac{\bar{n}_s^2}{N\lambda_s}\right) + \frac{1}{N} S\left(\frac{\bar{n}_s}{\lambda_s}\right) \quad \text{..................(ix)}.$$

In the particular case of the contingency table

$$1 + \bar{\phi}^2 = \left(1 - \frac{1}{N}\right) S\left(\frac{\bar{n}_{uv}^2}{\bar{n}_u \bar{n}_v}\right) + \frac{1}{N} S\left(\frac{N\bar{n}_{uv}}{\bar{n}_u \bar{n}_v}\right) \text{..................(x)}.$$

Now $S\left(\frac{\bar{n}_{uv}^2}{\bar{n}_u \bar{n}_v}\right) = 1 + \phi_p^2$, where ϕ_p^2 is the mean square contingency for the whole population, so that the mean value of ϕ^2 as determined from a large number of samples is in excess of the true mean square contingency by

$$\frac{1}{N}\left(S\left(\frac{N\bar{n}_{uv}}{\bar{n}_u \bar{n}_v}\right) - (1 + \phi_p^2)\right).$$

(5) *Standard Deviation of ϕ^2. Non-approximative Formulae.*

From the equations

$$1 + \bar{\phi}^2 + \delta\phi^2 = S\left(\frac{\bar{n}_s^2}{N\lambda_s}\right) + \frac{2}{N} S\left(\frac{\bar{n}_s \delta n_s}{\lambda_s}\right) + \frac{1}{N} S\left(\frac{(\delta n_s)^2}{\lambda_s}\right),$$

and

$$1 + \bar{\phi}^2 = S\left(\frac{\bar{n}_s^2}{N\lambda_s}\right) + \chi_1 \frac{1}{N} S\left\{\frac{\bar{n}_s}{\lambda_s}\left(1 - \frac{\bar{n}_s}{N}\right)\right\},$$

we have

$$N\delta\phi^2 = S\left(\frac{(\delta n_s)^2}{\lambda_s}\right) + 2S\left(\frac{\bar{n}_s \delta n_s}{\lambda_s}\right) - \chi_1 S\left\{\frac{\bar{n}_s}{\lambda_s}\left(1 - \frac{\bar{n}_s}{N}\right)\right\} \quad \text{.........(xi)}.$$

Squaring, summing for a large number of samples and dividing by the number of samples, we have

$$\begin{aligned} N^2\sigma^2_{\phi^2} &= \text{Mean } N^2 (\delta\phi^2)^2 \\ &= \text{Mean}\Bigg[\left(S\left(\frac{(\delta n_s)^2}{\lambda_s}\right)\right)^2 + 4\left(S\left(\frac{\bar{n}_s \delta n_s}{\lambda_s}\right)\right)^2 + \chi_1^2\left(S\left\{\frac{\bar{n}_s}{\lambda_s}\left(1 - \frac{\bar{n}_s}{N}\right)\right\}\right)^2 \\ &\qquad + 4S\left(\frac{(\delta n_s)^2}{\lambda_s}\right) . S\left(\frac{\bar{n}_s \delta n_s}{\lambda_s}\right) - 2\chi_1 S\left\{\frac{\bar{n}_s}{\lambda_s}\left(1 - \frac{\bar{n}_s}{N}\right)\right\} . S\left(\frac{\delta n_s^2}{\lambda_s}\right)\Bigg] \end{aligned}$$

$$
\begin{aligned}
= \text{Mean}\Bigg[&\underset{s}{S}\left(\frac{(\delta n_s)^4}{\lambda_s^2}\right) + \underset{s\ s'}{S\,S}\left(\frac{(\delta n_s)^2\,(\delta n_{s'})^2}{\lambda_s\lambda_{s'}}\right) + 4\underset{s}{S}\left(\frac{\bar{n}_s^{\,2}\,(\delta n_s)^2}{\lambda_s^2}\right)\\
&+ 4\underset{s\ s'}{S\,S}\left(\frac{\bar{n}_s\bar{n}_{s'}\,\delta n_s\,\delta n_{s'}}{\lambda_s\lambda_{s'}}\right) + 4\underset{s}{S}\left(\frac{(\delta n_s)^3\,\bar{n}_s}{\lambda_s^2}\right) + 4\underset{s\ s'}{S\,S}\left(\frac{(\delta n_s)^2\,\bar{n}_{s'}\,\delta n_s}{\lambda_s\lambda_{s'}}\right)\\
&- \chi_1^2\underset{s}{S}\left(\frac{\bar{n}_s^{\,2}}{\lambda_s^2}\,\frac{(N-\bar{n}_s)^2}{N^2}\right) - \chi_1^2\underset{s\ s'}{S\,S}\left(\frac{\bar{n}_s\bar{n}_{s'}}{\lambda_s\lambda_{s'}}\,\frac{(N-\bar{n}_s)\,(N-\bar{n}_{s'})}{N^2}\right)\Bigg],
\end{aligned}
$$

where $\underset{s}{S}$ denotes the same as S, i.e. summation for all values of s, and $\underset{s\ s'}{S\,S}$ denotes summation for all values of s' except $s = s'$ followed by summation for all values of s*.

Substituting from the equations (*a*), (*b*), (*c*), (*d*), (*e*), (*f*), we have

$$
\begin{aligned}
N^2\sigma^2_{\phi^2} = &\underset{s}{S}\left\{\frac{1}{\lambda_s^2}\,\chi_1\,\frac{\bar{n}_s(N-\bar{n}_s)}{N}\left(3\chi_3\,\frac{\bar{n}_s(N-\bar{n}_s)}{N}+\chi_4\right)\right\}\\
&\qquad + \underset{s\ s'}{S\,S}\left\{\frac{1}{\lambda_s\lambda_{s'}}\,\chi_1\,\frac{\bar{n}_s\bar{n}_{s'}}{N}\left(\chi_3 N\left(1-\frac{\bar{n}_s+\bar{n}_{s'}}{N}+\frac{3\bar{n}_s\bar{n}_{s'}}{N^2}\right)+\chi_4\right)\right\}\\
&+ 4\underset{s}{S}\left\{\frac{\bar{n}_s^{\,2}}{\lambda_s^2}\,\chi_1\,\frac{\bar{n}_s(N-\bar{n}_s)}{N}\right\} - 4\underset{s\ s'}{S\,S}\left\{\frac{\bar{n}_s\bar{n}_{s'}}{\lambda_s\lambda_{s'}}\,\chi_1\,\frac{\bar{n}_s\bar{n}_{s'}}{N}\right\}\\
&+ 4\underset{s}{S}\left\{\frac{\bar{n}_s}{\lambda_s^2}\,\chi_1\chi_2\,\frac{\bar{n}_s(N-\bar{n}_s)(N-2\bar{n}_s)}{N^2}\right\} - 4\underset{s\ s'}{S\,S}\left\{\frac{\bar{n}_{s'}}{\lambda_s\lambda_{s'}}\,\chi_1\chi_2\,\frac{\bar{n}_s\bar{n}_{s'}(N-2\bar{n}_s)}{N^2}\right\}\\
&- \chi_1^2\underset{s}{S}\left\{\frac{\bar{n}_s^{\,2}}{\lambda_s^2}\,\frac{(N-\bar{n}_s)^2}{N^2}\right\} - \chi_1^2\underset{s\ s'}{S\,S}\left\{\frac{\bar{n}_s\bar{n}_{s'}}{\lambda_s\lambda_{s'}}\,\frac{(N-n_s)(N-\bar{n}_{s'})}{N^2}\right\}\\
= &\underset{s}{S}\left\{\frac{\chi_1}{\lambda_s^2}\left[\chi_4\bar{n}_s + \left(3\chi_3+4\chi_2-\chi_1-\frac{\chi_4}{N}\right)\bar{n}_s^{\,2} + \left(4-(6\chi_3+12\chi_2-2\chi_1)\frac{1}{N}\right)\bar{n}_s^{\,3}\right.\right.\\
&\qquad\qquad\left.\left. + \left(-\frac{4}{N}+(3\chi_3+8\chi_2-\chi_1)\frac{1}{N^2}\right)\bar{n}_s^{\,4}\right]\right\}\\
&+ \underset{s\ s'}{S\,S}\left\{\frac{\chi_1}{\lambda_s\lambda_{s'}}\left[\left(\chi_3-\chi_1+\frac{\chi_4}{N}\right)\bar{n}_s\bar{n}_{s'} + (\chi_1-\chi_3-2\chi_2)\,\frac{\bar{n}_s\bar{n}_{s'}(\bar{n}_s+\bar{n}_{s'})}{N}\right.\right.\\
&\qquad\qquad\left.\left. + \left(-\frac{4}{N}+(3\chi_3+8\chi_2-\chi_1)\frac{1}{N^2}\right)\bar{n}_s^{\,2}\bar{n}_{s'}^{\,2}\right]\right\},
\end{aligned}
$$

after expansion and rearrangement.

Now it is evident that in numerical work the double summation would involve much extra labour, but we can get rid of it by using the identities

$$
\begin{aligned}
\left(\underset{s}{S}\left(\frac{\bar{n}_s}{\lambda_s}\right)\right)^2 &= \underset{s}{S}\left(\frac{\bar{n}_s^{\,2}}{\lambda_s^2}\right) + \underset{s\ s'}{S\,S}\left(\frac{\bar{n}_s\bar{n}_{s'}}{\lambda_s\lambda_s}\right),\\
\underset{s}{S}\left(\frac{\bar{n}_s}{\lambda_s}\right)^2\underset{s}{S}\left(\frac{\bar{n}_s}{\lambda_s}\right) &= \underset{s}{S}\left(\frac{\bar{n}_s^{\,3}}{\lambda_s^2}\right) + \underset{s\ s'}{S\,S}\left(\frac{\bar{n}_s^{\,2}\bar{n}_{s'}}{\lambda_s\lambda_{s'}}\right) = \underset{s}{S}\left(\frac{\bar{n}_s^{\,3}}{\lambda_s^2}\right) + \tfrac{1}{2}\underset{s\ s'}{S\,S}\left(\frac{\bar{n}_s\bar{n}_{s'}}{\lambda_s\lambda_{s'}}(\bar{n}_s+\bar{n}_{s'})\right),\\
\left(\underset{s}{S}\left(\frac{\bar{n}_s^{\,2}}{\lambda_s}\right)\right)^2 &= \underset{s}{S}\left(\frac{\bar{n}_s^{\,4}}{\lambda_s^2}\right) + \underset{s\ s'}{S\,S}\left(\frac{\bar{n}_s^{\,2}\bar{n}_{s'}^{\,2}}{\lambda_s\lambda_{s'}}\right),
\end{aligned}
$$

and so reducing all to single summations.

---

* As this notation may be somewhat unusual, it may be better to make it clear by taking a case with three variates only, for example:

$$(\underset{s}{S}n_s)^2 = (n_1+n_2+n_3)^2 = n_1^2+n_2^2+n_3^2+(n_2+n_3)\,n_1+(n_3+n_1)\,n_2+(n_1+n_2)\,n_3 = \underset{s}{S}n_s^2+\underset{s\ s'}{S\,S}n_s n_{s'}.$$

This leads to

$$\begin{aligned} N^2\sigma^2_{\phi^2} = \chi_1\chi_4 S\left(\frac{\bar{n}_s}{\lambda_s^2}\right) &+ \chi_1\left(\chi_3 - \chi_1 + \frac{\chi_4}{N}\right)\left\{S\left(\frac{\bar{n}_s}{\lambda_s}\right)\right\}^2 \\ &+ \chi_1\left(2\chi_3 + 4\chi_2 - \frac{2\chi_4}{N}\right) S\left(\frac{\bar{n}_s^2}{\lambda_s^2}\right) \\ &+ \chi_1\left(\frac{2\chi_1 - 2\chi_3 - 4\chi_2}{N}\right) S\left(\frac{\bar{n}_s^2}{\lambda_s}\right) S\left(\frac{\bar{n}_s}{\lambda_s}\right) \\ &+ \chi_1\left(4 - \frac{(4\chi_3 + 8\chi_2)}{N}\right) S\left(\frac{\bar{n}_s^3}{\lambda_s^2}\right) \\ &+ \chi_1\left(-\frac{4}{N} + (3\chi_3 + 8\chi_2 - \chi_1)\frac{1}{N^2}\right)\left\{S\left(\frac{\bar{n}_s^2}{\lambda_s}\right)\right\}^2 \quad \ldots\text{(xii)}. \end{aligned}$$

(6) *Standard Deviation of ϕ^2. Approximate Formulae.*

The result of the preceding section is an exact one since we have neglected no terms in arriving at it, but as mentioned before we can usually take M to be very large compared with N and make $\chi_1 = \chi_2 = \chi_3 = \chi_4 = 1$. With this simplification equation (xii) becomes

$$\begin{aligned} \sigma^2_{\phi^2} = \frac{4}{N}&\left[S\left(\frac{\bar{n}_s^3}{N\lambda_s^2}\right) - \left\{S\left(\frac{\bar{n}_s^2}{N\lambda_s}\right)\right\}^2\right] \\ &+ \frac{1}{N^2}\left[6S\left(\frac{\bar{n}_s^2}{\lambda_s^2}\right) - 4S\left(\frac{\bar{n}_s^2}{N\lambda_s}\right) S\left(\frac{\bar{n}_s}{\lambda_s}\right) - 10\left\{S\left(\frac{\bar{n}_s^2}{N\lambda_s}\right)\right\}^2 - 12S\left(\frac{\bar{n}_s^3}{N\lambda_s^2}\right)\right] \\ &+ \frac{1}{N^3}\left[S\left(\frac{N\bar{n}_s}{\lambda_s^2}\right) + \left\{S\left(\frac{\bar{n}_s}{\lambda_s}\right)\right\}^2 - 2S\left(\frac{\bar{n}_s^2}{\lambda_s^2}\right)\right] \ldots\ldots\ldots\ldots\text{(xiii)}. \end{aligned}$$

In the great majority of cases it will be impossible to make rigorous use of this formula since we have no other knowledge of the whole population than what is given by the sample. In particular the $\bar{n}$'s are usually unknown and we must simply make use of the approximations at our disposal, namely the n's of the observed sample.

Again, it will usually happen that while $\bar{n}_s$ may be fairly large $\bar{n}_s - \lambda_s$ will be small and it will give formulae which are much more convenient for computation if we write $\psi_s = \bar{n}_s - \lambda_s$ and substitute $\psi_s + \lambda_s$ for $\bar{n}_s$ in equation (xiii), remembering that $S(\lambda_s) = N$.

After some reduction the formula becomes

$$\begin{aligned} \sigma^2_{\phi^2} = \frac{4}{N}&\left[S\left(\frac{\psi_s^3}{N\lambda_s^2}\right) + \phi^2 - \phi^4\right] \\ &+ \frac{1}{N^2}\left[6S\left(\frac{\psi_s^2}{\lambda_s^2}\right) + (8 - 4\phi^2)S\left(\frac{\psi_s}{\lambda_s}\right) - 12S\left(\frac{\psi_s^3}{N\lambda_s^2}\right) + (2 - 4\phi^2)c - 22 - 56\phi^2 - 10\phi^4\right] \\ &+ \frac{1}{N^3}\left[S\left(\frac{N\psi_s}{\lambda_s^2}\right) + S\left(\frac{N}{\lambda_s}\right) + \left\{S\left(\frac{\psi_s}{\lambda_s}\right)\right\}^2 - 2S\left(\frac{\psi_s^2}{\lambda_s^2}\right) + 2(c-2)S\left(\frac{\psi_s}{\lambda_s}\right) + c(c-2)\right] \\ &\qquad\ldots\ldots\ldots\ldots\text{(xiv)}, \end{aligned}$$

where c is the number of classes or categories in the population in question.

(7) *First Application. Contingency.*

As mentioned in (1), if we regard the sth division as the (u, v) cell of a contingency-table and if we take

$$\lambda_s = N\frac{m_u m_v}{M^2} = \frac{\bar{n}_u \bar{n}_v}{N},$$

then

$$1 + \phi^2 = S\left(\frac{n^2_{uv}}{\bar{n}_u \bar{n}_v}\right),$$

and

$$\phi^2 = S\left\{\frac{\left(n_{uv} - \frac{\bar{n}_u \bar{n}_v}{N}\right)^2}{\bar{n}_u \bar{n}_v}\right\} = \text{Mean square contingency.}$$

Accordingly, with the notation

$$\lambda_s = \frac{\bar{n}_u \bar{n}_v}{N},$$

$$\psi_s = n_{uv} - \lambda_s = n_{uv} - \frac{\bar{n}_u \bar{n}_v}{N},$$

and

$$\phi^2 = S\left(\frac{\psi_s^2}{N\lambda_s}\right) = S\left\{\frac{\left(n_{uv} - \frac{\bar{n}_u \bar{n}_v}{N}\right)^2}{\bar{n}_u \bar{n}_v}\right\},$$

the equation (xiv) gives the standard deviation of the mean square contingency, when M is very large as compared with N.

The terms enclosed in the first bracket of (xiv) are exactly those of Pearson's 1914 paper in *Biometrika*, so that the second and third brackets contain the terms arising from the squares and higher products of δn_s.

Of the total correction due to the higher approximation it is of interest to find how much is due to the change of mean and consequently the change of origin of $\delta\phi^2$, when the square of δn_s is not neglected. The true mean is given by

$$1 + \bar{\phi}^2 = \frac{1}{N} S\left(\frac{\bar{n}_s^2}{\lambda_s}\right) + \frac{1}{N} S\left\{\frac{\bar{n}_s}{\lambda_s}\left(1 - \frac{\bar{n}_s}{N}\right)\right\},$$

and using the observed values of n_s as the best approximation available for $\bar{n}_s$

$$1 + \bar{\phi}^2 = (1 + \phi^2)\left(1 - \frac{1}{N}\right) + \frac{1}{N} S\left(\frac{\psi_s + \lambda_s}{\lambda_s}\right)$$

$$= (1 + \phi^2) + \frac{1}{N}\left\{S\left(\frac{\psi_s}{\lambda_s}\right) - \phi^2 + c - 1\right\},$$

so that the difference between the true mean and the approximate mean obtained by neglecting squares of δn_s is

$$\bar{\phi}^2 - \phi^2 = \frac{1}{N}\left\{S\left(\frac{\psi_s}{\lambda_s}\right) - \phi^2 + c - 1\right\}.$$

In accordance, then, with the formula for change of second moment with change of origin we get the effect of the change of mean on $\sigma^2_{\phi^2}$ by subtracting

$$\frac{1}{N^2}\left\{S\left(\frac{\psi_s}{\lambda_s}\right) - \phi^2 + c - 1\right\}^2$$

from the approximate value.

In the examples given below it will be seen that this is only a small part of the total correction and thus the main part of the correction is due to the retention of the squares and products in the value of $(\delta\phi^2)^2$ used in (5).

(8) *Numerical Illustrations.*

I. Contingency between Handwriting and Intelligence in Girls.

The probable errors of the contingency constants in this table have been worked out both in the 1905 and in the 1914 papers and below is given a table showing the effect of the corrective terms of the present discussion.

The new summations required are found to be

$$S\left(\frac{\psi_s}{\lambda_s}\right) = 12{\cdot}788, \qquad S\left(\frac{N}{\lambda_s}\right) = 15982,$$

$$S\left(\frac{\psi_s^2}{\lambda_s^2}\right) = 93{\cdot}144, \qquad S\left(\frac{N\psi_s}{\lambda_s^2}\right) = 57270,$$

and in the following equation the numerical values of the various terms of equation (xiv) are given in the same order as their corresponding algebraic terms:

$$\begin{aligned}\sigma^2_\phi &= \frac{4}{1801}[{\cdot}14865 + {\cdot}09580 - {\cdot}00918] \\ &+ \frac{1}{(1801)^2}[558{\cdot}864 + 97{\cdot}404 - 1{\cdot}784 + 58{\cdot}205 - 22 - 5{\cdot}365 - {\cdot}092] \\ &+ \frac{1}{(1801)^3}[57270 + 15982 + 164 - 186 + 870 + 1224].\end{aligned}$$

The other calculations are summarised in the table below:

TABLE I.

$$\phi^2 = {\cdot}09580, \quad C_2 = \sqrt{\frac{\phi^2}{1+\phi^2}} = {\cdot}2957.$$

	Various formulae used			
	Blakeman and Pearson (1905)	1st Term of (xiv) or Pearson (1914)	1st and 2nd Terms of (xiv)	All Terms of (xiv)
σ_{ϕ^2}	·02023	·02286	·02709	·02729
Probable error of ϕ^2	·01365*	·01542	·01827	·01841
σ_C	·0285	·03219	·03815	·03844
Probable error of C	·0192	·02171	·02573	·02593

In this table the work has been carried out to four significant figures with a view to showing the corrective effects of the various terms. It is apparent that the fineness of approximation given by (xiv) in full is more than is required in practice, but there is a considerable difference between the values given according as the second term is used or not and it seems that in some cases it would be advisable to calculate this term or at least the most important terms in it, viz.

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* Incorrectly given as ·0042 in Blakeman and Pearson's paper, *loc. cit.* footnote p. 196.

$$\frac{1}{N^2}\left(6S\left(\frac{\psi_s^2}{\lambda_s^2}\right)+8S\left(\frac{\psi_s}{\lambda_s}\right)+2c\right)\text{..........................(xv).}$$

Using this approximation in the above case, we obtain $\sigma_{\phi^2} = \cdot 02730$, a result which—by a mere chance, of course—is almost exactly that given by the full expression in (xiv).

II. Contingency between the Hair-colours of Pairs of Female Cousins.

In the example just given the total number N in the sample was fairly large, viz. 1801, and it might be expected that in smaller samples the corrective terms would be of increased importance. With a view to testing this a contingency-table given by Miss Elderton in her Memoir on "The Measure of the Resemblance of First Cousins"* was selected. There are 36 cells in this table and the total number in the sample is only 218, there being several cells with zero or very small content.

The Table is given in full on p. 226 along with the quantities required for the calculation of ϕ^2 and σ_{ϕ^2}; it is there evident from the figures how large a proportion of the variability depends on the cells of small content. This is of course to be expected but the importance of having large numbers in all the cells is not always appreciated. In this particular case physiological reasons would prevent us from clubbing together the "Fairs" and "Reds" and with the fewness of the observations at our disposal we must use the table simply as it stands.

The scheme followed in each cell of the table is shown in the last column, and in the marginal totals are given the values of all the summations required for (xiv). These are

$$\phi^2 = S\left(\frac{\psi_s^2}{N\lambda_s}\right) = \cdot 14895, \qquad S\left(\frac{N\psi_s}{\lambda_s^2}\right) = -2336\cdot 8,$$

$$S\left(\frac{\psi_s}{\lambda_s}\right) = -1\cdot 8481, \qquad S\left\{\left(\frac{\psi_s}{\lambda_s}\right)^2\right\} = 19\cdot 162,$$

$$S\left(\frac{N}{\lambda_s}\right) = 5170\cdot 9, \qquad S\left(\frac{\psi_s^3}{N\lambda_s^2}\right) = \cdot 08277.$$

When these are substituted in equation (xiv), we have, preserving the algebraic order as before,

$$\begin{aligned}\sigma^2_{\phi^2} = {} & \frac{4}{218}[\cdot 08277 + \cdot 14895 - \cdot 02218] \\ & + \frac{1}{(218)^2}[114\cdot 9714 - 13\cdot 6837 - \cdot 9932 + 50\cdot 5515 - 22 - 8\cdot 3411 - \cdot 2219] \\ & + \frac{1}{(218)^3}[-2336\cdot 8 + 5170\cdot 9 + 3\cdot 4 - 38\cdot 3 + 125\cdot 7 + 1224],\end{aligned}$$

and the whole work is again summarised in Table III (p. 227):

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* *Eugenics Laboratory Memoirs*, IV. Cambridge University Press 1907.

TABLE II. *Contingency between the Hair-colours of Female Cousins*

First Female Cousin

Second Female Cousin	Tints	Very Dark	Dark Brown	Brown	Light Brown	Fair	Red	Totals
	Very Dark	**9** 5·945 36·67 ·5139 ·2641 18·84 ·00720 ·00370	**11** 9·248 23·57 ·1894 ·0359 4·46 ·00152 ·00029	**3·5** 6·523 33·42 −·4634 ·2147 −15·49 ·00642 −·00298	**11** 9·495 22·96 ·1585 ·0251 3·64 ·00109 ·00017	**1·5** 3·963 55·00 −·6214 ·3861 −34·18 ·00702 −·00436	**0** 0·826 263·93 −1·0000 1·0000 −263·93 ·00379 −·00379	**36** $=Sn_s$ — (λ_s) 435·55 $=S(N/\lambda_s)$ −1·2230 $=S(\psi_s/\lambda_s)$ 1·9259 $=S(\psi_s^2/\lambda_s^2)$ −286·66 $=S(N\psi_s/\lambda_s^2)$ ·02705 $=S(\psi_s^2/N\lambda_s)$ −·00697 $=S(\psi_s^3/N\lambda_s^2)$
	Dark Brown	**11** 9·248 23·57 ·1894 ·0359 4·46 ·00152 ·00029	**13** 14·385 15·16 −·0963 ·0093 −1·46 ·00061 −·00006	**12** 10·147 21·48 ·1826 ·0333 3·92 ·00155 ·00028	**10** 14·771 14·76 −·3230 ·1043 −4·77 ·00707 −·00228	**9** 6·165 35·36 ·4598 ·2114 16·26 ·00598 ·00275	**1** 1·284 169·78 −·2212 ·0489 −37·56 ·00029 −·00006	**56** — 280·11 ·1913 ·4431 −19·15 ·01702 ·00092
	Brown	**3·5** 6·523 33·42 −·4634 ·2147 −15·49 .00642 −·00298	**12** 10·147 21·48 ·1826 ·0333 3·92 ·00155 ·00028	**8** 7·157 30·45 ·1178 ·0139 3·59 ·00046 ·00005	**9·75** 10·419 20·92 −·0642 ·0041 −1·34 ·00020 −·00001	**3·25** 4·349 50·12 −·2527 ·0639 −12·67 ·00127 −·00032	**3** 0·906 240·61 2·3111 5·3412 556·07 ·02220 ·05131	**39·5** — 397·00 1·8312 5·6711 534·08 ·03210 ·04833
	Light Brown	**11** 9·495 22·96 ·1585 ·0251 3·64 ·00109 ·00017	**10** 14·771 14·76 −·3230 ·1043 −4·77 ·00707 −·00228	**9·75** 10·419 20·92 −·0642 ·0041 −1·34 ·00020 −·00001	**16·5** 15·166 14·37 ·0880 ·0077 1·26 ·00054 ·00005	**9·25** 6·330 34·44 ·4614 ·2129 15·89 ·00618 ·00285	**1** 1·319 165·29 −·2419 ·0585 −39·98 ·00035 −·00009	**57·5** — 272·74 ·0788 ·4126 −25·30 ·01543 ·00069
	Fair	**1·5** 3·963 55·00 −·6214 ·3861 −34·18 ·00702 −·00436	**9** 6·165 35·36 ·4598 ·2114 16·26 ·00598 ·00275	**3·25** 4·349 50·12 −·2527 ·0639 −12·67 ·00127 −·00032	**9·25** 6·330 34·44 ·4614 ·2129 15·89 ·00618 ·00285	**1** 2·642 82·51 −·6215 ·3863 −51·28 ·00468 −·00291	**0** 0·550 396·38 −1·0000 1·0000 −396·38 ·00252 −·00252	**24** — 653·81 −1·5744 2·2606 −462·36 ·02766 −·00452
	Red	**0** 0·826 263·93 −1·0000 1·0000 −263·93 ·00379 −·00379	**1** 1·284 169·78 −·2212 ·0489 −37·56 ·00029 −·00006	**3** 0·906 240·61 2·3111 5·3412 556·07 ·02220 ·05131	**1** 1·319 165·29 −·2419 ·0585 −39·98 ·00035 −·00009	**0** 0·550 396·38 −1·0000 1·0000 −396·38 ·00252 −·00252	**0** 0·115 1895·66 −1·0000 1·0000 −1895·66 ·00053 −·00053	**5** — 3131·65 −1·1520 8·4486 −2077·44 ·02968 ·04431
	Totals	Marginal Totals are the same as for vertical margin						$N=$ **218** $\phi^2=$·14895

$S(N/\lambda_s)=5170{\cdot}9$; $S(\psi_s/\lambda_s)=-1{\cdot}8481$; $S(\psi_s^2/\lambda_s^2)=19{\cdot}162$; $S(N\psi_s/\lambda_s^2)=-2336{\cdot}8$; $S(\psi_s^3/N\lambda_s^2)={\cdot}08277$.

TABLE III.

$$\phi^2 = \cdot 14895, \qquad C_2 = \sqrt{\frac{\phi^2}{1+\phi^2}} = \cdot 36005.$$

	Various formulae used		
	1st Term of equation (xiv)	1st and 2nd Terms of equation (xiv)	All Terms of equation (xiv)
σ_{ϕ^2}	·062006	·079848	·082317
Probable error of ϕ^2	·041822	·053857	·055522
σ_C	·16902	·21765	·22438
Probable error of C	·11400	·14680	·15134

The relative importance of the three terms of equation (xiv) is not markedly different in this table of small total content from what it was in the case of the Handwriting-Intelligence table and we cannot base different conclusions on the two cases.

Again, using the approximation given by selecting the large terms from the second bracket of (xiv), viz.

$$6S\left(\frac{\psi_s^2}{\lambda_s^2}\right) + 8S\left(\frac{\psi_s}{\lambda_s}\right) + 2c,$$

we obtain

$$\sigma_{\phi^2} = \cdot 0864,$$

which as in the previous example is a reasonable approximation to the full expression result.

(9) *Second Application. Test for Zero Contingency.*

Suppose that we may expect in the sampled population an absence of contingency or correlation between the variates considered. In other words we will suppose

$$\lambda_s = \frac{\bar{n}_u \bar{n}_v}{N} = \bar{n}_s.$$

If, however, we take a *sample* from that population, the quantity

$$\phi^2 = S\frac{(n_s - \bar{n}_s)^2}{N\bar{n}_s},$$

which is the mean square contingency in the case of a population with zero correlation, would certainly not vanish. The problem then arises: How great may the quantity ϕ^2 be without making it highly improbable that the sample in question is really a sample from a population of uncorrelated material?

First of all, the mean value of ϕ^2 as determined from a large number of samples would be

$$\bar{\phi}^2 = \chi_1 \frac{1}{N}(c-1) \quad \text{..............................(xvi)},$$

as is obtained by substitution of $\lambda_s = \bar{n}_s$ in (viii), or, if $\chi_1 = 1$,

$$\bar{\phi}^2 = \frac{1}{N}(c-1).$$

In the same way we derive from equation (xiii)

$$\sigma^2_{\phi^2} = \frac{\chi_1}{N^2}\left\{\chi_4\left(\frac{c}{H} + \frac{c(c-2)}{N}\right) + \chi_3(c^2-1) - \chi_1(c-1)^2\right\} \quad \text{......(xvii)},$$

where H is the harmonic mean of the mean cell contents, or for the usual particular case when M is very large as compared with N

$$\sigma^2_{\phi^2} = \frac{1}{N^2}\left\{\frac{c}{H} + \frac{c(c-2)}{N} + 2(c-1)\right\} \quad \text{..................(xviii)},$$

an expression which is very easily calculated especially as $\frac{1}{H}$ will usually be small compared with c and hence a good rough approximation for a fairly large table will be got from

$$\sigma^2_{\phi^2} = \frac{2c}{N^2} \quad \text{....................................(xix)}.$$

Thus if we take twice the standard deviation as a limit to the probability of a deviation being that of a random sample, we have as a rough upper limit to the value, which ϕ^2 may be expected to take in any sample,

$$\frac{1}{N}(c-1) + 2\frac{\sqrt{2c}}{N}.$$

(10) *Numerical Illustration.*

In the example of the Contingency-table for Handwriting and Intelligence in Girls

$$\bar{\phi}^2 = \frac{1}{N}(c-1) = \cdot 01943,$$

and when calculated from the more exact formula (xviii)

$$\sigma_{\phi^2} = \cdot 004879,$$

the approximation given by (xix) being ·0046.

Hence in accordance with our assertion above, we should regard any observed value of ϕ^2 which exceeds $\cdot 01943 + 2 \times \cdot 00488$, i.e. ·02919 or, say, ·03, as being incompatible with zero contingency. The observed value of $\phi^2 = \cdot 0958$.

The corresponding mean value of C—the coefficient of contingency—is

$$\sqrt{\frac{\cdot 01943}{1\cdot 01943}} = \cdot 13806,$$

and the upper limit for C according to our assertion is ·1684 or, say, ·17. The observed value of C is ·2957. Clearly there is definite association between intelligence and handwriting.

(11) *Summary of Formulae.*

It will be convenient for purposes of reference to have all the formulae collected into one section.

General Formulae.

For ϕ^2 defined by

$$\left.\begin{array}{ll} & 1+\phi^2 = S\left(\frac{n_s}{N\lambda_s}\right) \\ \text{or} & \phi^2 = S\left\{\frac{(n_s-\lambda_s)^2}{N\lambda_s}\right\} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\text{A}),$$

where N is the number in a sample, n_s is the number in the sth division of that sample and λ_s is a number connected with the sth division satisfying the condition $S(\lambda_s) = S(n_s) = N$, we have proved that for an "infinite" sampled population

$$1+\bar{\phi}^2 = \left(1-\frac{1}{N}\right) S\left(\frac{\bar{n}_s^{\,2}}{N\lambda_s}\right) + \frac{1}{N} S\left(\frac{\bar{n}_s}{\lambda_s}\right) \ldots\ldots\ldots\ldots\ldots\ldots (\text{B})^*,$$

and

$$\begin{aligned} \sigma^2{}_{\phi^2} = {} & \frac{4}{N}\left[S\left(\frac{\psi_s^{\,3}}{N\lambda_s^{\,2}}\right) + \phi^2 - \phi^4\right] \\ & + \frac{1}{N^2}\left[6S\left(\frac{\psi_s^{\,2}}{\lambda_s^{\,2}}\right) + (8-4\phi^2)\,S\left(\frac{\psi_s}{\lambda_s}\right) - 12S\left(\frac{\psi_s^{\,3}}{N\lambda_s^{\,2}}\right) + (2-4\phi^2)c - 22 - 56\phi^2 - 10\phi^4\right] \\ & + \frac{1}{N^3}\left[S\left(\frac{N\psi_s}{\lambda_s^{\,2}}\right) + S\left(\frac{N}{\lambda_s}\right) + \left\{S\left(\frac{\psi_s}{\lambda_s}\right)\right\}^2 - 2S\left(\frac{\psi_s^{\,2}}{\lambda_s^{\,2}}\right) + 2(c-2)\,S\left(\frac{\psi_s}{\lambda_s}\right) + c(c-2)\right] \\ & \ldots\ldots\ldots\ldots\ldots (\text{C}), \end{aligned}$$

where c is the number of divisions or cells and $\psi_s = \bar{n}_s - \lambda_s$, or, with a fair amount of approximation,

$$\sigma^2{}_{\phi^2} = \frac{4}{N}\left[S\left(\frac{\psi_s^{\,3}}{N\lambda_s^{\,2}}\right) + \phi^2 - \phi^4\right] + \frac{1}{N^2}\left[6S\left(\frac{\psi_s^{\,2}}{\lambda_s^{\,2}}\right) + 8S\left(\frac{\psi_s}{\lambda_s}\right) + 2c\right] \ldots (\text{D}).$$

Contingency.

In the case of a contingency-table the sth division may be taken to be the (u, v) cell and $\lambda_s = \frac{\bar{n}_u \bar{n}_v}{N}$, where n_u and n_v are the marginal totals of the uth row and the vth column. Formulae (A), (B), (C), (D) are then directly applicable.

Test for zero contingency.

When there is zero contingency in the total population

$$\lambda_s = \frac{\bar{n}_u \bar{n}_v}{N} = \bar{n}_s,$$

and (B) reduces to

$$\bar{\phi}^2 = \frac{1}{N}(c-1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\text{B}'),$$

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* As usual the bar over a letter denotes "the mean value of." It is to be noted that usually there is only one sample and the value of n_s in that sample has to be taken as $\bar{n}_s$.

and (C) to

$$\sigma^2_{\phi^2} = \frac{1}{N^2}\left\{\frac{c}{H} + \frac{c(c-2)}{N} + 2(c-1)\right\} \text{.....................(C}'\text{)},$$

where c is the number of cells in the table and H is the harmonic mean of the cell contents.

Rough approximations in the case of zero contingency are given by

$$\bar{\phi}^2 = \frac{c}{N},$$

and

$$\sigma^2_{\phi^2} = \frac{2c}{N^2},$$

and from these we can derive as a rough *upper limit* to the value of ϕ^2 given by a random sample from a population of zero contingency

$$\frac{c}{N} + 2\frac{\sqrt{2c}}{N} = \frac{c}{N}\left(1 + \frac{2{\cdot}8284}{\sqrt{c}}\right).$$

ON SOME NOVEL PROPERTIES OF PARTIAL AND MULTIPLE CORRELATION COEFFICIENTS IN A UNIVERSE OF MANIFOLD CHARACTERISTICS.

By KARL PEARSON, F.R.S.

(1) Let the universe consist of N individuals each the bearer of n characteristics symbolised by the numbers $1, 2, 3, \ldots s, s', s'', \ldots n$ respectively. Let $r_{ss'}$ be the correlation coefficient of the s and s' characteristics, and let the whole system of total correlations be provided by the determinant Δ, where

$$\Delta = \begin{vmatrix} 1, & r_{12} & \cdots & r_{1s'} & \cdots & r_{1n} \\ r_{21}, & 1, & \cdots & r_{2s'} & \cdots & r_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{s1}, & r_{s2} & \cdots & r_{ss'} & \cdots & r_{sn} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{n1}, & r_{n2} & \cdots & r_{ns'} & \cdots & 1 \end{vmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}.$$

This determinant being symmetrical because $r_{ss'} = r_{s's}$.

We shall use $\Delta_{ss'}$ for the first minor corresponding to the constituent in the sth row and s'th column, and $\Delta_{ss's''s'''}$ for the second minor corresponding to the first minor of $\Delta_{ss'}$ which is associated with the constituent in the s''th row and s'''th column.

Then if $R_{s.123\ldots(s)\ldots n}$ denote the multiple correlation coefficient of the sth characteristic on the other $n-1$ characteristics, i.e. the n without s, and ${}_{ss'}\rho_{123\ldots(s)\ldots(s')\ldots n}$ denote the partial correlation coefficient of the sth and s'th characteristics for the remaining $n-2$ characteristics constant, the following results are fundamental and well-known:

$$R^2_{s.123\ldots(s)\ldots n} = 1 - \frac{\Delta}{\Delta_{ss}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)},$$

$${}_{ss'}\rho_{123\ldots(s)\ldots(s')\ldots n} = -\frac{\Delta_{ss'}}{\sqrt{\Delta_{ss}\Delta_{s's'}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(iii)}.$$

To abbreviate the subscripts we shall write these

$$R_{s.1-n} \text{ and } {}_{ss'}\rho_{1-n},$$

but where others of the variates are to be left out of account we shall be obliged to introduce the bracket system to mark partial or multiple correlation coefficients of lower orders. Thus $R_{s.1-(s'')-(s''')-n}$ or simply $R_{s.1-(s''s''')-n}$ would signify the multiple correlation coefficient of the sth characteristic for $n-3$ other characteristics, s'', s''' and of course s being excluded. Similarly ${}_{ss'}\rho_{1-(s''s'''s^{\text{iv}})-n}$ would be the partial correlation of the sth and s'th characteristics for $n-5$ other characteristics supposed constant, i.e. the n variates with the exception of s, s', s'', s''' and s^{iv}.

(2) *Preliminary Propositions. To express First Minors in Terms of Second Minors.*

While values of the second minors in terms of the first have long been known, i.e. the equations of the form

$$\Delta_{sss's'} = \frac{\Delta_{ss}\Delta_{s's'} - \Delta^2_{ss'}}{\Delta} \quad\ldots\ldots\ldots\ldots\ldots\ldots\text{(iv)},$$

$$\Delta_{sss's''} = \frac{\Delta_{ss}\Delta_{s's''} - \Delta_{ss'}\Delta_{ss''}}{\Delta} \quad\ldots\ldots\ldots\ldots\ldots\ldots\text{(v)},$$

the reversed results, i.e. the expressions for first minors in terms of second, seem to me less familiar, at any rate I have not come across them in the literature with which I am acquainted.

The forms to be demonstrated are

$$\Delta_{ss} = \frac{\Delta^2}{\nabla_{ss's''}}\left(\Delta_{sss's'}\Delta_{sss''s''} - \Delta^2_{sss's''}\right) \quad\ldots\ldots\ldots\ldots\ldots\ldots\text{(vi)},$$

$$\Delta_{s's''} = \frac{\Delta^2}{\nabla_{ss's''}}\left(\Delta_{s's's''s''}\Delta_{sss's''} + \Delta_{s's'ss''}\Delta_{s''s''ss'}\right) \ldots\ldots\ldots\ldots\ldots\ldots\text{(vii)},$$

where

$$\nabla_{ss's''} = \begin{vmatrix} \Delta_{ss}, & \Delta_{ss'}, & \Delta_{ss''} \\ \Delta_{s's}, & \Delta_{s's'}, & \Delta_{s's''} \\ \Delta_{s''s}, & \Delta_{s''s'}, & \Delta_{s''s''} \end{vmatrix} \quad\ldots\ldots\ldots\ldots\ldots\ldots\text{(viii)},$$

and

$$\nabla^2_{ss's''} = \Delta^3 \begin{vmatrix} \Delta_{s's's''s''}, & -\Delta_{s''s''ss'}, & -\Delta_{s's's''s} \\ -\Delta_{s''s''ss'}, & \Delta_{s''s''ss}, & -\Delta_{sss's''} \\ -\Delta_{s's's''s}, & -\Delta_{sss's''}, & \Delta_{sss's'} \end{vmatrix} \quad\ldots\ldots\ldots\text{(ix)}.$$

To prove these results I start from (iv) and (v) to express the value of

$$\Delta_{s's's''s''}\Delta_{sss's''} + \Delta_{s's'ss''}\Delta_{s''s''ss'}.$$

We find

$$\begin{aligned}\Delta_{s's's''s''}\Delta_{sss's''} + \Delta_{s's'ss''}\Delta_{s''s''ss'} &= \frac{(\Delta_{s's'}\Delta_{s''s''} - \Delta^2_{s's''})}{\Delta} \times \frac{(\Delta_{ss}\Delta_{s's''} - \Delta_{ss'}\Delta_{ss''})}{\Delta} \\ &\quad + \frac{(\Delta_{s's'}\Delta_{s''s} - \Delta_{s's''}\Delta_{s's})}{\Delta} \times \frac{(\Delta_{s''s''}\Delta_{ss'} - \Delta_{s''s}\Delta_{s''s'})}{\Delta} \\ &= \frac{\Delta_{s's''}}{\Delta^2}\left(\Delta_{ss}\Delta_{s's'}\Delta_{s''s''} - \Delta_{ss}\Delta^2_{s's''} - \Delta_{s's'}\Delta^2_{s''s} - \Delta_{s''s''}\Delta^2_{ss'} + 2\Delta_{s's''}\Delta_{s''s}\Delta_{ss'}\right) \\ &= \frac{\Delta_{s's''}\,.\,\nabla_{ss's''}}{\Delta^2},\end{aligned}$$

which gives us at once (vii).

Similarly we have by (iv) and (v)

$$\begin{aligned}\Delta_{sss's'}\Delta_{sss''s''}-\Delta^2_{sss's''}&=\frac{\Delta_{ss}\Delta_{s's'}-\Delta^2_{ss'}}{\Delta}\times\frac{\Delta_{ss}\Delta_{s''s''}-\Delta^2_{ss''}}{\Delta}-\frac{(\Delta_{ss}\Delta_{s's''}-\Delta_{ss}\Delta_{ss''})^2}{\Delta^2}\\&=\frac{\Delta_{ss}}{\Delta^2}(\Delta_{ss}\Delta_{s's'}\Delta_{s''s''}-\Delta_{ss}\Delta^2_{s's''}-\Delta_{s's'}\Delta^2_{s''s}-\Delta_{s''s''}\Delta^2_{ss'}+2\Delta_{s's''}\Delta_{s''s}\Delta_{ss'})\\&=\frac{\Delta_{ss}}{\Delta^2}\nabla_{ss's''},\end{aligned}$$

which gives at once the result (vi).

We shall now substitute values like (vi) and (vii) in our definition (viii) of $\nabla_{ss's''}$ and so deduce (ix).

We have

$$\nabla_{ss's''}=\frac{\Delta^6}{(\nabla_{ss's''})^3}\begin{vmatrix}\begin{pmatrix}\Delta_{sss's'}\Delta_{sss''s''}\\-\Delta^2_{sss's''}\end{pmatrix}, & \begin{pmatrix}\Delta_{sss's'}\Delta_{s''s''ss'}\\+\Delta_{sss''s'}\Delta_{s's's''s}\end{pmatrix}, & \begin{pmatrix}\Delta_{sss''s''}\Delta_{s's'ss''}\\+\Delta_{sss's''}\Delta_{s''s''s's}\end{pmatrix}\\ \begin{pmatrix}\Delta_{sss's'}\Delta_{s''s''ss'}\\+\Delta_{sss''s'}\Delta_{s's's''s}\end{pmatrix}, & \begin{pmatrix}\Delta_{s's's''s''}\Delta_{s's'ss}\\-\Delta^2_{s's's''s}\end{pmatrix}, & \begin{pmatrix}\Delta_{s's's''s''}\Delta_{sss's''}\\+\Delta_{s's'ss''}\Delta_{s''s''ss'}\end{pmatrix}\\ \begin{pmatrix}\Delta_{sss''s''}\Delta_{s's'ss''}\\+\Delta_{sss's''}\Delta_{s''s''s's}\end{pmatrix}, & \begin{pmatrix}\Delta_{s's's''s''}\Delta_{sss's''}\\+\Delta_{s's'ss''}\Delta_{s''s''ss'}\end{pmatrix}, & \begin{pmatrix}\Delta_{s''s''ss}\Delta_{s''s''s's'}\\-\Delta^2_{s''s''ss'}\end{pmatrix}\end{vmatrix},$$

but the determinant on the right is the square of the determinant

$$\begin{vmatrix}\Delta_{s's's''s''}, & -\Delta_{s''s''ss'}, & -\Delta_{s's's''s}\\ -\Delta_{s''s''ss'}, & \Delta_{s''s''ss}, & -\Delta_{sss's''}\\ -\Delta_{s's's''s}, & -\Delta_{sss's''}, & \Delta_{sss's'}\end{vmatrix},$$

whence by taking the square root we have the result (ix).

(3) *Application to Multiple and Partial Correlation Coefficients.*

Equation (viii) may be put into the form

$$\begin{aligned}\nabla_{ss's''}&=\Delta_{ss}\Delta_{s's'}\Delta_{s''s''}\begin{vmatrix}1, & \dfrac{\Delta_{ss'}}{\sqrt{\Delta_{ss}\Delta_{s's'}}}, & \dfrac{\Delta_{ss''}}{\sqrt{\Delta_{ss}\Delta_{s''s''}}}\\ \dfrac{\Delta_{ss'}}{\sqrt{\Delta_{ss}\Delta_{s's'}}}, & 1, & \dfrac{\Delta_{s's''}}{\sqrt{\Delta_{s's'}\Delta_{s''s''}}}\\ \dfrac{\Delta_{ss''}}{\sqrt{\Delta_{ss}\Delta_{s''s''}}}, & \dfrac{\Delta_{s's''}}{\sqrt{\Delta_{s's'}\Delta_{s''s''}}}, & 1,\end{vmatrix}\\&=\frac{\Delta^3}{(1-R^2_{s\cdot 1-n})(1-R^2_{s'\cdot 1-n})(1-R^2_{s''\cdot 1-n})}\begin{vmatrix}1, & -{}_{ss'}\rho_{1-n}, & -{}_{ss''}\rho_{1-n}\\ -{}_{ss'}\rho_{1-n}, & 1, & -{}_{s's''}\rho_{1-n}\\ -{}_{ss''}\rho_{1-n}, & -{}_{s's''}\rho_{1-n}, & 1\end{vmatrix}\end{aligned}$$

............(x).

Turning to the expression (ix) we may write it

$$\nabla^2_{ss's''} = \Delta^3 \Delta_{s's's''s''} \Delta_{s''s''ss} \Delta_{sss's'} \begin{vmatrix} 1, & \dfrac{-\Delta_{s''s''ss'}}{\sqrt{\Delta_{s's's''s''}\Delta_{s''s''ss}}}, & \dfrac{-\Delta_{s's's''s}}{\sqrt{\Delta_{s's's''s''}\Delta_{sss's'}}} \\ \dfrac{-\Delta_{s''s''ss'}}{\sqrt{\Delta_{s's's''s''}\Delta_{s''s''ss}}}, & 1, & \dfrac{-\Delta_{sss's''}}{\sqrt{\Delta_{s''s''ss}\Delta_{sss's'}}} \\ \dfrac{-\Delta_{s's's''s}}{\sqrt{\Delta_{s's's''s''}\Delta_{sss's'}}}, & \dfrac{-\Delta_{sss's''}}{\sqrt{\Delta_{s''s''ss}\Delta_{sss's'}}}, & 1 \end{vmatrix}.$$

The determinant on the right can be expressed in terms of partial correlation coefficients of one lower order, i.e. it equals

$$\begin{vmatrix} 1, & {}_{ss'}\rho_{1-(s'')-n}, & {}_{ss''}\rho_{1-(s')-n} \\ {}_{ss'}\rho_{1-(s'')-n}, & 1, & {}_{s's''}\rho_{1-(s)-n} \\ {}_{ss''}\rho_{1-(s')-n}, & {}_{s's''}\rho_{1-(s)-n}, & 1 \end{vmatrix}.$$

Further:

$$\Delta^3 \Delta_{s's's''s''} \Delta_{s''s''ss} \Delta_{sss's'} = \Delta^6 \cdot \frac{\Delta_{ss}}{\Delta} \cdot \frac{\Delta_{s's'}}{\Delta} \cdot \frac{\Delta_{s''s''}}{\Delta} \cdot \frac{\Delta_{s's's''s''}}{\Delta_{s's'}} \cdot \frac{\Delta_{s''s''ss}}{\Delta_{s''s''}} \cdot \frac{\Delta_{sss's'}}{\Delta_{ss}}.$$

Also:

$$= \Delta^6 \cdot \frac{\Delta_{ss}}{\Delta} \cdot \frac{\Delta_{s's'}}{\Delta} \cdot \frac{\Delta_{s''s''}}{\Delta} \cdot \frac{\Delta_{s's's''s''}}{\Delta_{s''s''}} \cdot \frac{\Delta_{s''s''ss}}{\Delta_{ss}} \cdot \frac{\Delta_{sss's'}}{\Delta_{s's'}}$$

$$= \Delta^6 \cdot \frac{1}{(1 - R^2_{s.1-n})(1 - R^2_{s'.1-n})(1 - R^2_{s''.1-n})}$$

$$\times \frac{1}{(1 - R^2_{s.1-(s'')-n})(1 - R^2_{s'.1-(s)-n})(1 - R^2_{s''.1-(s')-n})}.$$

Thus:

$$(1 - R^2_{s.1-(s'')-n})(1 - R^2_{s'.1-(s)-n})(1 - R^2_{s''.1-(s')-n})$$
$$= (1 - R^2_{s.1-(s')-n})(1 - R^2_{s'.1-(s'')-n})(1 - R^2_{s''.1-(s)-n}) \quad \text{......(xi)}.$$

This is a relation between sets of three multiple correlation coefficients of the $(n-2)$th order.

Clearly
$$\nabla^2_{ss's''} = \Delta^6 \cdot \frac{1}{(1 - R^2_{s.1-n})(1 - R^2_{s'.1-n})(1 - R^2_{s''.1-n})}$$
$$\times \frac{1}{(1 - R^2_{s.1-(s')-n})(1 - R^2_{s'.1-(s'')-n})(1 - R^2_{s''.1-(s)-n})}$$
$$\times \begin{vmatrix} 1, & {}_{ss'}\rho_{1-(s'')-n}, & {}_{ss''}\rho_{1-(s')-n} \\ {}_{ss'}\rho_{1-(s'')-n}, & 1, & {}_{s's''}\rho_{1-(s)-n} \\ {}_{ss''}\rho_{1-(s')-n}, & {}_{s's''}\rho_{1-(s)-n}, & 1 \end{vmatrix} \quad \text{.....................(xii)}.$$

Squaring (x) and combining with (xii) we find

$$\frac{\begin{vmatrix} 1, & -{}_{ss'}\rho_{1-n}, & -{}_{ss''}\rho_{1-n} \\ -{}_{ss'}\rho_{1-n}, & 1, & -{}_{s's''}\rho_{1-n} \\ -{}_{ss''}\rho_{1-n}, & -{}_{s's''}\rho_{1-n}, & 1 \end{vmatrix}^2}{(1 - R^2_{s.1-n})(1 - R^2_{s'.1-n})(1 - R^2_{s''.1-n})}$$

$$= \frac{\begin{vmatrix} 1, & {}_{ss'}\rho_{1-(s'')-n}, & {}_{ss''}\rho_{1-(s')-n} \\ {}_{ss'}\rho_{1-(s'')-n}, & 1, & {}_{s's''}\rho_{1-(s)-n} \\ {}_{ss''}\rho_{1-(s')-n}, & {}_{s's''}\rho_{1-(s)-n}, & 1 \end{vmatrix}}{(1 - R^2_{s.1-(s')-n})(1 - R^2_{s'.1-(s'')-n})(1 - R^2_{s''.1-(s)-n})} \quad \text{.........(xiii)}.$$

The denominator on the right-hand side of (xiii) can be replaced by the left-hand side of (xi), or by the square root of the product of both sides of (xi). This is a relation between partial correlation coefficients of the $(n-2)$th order with multiple coefficients of the $(n-1)$th order and partial correlation coefficients of the $(n-3)$th order with multiple coefficients of the $(n-2)$th order. In the particular case of three variates 1, 2, 3 with total correlations r_{12}, r_{23}, r_{31} (xiii) reduces to

$$\begin{vmatrix} 1, & -{}_{12}\rho_3, & -{}_{13}\rho_2 \\ -{}_{21}\rho_3, & 1, & -{}_{23}\rho_1 \\ -{}_{31}\rho_2, & -{}_{32}\rho_1, & 1 \end{vmatrix}^2 = \frac{(1-R^2{}_{1.23})(1-R^2{}_{2.31})(1-R^2{}_{3.12})}{(1-r^2{}_{23})(1-r^2{}_{31})(1-r^2{}_{12})} \begin{vmatrix} 1, & r_{12}, & r_{13} \\ r_{21}, & 1, & r_{23} \\ r_{31}, & r_{32}, & 1 \end{vmatrix} \quad \text{............(xiv),}$$

a result which might possibly be of practical value in testing the accuracy of the determination of the first order partial and multiple correlation coefficients.

I now return to equations (vi) and (vii), and I divide (vii) by the square root of the product of $\Delta_{s's'}$ and $\Delta_{s''s''}$ obtained by cyclical interchange from (vi). We find

$$\frac{\Delta_{s's''}}{\sqrt{\Delta_{s's'}\Delta_{s''s''}}} = \frac{\Delta_{s's's''s''}\Delta_{sss's''} + \Delta_{s's'ss''}\Delta_{s''s''ss'}}{\sqrt{\Delta_{s's's''s''}\Delta_{s's'ss} - \Delta^2{}_{s's's''s}}\sqrt{\Delta_{s''s''ss}\Delta_{s''s''s's'} - \Delta^2{}_{s''s''ss'}}}$$

$$= \frac{\dfrac{\Delta_{sss's''}}{\sqrt{\Delta_{s's'ss}\Delta_{s''s''ss}}} + \dfrac{\Delta_{s's'ss''}}{\sqrt{\Delta_{s's'ss}\Delta_{s's's''s''}}}\,\dfrac{\Delta_{s''s''ss'}}{\sqrt{\Delta_{s''s''ss}\Delta_{s''s''s's'}}}}{\sqrt{1 - \dfrac{\Delta^2{}_{s's'ss''}}{\Delta_{s's'ss}\Delta_{s's's''s''}}}\sqrt{1 - \dfrac{\Delta^2{}_{s''s''ss'}}{\Delta_{s''s''ss}\Delta_{s''s''s's'}}}},$$

or, changing sign throughout,

$${}_{s's''}\rho_{1-n} = \frac{{}_{s's''}\rho_{1-(s)-n} - {}_{ss''}\rho_{1-(s')-n} \times {}_{ss'}\rho_{1-(s'')-n}}{\sqrt{1 - {}_{ss''}\rho^2{}_{1-(s')-n}}\sqrt{1 - {}_{ss'}\rho^2{}_{1-(s'')-n}}} \quad \text{..............(xv).}$$

(xv) is the familiar result for obtaining a partial correlation coefficient of the $(n-2)$th order from those of the $(n-3)$th order, but the proofs usually given of it seem to be based on some appeal to general analogy rather than to the definite algebraical form of the coefficients concerned. It was, indeed, a lecture proof of the relation (xv) from the determinantal forms of the coefficients which led me to the results (vi) and (vii), as apparently novel determinantal relations.

We can next consider results (vi) and (vii) individually. From (vi) we find

$$\frac{\Delta_{ss}}{\Delta} = \frac{\Delta^3}{\nabla_{ss's''}}\,\frac{\Delta_{s's'}}{\Delta}\,\frac{\Delta_{s''s''}}{\Delta}\,\frac{\Delta_{sss's'}}{\Delta_{s's'}}\,\frac{\Delta_{sss''s''}}{\Delta_{s''s''}}\left(1 - \frac{\Delta^2{}_{sss's''}}{\Delta_{sss's'}\Delta_{sss''s''}}\right),$$

or, converting into partials and multiples, and writing

$${}^{\text{‘}}\mathrm{P}_{n-2} = \begin{vmatrix} 1, & -{}_{ss'}\rho_{1-n}, & -{}_{ss''}\rho_{1-n} \\ -{}_{ss'}\rho_{1-n}, & 1, & -{}_{s's''}\rho_{1-n} \\ -{}_{ss''}\rho_{1-n}, & -{}_{s's''}\rho_{1-n}, & 1 \end{vmatrix},$$

we have

$$\frac{1}{1-R^2_{s.1-n}}=\frac{(1-R^2_{s.1-n})(1-R^2_{s'.1-n})(1-R^2_{s''.1-n})}{{}^{\backprime}\mathrm{P}_{n-2}}\,\frac{1}{(1-R^2_{s'.1-n})(1-R^2_{s''.1-n})}$$
$$\times\frac{1}{(1-R^2_{s.1-(s')-n})(1-R^2_{s.1-(s'')-n})}\times(1-{}_{s's''}\rho^2_{1-(s)-n}).$$

Hence

$$(1-R^2_{s.1-n})^2=\frac{(1-R^2_{s.1-(s')-n})(1-R^2_{s.1-(s'')-n})\,{}^{\backprime}\mathrm{P}_{n-2}}{1-{}_{s's''}\rho^2_{1-(s)-n}}\text{.........(xvi).}$$

This is an expression for a multiple coefficient of the $(n-1)$th order in terms of multiple coefficients of the $(n-2)$th order and partial coefficients of the $(n-2)$th and $(n-3)$th orders.

Equation (xvi) just obtained may be put into another form by aid of the relations*

$$1-{}_{ss'}\rho^2_{1-n}=\frac{1-R^2_{s.1-n}}{1-R^2_{s.1-(s')-n}},$$
$$1-{}_{ss''}\rho^2_{1-n}=\frac{1-R^2_{s.1-n}}{1-R^2_{s.1-(s'')-n}},$$

leading to

$$(1-{}_{ss'}\rho^2_{1-n})(1-{}_{ss''}\rho^2_{1-n})=\frac{(1-R^2_{s.1-n})^2}{(1-R^2_{s.1-(s')-n})(1-R^2_{s.1-(s'')-n})}$$
$$=\frac{{}^{\backprime}\mathrm{P}_{n-2}}{1-{}_{s's''}\rho^2_{1-(s)-n}}\quad\text{by (xvi).}$$

Hence

$$1-{}_{s's''}\rho^2_{1-(s)-n}=\frac{{}^{\backprime}\mathrm{P}_{n-2}}{(1-{}_{ss'}\rho^2_{1-n})(1-{}_{ss''}\rho^2_{1-n})}\text{............(xvi)}^{\text{bis}}.$$

Thus (xvi)$^{\text{bis}}$ is the reverse of (xv), giving a partial correlation of the $(n-3)$th order in terms of those of the $(n-2)$th order. For example, if there be three variates, 1, 2, 3,

$$1-r^2_{23}=\frac{1-{}_{32}\rho_1^{\,2}-{}_{13}\rho_2^{\,2}-{}_{21}\rho_3^{\,2}+2\,{}_{32}\rho_1\cdot{}_{13}\rho_2\cdot{}_{21}\rho_3}{(1-{}_{12}\rho_3^{\,2})(1-{}_{13}\rho_2^{\,2})},$$

leading to

$$r_{23}=\frac{{}_{32}\rho_1+{}_{12}\rho_3\cdot{}_{13}\rho_2}{\sqrt{1-{}_{12}\rho_3^{\,2}}\sqrt{1-{}_{13}\rho_2^{\,2}}},$$

which can be easily verified by substitution of the values of the partial correlations, or be seen at once from the polar triangle.

For the particular case of three variates we may use (xvi) instead of (xvi)$^{\text{bis}}$, writing it

$$(1-R^2_{1.23})^2=\frac{(1-r^2_{12})(1-r^2_{13})}{1-r^2_{23}}\begin{vmatrix}1, & -{}_{12}\rho_3, & -{}_{13}\rho_2\\ -{}_{12}\rho_3, & 1, & -{}_{23}\rho_1\\ -{}_{13}\rho_2, & -{}_{23}\rho_1, & 1\end{vmatrix}\text{...(xvii),}$$

which will be found on substitution of the values of the partial coefficients on the right and the multiple on the left to reduce to the familiar result that the square of a 3 by 3 determinant is equal to the determinant formed by its minors. In this case of three variates, if r_{23}, r_{31}, r_{12} be taken as the cosines of the sides of a spherical triangle, then ${}_{23}\rho_1$, ${}_{31}\rho_2$, ${}_{12}\rho_3$ are the cosines of the angles, and $R_{2.31}$, $R_{1.23}$, $R_{3.12}$ are the cosines of the perpendiculars from the angles on the opposite sides. If a, b, c are the sides, A, B, C the opposite angles, pa, pb, pc the perpendiculars on the sides, then the above relation is

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* *R. S. Proc.* A, Vol. XCI. p. 496.

$$\sin^4 pa = \frac{\sin^2 b \sin^2 c}{\sin^2 a} \begin{vmatrix} 1, & -\cos C, & -\cos B \\ -\cos C, & 1 & -\cos A \\ -\cos B, & -\cos A, & 1 \end{vmatrix}.$$

It is greatly to be desired that the "trigonometry" of higher dimensioned plane space should be fully worked out, for all our relations between multiple correlation and partial correlation coefficients of n variates are properties of the "angles," "edges" and "perpendiculars" of sphero-polyhedra in multiple space. It would be a fine task for an adequately equipped pure mathematician to write a treatise on "spherical polyhedrometry"; he need not fear that his results would be without practical application for they embrace the whole range of problems from anatomy to medicine and from medicine to sociology and ultimately to the doctrine of evolution.

Lastly we may turn to (vii), and express it also in multiple and partial correlation coefficients. We have

$$\frac{\Delta_{s's''}}{\sqrt{\Delta_{s's'}\Delta_{s''s''}}} = \frac{\Delta^3}{\nabla_{ss's''}} \frac{\Delta_{s's's''s''}}{\sqrt{\Delta_{s's'}\Delta_{s''s''}}} \sqrt{\frac{\Delta_{s's'ss}}{\Delta_{s's'}} \frac{\Delta_{s''s''ss}}{\Delta_{s''s''}}} \sqrt{\frac{\Delta_{s's'}}{\Delta} \frac{\Delta_{s''s''}}{\Delta}}$$
$$\times \left(\frac{\Delta_{sss's''}}{\sqrt{\Delta_{s's'ss}\Delta_{s''s''ss}}} + \frac{\Delta_{s's'ss''}}{\sqrt{\Delta_{s's'ss}\Delta_{s's's''s''}}} \times \frac{\Delta_{s''s''ss'}}{\sqrt{\Delta_{s''s''ss}\Delta_{s''s''s's'}}}\right),$$

or using the correlation symbols

$${}_{s's''}\rho_{1-n} = \frac{(1 - R^2{}_{s.1-n})(1 - R^2{}_{s'.1-n})(1 - R^2{}_{s''.1-n})}{{}^{\backprime}\mathrm{P}_{n-2}} \frac{1}{\sqrt{(1 - R^2{}_{s''.1-(s')-n})(1 - R^2{}_{s'.1-(s'')-n})}}$$
$$\times \frac{1}{\sqrt{(1 - R^2{}_{s.1-(s')-n})(1 - R^2{}_{s.1-(s'')-n})}} \frac{1}{\sqrt{(1 - R^2{}_{s'.1-n})(1 - R^2{}_{s''.1-n})}}$$
$$\times \left({}_{s's''}\rho_{1-(s)-n} - {}_{ss''}\rho_{1-(s')-n} \times {}_{ss'}\rho_{1-(s'')-n}\right).$$

Or,

$${}_{s's''}\rho_{1-n} = (1 - R^2{}_{s.1-n})$$
$$\times \sqrt{\frac{(1 - R^2{}_{s'.1-n})(1 - R^2{}_{s''.1-n})}{(1 - R^2{}_{s.1-(s')-n})(1 - R^2{}_{s.1-(s'')-n})(1 - R^2{}_{s''.1-(s')-n})(1 - R^2{}_{s'.1-(s'')-n})}}$$
$$\times \frac{{}_{s's''}\rho_{1-(s)-n} - {}_{ss''}\rho_{1-(s')-n} \times {}_{ss'}\rho_{1-(s'')-n}}{{}^{\backprime}\mathrm{P}_{n-2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(xviii)}.$$

This is a complicated form and unlikely to be of material service.

If we use the symbol ${}^{\backprime}\mathrm{P}_{n-3}$ to represent the determinant

$$\begin{vmatrix} 1, & {}_{ss'}\rho_{1-(s'')-n}, & {}_{ss''}\rho_{1-(s')-n} \\ {}_{ss'}\rho_{1-(s'')-n}, & 1, & {}_{s's''}\rho_{1-(s)-n} \\ {}_{ss''}\rho_{1-(s')-n}, & {}_{s's''}\rho_{1-(s)-n}, & 1 \end{vmatrix},$$

we can throw (xviii) into the form

$$ {}_{s's''}\rho_{1-n} = \sqrt{1 - R^2{}_{s.1-n}} \sqrt{\frac{1 - R^2{}_{s''.1-(s)-n}}{\left(1 - R^2{}_{s.1-(s'')-n}\right)\left(1 - R^2{}_{s''.1-(s')-n}\right)}} $$

$$ \times \frac{{}_{s's''}\rho_{1-(s)-n} - {}_{ss''}\rho_{1-(s')-n} \times {}_{ss'}\rho_{1-(s'')-n}}{\sqrt{{}'\mathrm{P}_{n-3}}} \quad \text{......................(xviii).} $$

Here on the right we have only partial correlations of the order $n-3$, but the expression involves a multiple of the order $n-1$ as well as those of order $n-2$. Of course in the second radical s'' and s' may be interchanged owing to the existence of (xi). On the whole it appears best to confine our attention to (xv) and (xvi) as the fittest representatives of (vi) and (vii) expressed in correlation forms. (xv) has long been in use either to determine the partial correlation coefficients of higher orders by repetitional processes, or better to verify results given by (iii). The calculation of the higher multiple correlation coefficients by (ii) can be verified by the aid of (xvi) if the partials have also been found. But if once the value of Δ_n has been determined the continuous product formula may often be advantageously used. Looked at from the determinantal standpoint this may be reached as follows:

$$ \Delta_n = \frac{\Delta_n}{\Delta_{ss}} \cdot \frac{\Delta_{ss}}{\Delta_{sss's'}} \cdot \frac{\Delta_{sss's'}}{\Delta_{sss's's''s''}} \cdot \frac{\Delta_{sss's's''s''}}{\Delta_{sss's's''s''s'''s'''}} \cdots \frac{\Delta_{ss} \text{ to } n-1 \text{ terms}}{1} $$

$$ = \left(1 - R^2{}_{s.1-n}\right)\left(1 - R^2{}_{s'.1-(s)-n}\right)\left(1 - R^2{}_{s''.1-(ss')-n}\right)\left(1 - R^2{}_{s'''.1-(ss's'')-n}\right)\dots\left(1 - r^2pq\right), $$

if the pth and qth variates be the last to be excluded.

Hence*

$$ 1 - R^2{}_{s.1-n} = \frac{\Delta_n}{\left(1 - R^2{}_{s'.1-(s)-n}\right)\left(1 - R^2{}_{s''.1-(ss')-n}\right)\left(1 - R^2{}_{s'''.1-(ss's'')-n}\right)\dots\left(1 - r^2pq\right)} $$

............(xix).

The trouble of this method is that Δ_n has to be calculated at each stage, if we deduce $R_{s.1-n}$ by a repetitional process. If we use it merely as a verification process for (ii) we shall not verify Δ_n unless it is worked out twice. Of course Δ_n, if n be at all large, is more troublesome to calculate than the 3×3 determinants of formula (xvi).

However the primary object of the present paper is not so much to provide verification formulae as to show by direct determinantal analysis certain relations known and unknown between the higher multiple and partial correlation coefficients.

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* This must not be confused with the formula

$$ 1 - {}_{ss'}\rho^2{}_{12\cdots n} = \frac{1 - R^2{}_{s.1-n}}{\left(1 - {}_{ss''}\rho^2{}_{1-(s')-n}\right)\left(1 - {}_{ss'''}\rho^2{}_{1-(s's'')-n}\right)\left(1 - {}_{ss''''}\rho^2{}_{1-(s's''s''')-n}\right)\dots\left(1 - r^2pq\right)}, $$

see Yule, *R. S. Proc.* A, Vol. LXXIX. p. 189, and Pearson, *R. S. Proc.* A, Vol. XCI. p. 496.

ON THE APPLICATION OF "GOODNESS OF FIT" TABLES TO TEST REGRESSION CURVES AND THEORETICAL CURVES USED TO DESCRIBE OBSERVATIONAL OR EXPERIMENTAL DATA.

By KARL PEARSON, F.R.S.

Let us suppose that a sample of size N with class groups n_{qp} is taken out of an indefinitely large population of size M with class groups ν_{qp}, these classes being arranged according to two variates x and y. Then the mean of any array of x's for a given range of y variates connoted by the centre, y_p, of this (usually small) range will be

$$m_p = \frac{S\,(n_{qp} x_q)}{n_p} \quad \text{......................................(i).}$$

Here n_{qp}, the number in the x_q, y_p class, and n_p, the total number in the pth array of x's, will vary from sample to sample. But x_q and y_p will remain of course the same. Now let $\bar{m}_p$ be the mean value of m_p found from a large number λ of samples and let us measure $m_p = \bar{m}_p + \delta m_p$ from $\bar{m}_p$ and n_{qp} from $\bar{n}_{qp} = N\nu_{qp}/M$, and n_p from $N\nu_p/M$, or take $n_{pq} = \bar{n}_{pq} + \delta\bar{n}_{pq}$, and $n_p = \bar{n}_p + \delta n_p$. Here the differentials are statistical differences and do not at present denote that we are going to neglect their higher powers. From (i) we have

$$\bar{m}_p + \delta m_p = S\left(\frac{\bar{n}_{qp} x_q}{\bar{n}_p}\right)\left\{1 - \frac{\delta n_p}{\bar{n}_p} + \left(\frac{\delta n_p}{\bar{n}_p}\right)^2 - \left(\frac{\delta n_p}{\bar{n}_p}\right)^3 + \dots\right\}$$
$$+ S\left(\frac{\delta n_{qp} x_q}{\bar{n}_p}\right)\left\{1 - \frac{\delta n_p}{\bar{n}_p} + \left(\frac{\delta n_p}{\bar{n}_p}\right)^2 - \left(\frac{\delta n_p}{\bar{n}_p}\right)^3 + \dots\right\}.$$

Now we shall sum this for all λ samples (dividing by λ) and suppose that third order powers and products of $\frac{\delta n_{qp}}{\bar{n}_p}$ and $\frac{\delta n_p}{\bar{n}_p}$ are negligible as compared with lower order powers and products*. If Σ denote a summation for all λ samples

$$\frac{\Sigma\,(\delta m_p)}{\lambda} = \frac{\Sigma\,(\delta n_{qp})}{\lambda} = \frac{\Sigma\,(\delta n_p)}{\lambda} = 0,$$

since all these quantities are measured from their mean values. Thus we find

$$\bar{m}_p = S\left(\frac{\bar{n}_{qp} x_q}{\bar{n}_p}\right)\left(1 + \frac{\Sigma\,(\delta n_p)^2}{\lambda \bar{n}_p{}^2}\right) - S\left(\frac{x_q}{\bar{n}_p{}^2}\,\frac{\Sigma\,(\delta n_{qp}\delta n_p)}{\lambda}\right).$$

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* Actually terms of the third order also vanish. I have not investigated whether this be true for terms of the fourth and higher orders.

But*
$$\frac{\Sigma\,(\delta n_p)^2}{\lambda} = \bar{n}_p\left(1 - \frac{\bar{n}_p}{N}\right),\quad \frac{\Sigma\,(\delta n_{qp}\delta n_p)}{\lambda} = \bar{n}_{qp}\left(1 - \frac{\bar{n}_p}{N}\right)\qquad\text{(ii)},$$
and accordingly
$$\begin{aligned}\bar{m}_p &= S\left(\frac{\bar{n}_{qp}x_q}{\bar{n}_p}\right)\left\{1 + \frac{1}{\bar{n}_p}\left(1 - \frac{\bar{n}_p}{N}\right) - \frac{1}{\bar{n}_p}\left(1 - \frac{\bar{n}_p}{N}\right)\right\}\\ &= \frac{S\,(\bar{n}_{qp}x_q)}{\bar{n}_p} = \frac{S\,(\nu_{qp}x_q)}{\nu_p}\\ &= \text{mean of the array in the sampled population.}\end{aligned}$$

Thus to a high order of approximation at least the mean of the array means is the mean of the corresponding array in the sampled population. This result cannot be taken as obvious, as the size of the array in the sample varies†.

We can now write to a second approximation:
$$\begin{aligned}\delta m_p &= \frac{S\,(\delta n_{qp}x_q)}{\bar{n}_p}\left(1 - \frac{\delta n_p}{\bar{n}_p}\right) - \frac{\bar{m}_p\delta n_p}{\bar{n}_p}\left(1 - \frac{\delta\bar{n}_p}{\bar{n}_p}\right)\\ &= \frac{S\,\{\delta n_{qp}\,(x_q - \bar{m}_p)\}}{\bar{n}_p}\left(1 - \frac{\delta n_p}{\bar{n}_p}\right)\\ &= \frac{S\,\{\delta n_{qp}\,(x_q - \bar{m}_p)\}}{\bar{n}_p} - \frac{S\,\{\delta n_{qp}\delta n_p\,(x_q - \bar{m}_p)\}}{\bar{n}_p{}^2}\qquad\text{(iii)},\end{aligned}$$
since
$$\delta n_p = S\,(\delta n_{qp}).$$

We shall now find the mean value of $(\delta m_p)^2$ as far as third order terms. Let O_2 and O_3 give the second and third order terms; let Σ denote a summation for all samples and λ their number; write $\tilde{x}_q$ for $x_q - \bar{m}_p$. Then
$$\begin{aligned}O_2 &= \frac{1}{\lambda}\frac{\Sigma\,\{S\,(\delta n_{qp}\tilde{x}_q)\}^2}{\bar{n}_p{}^2}\\ &= \frac{1}{\lambda}\frac{S\,\{\Sigma\,(\delta n_{qp})^2\,\tilde{x}_q{}^2\}}{\bar{n}_p{}^2} + \frac{2}{\lambda}\frac{S_1\,\{\Sigma\,(\delta n_{q'p}\delta n_{q''p})\,\tilde{x}_{q'}\tilde{x}_{q''}\}}{\bar{n}_p{}^2}.\end{aligned}$$

But
$$\left.\begin{aligned}\frac{\Sigma\,(\delta n_{qp})^2}{\lambda} &= n_{qp}\left(1 - \frac{n_{qp}}{N}\right)\\ \frac{\Sigma\,(\delta n_{q'p}\delta n_{q''p})}{\lambda} &= -\frac{n_{q'p}n_{q''p}}{N}\end{aligned}\right\}\qquad\text{(iv)}.$$

Thus
$$\begin{aligned}O_2 &= \frac{S\,(\bar{n}_{qp}\tilde{x}_q{}^2)}{\bar{n}_p{}^2} - \frac{S\,(\bar{n}^2{}_{qp}\tilde{x}_q{}^2)}{\bar{n}_p{}^2N} - \frac{2S_1\,(\bar{n}_{q'p}\bar{n}_{q''p}\tilde{x}_{q'}\tilde{x}_{q''})}{\bar{n}_p{}^2N}\\ &= \frac{\sigma^2{}_{\bar{n}_p}}{\bar{n}_p} - \frac{1}{N}\left\{\frac{S\,(\bar{n}_{qp}\tilde{x}_q)}{\bar{n}_p}\right\}^2\\ &= \frac{\sigma^2{}_{\bar{n}_p}}{\bar{n}_p},\ \text{since } S\,(\bar{n}_{qp}\tilde{x}_q) = 0\qquad\text{(v)}.\end{aligned}$$

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* *Biometrika*, Vol. IX. p. 2.

† The assumption that the mean value of a character in a number of samples is the value in the sampled population is often made, but is nearly as often erroneous. Thus the mean value of the correlation of a character in samples is *not* the correlation of these characters in the sampled population.

Hence to this order we have $\sigma_{m_p} = \sigma_{\bar{n}_p}/\sqrt{\bar{n}_p}$, precisely the value of the standard deviation of the mean on the hypothesis that the array is of constant size equal to the mean size $\bar{n}_p$. We next turn to the third order terms O_3. Here

$$O_3 = -\frac{2}{\lambda}\Sigma\left\{\frac{S(\delta n_{qp}\tilde{x}_q) \times S(\delta n_{qp}\delta n_p\tilde{x}_q)}{\bar{n}_p^{\,3}}\right\}$$

$$= -2\left[\frac{S\,\Sigma(\delta n^2_{qp}\delta n_p\tilde{x}_q^{\,2})}{\lambda\bar{n}_p^{\,3}} + 2\frac{S\,\Sigma(\delta n_{q'p}\delta n_{q''p}\delta n_p x_{q'}x_{q''})}{\lambda\bar{n}_p^{\,3}}\right].$$

But the summations marked by Σ can be expressed by aid of (*e*) and (*f*) on p. 244 below. We have

$$O_3 = -\frac{2}{\bar{n}_p^{\,3}}\left[S\left\{\bar{n}_{qp}\left(1-\frac{2\bar{n}_{qp}}{N}\right)\left(1-\frac{\bar{n}_p}{N}\right)\tilde{x}_q^{\,2}\right\} - 2S\left\{\frac{\bar{n}_{q'p}\bar{n}_{q''p}}{N}\left(1-\frac{\bar{n}_p}{N}\right)\tilde{x}_{q'}\tilde{x}_{q''}\right\}\right]$$

$$= -\frac{2}{\bar{n}_p^{\,2}}\left(1-\frac{\bar{n}_p}{N}\right)\left[\frac{S(\bar{n}_{qp}\tilde{x}_q^{\,2})}{\bar{n}_p} - \frac{2\bar{n}_p}{N}\left\{\frac{S(\bar{n}_{qp}\tilde{x}_q)}{\bar{n}_p}\right\}^2\right]$$

$$= -\frac{\sigma^2_{\bar{n}_p}}{\bar{n}_p} \times \frac{2}{\bar{n}_p}\left(1-\frac{\bar{n}_p}{N}\right), \text{ since } S(\bar{n}_{qp}\tilde{x}_q) = 0.$$

Accordingly
$$\sigma^2_{m_p} = O_2 + O_3 = \frac{\sigma^2_{\bar{n}_p}}{\bar{n}_p}\left\{1 - \frac{2}{\bar{n}_p}\left(1-\frac{\bar{n}_p}{N}\right)\right\} \quad \text{..................(vi).}$$

The corrective term O_3 is, I think, of marked importance for it indicates that the standard deviation of the mean of an array in a sample is not the same thing as the standard deviation of an array of *constant* size. It is not therefore legitimate to assume, as some authors have done, that the standard deviation of the mean of an array is given by the same formula, i.e. $\sigma_{\bar{n}_p}/\sqrt{\bar{n}_p}$, as for an array of constant size equal to the mean number in that array for a large number of samples.

Had we included the terms of the next order in (vi) we should have obtained in the curled brackets terms of the order $1/\bar{n}_p^{\,2}$. For a small array this might be equally important with the term already given in $1/N$. Hence some caution must be exercised in retaining that term and dropping terms in $1/\bar{n}_p^{\,2}$; at the same time for the larger arrays $\bar{n}_p/N$ may be commensurable with unity.

We may throw (vi) into the form

$$\sigma^2_{m_p} = \frac{\sigma^2_{\bar{n}_p}}{\bar{n}_p\left(1-\dfrac{2}{N}\right)+2} \quad \text{..........................(vi)}^{\text{bis}}$$

$$= \frac{\sigma^2_{\bar{n}_p}}{\bar{n}_p+2}, \text{ for many purposes.}$$

This result agrees only with that given by me in 1905*, i.e.

$$\sigma_{m_p} = \frac{\sigma_{\bar{n}_p}}{\sqrt{\bar{n}_p}},$$

to a *first* approximation, i.e. when we may neglect $2/N$ as compared with unity and 2 as compared with the mean number in the array. To assume without proof the above value to be true is not legitimate as there is no *a priori* reason why the standard deviation of the mean of an array of *variable size* should take, even to a first approximation, the value of the standard deviation of the mean of arrays of constant size, that constant size being the reduced frequency (i.e. $\bar{n}_p = \nu_p N/M$) of the sampled population. Actually a better approximation to σ_{m_p} appears to be obtained if we add two units to the mean frequency—a correction which may be of considerable importance for small arrays.

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* "On the General Theory of Skew Correlation," p. 14. *Drapers' Company Research Memoirs*, Cambridge University Press.

We next turn to the correlation of means and we have to evaluate the product of two expressions like (iii) for δm_p and $\delta m_{p'}$ where p is not equal to p'. We shall obtain, summing for λ samples and dividing by λ, the value of $\sigma_{m_p}\sigma_{m_{p'}}R_{m_p m_{p'}}$, where $R_{m_p m_{p'}}$ is the correlation of the means. We shall again treat the square and cubic order terms separately:

Square order terms in $\sigma_{m_p}\sigma_{m_{p'}}R_{m_p m_{p'}}$

$$\begin{aligned}
&= S\left\{\frac{\Sigma\,(\delta n_{qp}\delta n_{q'p'})\,x_q x_{q'}}{\lambda\bar{n}_p\bar{n}_{p'}}\right\} - m_{p'}S\left\{\frac{\Sigma\,(\delta n_{qp}\delta n_{p'}x_q)}{\lambda\bar{n}_p\bar{n}_{p'}}\right\} \\
&\qquad\qquad - m_p S\left\{\frac{\Sigma\,(\delta n_{q'p'}\delta n_p x_{q'})}{\lambda\bar{n}_p\bar{n}_{p'}}\right\} + \frac{m_p m_{p'}}{n_p n_{p'}}\frac{\Sigma\,(\delta n_p \delta n_{p'})}{\lambda} \\
&= -\frac{S\,(\bar{n}_{qp}\bar{n}_{q'p'}x_q x_{q'})}{N\bar{n}_p\bar{n}_{p'}} + m_{p'}\frac{S\,(\bar{n}_{qp}\bar{n}_{p'}x_q)}{N\bar{n}_p\bar{n}_{p'}} \\
&\qquad\qquad + m_p\frac{S(\bar{n}_{q'p'}\bar{n}_p x_{q'})}{N\bar{n}_p\bar{n}_{p'}} - \frac{m_p m_{p'}}{\bar{n}_p\bar{n}_{p'}}\frac{\bar{n}_p\bar{n}_{p'}}{N} \\
&= -\frac{m_p m_{p'}}{N} + \frac{m_{p'}m_p}{N} + \frac{m_p m_{p'}}{N} - \frac{m_p m_{p'}}{N} \\
&= 0.
\end{aligned}$$

Thus we see that as far as square order terms are concerned:

$$R_{m_p m_{p'}} = 0 \quad \text{.....................................(vii)},$$

notwithstanding that there exists a correlation between the numbers in the two arrays on which the means are based. So far this result is only true to a *first* approximation*. But we now turn to the cubic terms given by the product of the square terms in δm_p with the linear terms in $\delta m_{p'}$. There results the four separate terms:

$$\begin{aligned}
&-\frac{\Sigma\,\{S\,(\delta n_{qp'}x_q)\,S\,(\delta n_{qp}\delta n_p x_q)\}}{\lambda\bar{n}_p{}^2\bar{n}_{p'}} + \frac{\bar{m}_p S\,\{\Sigma\,(\delta n_{qp}\delta n_p\delta n_{p'}x_q)\}}{\lambda\bar{n}_p{}^2\bar{n}_{p'}} \\
&\qquad + \frac{\bar{m}_{p'}S\,\{\Sigma\,(\delta n_{qp'}\,(\delta n_p)^2)\,x_q\}}{\lambda\bar{n}_p{}^2\bar{n}_{p'}} - \frac{\bar{m}_p\bar{m}_{p'}\Sigma\,\{\delta n_{p'}\,(\delta n_p)^2\}}{\lambda\bar{n}_p{}^2\bar{n}_{p'}}.
\end{aligned}$$

I have evaluated each of these terms in succession by the use of formulae similar to those on p. 244, and all these terms give the same result, i.e. their sum with proper signs to its constituents

$$= \frac{\bar{m}_p\bar{m}_{p'}}{N\bar{n}_p}\left(1 - \frac{2\bar{n}_p}{N}\right) - \frac{\bar{m}_p\bar{m}_{p'}}{N\bar{n}_p}\left(1 - \frac{2\bar{n}_p}{N}\right) - \frac{\bar{m}_p\bar{m}_{p'}}{N\bar{n}_p}\left(1 - \frac{2\bar{n}_p}{N}\right) + \frac{\bar{m}_p\bar{m}_{p'}}{N\bar{n}_p}\left(1 - \frac{2\bar{n}_p}{N}\right) = 0.$$

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* I have given this result in memoir just cited, p. 13.

Interchanging p and p' we obtain precisely the same result for all the four terms in the product of the square terms in $\delta m_{p'}$ with the linear terms in δm_p. Thus the cubic terms in the values of $\sigma_{m_p}\sigma_{m_{p'}}R_{m_p m_{p'}}$ are also zero. We have thus established to a high order of approximation that (i) $\overline{m}_p =$ mean of array in sampled population, (ii) that there is no correlation between the means in any two different arrays, while (iii) to a lower order of approximation only $\sigma_{m_p} = \sigma_{\overline{n}_p}/\sqrt{\overline{n}_p}$.

Now we know that the distribution of means of samples *of constant size* taken from a population following any law of frequency approximates very rapidly as the size of the sample increases to the normal law. How far may we extend this result to the present case of the means of arrays the total frequency of which varies from sample to sample?

With the view of considering the approach to a normal distribution, let us investigate the third moment coefficient of m_p for the samples of the pth array. From (iii) keeping only lowest order terms we have

$$\delta m_p = \frac{S\,(\delta n_{qp}\tilde{x}_q)}{\overline{n}_p},$$

and accordingly

$$\frac{\Sigma\,(\delta m_p)^3}{\lambda} = \frac{1}{\overline{n}_p{}^3}\left[S\left\{\frac{\Sigma\,(\delta n_{qp})^3\tilde{x}_q{}^3}{\lambda}\right\} + 3S\left\{\frac{\Sigma\,(\delta n_{qp})^2\delta n_{q'p}\tilde{x}_q{}^2\tilde{x}_{q'}}{\lambda}\right\}\right.$$
$$\left. + 6S\left\{\frac{\Sigma\,(\delta n_{qp}\delta n_{q'p}\delta n_{q''p})\,\tilde{x}_q\tilde{x}_{q'}\tilde{x}_{q''}}{\lambda}\right\}\right],$$

or, using (*a*), (*b*) and (*c*) on p. 244,

$$\frac{\Sigma\,(\delta m_p)^3}{\lambda} = \frac{1}{\overline{n}_p{}^3}\left[S\left\{\overline{n}_{qp}\left(1-\frac{\overline{n}_{qp}}{N}\right)\left(1-\frac{2\overline{n}_{qp}}{N}\right)\tilde{x}_q{}^3\right\}\right.$$
$$\left. - 3S\left\{\overline{n}_{qp}\left(1-\frac{2\overline{n}_{qp}}{N}\right)\frac{\overline{n}_{q'p}}{N}\tilde{x}_q{}^2\tilde{x}_{q'}\right\} + 12S\left\{\frac{\overline{n}_{qp}\overline{n}_{q'p}\overline{n}_{q''p}}{N^2}\tilde{x}_q\tilde{x}_{q'}\tilde{x}_{q''}\right\}\right]$$
$$= \frac{1}{\overline{n}_p{}^3}\left[S\,(\overline{n}_{qp}\tilde{x}_q{}^3) - \frac{3}{N}S\,(\overline{n}_{qp}\tilde{x}_q{}^2)\,S\,(\overline{n}_{qp}\tilde{x}_q) + \frac{2}{N^2}\{S\,(\overline{n}_{qp}\tilde{x}_q)\}^3\right].$$

The last two terms vanish with the factor $S\,(\overline{n}_{qp}\tilde{x}_q)$. Thus

$$\frac{\Sigma\,(\delta m_p)^3}{\lambda} = \frac{1}{\overline{n}_p{}^2}\,{}_a\mu_3,$$

where ${}_a\mu_3$ is the third moment coefficient of the array about its own mean in the population sampled. We have seen also that to the same degree of approximation $\dfrac{\Sigma\,(\delta m_p)^2}{\lambda} = \dfrac{{}_a\mu_2}{\overline{n}p}$, where ${}_a\mu_2 = \sigma^2{}_{\overline{n}_p}$. Accordingly if ${}_pB_1$ be the value of the first β-coefficient for the distribution of the means of the pth array in samples:

$${}_pB_1 = \left\{\frac{\Sigma\,(\delta m_p)^3}{\lambda}\right\}^2 \Big/ \left\{\frac{\Sigma\,(\delta m_p)^2}{\lambda}\right\}^3$$
$$= \frac{1}{\overline{n}_p}\frac{{}_a\mu_3{}^2}{({}_a\mu_2)^3} = \frac{1}{\overline{n}_p}\,{}_p\beta_1,$$

where ${}_p\beta_1$ is the first β-coefficient of the pth array in the sampled population. Hence whatever be the nature of the frequency distribution in that population, if the number in the array be not too small, the distribution of the means of arrays will approach symmetry, i.e. ${}_pB_1$ will be small*.

The following values have been used in the previous investigations, some of which have and some of which have not, as far as I am aware, yet been published†. The remainder will be required in investigating ${}_pB_2$, or the value of $\Sigma\,(\delta m_p)^4/\lambda$.

Third Order Mean Products for Random Samples in the case of an indefinitely large Sampled Population.

(*a*)
$$\frac{\Sigma\,(\delta n_{qp})^3}{\lambda} = \bar{n}_{qp}\left(1 - \frac{\bar{n}_{qp}}{N}\right)\left(1 - \frac{2\bar{n}_{qp}}{N}\right),$$

(*b*)
$$\frac{\Sigma\,\{(\delta n_{qp})^2\,\delta n_{q'p}\}}{\lambda} = -\,\frac{\bar{n}_{qp}\bar{n}_{q'p}}{N}\left(1 - \frac{2\bar{n}_{qp}}{N}\right),$$

(*c*)
$$\frac{\Sigma\,(\delta n_{qp}\delta n_{q'p}\delta n_{q''p})}{\lambda} = \frac{2\bar{n}_{qp}\bar{n}_{q'p}\bar{n}_{q''p}}{N^2},$$

(*d*)
$$\frac{\Sigma\,(\delta n_p{}^2\delta n_{qp})}{\lambda} = \bar{n}_{qp}\left(1 - \frac{\bar{n}_p}{N}\right)\left(1 - \frac{2\bar{n}_p}{N}\right),$$

(*e*)
$$\frac{\Sigma\,(\delta n_p\delta n_{qp}\delta n_{q'p})}{\lambda} = -\,\frac{2\bar{n}_{qp}\bar{n}_{q'p}}{N}\left(1 - \frac{\bar{n}_p}{N}\right),$$

(*f*)
$$\frac{\Sigma\,\{\delta n_p\,(\delta n_{qp})^2\}}{\lambda} = \bar{n}_{qp}\left(1 - \frac{2\bar{n}_{qp}}{N}\right)\left(1 - \frac{\bar{n}_p}{N}\right),$$

(*g*)
$$\frac{\Sigma\,(\delta n_p)^3}{\lambda} = \bar{n}_p\left(1 - \frac{\bar{n}_p}{N}\right)\left(1 - \frac{2\bar{n}_p}{N}\right),$$

(*h*)
$$\frac{\Sigma\,\{\delta m_p\,(\delta n_p)^2\}}{\lambda} = 0,$$

(*i*)
$$\frac{\Sigma\,(\delta m_p\delta n_{pq}\delta n_p)}{\lambda} = \frac{\bar{n}_{qp}}{\bar{n}_p}\left(1 - \frac{\bar{n}_p}{N}\right)(x_q - \bar{m}_p),$$

(*j*)
$$\frac{\Sigma\,\{(\delta m_p)^2\,\delta n_p\}}{\lambda} = \frac{\sigma^2{}_{\bar{n}_p}}{\bar{n}_p}\left(1 - \frac{\bar{n}_p}{N}\right),$$

(*k*)
$$\frac{\Sigma\,\{(\delta m_p)^2\,\delta n_{qp}\}}{\lambda} = \frac{\bar{n}_{qp}}{\bar{n}_p{}^2}\left\{(x_q - \bar{m}_p)^2 - \frac{\bar{n}_p}{N}\,\sigma^2{}_{n_p}\right\},$$

(*l*)
$$\frac{\Sigma\,\{\delta m_p\,(\delta n_{qp})^2\}}{\lambda} = \frac{\bar{n}_{qp}}{\bar{n}_p}(x_q - \bar{m}_p)\left(1 - \frac{2\bar{n}_{qp}}{N}\right),$$

(*m*)
$$\frac{\Sigma\,(\delta m_p\delta n_{qp}\delta n_{q'p})}{\lambda} = -\,\frac{\bar{n}_{qp}\bar{n}_{q'p}}{\bar{n}_pN}\{x_q - \bar{m}_p + x_{q'} - \bar{m}_p\}.$$

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* Even if the array diverged considerably from the Gaussian value, i.e. ${}_p\beta_1 = \cdot 2$, say, then ${}_pB_1$ for an array of even 10 only would be but ·02 and the asymmetry very slight.

† (*a*) and (*g*), *Phil. Trans.*, Vol. 186, A, p. 347 (Pearson, 1894); (*b*) and (*c*), *Biometrika*, Vol. IX. p. 95 (Soper, 1913); (*d*)–(*m*) probably here for the first time.

The following fourth order mean products are deduced from the more general values given by Isserlis*:

$$(n)\quad \frac{\Sigma\,(\delta n_{qp}\delta n_{q'p}\delta n_{q''p}\delta n_{q'''p})}{\lambda} = 3\left(1-\frac{2}{N}\right)\frac{\bar{n}_{qp}\bar{n}_{q'p}\bar{n}_{q''p}\bar{n}_{q'''p}}{N^2},$$

$$(o)\quad \frac{\Sigma\,\{(\delta n_{qp})^2\,\delta n_{q'p}\delta n_{q''p}\}}{\lambda} = -\left(1-\frac{2}{N}\right)\frac{\bar{n}_{qp}\bar{n}_{q'p}\bar{n}_{q''p}}{N}\left(1-\frac{3\bar{n}_{qp}}{N}\right),$$

$$(p)\quad \Sigma\,\{(\delta n_{qp})^3\,\delta n_{q'p}\} = -\frac{\bar{n}_{qp}\bar{n}_{q'p}}{N}\left\{3\left(1-\frac{2}{N}\right)\bar{n}_{qp}\left(1-\frac{\bar{n}_{qp}}{N}\right)+1\right\},$$

$$(q)\quad \frac{\Sigma\,\{(\delta n_{qp})^2\,(\delta n_{q'p})^2\}}{\lambda} = \bar{n}_{qp}\bar{n}_{q'p}\left\{\left(1-\frac{2}{N}\right)\left(1-\frac{\bar{n}_{qp}}{N}-\frac{\bar{n}_{q'p}}{N}+\frac{3\bar{n}_{qp}\bar{n}_{q'p}}{N^2}\right)+\frac{1}{N}\right\},$$

while

$$(r)\quad \frac{\Sigma\,(\delta n_{qp})^4}{\lambda} = n_{qp}\left(1-\frac{\bar{n}_{qp}}{N}\right)\left\{3\left(1-\frac{2}{N}\right)\bar{n}_{qp}\left(1-\frac{\bar{n}_{qp}}{N}\right)+1\right\}$$

is due to Pearson†.

I shall now use (n) to (r) to determine $\Sigma\,(\delta m_p)^4/\lambda$ to the first order terms.

$$\begin{aligned}\Sigma\,(\delta m_p)^4/\lambda &= \frac{\Sigma}{\lambda}\left\{\frac{S\,(\delta n_{qp}\tilde{x}_q)}{\bar{n}_p}\right\}^4\\ &= \frac{1}{\bar{n}_p{}^4}\left[S\left(\frac{\Sigma\,(\delta n_{qp})^4\,\tilde{x}_q{}^4}{\lambda}\right)+4S\left(\frac{\Sigma\,(\delta n_{qp})^3\,\delta n_{q'p}\tilde{x}_q{}^3\tilde{x}_{q'}}{\lambda}\right)\right.\\ &\qquad +6S\left(\frac{\Sigma\,(\delta n_{qp})^2\,(\delta n_{q'p})^2\tilde{x}_q{}^2\tilde{x}_{q'}{}^2}{\lambda}\right)+12S\left(\frac{\Sigma\,(\delta n_{qp})^2\delta n_{q'p}\delta n_{q''p}\tilde{x}_q{}^2\tilde{x}_{q'}\tilde{x}_{q''}}{\lambda}\right)\\ &\qquad \left.+24S\left(\frac{\Sigma\delta n_{qp}\delta n_{q'p}\delta n_{q''p}\delta n_{q'''p}\tilde{x}_q\tilde{x}_{q'}\tilde{x}_{q''}\tilde{x}_{q'''}}{\lambda}\right)\right].\end{aligned}$$

Substituting from (n)–(q) above we obtain the following result:

$$\begin{aligned}\frac{\Sigma\,(\delta m_p)^4}{\lambda} &= \frac{1}{\bar{n}_p{}^4}\left[S\left\{\bar{n}_{qp}\left(1-\frac{\bar{n}_{qp}}{N}\right)\left(3\left(1-\frac{2}{N}\right)\bar{n}_{qp}\left(1-\frac{\bar{n}_{qp}}{N}\right)+1\right)\tilde{x}_q\right\}\right.\\ &\qquad -4S\left\{\frac{\bar{n}_{qp}\bar{n}_{q'p}}{N}\left(3\left(1-\frac{2}{N}\right)\bar{n}_{qp}\left(1-\frac{\bar{n}_{qp}}{N}\right)+1\right)\tilde{x}_q{}^3\tilde{x}_{q'}\right\}\\ &\qquad +6S\left\{\bar{n}_{qp}\bar{n}_{q'p}\left(\left(1-\frac{2}{N}\right)\left(1-\frac{\bar{n}_{qp}}{N}-\frac{\bar{n}_{q'p}}{N}+\frac{3\bar{n}_{qp}\bar{n}_{q'p}}{N^2}\right)+\frac{1}{N}\right)\tilde{x}_q{}^2\tilde{x}_{q'}{}^2\right\}\\ &\qquad -12S\left\{\frac{\bar{n}_{qp}\bar{n}_{q'p}\bar{n}_{q''p}}{N}\left(1-\frac{2}{N}\right)\left(1-\frac{3\bar{n}_{qp}}{N}\right)\tilde{x}_q{}^2\tilde{x}_{q'}\tilde{x}_{q''}\right\}\\ &\qquad \left.+24S\left\{\frac{3\bar{n}_{qp}\bar{n}_{q'p}\bar{n}_{q''p}\bar{n}_{q'''p}}{N^2}\left(1-\frac{2}{N}\right)\tilde{x}_q\tilde{x}_{q'}\tilde{x}_{q''}\tilde{x}_{q'''}\right\}\right].\end{aligned}$$

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* *R. S. Proc.* Vol. 92, A, pp. 28–29 (1915).

† *Phil. Trans.* Vol. 186, A, p. 347 (1894).

I now rearrange this into terms not involving the factor $1 - \frac{2}{N}$ and those involving it. The former are:

$$\frac{1}{\bar{n}_p{}^4}\left[S\left\{\bar{n}_{qp}\left(1 - \frac{\bar{n}_{qp}}{N}\right)\tilde{x}_q{}^4\right\} - 4S\left\{\frac{\bar{n}_{qp}\bar{n}_{q'p}}{N}\tilde{x}_q{}^3\tilde{x}_{q'}\right\} + 6S\left\{\frac{\bar{n}_{qp}\bar{n}_{q'p}}{N}\tilde{x}_q{}^2\tilde{x}_{q'}{}^2\right\}\right]$$

$$= \frac{1}{\bar{n}_p{}^4}\left[S(\bar{n}_{qp}\tilde{x}_q{}^4) + 3\{S(\bar{n}_{qp}\tilde{x}_q{}^2)\}^2/N - 4S(\bar{n}_{qp}\tilde{x}_q{}^3) \times S(\bar{n}_{qp}\tilde{x}_q)/N\right]$$

$$= \frac{1}{\bar{n}_p{}^3}\left[{}_p\mu_4 + \frac{3\bar{n}_p}{N}\,{}_p\mu_2{}^2\right],$$

since $S(\bar{n}_{qp}\tilde{x}_q) = 0$. Here ${}_p\mu_s$ is the sth moment coefficient of the pth array about the mean. I now take the terms involving the factor $1 - \frac{2}{N}$. They give

$$\frac{1}{\bar{n}_p{}^4}\left(1 - \frac{2}{N}\right)\left[3S\left\{\bar{n}^2{}_{qp}\left(1 - \frac{2\bar{n}_{qp}}{N} + \frac{\bar{n}^2{}_{qp}}{N^2}\right)\tilde{x}_q{}^4\right\} - 12S\left\{\frac{\bar{n}^2{}_{qp}\bar{n}_{q'p}}{N}\left(1 - \frac{\bar{n}_{qp}}{N}\right)\tilde{x}_q{}^3\tilde{x}_{q'}\right\}\right.$$
$$+ 6S\left\{\bar{n}_{qp}\bar{n}_{q'p}\left(1 - \frac{\bar{n}_{qp}}{N} - \frac{\bar{n}_{q'p}}{N} + \frac{3\bar{n}_{qp}\bar{n}_{q'p}}{N^2}\right)\tilde{x}_q{}^2\tilde{x}_{q'}{}^2\right\}$$
$$- 12S\left\{\frac{\bar{n}_{qp}\bar{n}_{q'p}\bar{n}_{q''p}}{N}\left(1 - \frac{3\bar{n}_{qp}}{N}\right)\tilde{x}_q{}^2\tilde{x}_{q'}\tilde{x}_{q''}\right\}$$
$$\left.+ 72S\left\{\frac{\bar{n}_{qp}\bar{n}_{q'p}\bar{n}_{q''p}\bar{n}_{q'''p}}{N^2}\tilde{x}_q\tilde{x}_{q'}\tilde{x}_{q''}\tilde{x}_{q'''}\right\}\right]$$

$$= \frac{3}{\bar{n}_p{}^4}\left(1 - \frac{2}{N}\right)\left[\{S(\bar{n}_{qp}\tilde{x}_q{}^2)\}^2 - \frac{2S(\bar{n}_{qp}\tilde{x}_q{}^2) \times \{S(\bar{n}_{qp}\tilde{x}_q)\}^2}{N} + \frac{\{S(\bar{n}_{qp}\tilde{x}_q)\}^4}{N^2}\right],$$

where the equivalence is most easily verified by expanding the latter expression and ticking off the corresponding terms in the previous one.

Since $S(\bar{n}_{qp}\tilde{x}_q) = 0$, we are left with the single term

$$\frac{3}{\bar{n}_p{}^2}\left(1 - \frac{2}{N}\right){}_p\mu_2{}^2.$$

Thus we deduce combining our two sets of terms:

$$\frac{\Sigma(\delta m_p)^4}{\lambda} = \frac{{}_p\mu_4}{\bar{n}_p{}^3} + 3\left(1 - \frac{1}{N}\right)\frac{{}_p\mu_2{}^2}{\bar{n}_p{}^2},$$

$${}_pB_2 \times \left\{\frac{\Sigma(\delta m_p)^2}{\lambda}\right\}^2 = \frac{{}_p\mu_2{}^2}{\bar{n}_p{}^2}\left\{\frac{{}_p\beta_2}{\bar{n}_p} + 3\left(1 - \frac{1}{N}\right)\right\},$$

where ${}_pB_2$ is the second β-coefficient of the distribution of m_p for the pth array, and ${}_p\beta_2$ is the second β-coefficient for the pth array itself in the sampled population.

Now to the degree of approximation needful

$$\frac{\Sigma(\delta m_p)^2}{\lambda} = \frac{{}_p\mu_2}{\bar{n}_p}\left\{1 - \frac{2}{\bar{n}_p}\left(1 - \frac{\bar{n}_p}{N}\right)\right\}.$$

Accordingly

$${}_pB_2 = \frac{{}_p\beta_2 - 3}{\bar{n}_p} + 3\left\{1 + \left(\frac{1}{\bar{n}_p} - \frac{1}{N}\right)\right\}\left\{1 + \frac{4}{\bar{n}_p}\left(1 - \frac{\bar{n}_p}{N}\right)\right\},$$

or
$$ {}_pB_2 - 3 = \frac{{}_p\beta_2 - 3}{\bar{n}_p} + 15\left(\frac{1}{\bar{n}_p} - \frac{1}{N}\right)^*. $$

Thus we see that the condition for normality of the array mean, i.e. ${}_pB_2 = 3$, is by no means so nearly satisfied as is the condition for symmetry of the array mean, ${}_pB_1 = 0$. For, while the latter might be fairly closely approximated to by an array of 15, even if the array distribution were not normal, yet the former even in the case of normality and of a big sample would give ${}_pB_2 = 4$, a wide deviation from the normal. Thus the array must be fairly considerable for the distribution of its means to become reasonably Gaussian. As a rule the distribution will be reasonably symmetrical but is leptokurtic, i.e. ${}_pB_2 > 3$, and therefore is to be described by a curve of my Type VII or one of the form

$$ y = y_0\left(1 + \frac{x^2}{a^2}\right)^{-s}, $$

rather than by a curve of the normal type[†]. Practically multiple frequency of this form is at present undiscussed and we are thrown back on treating the arrays as giving normal distributions of their means. Thus the assumption made by Slutsky in the paper cited p. 248 can only be considered as approximative, and that assumption was not legitimate until its degree of approximation had been investigated. It is singular that the goodness of fit theory can actually be applied with greater accuracy to test physical laws than to test regression lines.

It is clearly only in the large arrays that the kurtosis $({}_pB_2 - 3)$ approaches the normal value zero. What must be understood by "large" can easily be estimated roughly. For

$$ {}_pB_2 - 3 = \frac{{}_p\beta_2 + 12}{\bar{n}_p} \text{ nearly } = 15/\bar{n}_p \text{ roughly.} $$

Hence if $\bar{n}_p = 75$, ${}_pB_2$ would equal 3·2, which is certainly a limit to what may be roughly treated as a normal distribution. Accordingly when we assume a normal distribution for the means of an array, we must remember that this is really very rough in the case of the smaller arrays, and that we only do it in default of a better theory. At the same time it must be noted that the small arrays will have less weight than the larger, and the error made in assuming their distribution normal will be of far less significance for the *same* deviation[‡]. Thus far we have shown that (i) the means of different arrays are uncorrelated, (ii) that the standard deviations of these means are given by $\sigma_{\bar{n}_p}/\sqrt{\bar{n}''_p}$, where $\bar{n}''_p = \bar{n}_p\left(1 - \frac{2}{N}\right) + 2$,

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[*] This agrees with the result given by Isserlis (*loc. cit.* p. 31) only when $\bar{n}_p = N$, i.e. the "array" is as in his case a marginal total.

[†] For the curve to be of normal type s must be large. If the array were normal in the sampled population and the sample large, then $s = (2\bar{n}_p + 25)/10$, and this would be only 8·5, if $\bar{n}_p$ were 30. Thus the application of Gaussian theory to samples with even minimum arrays of 30 can only be approximative.

[‡] The deviations in the means of the small arrays are likely, however, to be much more irregular and greater than in the case of the large arrays.

and (iii) that the distribution of the means of arrays is for the larger arrays approximately normal, but that for the smaller arrays the distribution is approximately symmetrical and markedly leptokurtic. The small arrays will, however, have small relative weight and this is the only and not very satisfactory reason for adopting a Gaussian system throughout. Slutsky* assumes the normality without I think adequate investigation, and for that reason I have been unable to consider his paper as adequate and final in this matter.

With the assumption stated above we can use for the multiple regression surface

$$z = z_0 e^{-\frac{1}{2} S \left\{\frac{\bar{n}''_p (m_p - \bar{m}_p)^2}{\sigma^2_{\bar{n}_p}}\right\}},$$

where

$$\bar{n}''_p = \bar{n}_p \left(1 - \frac{2}{N}\right) + 2.$$

Here m_p is the observed value of the mean of the pth array, and $\bar{m}_p$, $\sigma_{\bar{n}_p}$ and $\bar{n}_p$ are constants which if it were feasible ought to be obtained from the sampled population. S denotes a summation for all values of p from the first to the last array. Now if we suppose $\bar{m}_p$, $\sigma_{\bar{n}_p}$ and $\bar{n}_p$ known, we can calculate

$$\chi^2 = S \left\{\frac{\bar{n}''_p (m_p - \bar{m}_p)^2}{\sigma^2_{\bar{n}_p}}\right\},$$

for, say, the series of t arrays, and as there will be t independent variables all we have to do is to determine from this value of χ^2 the value of P, the probability, corresponding to it in the tables for "goodness of fit"† under the value $n' = t + 1$. This applies to any form of frequency surface giving any type of regression line, i.e. locus of $\bar{m}_p$.

Thus far the problem looks straightforward, but now arises the difficult question as to what values are to be given to $\bar{n}_p$, $\bar{m}_p$ and $\sigma_{\bar{n}_p}$, which represent the unknown sampled population. Usually in problems, as of probable error, where we have the unknown constants of the sampled population we replace them by the corresponding constants of the sample. But it appears a somewhat arbitrary course to do this in the present instance (as suggested by Slutsky‡) for $\bar{n}_p$ and $\sigma_{\bar{n}_p}$, but not for $\bar{m}_p$. I do not think it accordingly legitimate to substitute for $\bar{n}_p$ and $\sigma_{\bar{n}_p}$ the sample values and leave $\bar{m}_p$ to be determined from other considerations. Clearly since our object is to test the goodness of fit of the regression line we have to replace $\bar{m}_p$ by $f(y_p)$, where

$$\bar{m}_p = f(y_p)$$

gives the regression curve or mean value of the array of x's corresponding to the value y_p of the other variate y. Of course this regression curve is determined from the whole series of observations and not from an individual array. But

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* "On the Criterion of Goodness of Fit of the Regression Lines, etc." *Journal of the R. Statistical Society*, Vol. LXXVII. p. 79.

† *Tables for Statisticians*, p. 26.

‡ *loc. cit.* pp. 78–84.

$\bar{m}_p$ is nevertheless subject to the probable error of the random sampling. I have shown that the probable error of $\bar{m}_p$, if the regression be linear, is

$$\frac{\cdot 67449\sigma_x\sqrt{1-r^2_{xy}}}{\sqrt{N}}\left(1+\frac{(y_p-\bar{y})^2}{\sigma_y^2}\right)^{\frac{1}{2}}$$ [*].

But this of course is not the probable error of m_p, but of $\bar{m}_p$ as found from the regression line, and will generally be small as compared with the probable error of m_p found from a single array[†].

Just as we have found $\bar{m}_p$, however, from the whole system of observations, so it appears to me we ought to determine $\sigma^2_{\bar{n}_p}$ and $\bar{n}_p$ from the whole system and not from a single array. The centre of $\sigma_{\bar{n}_p}$ is $\bar{m}_p$ and not m_p and to calculate a value for $\sigma_{\bar{n}_p}$ with m_p as centre seems to be an erroneous step especially when, owing possibly to n_p differing considerably from $\bar{n}_p$, m_p is much displaced from $\bar{m}_p$. I hold that for satisfactory results it is just as needful to find $\bar{n}_p$ and $\sigma_{\bar{n}_p}$ from the whole range of data—not from the individual array—as it is to find $\bar{m}_p$. This means that we must have some knowledge of the form of the frequency surface, and until we have this we cannot apply the test for goodness of fit to the regression line. There are accordingly two separate factors remaining for solution after we have determined the regression line:

(i) We need to determine $\bar{n}_p$. This is the total frequency of the pth array of x's. It can clearly be determined, when we know the frequency of the y's in this group. That is to say, we must determine a theoretical distribution for the marginal distribution of the y-variate. In some cases it will be sufficient to assume it Gaussian and then the table of the probability integral will suffice. In other cases it will be advisable to determine a skew frequency curve. But as a rule it will certainly be needful to graduate the array frequencies by some process, and not to assume them given by the observed marginal frequencies[‡]. The bringing of the determination of $\bar{n}_p$ into line with that of $\bar{m}_p$ does not seem therefore to present great difficulties.

(ii) We need to determine $\sigma_{\bar{n}_p}$. If the frequency surface be homoscedastic, then $\sigma^2_{\bar{n}_p}=\bar{\sigma}_x{}^2(1-\bar{\eta}^2_{x.y})$, if $\bar{\eta}_{x.y}$ be the correlation ratio of x on y, and the regression be skew. But if the regression be both homoscedastic and linear, then $\sigma^2_{\bar{n}_p}=\bar{\sigma}_x{}^2(1-\bar{r}^2_{xy})$, where $\bar{r}_{xy}$ is the correlation coefficient. In these two cases we may write respectively

$$\chi^2=\frac{1}{\bar{\sigma}_x{}^2(1-\bar{\eta}^2_{x.y})}S\{\bar{n}''_p(m_p-\bar{m}_p)^2\},$$

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[*] *Biometrika*, Vol. IX. p. 10, with the necessary changes in notation to fit the notation of this paper.

[†] Extreme arrays here again form an exception.

[‡] A precisely similar difficulty arises in working the ordinary expression for mean square contingency, i.e. $1+\phi^2=S\left(\frac{n^2_{pq}}{\bar{n}_{p.}\,\bar{n}_{.q}}\right)$, where $\bar{n}_{p.}$ and $\bar{n}_{.q}$ are the marginal frequencies (reduced of course in the proportion of size of sample to population) in the sampled population, not in the sample, although we ultimately use their sample values. There is more justification, however, in this use, for contingency is usually applied to broad categories and in such cases we have, perhaps, 3 to 7 marginal groups only; there is thus relatively less fear of big irregularities in $\bar{n}_{p.}$ or $\bar{n}_{.q}$ such as arise with the small arrays of regression lines.

and
$$\chi^2 = \frac{1}{\bar{\sigma}_x{}^2 \left(1 - \bar{r}^2{}_{xy}\right)} S\left\{\bar{n}''{}_p \left(m_p - \bar{m}_p\right)^2\right\},$$
which admit of easy calculation when once we have determined that characters of the sampled population $\bar{m}_p$, $\bar{n}_p$, $\bar{\sigma}_x$, $\bar{r}_{xy}$ or $\bar{\eta}_{x.y}$ may be replaced by the like characters calculated not from a single array but from the whole material of the sample. I am inclined to believe that
$$\chi^2 = N \frac{(\eta_{x.y})^2 - (\bar{\eta}_{x.y})^2}{1 - \bar{\eta}^2{}_{x.y}}$$
would be an effective measure of χ^2, where, however, we must not put $\eta_{x.y} = \bar{\eta}_{x.y}$ in the numerator for we are actually measuring the improbability of the deviations of $\eta_{x.y}$ from $\bar{\eta}_{x.y}$, i.e. of m_p from $\bar{m}_p$. But such a form would be of little use unless we knew the sampled population. It indicates, however, how risky is the problem of replacing only *certain* of the sampled population values of the constants by those of the sample.

In any given case it appears best to draw the scedastic curve, i.e. plot σ_{n_p} to y_p. This curve may be fitted with either a straight line or parabola of the second order, according as the array variability decreases or increases in one or both directions from some fairly central array. This is perfectly easy, but care must be taken to weight the standard deviations of the arrays with their frequencies. Very rarely as I have pointed out previously are the data ample enough to justify anything but a linear scedastic curve. If this line be practically horizontal then we can take $\sigma^2_{\bar{n}_p} = \sigma_x{}^2 (1 - \eta^2{}_{x.y})$ throughout. I propose to illustrate the whole process on the two examples selected by Slutsky.

Illustration I. Prices of Rye in Samara.

The example is given in Slutsky's recent paper* and is as follows:

Correlation between prices of rye at monthly intervals.

Price of rye in Samara a month earlier

Price of rye in Samara

Copecks per pud	*25*	*30*	*35*	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*	Totals
25	3	5	1	—	—	—	—	—	—	—	—	9
30	6	13	2	—	—	—	—	—	—	—	—	21
35	—	3	3	—	2	—	—	—	—	—	—	8
40	—	—	1	2	1	—	—	—	—	—	—	4
45	—	—	1	2	10	2	—	—	—	—	—	15
50	—	—	—	—	2	19	4	1	—	—	—	26
55	—	—	—	—	—	3	2	2·5	1·5	—	—	9
60	—	—	—	—	—	—	2	5·5	5	—	—	12·5
65	—	—	—	—	—	1	1	3·5	5	1	—	11·5
70	—	—	—	—	—	—	—	—	1	4	1	6
75	—	—	—	—	—	—	—	—	—	1	1	2
Totals	9	21	8	4	15	25	9	12·5	12·5	6	2	124

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* "On the Criterion of Goodness of Fit of the Regression Lines and on the Best Method of Fitting them to the Data." *Journal of the R. Statistical Society*, Vol. LXXVII. p. 81.

The data are not very suitable, but I use them because Slutsky has done so. Their unsuitability arises (i) from the smallness of the total population, but especially (ii) from the marked signs of heterogeneity obvious in the marginal totals. If y be the vertical, x the horizontal variate I obtain, *assuming that the column headings represent central values*, the following constants for the distribution:

$$m_x = 47{\cdot}1572 \text{ copecks per pud}, \quad \sigma_x = 13{\cdot}7769,$$

$$m_y = 47{\cdot}0363 \text{ copecks per pud}, \quad \sigma_y = 13{\cdot}6852,$$

$$r_{xy} = {\cdot}953{,}545,$$

$$\text{Regression line: } y_x = {\cdot}947{,}205x + 2{\cdot}3687.$$

My values of $\sigma_x, \sigma_y, r_{xy}$, and of the regression line constants, differ very considerably from Slutsky's. I have not used Sheppard's correction which would, however, only have emphasised our differences. I have not reworked Slutsky's first method with these changes because I do not think it is the correct method, i.e. he uses observational values for $\sigma^2_{\bar{n}_p}$, and thus I cannot say whether the above values would improve his bad fit (i.e. $P = 0{\cdot}02$), but I have adopted my own value

$$\sigma^2_{\bar{n}_p} \Big/ \left(\bar{n}_p\left(1 - \frac{2}{N}\right) + 2\right),$$

using for $\sigma^2_{\bar{n}_p}$ the mean value $\sigma_y^2(1 - r^2_{xy}) = 16{\cdot}9965$. Slutsky takes for the value of $\sigma_{\bar{n}_p}$, on the assumption of homoscedastic arrays, the mean of the observed standard deviations of the arrays, i.e.

$$\bar{\sigma}_{n_p} = \frac{1}{N} S(n_p \sigma_{n_p}) = 3{\cdot}8022$$

according to his values, or he takes

$$\sigma^2_{\bar{n}_p} = 14{\cdot}4567.$$

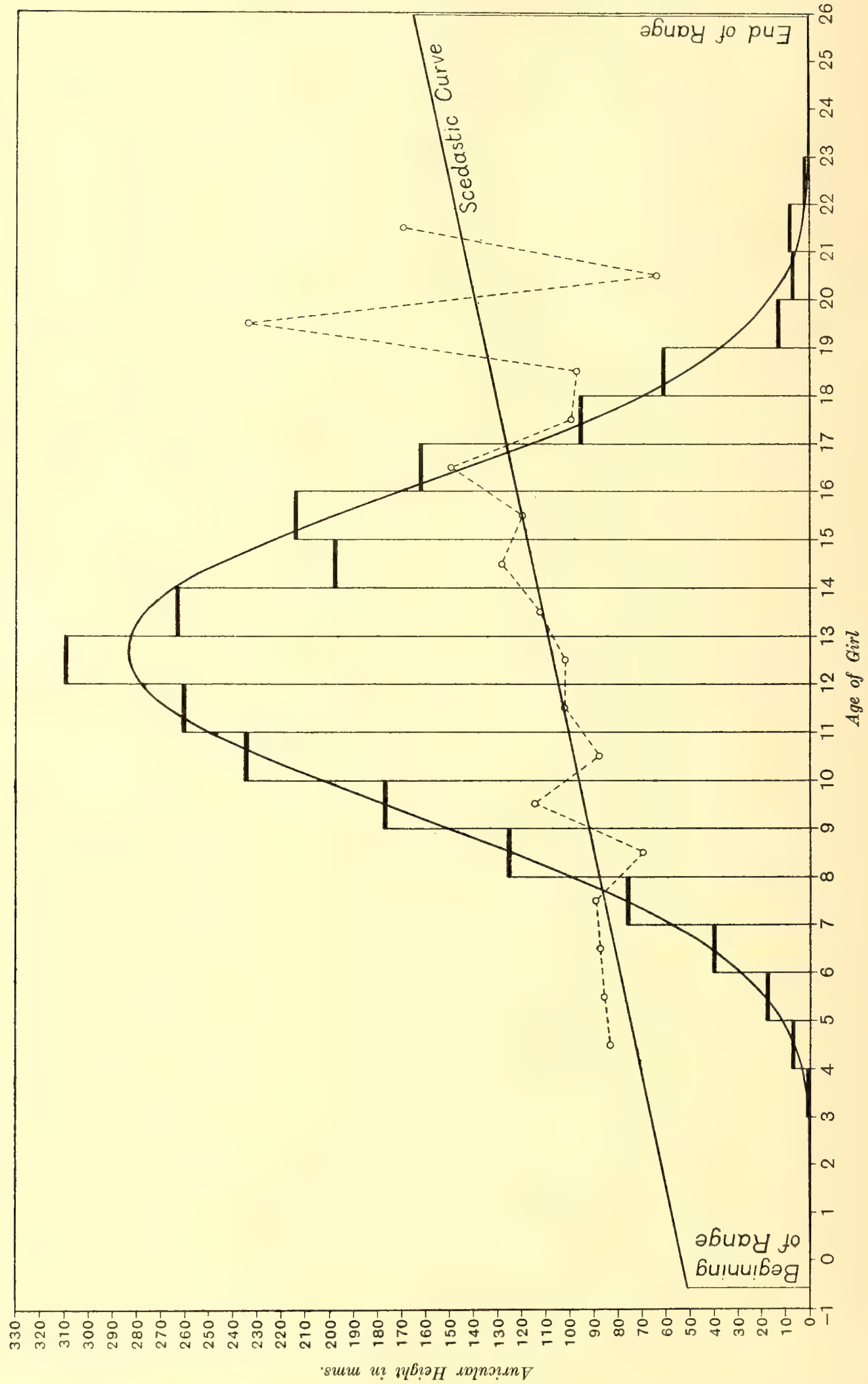
This value is, I think, theoretically incorrect, the mean value of $\sigma^2_{n_p} = \sigma_y^2(1 - r^2_{xy})$ and this must be the homoscedastic value. Clearly Slutsky's value is too small. The point remaining is the value of $\bar{n}_p$. I should naturally determine it from the frequency curve for the marginal x-totals, but the extreme irregularity of the marginal x-totals—due partly to paucity of data, but more to probable heterogeneity—makes any such process unsatisfactory. I have therefore taken $\bar{n}_p =$ to the observed array frequency—a result with which I am thoroughly dissatisfied, but which appears to be the only course. We have then the table on p. 253.

Thus $$\chi^2 = \frac{256{\cdot}81196}{16{\cdot}9965} = 15{\cdot}11.$$

Looking this out in the Tables for Goodness of Fit we find for $n' = 11 + 1 = 12$, $P = {\cdot}18$.

Thus we see that the fit is passable, although not brilliant, much better than Slutsky's $P = {\cdot}02$.

Slutsky also gives by his second method $\chi^2 = 15{\cdot}1$ and $P = {\cdot}18$ for the fit on the hypothesis of homoscedasticity, but I think this can only arise from a curious


Scedastic Curve
End of Range
Beginning of Range
Age of Girl
−1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
Auricular Height in mms.
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230 240 250 260 270 280 290 300 310 320 330

x array	$n''_p = n_p\left(1-\frac{2}{N}\right)+2$	Observed m_p	Theoretical $\overline{m}_p$	$(m_p-\overline{m}_p)^2$	$n''_p(m_p-\overline{m}_p)^2$
25	10·8548	28·333	26·049	5·2167	56·6262
30	22·6613	29·523	30·785	1·5926	36·0904
35	9·8710	34·375	35·521	1·3133	12·9636
40	5·9355	42·500	40·257	5·0310	29·8615
45	16·7581	44·000	44·993	·9860	16·5235
50	26·5967	50·800	49·729	1·1470	30·5064
55	10·8548	55·500	54·465	1·0712	11·6277
60	14·2984	59·600	59·201	·1592	2·27631
65	14·2984	62·200	63·937	3·0172	43·1411
70	7·9032	70·000	68·673	1·7609	13·9167
75	3·9677	72·500	73·409	·8263	3·2785

$S\{n''_p(m_p-\overline{m}_p)^2\} = 256{\cdot}81196.$

balancing of errors. For (i) his theoretical $\overline{m}_p$'s differ considerably from mine owing to the divergence in our values of standard deviations, correlation and regression, and (ii) he has used a different value to mine for $\bar{n}_p$, and what I believe to be an erroneous value of $\sigma^2_{\bar{n}_p}$—namely too low a value. It appears to be thus a mere chance that we should reach the same result.

Illustration II. Auricular Height of School Girls.

Slutsky takes this example from my memoir on Skew Correlation*, and it is a peculiarly good illustration for the following reasons:

(i) The regression line is distinctly skew. In my paper $\tilde{Y}_p$, the mean deviation in auricular height from the mean auricular height of the general population of girls who differ from the mean age of the general population by X_p, is given by the cubic

$$\tilde{Y}_p = {\cdot}296{,}076 + {\cdot}722{,}886 \times X_p - {\cdot}029{,}580 X_p^2 - {\cdot}002{,}223 X_p^3,$$

and the goodness of fit of this regression line is to be tested.

(ii) The frequency distribution of the ages of the 2272 girls is:

Age ...	*3—4*	*4—5*	*5—6*	*6—7*	*7—8*	*8—9*	*9—10*	*10—11*	*11—12*	*12—13*	*13—14*
Frequency	1	7	18	40	76	152	177	235	261	309	263

Age ...	*14—15*	*15—16*	*16—17*	*17—18*	*18—19*	*19—20*	*20—21*	*21—22*	*22—23*
Frequency	198	214	162	95	61	13	7	8	2

This is not normal. We find for its constants:

$$\text{Mean age} = 12{\cdot}7007 \text{ years}, \quad \beta_1 = {\cdot}001{,}335,$$
$$\sigma_x = 3{\cdot}064{,}819 \text{ years}, \quad \beta_2 = 2{\cdot}710{,}593.$$

The probable error of β_2 is about ·045 and thus β_2 cannot be the result of sampling from Gaussian material. β_1 is sufficiently near zero to mark the distribution as substantially symmetrical.

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* *Drapers' Company Research Memoirs.* Biometric Series II. p. 34. Cambridge University Press.

The distribution can accordingly be described by a symmetrical limited range curve of my Type II:

$$y = y_0 \left(1 - \frac{x^2}{b^2}\right)^m.$$

The values of the constants being found we have

$$y = 283{\cdot}7415 \left\{1 - \left(\frac{x}{13{\cdot}264{,}696}\right)^2\right\}^{7{\cdot}866{,}024}.$$

This curve was drawn on a large scale and the frequencies for each age integrated with the following results:

Age	Frequency		Age	Frequency	
	Observed	Calculated		Observed	Calculated
3—4	1	1·5	*13—14*	263	275
4—5	7	7·5	*14—15*	198	246
5—6	18	19	*15—16*	214	197
6—7	40	41·5	*16—17*	162	143·5
7—8	76	76	*17—18*	95	91·5
8—9	125	123·5	*18—19*	61	53
9—10	177	178	*19—20*	13	26
10—11	235	227·5	*20—21*	7	9
11—12	261	265·5	*21—22*	8	7
12—13	309	282·5	*22—23*	2	1·5

The goodness of fit of the calculated to the observed array frequencies was tested. I found $\chi^2 = 25{\cdot}4769$ giving $P = {\cdot}146$, or such a sample would occur about once in seven trials. The fit therefore is a fairly reasonable one, and the above values, not the observed ones, have been used for the array frequencies. Clearly they effectively smooth the random sampling.

(iii) The standard deviations of the arrays have been given by me*, and it has been shown that the arrays are very far from homoscedastic. The value of η is ·303,024, which combined with σ_x gives for the mean square standard deviation of the arrays in squared 2 mms. units

$$\bar{\sigma}_p{}^2 = 10{\cdot}835{,}433.$$

I now somewhat diverged from the plan of my memoir on skew regression. I sought the best fitting straight line to the weighted *squares* of the standard deviations, i.e. I made

$$u = S\{n_p(\sigma_p{}^2 - Ax - B)^2\}$$

a minimum, where n_p is the frequency of the pth array of auricular heights for girls of age x, and σ_p is the standard deviation of this array. In working this I omitted the first and last arrays as quite unreliable. I found $A = {\cdot}436{,}706$, giving for the line

$$\sigma_p{}^2 - \bar{\sigma}_p{}^2 = {\cdot}436{,}706\,(x - \bar{x}),$$

or

$$\sigma_p{}^2 = {\cdot}436{,}706x + 5{\cdot}290{,}970.$$

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* *Loc. cit.* p. 34, Table and Plate I, Diagram II.

From this equation the squared standard deviations of the arrays were calculated with the following results, the unit being 2 mms.:

Age array	Squared Standard Deviation		Age array	Squared Standard Deviation	
	Observed	Calculated		Observed	Calculated
3—4	[0?]	6·8194	*13—14*	11·2822	11·1865
4—5	8·3250	7·2561	*14—15*	12·8630	11·6232
5—6	8·5708	7·6929	*15—16*	12·0152	12·0599
6—7	8·7859	8·1296	*16—17*	14·9738	12·4966
7—8	8·9293	8·5663	*17—18*	10·0356	12·9333
8—9	6·9517	9·0030	*18—19*	9·7563	13·3700
9—10	11·4765	9·4397	*19—20*	23·4314	13·8067
10—11	8·7930	9·8764	*20—21*	6·4065	14·2434
11—12	10·2970	10·3131	*21—22*	17·1512	14·6801
12—13	10·2791	10·7498	*22—23*	[·9166?]	15·1169

The calculated results give a reasonable graduation of the somewhat erratic observed values, and these values were used in evaluating the formula

$$\chi^2 = S\left\{\frac{\bar{n}''_p\,(m_p - \bar{m}_p)^2}{\sigma^2_{\bar{n}_p}}\right\}.$$

The following table now gives all the required data for the calculation of χ^2, and we deduce $\chi^2 = 9·5845$ corresponding to $P = ·974$ for $n' = 20 + 1 = 21$ as argument.

Regression Curve for Girls' Auricular Height with Age.

Age group	$\bar{n}_p$	$\bar{n}''_p = \bar{n}_p\left(1 - \frac{2}{N}\right) + 2$	m_p*	$\bar{m}_p$*	$(m_p - \bar{m}_p)^2$*	$\sigma^2_{\bar{n}_p}$†	$\frac{\bar{n}''_p\,(m_p - \bar{m}_p)^2}{\sigma^2_{\bar{n}_p}}$
3—4	1·5	3·4987	115·25	116·90	2·7225	6·8194	1·3968
4—5	7·5	9·4934	116·96	117·66	·4900	7·2561	·6411
5—6	19	20·9833	117·47	118·42	·9025	7·6929	2·4617
6—7	41·5	43·4635	119·10	119·24	·0196	8·1296	·1048
7—8	76	77·9331	120·30	120·08	·0484	8·5663	·4403
8—9	123·5	125·3913	121·63	120·93	·4900	9·0030	6·8246
9—10	178	179·8433	121·72	121·78	·0036	9·4397	·0686
10—11	227·5	229·2997	122·82	122·62	·0400	9·8764	·9287
11—12	265·5	267·2663	123·14	123·42	·0784	10·3131	2·0318
12—13	282·5	284·2513	123·89	124·18	·0841	10·7498	2·2238
13—14	275	276·7579	124·86	124·88	·0004	11·1865	·0099
14—15	246	247·7834	125·71	125·52	·0361	11·6232	·7696
15—16	197	198·8266	126·16	126·07	·0081	12·0599	·1335
16—17	143·5	145·3737	126·53	126·52	·0001	12·4966	·0012
17—18	91·5	93·4195	126·91	126·87	·0016	12·9333	·0116
18—19	53	54·9533	127·02	127·09	·0049	13·3700	·0201
19—20	26	27·9771	129·56	127·18	5·6644	13·8067	11·4780
20—21	9	10·9921	123·82	127·11	10·8241	14·2434	8·3533
21—22	7	8·9938	126·50	126·88	·1444	14·6801	·0885
22—23	1·5	3·4987	125·25	126·48	1·5129	15·1169	·3502
	2272	—	—	—	—	Total $\chi'^2 = 38·3381$	

True $\chi^2 = \frac{1}{4}\chi'^2 = 9·5845$

* In millimetre units. For values of m_p and $\bar{m}_p$ (cubic (*c*)) see *loc. cit.* p. 37.

† In 2 millimetre units. Hence χ^2 as given above must be divided by *four*, to obtain true value.

We are now in a position to appreciate the influence of the various factors in the solution for this case. I hold that as $\overline{m}_p$ must be given the theoretical or calculated value, so also must the standard deviations of the arrays, i.e. σ_p^2. But as we do not know the sampled population in this case we must obtain σ_p^2 in precisely the same manner as we find $\overline{m}_p$, i.e. not by taking a value obtained from the individual array*, but by values graduated from the whole sample. Further it does not appear correct to take $\sigma_a^2 = \sigma^2_{\bar{n}_p}/\bar{n}_p$. The latter is only true when the number in the array is *a priori* fixed, that is to say is not itself provided by the random sampling. In the latter case we must take $\sigma_p^2 = \sigma^2_{\bar{n}_p}/\bar{n}''_p$, where

$$\bar{n}''_p = \bar{n}_p \left(1 - \frac{2}{N}\right) + 2,$$

to a second approximation and this modifies considerably the standard deviations of the small frequency arrays. If in the above example we use $\bar{n}_p$, the theoretical frequency of the array, instead of the value $\bar{n}''_p$, we find, still using the theoretical $\sigma^2_{\bar{n}_p}$, $\chi^2 = 8{\cdot}6166$ giving $P = {\cdot}986$. Slutsky*, who has used both the observed frequencies and the observed standard deviations† of the individual arrays, finds $\chi^2 = 9{\cdot}17$ and $P = {\cdot}980$. It will be seen that the correction for $\bar{n}''_p$ is the more important factor. In this case no marked changes are produced by using observed quantities instead of graduated values, but I think that in short series very fallacious results might be reached by this process, and the present paper is written to suggest caution at this point. $\bar{n}_p$ and $\sigma^2_{\bar{n}_p}$ are as definitely at $\overline{m}_p$ values in the *sampled* population, not in the sample itself.

Application of "Goodness of Fit" Theory to testing Physical, Technical or Astronomical Measurements.

In these cases there is no question in the ordinary sense of a frequency surface. The physicist makes a few measurements of a variate A for each of a series of values of a variate B. He plots the mean of his measurements for A to each value of his variate B, and he enquires whether the curve given by his series of mean values for A is closely approximate to some theoretical curve. It will be seen that his problem is very similar to that of the statistician. He has a number of means for the A variate, $m_1, m_2 \ldots m_p \ldots$, and he considers whether they are good fits to a theoretical curve—the statistician's regression curve. Obviously in this case these means are non-correlated as approximately in the statistical case. Further the variability of a mean will be given definitely by $\sigma_{\bar{n}_p}/\sqrt{\bar{n}_p}$—not now as an approximation. Here $\bar{n}_p$ is the number of observations in the array on which m_p depends while $\sigma_{\bar{n}_p}$ is the standard deviation (mean square error) of an indefinitely great number of measurements which he might make of A for the given value of B. He attributes these variations in A to "errors of observation," and he usually supposes such errors to obey the Gaussian law. This attribution is somewhat dogmatic. There is very little definite proof that errors of observation actually do obey the Gaussian law and secondly his "errors" are not in all probability solely due to observation. It is impossible to repeat each experimental measurement under precisely the same physical conditions for other variates $C, D, E, \ldots$ and changes in these variates may be as influential as personal errors of observation. Further it is far from certain that the value of B has remained without some variation and this alone would tend to cause some variation in A. The physicist aims at a constant value of B—it is by no means certain that he always reaches it. Without much more investigation than is easy, or is at all likely to be made, we probably can at present do no better than assume the distribution of A's for constant B to be Gaussian. Thus the distribution of means determined by the physicist will have for its distribution surface

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* *Loc. cit.* p. 81.

† In the case of the 3–4 array of *one* with observed $\sigma_a = 0$ Slutsky says this is due to random sampling and extrapolates a standard deviation; this is of course only a first slight step towards the proper graduation of the whole system of array variations. In the case of the array of *two* for 23–24, the observed value is $\sigma_a = 1{\cdot}9148$, which is just as much an inconsistency due to random sampling, but this value although about $\frac{1}{8}$ of the real value is retained and used by him.

$$z = z_0 e^{-\frac{1}{2}\chi^2},$$

where
$$\chi^2 = S\left\{\frac{n_p(m_p - \overline{m}_p)^2}{\sigma^2_{\overline{n}_p}}\right\},$$

and the corresponding value of the probability to be taken from the "goodness of fit" tables will be found by taking out P for the given value of χ^2 under the argument $n' + 1$, where n' is the number of arrays, for there are now n' independent variables, i.e. m_p's.

In the above value for χ^2, $\overline{m}_p$ is the theoretical value of A corresponding to the given B, m_p is the observed value and n_p is the number of observations on which it depends. The real difficulty arises in determining $\sigma_{\overline{n}_p}$ which is the standard deviation of the array for an indefinitely large number of observations and cannot be determined properly from the few observations made to determine the A corresponding to a given B.

If there are a considerable number of observations in an array and σ_{n_p} be the standard deviation of the array found from the observations themselves, then it is well known that the "best" or most probable value of $\sigma_{\overline{n}_p}$ is given by

$$\sigma^2_{\overline{n}_p} = \frac{n_p}{n_p - 1}\sigma^2_{n_p}.$$

In this case
$$\chi^2 = S\left\{\frac{(n_p - 1)(m_p - \overline{m}_p)^2}{\sigma^2_{n_p}}\right\},$$

a form which shows us that if we have arrays with only a single individual and use the observed σ_{n_p}'s, χ^2 will be indeterminate, for $n_p - 1 = 0$ and $\sigma^2_{n_p} = 0$. But the observed σ_{n_p} would be very risky for any system of small arrays, and this method of approaching the difficulty must I think be dropped unless the physicist be inclined to increase very much the number of measurements he makes of A for a given value of B. But the above method of approaching the subject indicates how a single value of A for each value of B cannot lead to any measure whatever of the "goodness of fit." I suggest the following method of determining a suitable value for $\sigma_{\bar{n}_p}$. Let us assume that the arrays are homoscedastic, i.e. that the physicist's difficulty in measuring A is the same for *all* values of B. This will not always be true, but is a fair working hypothesis for many cases. Then we have

$$\sigma^2_{\bar{n}_p} = \sigma_A{}^2 (1 - \eta^2{}_{A\cdot B}),$$

where σ_A is the standard deviation of *all* the measurements made on A (without regard to the value of B), a much more reliable quantity than the standard deviation of any array values of A. Further $\eta_{A\cdot B}$ is the correlation ratio of A on B or

$$\eta^2{}_{A\cdot B} = \frac{S\{n_p(m_p - \bar{m})^2\}}{N\sigma_A{}^2},$$

where $\bar{m}$ is the mean value of all measurements of A and N is their total number. Thus

$$\chi^2 = \frac{S\{n_p(m_p - \bar{m}_p)^2\}}{\sigma_A{}^2(1 - \eta^2{}_{A\cdot B})}$$

can be readily determined as soon as $\bar{m}$, σ_A and $\eta_{A\cdot B}$ have been found.

Of course it is needful for a test of this kind that the number of measurements of A should considerably exceed the number of values of B tested. It would fail entirely if only *one* value of A were taken for each value of B, however numerous the latter might be. We must have some basis on which to determine the error made in single determinations of A. This is a point I think often overlooked by the physicist. A fairly good determination—I mean a *quantitative* determination—of the goodness of fit of theory to observation could be made from 10 series of 8 observations of A corresponding to 10 values of B. But no measure of "goodness of fit" could be found from 80 observations of A corresponding to 80 values of B, and yet the latter system would probably make the greater appeal to most physicists. I do not see how quantitatively to obtain any measure of the goodness of fit of theory to observation in the latter method of procedure. It is not unusual to determine the mean square residual, i.e.

$$S(m_p - \bar{m}_p)^2/N,$$

but before we can really make use of this, i.e. find χ^2 and so P, the probability of as great or greater divergence between theory and observation, we must know $\sigma^2_{\bar{n}_p}$ and this can in no way be deduced from such a system of observations. In fact without a knowledge of $\sigma^2_{\bar{n}_p}$—the unit in which $(m_p - \bar{m}_p)^2$ is to be measured—the mean square residual is as delusive as the ocular comparison of a graph of the theoretical and observed results, where all turns on the arbitrary scale of the vertical ordinate.

Illustrations. As illustrations I will take some of the data connecting length of arc with loss of carbon per coulomb provided in a recent memoir by Professor Duffield*. My only reason for taking this material is that it is recent work and that I was as a mere statistician struck by the appearance of the curves as I turned over the pages of the *Proceedings*. I have considered the cases of 2 and 4 ampères for the anode, Diagram I. The data for individual experiments are given on Prof. Duffield's diagrams, but his tables only give mean values. I have therefore had to measure off individual values from his diagrams. The means obtained from these measurements agree in the main with the author's recorded means, but in a few cases it would appear that there is some discordance between table and diagram. On the diagrams curves are drawn presumably representing in each case the smoothed relation which the author considers to hold between loss of weight and length of arc. My object is to enquire how far these curves are probable representations of the phenomena in question. It will be seen that the observations are usually made in pairs, more rarely in triplicate or are even quadruple. There are a considerable number of isolated observations. We have thus rather slender material from which to determine the error of a single observation, and clearly but little conclusion could be drawn did we not assume that a measurement at each arc length was of equal weight*.

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* *R. S. Proc.* Vol. 92, A, p. 125.

The following table exhibits the work—first for the 4-ampère curve; the weight lost being per coulomb × 10^5 grammes.

Arc length	n_p	Weight lost, w_p	m_p	$\overline{m}_p$	$w_p - \overline{w}$	$m_p - \overline{w}$	$m_p - \overline{m}_p$	$S\{\overline{n}_p(m_p - \overline{m}_p)^2\}$
1	1	17·62	17·620	17·52	− 3·32	− 3·32	+ ·10	·0100
2 ,,	2	17·92 18·30	18·110	18·24	− 3·02 − 2·64	− 2·83	− ·13	·0338
3 ,,	2	19·19 19·62	19·405	19·15	− 1·75 − 1·32	− 1·54	+ ·25	·1250
4 ,,	2	19·70 20·32	20·010	20·12	− 1·24 − ·62	− ·93	− ·11	·0242
5	1	21·43	21·430	21·29	+ ·49	+ ·49	+ ·14	·0196
7	1	21·35	21·350	22·26	+ ·41	+ ·41	− ·91	·8281
8 ,,	2	22·40 22·63	22·515	22·51	+ 1·46 + 1·69	+ 1·57	·00	·0000
10 ,,	2	21·37 22·89	22·130	22·71	+ ·43 + 1·95	+ 1·19	− ·58	·6728
12 ,,	2	22·55 23·07	22·810	22·79	+ 1·61 + 2·13	+ 1·87	+ ·02	·0008
15	1	22·69	22·690	22·79	+ 1·75	+ 1·75	− ·10	·0100
20	1	22·89	22·890	22·79	+ 1·95	+ 1·95	+ ·10	·0100
Totals	17	355·94	—	—	Sum of squares = 57·2166	$S\{n_p(m_p - \overline{w})^2\}$ = 55·5422	—	1·7343 $= S\{n_p(m_p - \overline{m}_p)^2\}$
		$\overline{w} = 20·94$	—	—	$\sigma_A^2 = 3·3657$	$\Sigma^2 = 3·3672$	$\sigma_A^2(1 - \eta^2) = ·0985$	$\chi^2 = 17·607$
$n' = n + 1 = 11 + 1 = 12, \quad \therefore P = ·0920$					$\eta^2 = \dfrac{\Sigma^2}{\sigma_A^2} = ·9709$			

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* This is probably not true, but there is no evidence of multiple measurements at points where the measurements might be assumed to be critical.

Thus the probability, if the inscribed curve represented the phenomenon, that a system of observations as unfavourable or more unfavourable than the observed would arise is about 1 in 11, i.e. in eleven trials we should have had one result as bad. This is not very great odds (10 to 1) against the inscribed curve describing the facts, but it cannot be called a highly satisfactory concordance.

I now take the anode 2 ampère's data. The work is given in the following table:

Arc length	n_p	Weight lost, w_p	m_p	$\overline{m}_p$	$w_p - \overline{w}$	$m_p - \overline{w}$	$m_p - \overline{m}_p$	$n_p(m_p - \overline{m}_p)^2$
1 ,,	2	25·31 26·04	25·68	25·68	− 4·80 − 4·07	− 4·43	+ ·02	·0008
2 ,, ,,	3	26·04 26·77 26·95	26·59	26·46	− 4·07 − 3·34 − 3·16	− 3·52	+ ·13	·0507
3 ,,	2	26·83 28·30	27·57	27·68	− 3·28 − 1·81	− 2·54	− ·11	·0242
4 ,,	2	28·93 29·15	29·04	30·73	− 1·18 − ·96	− 1·07	− 1·69	5·7122
6 ,, ,, ,,	4	34·00 34·28 34·89 35·11	34·57	34·46	+ 3·89 + 4·17 + 4·78 + 5·00	+ 4·46	+ ·11	·0484
8 ,,	2	33·98 35·00	34·49	34·61	+ 3·87 + 4·89	+ 4·38	− ·12	·0288
Totals	15	451·58	—	—	Sum of squares = 212·92	$S\{n_p(m_p - \overline{w})^2\} = 209·51$	—	5·8651 $= S\{(n_p(m_p - \overline{m}_p)^2\}$
		$\overline{w} = 30·11$	—	—	$\sigma_A{}^2 = 14·1947$	$\Sigma^2 = 13·9673$	$\sigma_A{}^2(1-\eta^2) = ·2274$	$\chi^2 = 25·72$
$n' = n + 1 = 6 + 1 = 7, \quad \therefore P = ·0003$					$\eta^2 = \dfrac{\Sigma^2}{\sigma_A{}^2} = ·9839$			

Thus only three times in 10,000 trials if the inscribed curve actually represented the phenomenon would a series of observations so widely divergent as those observed arise. In other words the inscribed curve must be definitely rejected as a probable description of the series of phenomena. Now the great advantage of this "goodness of fit" method is that by the very working of it out the actual regions at which the theoretical results diverge with a maximum of improbability from the observations are indicated, and the investigator is able to say here are the points where discordance is greatest and where theory or observation needs modification. Clearly in this case the whole burden of the discordance falls on the observations at arc length 4. The special examples selected are of no real importance; the author probably laid no stress on his inscribed curves, and a little better draughtsmanship might have bettered them to some extent. They are used simply to illustrate that a new instrument is ready to the hand of the physicist. He must have felt very desirous at times in the past of being certain how far his observations were in accord with theory. How many times must he not have put to himself the question: What is the probability that my theory describes adequately my facts? Meanwhile the statistician who examines the physicist's diagrams has often been forced to smile by the degree of discordance which the physicist has allowed to pass as if some graph could demonstrate adequately the harmony of physical law and experiment. I look forward to the time when no physical paper will be considered complete unless it provides at the end of each series of experiments the value of *P*, i.e. the measure of the goodness of fit of observations recorded to theory adopted. There will be no excuse, there really is no excuse, now that tables are provided, for its omission. It is always possible in the course of an hour or so's arithmetic to measure the accordance between supposed law and recorded observation.

One of the important steps in the work given is undoubtedly the measurement by means of the correlation ratio of the mean square error of the physicist's determinations. I think that is undoubtedly the best way of finding it. The physicist is apt to use "mean errors" instead of mean square errors. He does not recognise that the probable error of a mean error is sensibly greater than that of a mean square error. But he may wish in the present case to have some test of the accuracy of the determination of σ_a, the standard deviation of an array, from other processes. One which it appears to me ought to be satisfactory is the following: Let the observations of A for a given value of B be taken in pairs, and let all these pairs of A values be formed and their differences $x_1 - x_2$ be taken in each case, x_1 being greater than x_2, then the following relation should hold if the distribution of errors be Gaussian:

$$\sigma_a{}^2 = \frac{\text{Mean } (x_1 - x_2)^2 - (\text{Mean } (x_1 - x_2))^2}{\cdot 72676}.$$

For Professor Duffield's 4-ampère curve this gives

$$\sigma_a{}^2 = \cdot 2444 \quad \text{against} \quad \sigma_A{}^2 (1 - \eta^2) = \cdot 0977,$$

and for his 2-ampère curve

$$\sigma_a{}^2 = \cdot 1987 \quad \text{against} \quad \sigma_A{}^2 (1 - \eta^2) = \cdot 2274.$$

The latter, considering that we deal only with 13 pairs, is fairly accordant; the agreement in the case of the former is very poor, but in this case there are only *six* pairs. I have purposely introduced this case, because no real verification of the η method could be obtained on the basis of six pairs only, and yet for the 4-ampère curve the whole question of whether the graph is a reasonable description of the data actually depends on the existence of six paired observations in the total of 17. We are really left without adequate material to determine effectively the probable error of any observation. This may be of no importance in the present case, but the absence of adequate repetition of the value of A for a given value of B in order to determine the probable error of observation and so the "goodness of fit" of observation to theory is characteristic of much current physical research.

I am much indebted to Mr Andrew W. Young for algebraical and arithmetical aid.

ON THE 'BEST' VALUES OF THE CONSTANTS IN FREQUENCY DISTRIBUTIONS.

By KIRSTINE SMITH.

(1) If we attempt to fit the normal or Gaussian curve to a system of observations, we almost invariably determine the constants $\bar{x}$ and σ of the equation

$$y = \frac{N}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\frac{(x-\bar{x})^2}{\sigma^2}}$$

by *the method of moments*. This method of moments has been extended by Thiele, Pearson, Lipps and others to obtain the constants involved in various skew frequency curves and series. It is an undoubtedly utile and accurate method; but the question of whether it gives the 'best' values of the constants has not been very fully studied. It is perfectly true that if we deal with *individual* observations then the method of moments gives, with a somewhat arbitrary definition of what is to be a maximum, the 'best' values for σ and $\bar{x}$ in the above equation to the Gaussian. Pearson* has shown that the method of moments agrees with the method of least squares in the case where the distribution is given by a high order parabola, and accordingly the method of moments is likely to give a very good result, when an expansion by Maclaurin's Theorem would closely give a frequency function. But the method of least squares itself can now-a-days hardly be spoken of as more than a utile and accurate method of fit, indeed its utility, owing to the cumbersome nature of the equations which frequently arise, is often far less than that of the method of moments.

Gauss' original proof that the probability of the observed individual results was a maximum when $\bar{x}$ and σ have been determined by moments has led to the extension of the conception that for *grouped* data, and for other results than the Gaussian curve, the 'best' values of the constants must be given by the lowest possible moments. This is of course not true For example, if we had as frequency curve

$$y = y_0 e^{-\frac{1}{4}\frac{(x-\bar{x})^4}{\sigma^4}},$$

and used individual observations, then the Gaussian 'best' value for $\bar{x}$ would be that found by determining the point for which the third moment coefficient vanished, and the 'best' value of σ would be determined by $\sigma = \sqrt[4]{\mu_4}$, where μ_4 is to be taken about the point for which $\mu_3 = 0$*.

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* *Biometrika*, Vol. I. pp. 267–70.

From another standpoint, however, the 'best values' of the frequency constants may be said to be those for which

$$\chi^2 = \frac{S(n_s - \bar{n}_s)^2}{\bar{n}_s}$$

is a minimum, where n_s is the observed frequency and $\bar{n}_s$ the theoretical frequency of the sth group†. For when χ^2 is a minimum then P, the probability of occurrence of a result as divergent as or more divergent than the observed, will be a maximum, or the frequency constants will have been so chosen as to make the probability P of results, as divergent from theory as the observed data occurring, a maximum.

It sounds somewhat paradoxical, but it is none the less true to say that the 'best value' of the mean is not necessarily the mean value, nor the 'best value' of the mean square deviation necessarily the mean square deviation‡. I shall illustrate this in the following five cases:

I. Fit of a normal curve to unilateral data.
II. Fit of a normal curve to bilateral data.
III. Fit of a Poisson limit to the binomial.
IV. Fit of a binomial to binomial data.
V. Fit of regression lines.

The general method is as follows. Suppose f to be any independent frequency constant; then χ^2 is to be a maximum with the variation of f. Accordingly we have from

$$1 + \chi^2 = S\left(\frac{n_s^2}{\bar{n}_s}\right)$$

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* University of London, Honours B.Sc., *Papers in Statistics*, Thursday, Oct. 28, 1915.

† *Phil. Mag.* Vol. L. p. 157, 1900.

‡ There is a point of some philosophical interest here which deserves further consideration. As is well known the Gaussian demonstration depends on making the product

$$P\left\{\frac{1}{\sqrt{2\pi}\,\sigma}e^{-\frac{1}{2}\frac{(x_s-\bar{x})^2}{\sigma^2}}\right\},$$

s being taken so as to include each individual observation, a maximum by varying σ and $\bar{x}$, the result being that the 'best' values are found from the first two moments. Now it will be observed that this is not the same idea as lies in the χ^2 test of goodness of fit. The conception of 'goodness' in that case is that we should measure the probability of a drawing from a certain population giving as divergent or *a more divergent result than that observed.* In other words while the Gaussian test makes a *single ordinate* of a generalised frequency surface a maximum, the χ^2 test makes a real probability, namely the *whole volume* lying outside a certain contour surface defined by χ^2 a maximum. Logically this seems the more reasonable, for the above product used in the Gaussian proof is not a probability at all. To make it a probability it must be multiplied by the product $\{\delta x_s\}$, and then the probability of the actually observed result, namely $x_1, x_2, \ldots x_s, \ldots x_q$, will of course be infinitely small, and what is made a maximum is an infinitely small probability. The exact meaning of $P\{\delta x_s\}$ when x_s is an actual observation is obscure, but it appears that the probability for constant indefinitely small ranges of the variates *in the neighbourhood* of the observed values is made a maximum. But probability means the frequency of recurrence in a repeated series of trials and this probability is in the case supposed *indefinitely small.* It seems far more reasonable to make a finite probability, i.e. the probability of a divergence as great *or greater* than the observed a maximum, i.e. to use the χ^2 test and not the Gaussian principle.

a number of equations of type

$$S\left(\frac{n_s^2}{\bar{n}_s^2}\frac{d\bar{n}_s}{df}\right) = 0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(1).$$

These equations will generally be far too involved to be directly solved. Accordingly we proceed thus: We suppose that the values of the frequency constants given by the method of moments are good starting-points, and we put, if $\bar{f}$ denote the moment value of a frequency constant, $f = f + \Delta f$. Accordingly if there be a number $f_1, f_2, \dots f_q$ of independent frequency constants, we shall have a series of equations to find Δf_1, $\Delta f_2, \dots \Delta f_q$ of the type

$$\begin{aligned} 0 = S\left\{\frac{n_s^2}{\bar{n}_s^2}\left[\frac{d\bar{n}_s}{df_1}\right]\right\} &+ S\left\{\frac{n_s^2}{\bar{n}_s^2}\left(\left[\frac{d^2\bar{n}_s}{df_1^2}\right] - \frac{2}{\bar{n}_s}\left[\left(\frac{d\bar{n}_s}{df_1}\right)^2\right]\right)\right\}\Delta f_1 \\ &+ S\left\{\frac{n_s^2}{\bar{n}_s^2}\left(\left[\frac{d^2\bar{n}_s}{df_1 df_2}\right] - \frac{2}{\bar{n}_s}\left[\frac{d\bar{n}_s}{df_1}\frac{d\bar{n}_s}{df_2}\right]\right)\right\}\Delta f_2 \\ &+ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &+ S\left\{\frac{n_s^2}{\bar{n}_s^2}\left(\left[\frac{d^2\bar{n}_s}{df_1 df_q}\right] - \frac{2}{\bar{n}_s}\left[\frac{d\bar{n}_s}{df_1}\frac{d\bar{n}_s}{df_q}\right]\right)\right\}\Delta f_q \quad \dots\dots(2\,a), \end{aligned}$$

where a square bracket round the differential coefficients signifies that the frequency constants $f_1, f_2, \dots f_q$ therein are to be given their moment values $\bar{f}_1, \bar{f}_2, \dots \bar{f}_q$. These values are of course also to be used in $\bar{n}_s$.

Since $S(\bar{n}_s) = N$, it is clear that

$$S\left(\frac{d\bar{n}_s}{df_1}\right) = S\left(\frac{d\bar{n}_s}{df_2}\right) = S\left(\frac{d^2\bar{n}_s}{df_1 df_2}\right) = \text{etc.} = 0.$$

Accordingly the above equations may be reduced to the type

$$\begin{aligned} 0 = S\left\{\frac{n_s^2 - \bar{n}_s^2}{\bar{n}_s^2}\left[\frac{d\bar{n}_s}{df_1}\right]\right\} &+ S\left\{\frac{n_s^2 - \bar{n}_s^2}{\bar{n}_s^2}\left[\frac{d^2\bar{n}_s}{df_1^2}\right] - \frac{2n_s^2}{\bar{n}_s^3}\left[\left(\frac{d\bar{n}_s}{df_1}\right)^2\right]\right\}\Delta f_1 \\ &+ S\left\{\frac{n_s^2 - \bar{n}_s^2}{\bar{n}_s^2}\left[\frac{d^2\bar{n}_s}{df_1 df_2}\right] - \frac{2n_s^2}{\bar{n}_s^3}\left[\frac{d\bar{n}_s}{df_1}\frac{d\bar{n}_s}{df_2}\right]\right\}\Delta f_2 \\ &+ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &+ S\left\{\frac{n_s^2 - \bar{n}_s^2}{\bar{n}_s^2}\left[\frac{d^2\bar{n}_s}{df_1 df_q}\right] - \frac{2n_s^2}{\bar{n}_s^3}\left[\frac{d\bar{n}_s}{df_1}\frac{d\bar{n}_s}{df_q}\right]\right\}\Delta f_q. \end{aligned}$$

It might reasonably be anticipated that terms involving the product of Δf and $(n_s^2 - \bar{n}_s^2)/\bar{n}_s^2$ could be neglected in the first place and accordingly that we should have as approximate type

$$\begin{aligned} \tfrac{1}{2}S\left\{\frac{n_s^2 - \bar{n}_s^2}{\bar{n}_s^2}\left[\frac{d\bar{n}_s}{df_1}\right]\right\} = S\left[\frac{n_s^2}{\bar{n}_s^3}\left(\frac{d\bar{n}_s}{df_1}\right)^2\right]\Delta f_1 &+ S\left\{\frac{n_s^2}{\bar{n}_s^3}\left[\frac{d\bar{n}_s}{df_1}\frac{d\bar{n}_s}{df_2}\right]\right\}\Delta f_2 \\ &+ \dots + S\left\{\frac{n_s^2}{\bar{n}_s^3}\left[\frac{d\bar{n}_s}{df_1}\frac{d\bar{n}_s}{df_q}\right]\right\}\Delta f_q \quad \dots\dots\dots\dots\dots(2\,b), \end{aligned}$$

but this approximation has not in every case numerically justified itself, and thus it cannot be invariably used as more than a reasonable starting-off point.

(2) *Fit of a Normal Curve.*

Differentiating $$\bar{n}_s = \frac{N}{\sqrt{2\pi}\sigma}\int_{x_s}^{x_{s+1}} e^{-\frac{1}{2}\frac{(x-m)^2}{\sigma^2}}\,dx,$$

and then putting

$$z_s = \frac{N}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x_s - m)^2}{\sigma^2}} \quad \text{and} \quad x_s/\sigma = h_s,$$

we have on substituting the differentials in (2 *a*):

$$\left.\begin{aligned}
0 &= \bar{\sigma} S \left\{\frac{n_s^2}{\bar{n}_s^2}[z_{s+1} - z_s]\right\} \\
&\quad + \Delta\bar{m}\left(S\left\{\frac{n_s^2}{\bar{n}_s^2}[h_{s+1}z_{s+1} - h_s z_s]\right\} + 2N\,S\left\{\frac{n_s^2}{\bar{n}_s^3}[z_{s+1} - z_s]^2\right\}\right) \\
&\quad + \Delta\bar{\sigma}\left(S\left\{\frac{n_s^2}{\bar{n}_s^2}[-z_{s+1} + z_s + h^2_{s+1}z_{s+1} - h_s^2 z_s]\right\} + 2N\,S\left\{\frac{n_s^2}{\bar{n}_s^2}[z_{s+1} - z_s][h_{s+1}z_{s+1} - h_s z_s]\right\}\right) \\
0 &= \bar{\sigma} S \left\{\frac{n_s^2}{\bar{n}_s^2}[h_{s+1}z_{s+1} - h_s z_s]\right\} \\
&\quad + \Delta\bar{m}\left(S\left\{\frac{n_s^2}{\bar{n}_s^2}[-z_{s+1} + z_s + h^2_{s+1}z_{s+1} - h_s^2 z_s]\right\} + 2N\,S\left\{\frac{n_s^2}{\bar{n}_s^3}[z_{s+1} - z_s][h_{s+1}z_{s+1} - h_s z_s]\right\}\right) \\
&\quad + \Delta\bar{\sigma}\left(S\left\{\frac{n_s^2}{\bar{n}_s^2}[(h^3_{s+1}z_{s+1} - h_s^3 z_s) - 2(h_{s+1}z_{s+1} - h_s z_s)]\right\} + 2N\,S\left\{\frac{n_s^2}{\bar{n}_s^2}[h_{s+1}z_{s+1} - h_s z_s]^2\right\}\right)
\end{aligned}\right\}\ldots(3),$$

the differential coefficients of χ^2 being

$$\left.\begin{aligned}
&\frac{d(\chi^2)}{dm} = \frac{N}{\bar{\sigma}} S\left\{\frac{n_s^2}{\bar{n}_s^2}(z_{s+1} - z_s)\right\} \\
\text{and}\quad &\frac{d(\chi^2)}{d\sigma} = \frac{N}{\bar{\sigma}} S\left\{\frac{n_s^2}{\bar{n}_s^2}(h_{s+1}z_{s+1} - h_s z_s)\right\}
\end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

Illustration I. Fit of Normal Curve to Unilateral Data.

Our first illustration treats a series of measurements by Bradley discussed by Bessel*. The mean of the observations is fixed, for in dealing with the observations Bessel has added positive and negative variations together.

TABLE I. *Measurements of Right Ascension.*

Limits	Observed	Gaussian curve by moments	Gaussian curve improved by minimum χ^2
0″·0—0″·1	114	101·61	98·63
0″·1—0″·2	84	84·12	82·59
0″·2—0″·3	53	57·65	57·91
0″·3—0″·4	24	32·71	34·00
0″·4—0″·5	14	15·36	16·72
0″·5—0″·6	6	5·974	6·881
0″·6—0″·7	3	1·923	2·372
0″·7—0″·8	1	·5122	·6843
0″·8—0″·9	1	·1370	·2053

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* Emanuel Czuber, *Theorie der Beobachtungsfehler*, p. 192. Search has been made in vain in the *Fundamenta Astronomiae* for the original data in order to remove the unilateral limitation.

$\bar{\sigma}$ was found equal to 2·282542 and the second formula of (3) gave the value 2·341735 for σ. As $\Delta\sigma$ was found so large that the approximation could not be expected to be very good, the following values of $\frac{d(\chi^2)}{d\sigma}$ were calculated from the second formula of (4):

σ	$\frac{d(\chi^2)}{d\sigma}$
2·282542	– 32·53
$\frac{1}{\cdot 43} = 2\cdot 325581$	– 11·81
$\frac{1}{\cdot 42} = 2\cdot 380952$	+ 8·06

By interpolation in this table $\sigma = 2\cdot 355860$ was found as the value for which $\frac{d(\chi^2)}{d\sigma}$ equals zero, and this is the σ of the improved Gaussian given above.

From χ^2 the 'goodness of fit' P was found:

	χ^2	P	$\frac{d(\chi^2)}{d\sigma}$
Gaussian ...	10·833	0·211	– 32·53
Impr. Gaussian	9·720	0·285	+ 0·20

As will be seen the better fit is obtained by making σ bigger than the Gaussian value, the improvement therefore cannot be looked upon as a correction for grouping. On the contrary the Sheppard correction would have given $\sigma = 2\cdot 264214$ and have raised χ^2 to 11·52. Thus we see that although the two methods give close values for P, the 'better value' is obtained as it should be from the lesser value of $d(\chi^2)/d\sigma$.

(3) *Illustration II. Fit of a Normal Curve to Bilateral Data.*

For the next illustration I have used a table giving frequencies of cephalic index in Bavarian skulls*. Both σ and m have here been varied. As the formulae (3) are somewhat laborious to work with, the approximations were used roughly suggested by the process on p. 264, but the results were not satisfactory† and these approximate results are therefore not given here. But $\frac{d(\chi^2)}{dm}$ and $\frac{d(\chi^2)}{d\sigma}$ for the two

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* J. Ranke, *Beiträge zur physischen Antropologie der Baiern*, München, 1883. The table includes the material from Tables I–VI and VIII–X inclusive which may be treated as typically 'Alt-Baierisch.'

† In fact the calculation of the exact value of $\frac{d^2(\chi^2)}{d\sigma^2}$ showed that the part of it neglected in formula (2 *b*) was about $\frac{1}{17}$ of the whole value. It essentially arose from the one tail group, this being $\frac{200}{207}$ of the whole neglected part. As $\frac{n_s{}^2 - \bar{n}_s{}^2}{\bar{n}_s{}^2}$ for this group was only as big as 1·0348, the approximate formula (2 *b*) cannot be expected to be of great value for the normal curve.

Gaussians found in this way were used for interpolation purposes and their constants are therefore given in the following table under (*b*) and (*c*). By (*a*) is indicated the Gaussian from which we started, namely that found by moments, Sheppard's correction being used.

Assuming $\frac{d(\chi^2)}{dm}$ and $\frac{d(\chi^2)}{d\sigma}$ to be linear functions of σ and m, we determined from the cases (*a*), (*b*) and (*c*) values of σ and m, given under (*d*), so as to make the differential coefficients zero. In the same way we found at last from the cases (*a*), (*b*) and (*d*) the constants of the Gaussian (*e*), the constants of which will be found in the following table. As will be seen we have succeeded in bringing the values of $\frac{d(\chi^2)}{dm}$ and $\frac{d(\chi^2)}{d\sigma}$ near to zero, certainly close enough for all practical purposes.

TABLE II.

	m	σ	χ^2	P	$\frac{d(\chi^2)}{dm}$	$\frac{d(\chi^2)}{d\sigma}$
(*a*)	83·06889	3·431833	10·205	·895	− ·57	+ 14·42
(*b*)	83·01498	3·358380	10·301	·891	− 10·56	− 9·97
(*c*)	82·98832	3·331365	11·048	·854	− 15·89	− 20·76
(*d*)	83·05329	3·349421	10·108	·899	− 4·59	− 12·10
(*e*)	83·07774	3·385991	9·858	·909	+ ·07	+ ·71

TABLE III.

	Observed	Gaussian curve by moments	Gaussian improved by minimum χ^2
75 and under	9·5	12·3387	11·3504
76	12·5	12·6842	12·0767
77	17	22·0702	21·3463
78	37	35·2942	34·6005
79	55	51·8794	51·4323
80	71·5	70·0925	70·1100
81	82	87·0421	87·6432
82	116	99·3519	100·4734
83	98	104·2329	105·6275
84	107	100·5128	101·8352
85	82	89·0879	90·0352
86	74	72·5781	72·9998
87	58	54·3468	54·2778
88	34·5	37·4049	37·0099
89	19	23·6625	23·1422
90	10	13·7588	13·2703
91	8	7·3532	6·9782
92 and over	9	6·3093	5·7910

(4) *Fit of a Poisson Limit to the Binomial.* For a Poisson limit with the general term $\frac{e^{-\bar{m}}\,\bar{m}^s}{s\,!}$ we find

$$\frac{d(\chi^2)}{dm} = S\left(\frac{n_s^2}{\bar{n}_s}\,\frac{\bar{m}-s}{\bar{m}}\right) \qquad \text{.................................(5)},$$

and putting $m = \overline{m} + \Delta m$,

$$\Delta m = \frac{S\left(\frac{n_s^2}{\overline{n}_s}(s-\overline{m})\right)}{S\left(\frac{n_s^2}{\overline{n}_s}((s-\overline{m})^2+s)\right)}\,\overline{m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6).$$

Starting with $\overline{m}$ equal to the mean of the observations I have found the improved values in the following two numerical examples.

Illustration III. The first table given by L. Whitaker* contains the number of deaths per day of women over 85 years, published in the *Times* newspaper during the years 1910–1912.

TABLE IV.

Number of deaths per day	Observed	Poisson by first moment	Poisson improved by minimum χ^2
0	364	336·250	331·133
1	376	397·302	396·334
2	218	234·720	237·186
3	89	92·446	94·630
4	33	27·308	28·316
5	13	6·4532	6·7782
6	2	1·2708	1·3521
7	1	0·2508	0·2715

The m, χ^2, P and $\frac{d(\chi^2)}{dm}$ calculated from (5) were determined for the two distributions as given in Table V.

TABLE V.

	m	χ^2	P	$\frac{d(\chi^2)}{dm}$
Poisson	1·181569	15·226	·0332	− 35·61
Poisson improved	1·196903	14·943	·0361	− 0·75

Illustration IV. As our second illustration we have taken a table of phagocytic frequencies published by Major McKendrick†.

TABLE VI.

No. of Deaths	Observed	Poisson by first moment	Poisson improved by minimum χ^2
0	620	605·924	600·676
1	282	303·568	306·164
2	79	76·044	78·026
3	16	12·699	13·257
4	2	1·5906	1·6892
5	1	·1738	·1881

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* *Biometrika*, Vol. x. p. 67.

† *Proceedings of the London Mathematical Society*, Vol. XIII. 1913, p. 401.

The numerical values of the constants of the series and of the 'goodness of fit' are

TABLE VII.

	m	χ^2	P	$\dfrac{d(\chi^2)}{dm}$
Poisson	·501000	6·865	·231	– 41·86
Poisson improved	·509700	6·672	·246	– 1·21

This table is of interest because it illustrates the apparent paradox, already seen in the case of the second Gaussian curve illustration, that the 'mean' is not necessarily the 'best value' of the constant termed the 'mean.'

(5) *Fit of a Binomial to Binomial Data.*

Let $\bar{n}_s$ be equal to the $(s+1)$th term of the binomial $(p+q)^l$, where $p+q=1$, or to

$$p^{n-s}(1-p)^s \frac{l(l-1)\dots(l-s+1)}{s!};$$

we then find

$$\frac{dn_s}{dp} = \bar{n}_s \frac{l-pl-s}{p(1-p)} = \bar{n}_s \frac{m-s}{p(1-p)},$$

where m is the mean or stand for $l(1-p)$,

$$\frac{d^2n_s}{dp^2} = \frac{n_s}{p^2(1-p)^2}\{(l-pl-s)^2-(l-pl-s)(1-2p)-lp(1-p)\}$$

$$= \frac{n_s}{p^2(1-p^2)}\{(m-s)^2+(m-s)(1-2p)+mp\},$$

$$\frac{dn_s}{dl} = \bar{n}_s\left(\log_e p+\frac{1}{l}+\frac{1}{l-1}+\dots+\frac{1}{l-s+1}\right),$$

$$\frac{d^2n_s}{dl^2} = \bar{n}_s\left\{\left(\log_e p+\frac{1}{l}+\frac{1}{l-1}+\dots+\frac{1}{l-s+1}\right)^2-\frac{1}{l^2}-\frac{1}{(l-1)^2}-\dots-\frac{1}{(l-s+1)^2}\right\},$$

$$\frac{d^2n_s}{dl\,dp} = \frac{\bar{n}_s}{p(1-p)}\left\{(m-s)\left(\log_e p+\frac{1}{l}+\frac{1}{l-1}+\dots+\frac{1}{l-s+1}\right)+(1-p)\right\}.$$

Hence we have

$$\left.\begin{aligned}\frac{d(\chi^2)}{dp} &= -S\left[\frac{n_s^2}{\bar{n}_s}\frac{m-s}{p(1-p)}\right]\\ \frac{d(\chi^2)}{dl} &= -S\left[\frac{n_s^2}{\bar{n}_s}\left(\log_e p+\frac{1}{l}+\frac{1}{l-1}+\dots+\frac{1}{l-s+1}\right)\right]\end{aligned}\right\}\dots\dots\dots\dots(7),$$

and the equations $(2\,a)$ take the form

$$\left.\begin{aligned}
&S\left(\frac{n_s{}^2}{\bar{n}_s}\,\frac{m-s}{p\,(1-p)}\right) = S\left(\frac{n_s{}^2}{\bar{n}_s\,p^2\,(1-p)^2}\left[(m-s)^2+(m-s)\,(1-2p)+mp\right]\right)\Delta p\\
&\qquad + S\left(\frac{n_s{}^2}{\bar{n}_s p\,(1-p)}\left[(m-s)\left(\log_e p+\frac{1}{l}+\frac{1}{l-1}+\ldots+\frac{1}{l-s+1}\right)-(1-p)\right]\right)\Delta l\\
&S\left(\frac{n_s{}^2}{\bar{n}_s}\left[\log_e p+\frac{1}{l}+\frac{1}{l-1}+\ldots+\frac{1}{l-s+1}\right]\right)\\
&\qquad = S\left(\frac{n_s{}^2}{\bar{n}_s p\,(1-p)}\left[(m-s)\left(\log_e p+\frac{1}{l}+\frac{1}{l-1}+\ldots+\frac{1}{l-s+1}\right)-(1-p)\right]\right)\Delta p\\
&\qquad + S\left(\frac{n_s{}^2}{\bar{n}_s}\left[\left(\log_e p+\frac{1}{l}+\frac{1}{l-1}+\ldots+\frac{1}{l-s+1}\right)^2+\frac{1}{l^2}+\frac{1}{(l+1)^2}+\ldots+\frac{1}{(l-s+1)^2}\right]\right)\Delta l
\end{aligned}\right\}\ (8),$$

while the approximate formulae of the type $(2\,b)$ are

$$\left.\begin{aligned}
&S\left(\frac{n_s{}^2}{\bar{n}_s}\,\frac{m-s}{p\,(1-p)}\right) = 2S\left(\frac{n_s{}^2}{\bar{n}_s\,p^2\,(1-p)^2}\,[m-s]^2\right)\Delta p\\
&\qquad + 2S\left(\frac{n_s{}^2}{\bar{n}_s\,p\,(1-p)}\left[(m-s)\left(\log_e p+\frac{1}{l}+\frac{1}{l-1}+\ldots+\frac{1}{l-s+1}\right)\right]\right)\Delta l\\
&S\left(\frac{n_s{}^2}{\bar{n}_s}\left[\log_e p+\frac{1}{l}+\frac{1}{l-1}+\ldots+\frac{1}{l-s+1}\right]\right)\\
&\qquad = 2S\left(\frac{n_s{}^2}{\bar{n}_s\,p\,(1-p)}\left[(m-s)\left(\log_e p+\frac{1}{l}+\frac{1}{l-1}+\ldots+\frac{1}{l-s+1}\right)\right]\right)\Delta p\\
&\qquad + 2S\left(\frac{n_s{}^2}{\bar{n}_s}\left[\log_e p+\frac{1}{l}+\frac{1}{l-1}+\ldots+\frac{1}{l-s+1}\right]\right)\Delta l
\end{aligned}\right\}\ (9).$$

Illustration V. Weldon's Dice Data.

For illustration are used the following data due to the late Professor W. F. R. Weldon*. They give the observed frequencies of dice with five or six points when a throw of twelve dice was made 26306 times.

TABLE VIII.

Number of dice in cast with 5 or 6 points	Observed frequency	Binomial by method of moments	Improved binomial (*a*) by χ^2 a minimum	Improved binomial (*b*) by χ^2 a minimum
0	185	189·679	190·651	190·659
1	1149	1154·441	1157·607	1157·600
2	3265	3223·426	3226·085	3225·959
3	5475	5461·01	5458·07	5457·78
4	6114	6253·64	6245·98	6245·71
5	5194	5101·31	5095·82	5095·79
6	3067	3041·04	3041·47	3041·69
7	1331	1335·82	1339·55	1339·81
8	403	429·627	432·815	432·984
9	105	98·865	100·351	100·419
10	14	15·5133	15·9413	15·9595
11	4	1·57640	1·65879	1·66210

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* *Phil. Mag.* July, 1900, p. 167.

Fitting the frequencies from the end by means of two moments we obtain the binomial

$$(\cdot 6658208 + \cdot 3341792)^{12\cdot 126379},$$

the terms of which are given in the table above under the head Binomial.

From these starting values of p and l we found by the equations (8) the constants of the improved binomial (*a*) $p = \cdot 6674922$ and $l = 12\cdot 188945$.

A comparison between the coefficients of the two sets of formulae (8) and (9) gave the result that they only diverged by between 1·4 and 5 per mille of their value. As $\frac{n_s{}^2 - \bar{n}_s{}^2}{\bar{n}_s{}^2}$ for the tail group was as big as 5·44, we are from this justified in expecting the approximate formulae (9) to be useful for binomial data.

Starting from the improved binomial (*a*) another improved binomial (*b*) was found by means of the formulae (9). As will be seen I only succeeded in diminishing $\frac{d(\chi^2)}{dp}$ by raising $\frac{d(\chi^2)}{dl}$, and χ^2 came out with exactly the same value as by the former formula. The constants for the improved binomial (*b*) are $p = \cdot 6675432$ and $l = 12\cdot 191141$.

The constants illustrating the 'goodness of fit' were found as follows:

TABLE IX.

	χ^2	P	$\frac{d(\chi^2)}{dp}$	$\frac{d(\chi^2)}{dl}$
Binomial	11·643	·390	159·47	− 8·15
Improved Binomial (*a*)	11·513	·401	26·02	− ·84
,, (*b*)	11·513	·401	− ·02	+ 1·96

It will be seen from the above illustrations that the probability of happening as determined by the χ^2 test of 'goodness of fit' being a maximum can always be made somewhat greater than the same probability deduced from a fit by the method of moments, which at any rate for the Gaussian curve is usually assumed to be the 'best.'

(6) *On the 'Best' Values of the Constants of Regression Curves.*

If we apply the test of 'goodness of fit' to regression curves as recently indicated by Pearson* modifying Slutsky's methods†, we shall experience the same divergence between the curves of regression found by the method of least squares and the curves calculated so as to make χ^2 a minimum, as we found when dealing with frequency distributions.

In the paper cited χ^2 for a regression curve is given as

$$\chi^2 = S\left(\frac{n_p\,(m_p - \overline{m}_p)^2}{\sigma^2_{\bar{n}_p}}\right) \quad \text{.............................(10)},$$

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* *Biometrika*, Vol. XI. pp. 239 *et seq.*

† *Journal of the Royal Statistical Society*, Vol. LXXVII. pp. 78–84.

where m is the mean of the pth array of the sample of size M from a population of size N, while $\bar{m}_p$ is the theoretical mean as found from the regression curve, $n_p, = \bar{n}_p M/N$, is the mean frequency and $\sigma_{\bar{n}_p}$ the mean standard deviation of the pth array in the samples. The difficulty in applying the 'goodness of fit' test lies in finding adequate values for n_p and $\sigma_{\bar{n}_p}$. Let us assume them to be found. The 'best' values of the constants $f_1, f_2, \ldots$ of the regression curve, i.e. the values which make χ^2 a minimum, will then be found from equations of the type

$$0 = -2S\left\{\frac{n_p}{\sigma^2_{\bar{n}_p}}(m_p - \bar{m}_p)\frac{d\bar{m}_p}{df_1}\right\} + S\left\{(m_p - \bar{m}_p)^2 \frac{d\left(\frac{n_p}{\sigma^2_{\bar{n}_p}}\right)}{df_1}\right\} \quad \ldots\ldots(11).$$

As will be seen these equations fall into the equations resulting from using the method of least squares if $\frac{n_p}{\sigma^2_{\bar{n}_p}}$ is independent of the constants of the regression curve and at the same time for the different arrays proportional to the n_p of the sample. Even if our sample be derived from truly Gaussian data, these conditions will only approximately be satisfied, the $\sigma_{\bar{n}_p}$, although constant, being dependent upon the constants of the regression curve and the n_p of the formula not being really the sample value.

Supposing $\frac{n_p}{\sigma^2_{\bar{n}_p}}$ to be independent of the constants of the regression line $\bar{m}_p = ax + b$, the equations (11) take the form

$$S\{v_p(m_p - ax - b)x\} = 0,$$
$$S\{v_p(m_p - ax - b)\} = 0,$$

when we put v_p for

$$\frac{\dfrac{n_p}{\sigma^2_{\bar{n}_p}}}{S\left(\dfrac{n_p}{\sigma^2_{\bar{n}_p}}\right)}.$$

From these equations we find

$$a = \frac{S(v_p m_p x) \,.\, S(v_p) - S(v_p m_p) S(x)}{S(v_p x^2) \,.\, S(v_p) - \{S(v_p x)\}^2}$$

and

$$b = \frac{S(v_p m_p)}{S(v_p)} - a\frac{S(v_p x)}{S(v_p)},$$

formulae agreeing with those derived from the method of least squares if v_p equals the marginal frequencies of the sample. But not agreeing with them if, for example, the material be heteroscedastic.

(7) *Illustration VI. Auricular Height of School Girls.*

This example was first used by Pearson in the memoir on skew correlation* and later as an illustration of the test of 'goodness of fit' of regression curves†.

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* *Drapers' Company Research Memoirs*, Biometric Series II. p. 34.

† *Biometrika*, Vol. XI. p. 253.

For the present use theoretical values of n_p and $\sigma_{\bar{n}_p}{}^2$ were determined, from which the values of v_p given in Table X are calculated. The n_p and v_p of the table represent the weights given to the means of arrays respectively by the method of least squares and by our method of making χ^2 a minimum. It will be seen that our method throws the weight more to the first half part of the groups of ages than the method of least squares. This is due to the heteroscedasticity of the material, the $\sigma^2_{\bar{n}_p}$ varying from 27·2776 in the youngest group to 60·4676 in the eldest. The two last columns of Table X contain the m_p calculated from our regression formula and from the usual formula; as might be expected our m_p's are closer to the means of the observations for the younger groups of ages and differ more for the higher ages than do the m_p values obtained by the method of least squares. The χ^2 calculated by (10) are for the two cases 18·45 and 18·67 and we have only raised the 'goodness of fit' *P* from ·543 to ·558 although the weighting in the two methods appeared sensibly different.

TABLE X.

Age	n_p observed	v_p	m_p observed	m_p from χ^2 a minimum	m_p from least squares
3—4	1	5·3790	115·25	117·76	117·95
4—5	7	13·7170	116·96	118·44	118·61
5—6	18	28·5973	117·47	119·13	119·27
6—7	40	56·0527	119·10	119·81	119·94
7—8	76	95·3828	120·30	120·49	120·60
8—9	125	146·023	121·63	121·17	121·26
9—10	177	199·783	121·72	121·86	121·92
10—11	235	243·414	122·82	122·54	122·59
11—12	261	271·704	123·14	123·22	123·25
12—13	309	277·232	123·89	123·90	123·91
13—14	263	259·386	124·86	124·59	124·58
14—15	198	223·505	125·71	125·27	125·24
15—16	214	172·851	126·16	125·95	125·90
16—17	162	121·965	126·53	126·63	126·57
17—18	95	75·7303	126·91	127·32	127·23
18—19	61	43·0926	127·02	128·00	127·89
19—20	13	21·2448	129·56	128·68	128·55
20—21	7	8·09110	123·82	129·36	129·22
21—22	8	6·42326	126·50	130·05	129·88
22—23	2	2·42653	125·25	130·73	130·54

The usual regression line is

$$m_p = 124·0467 + ·662979\,(x_p - 12·7007),$$

124·0467 and 12·7007 being the general means, and regression line from the χ^2 formula may be written

$$m_p = 124·0411 + ·682455\,(x_p - 12·7007)$$

from which is seen that it passes not far from the mean.

In a similar way I have treated the regression of ages on height of head. Also I have here calculated the heteroscedasticity and have had to use a parabola to

Diagram I. Comparison of Regression Straight Lines found by method of Least Squares and by χ^2 Test.

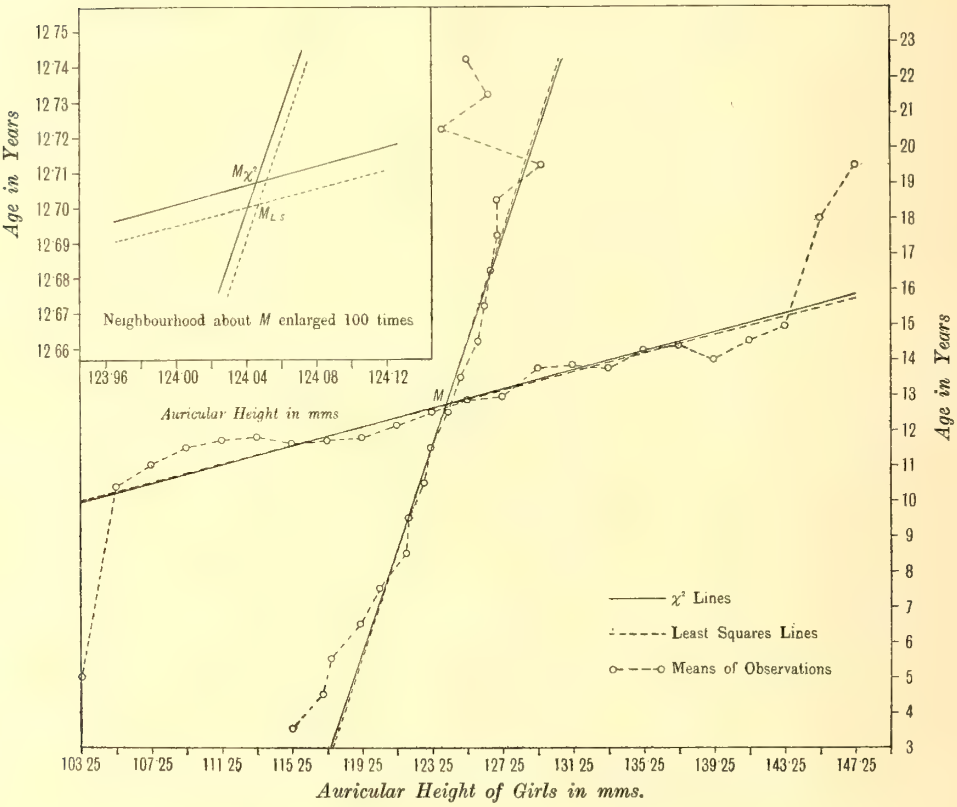
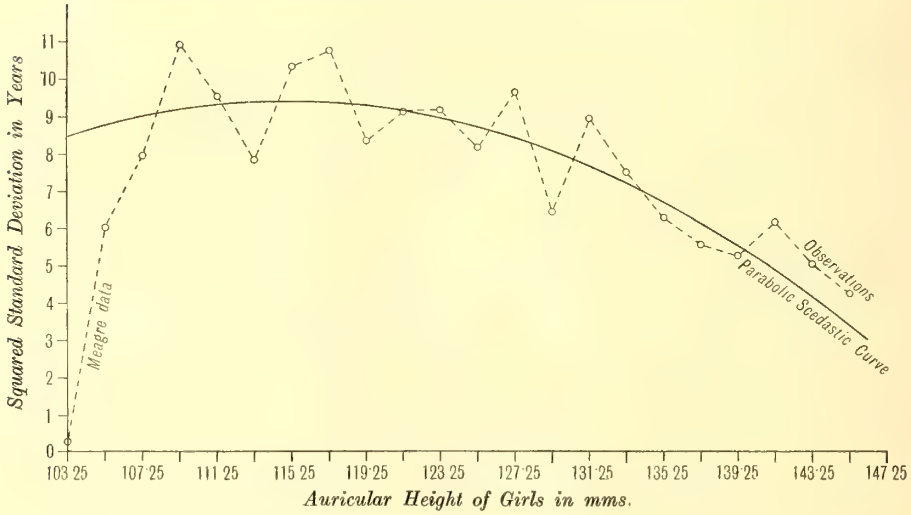


Diagram II. Scedastic Curve of Age on Auricular Height in Girls.



represent $\sigma^2_{\bar{n}_p}$, the squared standard deviation of the arrays of same height, to obtain a reasonable description; this is shown on the diagram. The marginal frequencies of the height variate could be expressed fairly well by a Gaussian curve. These theoretical values of $\sigma^2_{\bar{n}_p'}$ and n_p' are given in Table XI together with the weights

$$v' = \frac{\dfrac{n_p'}{\sigma^2_{\bar{n}_p'}}}{S\left(\dfrac{n_p'}{\sigma^2_{\bar{n}_p'}}\right)}$$

calculated from them.

TABLE XI.

Millims	$\sigma^2_{\bar{n}_p'}$ theoretical	n_p' theoretical	v'	n_p' observed	m_p' observed	m_p' from χ^2 minimum	m_p' from least squares
102·25—104·25	8·456	4·73	4·7123	2	5·00	9·92	9·99
104·25—106·25	8·748	6·62	6·3809	10	10·40	10·18	10·25
106·25—108·25	8·987	13·89	13·0282	10	11·10	10·45	10·51
108·25—110·25	9·172	26·80	24·6339	27	11·54	10·72	10·77
110·25—112·25	9·304	47·59	43·1217	56	11·71	10·99	11·03
112·25—114·25	9·382	77·76	69·8648	59	11·81	11·26	11·29
114·25—116·25	9·408	116·90	104·750	115	11·62	11·53	11·55
116·25—118·25	9·380	161·71	145·332	142	11·70	11·80	11·81
118·25—120·25	9·298	205·83	186·597	244	11·80	12·06	12·08
120·25—122·25	9·164	241·06	221·744	265	12·15	12·33	12·34
122·25—124·25	8·976	259·78	243·960	261	12·52	12·60	12·60
124·25—126·25	8·735	257·59	248·580	265	12·83	12·87	12·86
126·25—128·25	8·441	235·02	234·710	219	12·98	13·14	13·12
128·25—130·25	8·093	197·30	205·508	197	13·78	13·41	13·38
130·25—132·25	7·692	152·41	167·023	131	13·85	13·67	13·64
132·25—134·25	7·238	108·33	126·167	88	13·78	13·94	13·90
134·25—136·25	6·730	70·85	88·7361	77	14·28	14·21	14·16
136·25—138·25	6·170	42·64	58·2529	52	14·40	14·48	14·42
138·25—140·25	5·556	23·61	35·8204	20	14·05	14·75	14·69
140·25—142·25	4·888	12·03	20·7416	16	14·56	15·02	14·95
142·25—144·25	4·168	5·64	11·4040	11	14·95	15·29	15·21
144·25—146·25	3·394	2·43	6·0407	4	18·00	15·55	15·47
146·25—148·25	2·567	1·49	4·8835	1	19·50	15·82	15·73

The usual regression line is

$$m_p' = 12{\cdot}7007 + {\cdot}130489\,(y_p - 124{\cdot}0467),$$

and the line for which χ^2 is a minimum is

$$m_p' = 12{\cdot}7071 + {\cdot}1342345\,(y_p - 124{\cdot}0467).$$

For χ^2 were found in the two cases the values 44·411 and 44·109 and for the 'goodness of fit' P the values ·0047 and ·0051*.

---

* A case was purposely chosen in which the regression was known to be far from linear, in order to ascertain whether this fact itself would separate at all widely the least square and χ^2 regression lines.

The intersection point of the two χ^2 regression lines is $m = 124{\cdot}0453$, $m' = 12{\cdot}7070$, which is seen to be very near to the general means. Introducing that point into the equations of the lines, they take the form

$$m_p' = 12{\cdot}7070 + {\cdot}1342345\,(y_p - 124{\cdot}0453),$$

$$m_p = 124{\cdot}0453 + {\cdot}682455\,(x_p - 12{\cdot}7070).$$

From the slopes of the lines we find the value ·3027 for the correlation coefficient, whereas the method of least squares gives the value ·2941.

Although we have found the material to be decidedly heteroscedastic and the weighting of the two series of means rather different from that of the marginal frequencies, we nevertheless see that the resulting regression lines differ very little from the ordinary regression lines, both the deviations of the means and the correlation coefficient derived from them being less than their probable errors.

(8) The conclusions to be drawn from the present investigation are:

(i) The definition of 'best,' which leads to the method of moments being considered 'best' and incidentally to the method of least squares being considered 'best,' is undoubtedly somewhat arbitrary. If we use Pearson's 'Goodness of Fit' test, then the method of moments is not necessarily the 'best,' the best value of the constant termed the mean is not necessarily the mean, nor generally the best value of the correlation coefficient between two variates that calculated by the moments and product moment method.

(ii) On the other hand the present numerical illustrations appear to indicate that but little practical advantage is gained by a great deal of additional labour, the values of P are only slightly raised—probably always within their range of probable error. In other words the investigation justifies the method of moments as giving excellent values of the constants with nearly the maximum value of P or it justifies the use of the method of moments, if the definition of 'best' by which that method is reached must at least be considered somewhat arbitrary.

The present paper was worked out in the Biometric Laboratory and I have to thank Professor Pearson for his aid throughout the work.

MISCELLANEA.

Note on the Standard Deviations of Samples of Two or Three.

By ANDREW W. YOUNG, M.A.

In an "Editorial" contained in Vol. x of *Biometrika**, there is a discussion of the distribution of the values of the standard deviation of a population which are deduced from small samples of the population. It is there shown how the distribution approaches normality as the number, n, in the sample increases, a table of the characteristic constants of the frequency curves for various values of n being given. The smallest sample considered is that of $n = 4$, but samples of two and three are of occasional occurrence especially in physical work and now and again a value of the probable error of an experimental result is deduced from a set of two or of three observations. A knowledge of the theoretical distribution of the standard deviations for such small samples will give us some idea of the reliability of this procedure and it is the object of this note to supply this omission from the former paper.

"Student's" formula for the distribution of samples of standard deviation is

$$y = y_0 \Sigma^{n-2} e^{-\frac{1}{2}\frac{n\Sigma^2}{\sigma^2}},$$

where σ is the standard deviation of the whole population and Σ is the standard deviation given by a sample of size n. Thus the distribution for samples of two is

$$y = y_0 e^{-\frac{\Sigma^2}{\sigma^2}},$$

extending from $\Sigma = 0$ to $\Sigma = \infty$, i.e. the distribution is simply half of a normal curve; and the distribution for samples of three is

$$y = y_0 \Sigma e^{-\frac{3}{2}\frac{\Sigma^2}{\sigma^2}},$$

also, of course, extending from $\Sigma = 0$ to $\Sigma = \infty$.

It is easy to find by direct integration the moment coefficients of these curves.

Case of Samples of Two.

If we denote by N the total number of samples in the assumed distribution,

$$N = y_0 \int_0^\infty e^{-\frac{\Sigma^2}{\sigma^2}} d\Sigma = y_0 \sigma \frac{\sqrt{\pi}}{2}.$$

Taking the first four moment coefficients about the origin to be μ_1', μ_2', μ_3', μ_4', and the moment coefficients about the mean to be, as usual, μ_2, μ_3, μ_4, we have

$$\mu_1' = \text{Mean value of } \Sigma = \bar{\Sigma} = \frac{y_0}{N}\int_0^\infty \Sigma e^{-\frac{\Sigma^2}{\sigma^2}} d\Sigma = \frac{y_0 \sigma^2}{2N} = \frac{\sigma}{\sqrt{\pi}} = \cdot 5642\sigma.$$

$$\mu_2' = \frac{y_0}{N}\int_0^\infty \Sigma^2 e^{-\frac{\Sigma^2}{\sigma^2}} d\Sigma = \frac{y_0}{N}\frac{\sigma^3\sqrt{\pi}}{4} = \frac{\sigma^2}{2},$$

giving:
$$\mu_2 = \sigma^2\left(\frac{1}{2} - \frac{1}{\pi}\right) = \cdot 1817\sigma^2,$$

and the standard deviation of $\Sigma = \sigma_\Sigma = \cdot 4263\sigma$.

---

* "On the Distribution of the Standard Deviations of Small Samples." Appendix I to papers by "Student" and R. A. Fisher. *Biometrika*, Vol. x. p. 522, 1915.

$$\mu_3' = \frac{y_0}{N}\int_0^\infty \Sigma^3 e^{-\frac{\Sigma^2}{\sigma^2}} d\Sigma = \frac{y_0}{N}\frac{\sigma^4}{2} = \frac{\sigma^3}{\sqrt{\pi}},$$

giving:

$$\mu_3 = \frac{\sigma^3}{\sqrt{\pi}}\left(\frac{2}{\pi} - \frac{1}{2}\right) = \cdot 0771\sigma^3.$$

$$\mu_4' = \frac{y_0}{N}\int_0^\infty \Sigma^4 e^{-\frac{\Sigma^2}{\sigma^2}} d\Sigma = \frac{y_0}{N}\frac{3\sigma^5\sqrt{\pi}}{8} = \tfrac{3}{4}\sigma^4,$$

giving:

$$\mu_4 = \sigma^4\left(\frac{3}{4} + \frac{5}{\pi} - \frac{11}{\pi^2}\right) = \cdot 1277\sigma^4.$$

From these we derive

$$\beta_1 = \cdot 9906, \qquad \beta_2 = 3\cdot 8692.$$

Case of Samples of Three.

In the same way

$$\mu_1' = \bar{\Sigma} = \sqrt{\frac{\pi}{6}}\,.\,\sigma = \cdot 7236\sigma.$$

$$\mu_2' = \frac{2\sigma^2}{3},$$

giving:

$$\mu_2 = \sigma^2\left(\frac{2}{3} - \frac{\pi}{6}\right) = \cdot 1431\sigma^2,$$

and

$$\sigma_\Sigma = \cdot 3782.$$

$$\mu_3' = \sigma^3\sqrt{\frac{\pi}{6}},$$

giving:

$$\mu_3 = \sigma^3\sqrt{\frac{\pi}{6}}\left(\frac{\pi}{3} - 1\right) = \cdot 0342\sigma^3.$$

$$\mu_4' = \tfrac{8}{9}\sigma^4,$$

giving:

$$\mu_4 = \sigma^4\left(\frac{8}{9} - \frac{\pi^2}{12}\right) = \cdot 0664\sigma^4,$$

and

$$\beta_1 = \cdot 3983, \qquad \beta_2 = 3\cdot 2451.$$

Modal Values.

In the case of $n = 2$, the mode of the theoretical curve is at the origin $\Sigma = 0$, but it is to be borne in mind that it is the *areas* of strips of the frequency-curve which are to be used to estimate the probability. In practice, therefore, seeing that all measurements must be made in discrete amounts and cannot be mathematically continuous, we can only assert that the most frequently occurring values of Σ are those which are nearest zero—not actually zero. The example given below will make this clear.

For $n = 3$, the mode is obtained by differentiating the equation

$$y = y_0 \Sigma e^{-\frac{3}{2}\frac{\Sigma^2}{\sigma^2}}.$$

This gives, for the mode,

$$0 = \frac{1}{y}\frac{dy}{d\Sigma} = \frac{1}{\Sigma} - \frac{3\Sigma}{\sigma^2},$$

thus the modal value $\tilde{\Sigma}$ of Σ is given by

$$\tilde{\Sigma} = \frac{\sigma}{\sqrt{3}} = \cdot 5774\sigma.$$

Skewness.

The skewness of the distributions is for $n = 2$, 1·3236, and for $n = 3$, ·3867.

Thus whether we consider the mean or the modal values of the distributions, it is evident that the probable error determined from a set of three observations is very untrustworthy and that when there are only two observations it is very much worse.

With the preceding calculations we can now complete the Table on p. 529 of *Biometrika*, Vol. x, with the following

TABLE I.

Table of Values of the Constants of the Frequency Distributions of the Standard Deviations of Samples of Size 2 and 3 drawn at random from a Normal Population.

Size of sample	Mode $\tilde{\Sigma}/\sigma$	Mean $\bar{\Sigma}/\sigma$	Standard Deviation		Measures of Deviation from Normality		
			σ_{Σ}/σ	$\sigma_{\Sigma}/(\sigma/\sqrt{2n})$	Skewness	β_1	β_2
2	0·0	·5642	·4263	·8525	1·3236	·9906	3·8692
3	·5774	7236	·3782	·9265	·3867	·3983	3·2451

Experimental Verification for the Case of Samples of Two.

The frequency distribution of the number of stigmatic bands on the capsules of a growth of Shirley poppies is as follows*:

TABLE II.

No. of Stigmatic Bands ...	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	Total
No. of Capsules	1	11	32	56	148	363	628	925	954	709	397	155	51	12	1	4443

the standard deviation of the distribution, as calculated by the ordinary method, being 1·8977.

We will examine the frequency distribution of the values of the standard deviation of this series which are given by samples of two capsules taken at random. Now the standard deviation of two measurements x_1 and x_2 is easily shown to be $\frac{x_1 - x_2}{2}$, taken with the plus sign, so that in this case, the variate being measurable only in units, the possible standard deviations will be 0, ·5, 1, 1·5, ..., and we can find the theoretical frequencies (column (*b*) of Table III) of these values by taking the areas of the strips of the half-Gaussian

$$y = \frac{2N}{1{\cdot}8977\sqrt{\pi}} e^{-\left(\frac{\Sigma}{1{\cdot}8977}\right)^2},$$

whose bounding ordinates cut the axis of Σ at 0, ·25; ·25, ·75; ·75, 1·25; .... Thus the breadth of the first strip is only half that of the others and it will be found that it is the value ·5 which is the real mode of the probability.

In the case of such a distribution we can find the frequencies of the standard deviations of samples of two by actual calculation. For if we denote the chance of a capsule occurring with 5, 6, 7, 8, 9, ... stigmatic bands by *a*, *b*, *c*, *d*, *e*, ... respectively, it is clear that the chance of the value 0 of the standard deviation occurring is $a^2 + b^2 + c^2 + d^2 + \ldots$, and the chance of the value ·5 occurring is the chance of a sample of two flowers whose numbers of stigmatic bands differ by 1, i.e. is

$$ab + b\,(a + c) + c\,(b + d) + \ldots = 2\,(ab + bc + cd + \ldots),$$

and similarly for $\Sigma = {\cdot}1$ the chance is

$$ac + bd + c\,(a + e) + d\,(b + f) + \ldots = 2\,(ac + bd + ce + \ldots),$$

and so on.

By this means the theoretical frequencies given in column (*a*) of Table III were calculated. A histogram of these values will be found to be well in accord with the half-Gaussian given above.

---

[*] Pearson, *Phil. Trans.* Vol. 197 A, p. 314, Hampden Series.

With a view to confirming the theoretical results, a practical random sampling was made in the following way. A circle, about 20 inches in diameter, was carefully divided into sectors whose angles (and therefore areas) were proportional to the class-frequencies given in Table II. For each sampling a pointer was placed at random inside the circle and the number of the sector in which it was placed was noted. It appeared that, if the observer kept his eyes shut and rotated the circle for a time between each pointing, the method would give quite satisfactorily random sampling. In this way an ordered series of 648 samples was made and by taking samples of two by pairing in accordance with three different rules, the series of numbers given in Table III were obtained. Alongside of each series in the Table is given the ratio of the number in each set to the total. These ratios are to be compared with the "theoretical chances." In all three samples the number with standard deviation 0 is considerably less than the theoretical numbers, but from the values of the Goodness of Fit "P" given at the ends of the columns it appears that this can be accounted for by the variations of random sampling.

TABLE III.

Frequency distribution of Standard Deviations of Samples of Two Capsules from the Shirley Poppy Series.

Standard Deviation	Theoretical Chance		Experimental Results					
			I		II		III	
	(*a*)	(*b*)	Number of samples	Chance	Number of samples	Chance	Number of samples	Chance
0	·1522	·1478	39	·1204	38	·1172	37	·1142
·5	·2820	·2760	102	·3148	99	·3056	109	·3364
1	·2249	·2246	75	·2315	75	·2315	74	·2284
1·5	·1553	·1594	55	·1698	58	·1790	44	·1358
2·0	·0937	·0986	26	·0802	26	·0802	31	·0957
2·5	·0503	·0532	7	·0216	15	·0463	15	·0463
3·0	·0243	·0250	12	·0370	5	·0154	8	·0247
3·5	·0108	·0102	5	·0154	5	·0154	5	·0154
4·0	·0043	·0037	3	·0093	2	·0062	0	·0000
4·5	·0015	·0011	0	·0000	1	·0031	1	·0031
5·0	·0005	·0003	0	·0000	0	·0000	0	·0000
5·5	·0001	·0001	0	·0000	0	·0000	0	·0000
Using Theoretical Chance (*a*)			$\chi^2=15{\cdot}0$,	$P={\cdot}18$	$\chi^2=8{\cdot}3$,	$P={\cdot}69$	$\chi^2=10{\cdot}5$,	$P={\cdot}49$
Using Theoretical Chance (*b*)			$\chi^2=16{\cdot}6$,	$P={\cdot}12$	$\chi^2=8{\cdot}9$,	$P={\cdot}63$	$\chi^2=11{\cdot}3$,	$P={\cdot}42$

The mean standard deviations given by these sets are for I, 1·06, for II, 1·07, for III, 1·05, while those derived from the half-Gaussian and from the "theoretical chances" are 1·071 and 1·050 respectively. It will be noticed also that although the calculated chances (*a*) appear to give a better fit than the half-Gaussian, the differences in the "P's" are very slight, and the half-Gaussian would suffice for most purposes. These results form a strong confirmation of the correctness of the theory.

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W. R. MACDONELL (1852—1916)

Contributor to and Assistant Editor of *Biometrika*

VOLUME XI MAY, 1917 No. 4

Smithsonian Institu
JUL 21
National Mus

W. R. MACDONELL.

Born October 16, 1852. Died May 15, 1916.

FOR the third time the hand of death has fallen heavily on this Journal. Dr W. R. Macdonell died on May 15 at the relatively early age of 63. On his retirement from business in 1899, at the suggestion of W. H. Macaulay he joined the Biometric Laboratory to do research work, and his patient labour, his wise counsel and lovable disposition soon made him an essential part of the place. It would be difficult to appraise adequately the help he gave to the Biometric School in its early days. It was not only in material and apparatus, it was not only in resourceful suggestion to his fellow-workers, but it was especially in the general sense of courage and in the spirit of readiness to undertake the tedious because it meant profit to science in the future, which he diffused around him that his help was so invaluable. And there was need of the heartening which Macdonell gave! The first greeting that Fawcett's paper on the Naqada crania, involving years of work, received from anthropologist and anatomist was anything but favourable, but Macdonell persisted on the lines thus laid down in his great memoirs on the 17th century English crania, and it would have gladdened him to have seen Fawcett's memoir now described by a distinguished anthropologist as Fawcett's "classical memoir," and the anatomical author of the most recent British cranial research stating that he "cannot do better than follow the scheme adopted by Fawcett in her study of the Naqada crania and also by Macdonell in his study of the Whitechapel English crania and other series of English skulls." It takes a long while to reform any branch of science, but when the history of craniometry comes to be written, those early workers in the Biometric Laboratory will be remembered, and not the least Macdonell, who gave heart to them all.

When *Biometrika* was founded Macdonell joined the little group of men who provided the guarantee fund—there is small harm now in mentioning their names: Francis Galton, W. F. R. Weldon, W. R. Macdonell, Lord Parker of Waddington, and the present Editor—and the aid then provided carried this Journal through the troublous days of infancy to the completely self-supporting stage which preceded the war. It is a sad element in the fate of this Journal that it has to meet a new crisis in its existence with many new friends it is true, but without the majority of the old supporters. From the first issue until this very year, Macdonell acted as assistant editor of *Biometrika*, and his name appeared on the title page as such for several years after the death of Weldon. Our readers were, however, very little conscious of all the labour Macdonell put into proof-reading and many a contributor has profited in a way he little realised by Macdonell's assiduity and not infrequently by his suggestions. Scotland has preserved its knowledge and appreciation of grammar long after grammar has been discarded in England, and few abler proof-readers can be found than a Scotsman trained in Oxford, especially if he has graduated in science, and tempered his science with modern European literature as a hobby. The width of Macdonell's studies and of his interests was effectively demonstrated by his memoir "On the Expectation of Life in Ancient Rome and in the Provinces of Hispania and Lusitania and Africa." Few men could have been found to combine the necessary biometric with the still more needful literary training requisite for a study of this character, and fewer still would have concluded it with such words of modesty as Macdonell did. With his death biometry loses a sturdy champion, our subscribers and contributors more than they yet realise, and some of us a close and most trustworthy friend. Not least will this be recognised by the little band of pupils he formed around him during his occupancy of the lectureship on biometry in the University of Aberdeen. He acted straightly and he advised wisely both in his mercantile and in his scientific career. He had the reticence which flows from strength, and the persistency and courage of the strong who are reticent. Life seemed a spacious thing in his handling. What more could we have desired for him?

We owe to the kindness of Professor W. Paton Ker the following brief notes on Macdonell which, emphasising the width of his interests and his genial personality, will enable those of our readers who only know him from his memoirs in *Biometrika* to understand better another of the early members of the Biometric School.

I first met Macdonell in my freshman's term at Balliol: I succeeded him in rooms possibly the worst in College, and he came in a few days later to ask if I had seen his copy of Victor Hugo's poems lying about. This I had already found and now restored to him; it provided something to talk about afterwards. He was a Mathematical Scholar of Balliol and did not neglect his subject, but he always seemed more interested in other things, especially Greek. He had been at Aberdeen along with R. A. Neil of Pembroke, Cambridge, always one of his greatest friends; at Balliol he found another Greek scholar, William Gunion Rutherford. Rutherford and he used to read Greek together for a time. Then Macdonell left Oxford to go into business, and for a year or two I saw nothing of him. But I found him in London when I went there in 1879 after taking my degree; Rutherford was there also, a master at St Paul's School, and we made it a regular thing to meet on Saturday evenings, the three of us, at Rutherford's rooms in Mitre Court. It was a good time while it lasted, but soon we went different ways —Macdonell to India—and I lost sight of him and did not hear much, for several years. Soon after he got to Bombay he wrote to me, asking for the *Journal of Philology* to be sent to him; he was keeping up his Greek and Latin; he was also amused at the manners of scholars in disputation. At Bombay he was once asked by the Danish merchants there to choose for himself a present which they wished to give him in acknowledgement of services rendered by him in connexion with their business. He chose, as the best thing obtainable from Copenhagen, Madvig's edition of Cicero *De Finibus*, and thought that they were pleased with his choice.

Macdonell was chairman of the Chamber of Commerce in Bombay, and additional member of Council there, 1893–1895. Then he came home, and for some time worked in the London office of his firm. He retired from business in 1899. He had carried out his plans, and earned his leisure. Probably few men are to be trusted with leisure at the age of 47, but Macdonell made the most of his life. He never lost interest in any study that once had engaged his attention, though he had so many interests—in books, pictures, music and travelling—that naturally he could not always attend to everything. He never forgot a friend. He had been six years married when he came home from India, and the friend that was made welcome by his wife, and his children, as well as himself in his house at Enfield has much to thank him for. He made new friends as he went on, through common interests for the most part, and particularly through the investigations in which he took part along with Weldon and Pearson. He felt very deeply the loss of friends; he could scarcely bear to speak of the death of Neil of Pembroke.

He inherited his love of books; one of his early recollections was the farmhouse in the Highlands where it was the custom to read aloud in the evenings, usually from some of the English classics but sometimes *Don Quixote* to the farm-servants; light being provided for the reader from a stock of pine splinters, lighted in succession one from another: one of the party was employed in cutting these brands, and a boy held up the light over the book, and renewed it from the supply that was passed to him. Macdonell's people were Catholics, and his study of the humanities may have owed something to the traditions of the Church. Among his friends was Monsignor Fraser, Rector of the Scots College in Rome. I remember Macdonell bringing back from a visit to Rome a story of a pibroch heard at night there, and how the music was traced to the roof of the College—a story that would have delighted Sir Walter Scott.

No weakness of the human race gave Macdonell more amusement than the vanity of biographers, and I can almost see him looking over these notes of mine, and making disrespectful remarks about them. But I am glad to have been allowed to write them, such as they are.

W. P. KER.

UNIVERSITY COLLEGE, LONDON.
3 *July* 1916.

FURTHER SUPPLEMENTARY TABLES FOR DETERMINING HIGH CORRELATIONS FROM TETRACHORIC GROUPINGS.

BY ALICE LEE, D.Sc.

THE difficulty of determining correlations between ·80 and 1·00 by the tetrachoric method, owing to the slow convergency of the terms of the fundamental equation for 'tetrachoric r' has long been recognised. In 1912 Everitt* published "Supplementary Tables for Determining High Correlations from Tetrachoric Groupings." These tables much simplified the work within the field in which it is really possible to determine accurately a high correlation—beyond certain values of 'h and k' such determination is impossible owing to the influence of random sampling on a quadrant category which in most practical cases will only contain an isolated unit or two. Everitt's tables covered the values of r from $+ {\cdot}80$ to $+ 1{\cdot}00$ for values of the dichotomic planes given by h and k varying from $+ {\cdot}0$ to $+ 2{\cdot}6$. They admitted at once of our dealing with those cases of negative values of r, for which *either* h or k was negative, but not with cases in which r was negative and both h and k remained of the same sign. The present tables provide for this omitted portion of the possible field and thus complete Everitt's work.

I have followed his method of quadrature in evaluating my integrals. But I have preserved more decimal places than he has done, partly because my significant figures are thrown into higher decimal places than his by the nature of the case, and partly because recent experience in other fields has shown workers in the Biometric Laboratory, that tables are often of service for purposes other than those for which they were originally calculated, and that it is worth while preserving every reliable figure. I think my results are always correct to six figures and generally to the actual number tabulated. While many of the entries would only be of service in the case of total populations approaching the magnitude of census populations, their indication of the high decimal place in which the first significant figure can occur in the 'd' quadrant, will help to dispel the illusion that absence of frequency in a quadrant is necessarily indicative of absolute association, when the table is based on a limited sample. A further reason for the number of figures I have preserved may be found in what I believe to be one of the principal uses of tables of this type. They are not only valuable for the calculation of the correlation in the case of given data, but also in many theoretical investigations where the correlation is supposed known *a priori* and we require to ascertain how many individuals may be expected to possess a combination of characters each of which exceeds a given value. I have adopted throughout Pearson's original notation in his memoir on the fourfold table*, i.e. d is the frequency of the quadrant diagonally opposed to that in which the mean falls, i.e.:

---

* *Biometrika*, Vol. VIII. p. 385.

$$\frac{d}{N}=\frac{1}{2\pi\sqrt{1-r^2}}\int_h^\infty\int_k^\infty e^{-\frac{1}{2}\frac{1}{1-r^2}(x^2-2rxy+y^2)}\,dx\,dy,$$

where in our case r is given values from $-\cdot8$ to $-1\cdot0$. This is the integral I have tabled. For the case of $r=-1\cdot0$, d/N will be zero for all values of h and k, so that it is not needful to provide a table for $r=-1\cdot0$ and interpolation between $-\cdot95$ and $-1\cdot0$ must be between the values given in the table for $-\cdot95$ and zero values.

Illustration I. An appreciation of sex was made by two different observers on 216 femora. It is required to find a measure of the association in judgment between the two observers.

In appreciating sex by the examination of a bone the observers opinion varies from practical certainty of maleness through every shade of doubt to practical certainty of femaleness. The strength of the judgment is therefore a continuous character, although the actual sex forms a rigid categorical differentiation. It is with the judgment, and not with the actual sex that we are here concerned, and we have simplified those judgments down to unique categories ♂ and ♀ classifying under each all queried values.

The table is:

First observer

Second observer		♂	♀	Totals
	♂	98	16	114
	♀	6	96	102
	Totals	104	112	216

Mean

Mean

Rearranging in standard form:

16	98	114
96	6	102
112	104	216

we see that h and k will now both be positive and r will be negative.

---

* *Phil. Trans.* Vol. 195, A, p. 1.

We have $$d/N = 6/216 = \cdot02778,$$

$$\tfrac{102}{216} = \tfrac{1}{2}(1-\alpha_1) = \cdot47222, \qquad \tfrac{104}{216} = \tfrac{1}{2}(1-\alpha_2) = \cdot48148,$$

or $$h = \cdot06969, \qquad k = \cdot04644.$$

From the Tables for $r = -\cdot95$

$$\left.\begin{array}{l}\left.\begin{array}{l}h=0\\k=0\end{array}\right\}\dfrac{d}{N}=\cdot050{,}542\\[1ex]\left.\begin{array}{l}h=\cdot1\\k=0\end{array}\right\}\dfrac{d}{N}=\cdot033{,}024\end{array}\right\},\quad\text{hence }\left.\begin{array}{l}h=\cdot06969\\k=0\end{array}\right\}\dfrac{d}{N}=\cdot038{,}334,$$

$$\left.\begin{array}{l}\left.\begin{array}{l}h=0\\k=\cdot1\end{array}\right\}\dfrac{d}{N}=\cdot033{,}024\\[1ex]\left.\begin{array}{l}h=\cdot1\\k=\cdot1\end{array}\right\}\dfrac{d}{N}=\cdot020{,}318\end{array}\right\},\quad\text{hence }\left.\begin{array}{l}h=\cdot06969\\k=\cdot1\end{array}\right\}\dfrac{d}{N}=\cdot024{,}169.$$

From the last column on right we deduce:

$$h = \cdot06969, \qquad k = \cdot04644, \qquad \frac{d}{N} = \cdot031{,}756.$$

For the given values of h and k we have accordingly:

$$r = 1\cdot00, \qquad r = -\cdot95,$$
$$d/N = 0, \qquad d/N = \cdot031{,}756.$$

Thus for $d/N = \cdot02778$, we have $r = -\cdot9562$.

Accordingly for the table returned to its original form, we conclude that the association between judgments of sex made by two independent and competent observers from the femur is measured by a tetrachoric correlation of $+\cdot956$.

Illustration II. The following data are for the French long bones in the male, maximum lengths:

Femur: Mean = 452·28 mm. *Humerus*: Mean = 330·10 mm.

Standard deviation = 23·72 mm. Standard deviation = 15·38 mm.

Correlation of femur and humerus = ·8421.

(*a*) Find the percentage of cases in which a humerus of under 300 mm. will be combined in the same individual with a femur of over 480 mm.

Here $h = 27\cdot72/23\cdot72 = 1\cdot16863$; $k = -30\cdot10/15\cdot38 = -1\cdot95709$.

Since h is positive, k negative and r positive, we must replace our system by

$$h = 1\cdot16863, \quad k = 1\cdot95709, \quad \text{and} \quad r = -\cdot8421.$$

Our tables for $r = -\cdot80$ and $-\cdot85$ show that for the given values of h and k the required frequency would probably be less than 1 in 50,000,000. We may conclude therefore that no such individuals would occur in the total French male population. This is a result whose order would hardly be appreciated without an examination of the present tables.

(*b*) Find the percentage of cases in which a humerus of under 320 mm. will be associated with a femur of over 460 mm.

Here $h = 7{\cdot}72/23{\cdot}72 = {\cdot}32546$; $k = -\, 10{\cdot}10/15{\cdot}38 = -\, {\cdot}65670$. Accordingly our system is

$$h = {\cdot}32546, \qquad k = {\cdot}65670, \qquad r = -\, {\cdot}8421.$$

Our Tables give for $r = -\, {\cdot}80$

$$\left.\begin{matrix} h = {\cdot}3 \\ k = {\cdot}6 \end{matrix}\right\} d/N = {\cdot}009{,}0146; \qquad \left.\begin{matrix} h = {\cdot}3 \\ k = {\cdot}7 \end{matrix}\right\} d/N = {\cdot}006{,}2334.$$

Thus, for $h = {\cdot}3$, $k = {\cdot}65670$, $d/N = {\cdot}007{,}4377$.

Again $$\left.\begin{matrix} h = {\cdot}4 \\ k = {\cdot}6 \end{matrix}\right\} d/N = {\cdot}006{,}3352; \qquad \left.\begin{matrix} h = {\cdot}4 \\ k = {\cdot}7 \end{matrix}\right\} d/N = {\cdot}004{,}3066.$$

Thus for $h = {\cdot}4$, $k = {\cdot}65670$, $d/N = {\cdot}005{,}1850$. Accordingly for

$$h = {\cdot}32546, \qquad k = {\cdot}65670,$$

we find $$d/N = {\cdot}006{,}8642.$$

We must now repeat this work for $r = -\, {\cdot}85$.

$$\left.\begin{matrix} h = {\cdot}3 \\ k = {\cdot}6 \end{matrix}\right\} d/N = {\cdot}004{,}6616; \qquad \left.\begin{matrix} h = {\cdot}3 \\ k = {\cdot}7 \end{matrix}\right\} d/N = {\cdot}002{,}9477.$$

Thus for $h = {\cdot}3$, $k = {\cdot}65670$, $d/N = {\cdot}003{,}6898$,

$$\left.\begin{matrix} h = {\cdot}4 \\ k = {\cdot}6 \end{matrix}\right\} d/N = {\cdot}002{,}9950; \qquad \left.\begin{matrix} h = {\cdot}4 \\ k = {\cdot}7 \end{matrix}\right\} d/N = {\cdot}001{,}8483.$$

Thus for $h = {\cdot}4$, $k = {\cdot}65670$, $d/N = {\cdot}002{,}3448$. Accordingly for

$$h = {\cdot}32546, \qquad k = {\cdot}65670,$$

we find $$d/N = {\cdot}003{,}3474.$$

We have accordingly for the given values of h and k:

$$r = -\, {\cdot}80, \qquad\qquad r = -\, {\cdot}85,$$

$$d/N = {\cdot}006{,}8642, \qquad d/N = {\cdot}003{,}3474,$$

or, for $$r = -\, {\cdot}8421, \qquad d/N = {\cdot}003{,}9006.$$

Accordingly the occurrence of individuals with femur and humerus within the limits given is about 0·4 per cent. of the male French population.

Many other illustrations will occur to the biometric reader of cases wherein these additional tables are likely to be of service. I have to record my very hearty thanks to Dr W. F. Sheppard for the loan of his manuscript tables of the probability integral which enabled me to work to a larger number of figures than his published tables, which go to fewer figures, would have permitted.

d/N for $r = -\cdot 80$.

	$h=0$	$h=\cdot 1$	$h=\cdot 2$	$h=\cdot 3$	$h=\cdot 4$	$h=\cdot 5$	$h=\cdot 6$	$h=\cdot 7$	$h=\cdot 8$
$k=$ 0	·1024,160	·0835,592	·0669,639	·0526,767	·0406,497	·0307,541	·0227,991	·0165,531	·0117,648
·1	·0835,592	·0673,159	·0532,349	·0413,006	·0314,152	·0234,157	·0170,936	·0122,156	·0085,420
·2	·0669,639	·0532,349	·0415,199	·0317,511	·0237,937	·0174,638	·0125,482	·0088,225	·0060,673
·3	·0526,767	·0413,006	·0317,511	·0239,210	·0176,518	·0127,520	·0090,146	·0062,334	·0042,143
·4	·0406,497	·0314,152	·0237,937	·0176,518	·0128,207	·0091,123	·0063,352	·0043,066	·0028,615
·5	·0307,541	·0234,157	·0174,638	·0127,520	·0091,123	·0063,694	·0043,534	·0029,084	·0018,986
·6	·0227,991	·0170,936	·0125,482	·0090,146	·0063,352	·0043,534	·0029,242	·0019,193	·0012,306
·7	·0165,531	·0122,156	·0088,225	·0062,334	·0043,066	·0029,084	·0019,193	·0012,373	·0007,790
·8	·0117,648	·0085,420	·0060,673	·0042,143	·0028,615	·0018,986	·0012,306	·0007,790	·0004,814
·9	·0081,817	·0058,423	·0040,797	·0027,849	·0018,578	·0012,107	·0007,705	·0004,788	·0002,904
1·0	·0055,652	·0039,068	·0026,812	·0017,982	·0011,782	·0007,539	·0004,710	·0002,873	·0001,710
1·1	·0037,010	·0025,535	·0017,217	·0011,342	·0007,297	·0004,583	·0002,810	·0001,682	·0000,982
1·2	·0024,056	·0016,307	·0010,799	·0006,985	·0004,412	·0002,720	·0001,637	·0000,961	·0000,550
1·3	·0015,278	·0010,172	·0006,615	·0004,200	·0002,604	·0001,575	·0000,930	·0000,535	·0000,301
1·4	·0009,477	·0006,196	·0003,956	·0002,465	·0001,499	·0000,890	·0000,515	·0000,291	·0000,160
1·5	·0005,741	·0003,685	·0002,309	·0001,412	·0000,842	·0000,490	·0000,278	·0000,154	·0000,083
1·6	·0003,395	·0002,139	·0001,315	·0000,789	·0000,462	·0000,264	·0000,147	·0000,080	·0000,042
1·7	·0001,960	·0001,211	·0000,731	·0000,430	·0000,247	·0000,138	·0000,075	·0000,040	·0000,021
1·8	·0001,104	·0000,669	·0000,396	·0000,229	·0000,129	·0000,071	·0000,038	·0000,020	·0000,010
1·9	0000,606	·0000,361	·0000,209	·0000,118	·0000,065	·0000,035	·0000,018	·0000,009	·0000,005
2·0	·0000,3252	·0000,1897	·0000,1079	·0000,0599	·0000,0324	·0000,0171	·0000,0088	·0000,0044	·0000,0021
2·1	·0000,1700	·0000,0972	·0000,0542	·0000,0295	·0000,0156	·0000,0081	·0000,0041	·0000,0020	·0000,0010
2·2	·0000,0867	·0000,0486	·0000,0266	·0000,0142	·0000,0074	·0000,0037	·0000,0018	·0000,0009	·0000,0004
2·3	·0000,0431	·0000,0237	·0000,0127	·0000,0066	·0000,0034	·0000,0017	·0000,0008	·0000,0004	·0000,0002
2·4	·0000,0209	·0000,0112	·0000,0059	·0000,0030	·0000,0015	·0000,0007	·0000,0003	·0000,0002	·0000,0001
2·5	·0000,0099	·0000,0052	·0000,0027	·0000,0013	·0000,0007	·0000,0003	·0000,0001	·0000,0001	·0000,0000
2·6	·0000,0045	·0000,0023	·0000,0012	·0000,0006	·0000,0003	·0000,0001	·0000,0001	·0000,0000	

	$h=\cdot 9$	$h=1\cdot 0$	$h=1\cdot 1$	$h=1\cdot 2$	$h=1\cdot 3$	$h=1\cdot 4$	$h=1\cdot 5$	$h=1\cdot 6$	$h=1\cdot 7$
$k=$ 0	·0081,817	·0055,652	·0037,010	·0024,056	·0015,278	·0009,477	·0005,741	0003,395	·0001,960
·1	·0058,423	·0039,068	·0025,535	·0016,307	·0010,172	·0006,196	·0003,685	·0002,139	·0001,211
·2	·0040,797	·0026,812	·0017,217	·0010,799	·0006,615	·0003,956	·0002,309	·0001,315	·0000,731
·3	·0027,849	·0017,982	·0011,342	·0006,985	·0004,200	·0002,465	·0001,412	·0000,789	·0000,430
·4	·0018,578	·0011,782	·0007,297	·0004,412	·0002,604	·0001,499	·0000,842	·0000,462	·0000,247
·5	·0012,107	·0007,539	·0004,583	·0002,720	·0001,575	·0000,890	·0000,490	·0000,264	·0000,138
·6	·0007,705	·0004,710	·0002,810	·0001,637	·0000,930	·0000,515	·0000,278	·0000,147	·0000,075
·7	·0004,788	·0002,873	·0001,682	·0000,961	·0000,535	·0000,291	·0000,154	·0000,080	·0000,040
·8	·0002,904	·0001,710	·0000,982	·0000,550	·0000,301	·0000,160	·0000,083	·0000,042	·0000,021
·9	·0001,719	·0000,993	·0000,559	·0000,307	·0000,165	·0000,086	·0000,044	·0000,022	·0000,011
1·0	·0000,993	·0000,562	·0000,311	·0000,167	·0000,088	·0000,045	·0000,023	·0000,011	·0000,005
1·1	·0000,559	·0000,311	·0000,168	·0000,089	·0000,046	·0000,023	·0000,011	·0000,005	·0000,002
1·2	·0000,307	·0000,167	·0000,089	·0000,046	·0000,023	·0000,011	·0000,005	·0000,003	·0000,001
1·3	·0000,165	·0000,088	·0000,046	·0000,023	·0000,012	·0000,006	·0000,003	·0000,001	·0000,0005
1·4	·0000,086	·0000,045	·0000,023	·0000,011	·0000,006	·0000,003	·0000,001	·0000,0005	·0000,0002
1·5	·0000,044	·0000,023	·0000,011	·0000,005	·0000,003	·0000,001	·0000,0005	·0000,0002	·0000,0001
1·6	·0000,022	·0000,011	·0000,005	·0000,003	·0000,001	·0000,0005	·0000,0002	·0000,0001	·0000,0000
1·7	·0000,011	·0000,005	·0000,002	·0000,001	·0000,0005	·0000,0002	·0000,0001	·0000,0000	
1·8	·0000,005	·0000,002	·0000,001	·0000,0005	·0000,0002	·0000,0001	·0000,0000		
1·9	·0000,002	·0000,001	·0000,0005	·0000,0002	·0000,0001	·0000,0000			
2·0	·0000,0010	·0000,0005	·0000,0002	·0000,0001	·0000,0000				
2·1	·0000,0004	·0000,0002	·0000,0001	·0000,0000					
2·2	·0000,0002	·0000,0001	·0000,0000						
2·3	·0000,0001	·0000,0000							
2·4	·0000,0000								

d/N for r = − ·80.

	h=1·8	*h*=1·9	*h*=2·0	*h*=2·1	*h*=2·2	*h*=2·3	*h*=2·4	*h*=2·5	*h*=2·6
k= 0	·0001,104	·0000,606	·0000,3252	·0000,1700	·0000,0867	·0000,0431	.0000,0209	·0000,0099	·0000,0045
·1	·0000,669	·0000,361	·0000,1897	·0000,0972	·0000,0486	·0000,0237	·0000,0112	·0000,0052	·0000,0023
·2	·0000,396	·0000,209	·0000,1079	·0000,0542	·0000,0266	·0000,0127	·0000,0059	·0000,0027	·0000,0012
·3	·0000,229	·0000,118	·0000,0599	·0000,0295	·0000,0142	·0000,0066	·0000,0030	·0000,0013	·0000,0006
·4	·0000,129	·0000,065	·0000,0324	·0000,0156	·0000,0074	·0000,0034	·0000,0015	·0000,0007	·0000,0003
·5	·0000,071	·0000,035	·0000,0171	·0000,0081	·0000,0037	·0000,0017	·0000,0007	·0000,0003	·0000,0001
·6	·0000,038	·0000,018	·0000,0088	·0000,0041	·0000,0018	·0000,0008	·0000,0003	·0000,0001	·0000,0001
·7	·0000,020	·0000,009	·0000,0044	·0000,0020	·0000,0009	·0000,0004	·0000,0002	·0000,0001	·0000,0000
·8	·0000,010	·0000,005	·0000,0021	·0000,0010	·0000,0004	·0000,0002	·0000,0001	·0000,0000	
·9	·0000,005	·0000,002	·0000,0010	·0000,0004	·0000,0002	·0000,0001	·0000,0000		
1·0	·0000,002	·0000,001	·0000,0005	·0000,0002	·0000,0001	·0000,0000			
1·1	·0000,001	·0000,0005	·0000,0002	·0000,0001	·0000,0000				
1·2	·0000,0005	·0000,0002	·0000,0001	·0000,0000					
1·3	·0000,0002	·0000,0001	·0000,0000						
1·4	·0000,0001	·0000,0000							
1·5	·0000,0000								

d/N for r = − ·85.

	h=0	*h*=·1	*h*=·2	*h*=·3	*h*=·4	*h*=·5	*h*=·6	*h*=·7	*h*=·8
k= 0	·0883,010	·0696,651	·0537,127	·0404,314	·0296,851	·0212,402	·0147,991	·0100,334	·0066,146
·1	·0696,651	·0539,919	·0408,547	·0301,542	·0216,905	·0151,935	·0103,559	·0068,639	·0044,211
·2	·0537,127	·0408,547	·0303,123	·0219,192	·0154,352	·0105,767	·0070,478	·0045,639	·0028,705
·3	·0404,314	·0301,542	·0219,192	·0155,166	·0106,889	·0071,604	·0046,616	·0029,477	·0018,094
·4	·0296,851	·0216,905	·0154,352	·0106,889	·0071,984	·0047,113	·0029,950	·0018,483	·0011,068
·5	·0212,402	·0151,935	·0105,767	·0071,604	·0047,113	·0030,109	·0018,680	·0011,246	·0006,566
·6	·0147,991	·0103,559	·0070,478	·0046,616	·0029,950	·0018,680	·0011,306	·0006,637	·0003,777
·7	·0100,334	·0068,639	·0045,639	·0029,477	·0018,483	·0011,246	·0006,637	·0003,797	·0002,106
·8	·0066,146	·0044,211	·0028,705	·0018,094	·0011,068	·0006,566	·0003,777	·0002,106	·0001,137
·9	·0042,377	·0027,658	·0017,526	·0010,778	·0006,428	·0003,717	·0002,083	·0001,131	·0000,595
1·0	·0026,368	·0016,797	·0010,383	·0006,226	·0003,620	·0002,039	·0001,113	·0000,589	·0000,301
1·1	·0015,927	·0009,897	·0005,966	·0003,487	·0001,975	·0001,084	·0000,576	·0000,297	·0000,148
1·2	·0009,335	·0005,656	·0003,323	·0001,893	·0001,044	·0000,558	·0000,289	·0000,145	·0000,070
1·3	·0005,306	·0003,134	·0001,794	·0000,995	·0000,535	·0000,278	·0000,140	·0000,068	·0000,032
1·4	·0002,924	·0001,682	·0000,938	·0000,507	·0000,265	·0000,134	·0000,066	·0000,031	·0000,014
1·5	·0001,561	·0000,875	·0000,475	·0000,250	·0000,127	·0000,063	·0000,030	·0000,014	·0000,006
1·6	·0000,808	·0000,441	·0000,233	·0000,119	·0000,059	0000,028	·0000,013	·0000,006	·0000,003
1·7	·0000,405	·0000,215	·0000,111	·0000,055	·0000,027	·0000,012	·0000,006	·0000,002	·0000,001
1·8	·0000,196	·0000,102	·0000,051	·0000,025	·0000,012	·0000,005	·0000,002	·0000,001	·0000,0004
1·9	·0000,092	·0000,046	·0000,023	·0000,011	·0000,005	·0000,002	·0000,001	·0000,0004	·0000,0001
2·0	·0000,0419	·0000,0205	·0000,0097	·0000,0045	·0000,0020	·0000,0009	·0000,0004	·0000,0001	·0000,0001
2·1	·0000,0184	·0000,0088	·0000,0040	·0000,0018	·0000,0008	·0000,0003	·0000,0001	·0000,0000	·0000,0000
2·2	·0000,0079	·0000,0036	·0000,0017	·0000,0007	·0000,0003	·0000,0001	·0000,0000		
2·3	·0000,0032	·0000,0015	·0000,0006	·0000,0003	·0000,0001	·0000,0000			
2·4	·0000,0013	·0000,0006	·0000,0002	·0000,0001	·0000,0000				
2·5	·0000,0005	·0000,0002	·0000,0001	·0000,0000					
2·6	·0000,0002	·0000,0001	·0000,0000						

d/N for $r = -\cdot 85$.

	$h = \cdot 9$	$h = 1\cdot 0$	$h = 1\cdot 1$	$h = 1\cdot 2$	$h = 1\cdot 3$	$h = 1\cdot 4$	$h = 1\cdot 5$	$h = 1\cdot 6$	$h = 1\cdot 7$
$k =$ 0	·0042,377	·0026,368	·0015,927	·0009,335	·0005,306	·0002,924	·0001,561	·0000,808	·0000,405
·1	·0027,658	·0016,797	·0009,897	·0005,656	·0003,134	·0001,682	·0000,875	·0000,441	·0000,215
·2	·0017,526	·0010,383	·0005,966	·0003,323	·0001,794	·0000,938	·0000,475	·0000,233	·0000,111
·3	·0010,778	·0006,226	·0003,487	·0001,893	·0000,995	·0000,507	·0000,250	·0000,119	·0000,055
·4	·0006,428	·0003,620	·0001,975	·0001,044	·0000,535	·0000,265	·0000,127	·0000,059	·0000,027
·5	·0003,717	·0002,039	·0001,084	·0000,558	·0000,278	·0000,134	·0000,063	·0000,028	·0000,012
·6	·0002,083	·0001,113	·0000,576	·0000,289	·0000,140	·0000,066	·0000,030	·0000,013	·0000,006
·7	·0001,131	·0000,589	·0000,297	·0000,145	·0000,068	·0000,031	·0000,014	·0000,006	·0000,002
·8	·0000,595	·0000,301	·0000,148	·0000,070	·0000,032	·0000,014	·0000,006	·0000,003	·0000,001
·9	·0000,303	·0000,149	·0000,071	·0000,033	·0000,015	·0000,006	·0000,003	·0000,001	·0000,0004
1·0	·0000,149	·0000,072	·0000,033	·0000,015	·0000,006	·0000,003	·0000,001	·0000,0004	·0000,0002
1·1	·0000,071	·0000,033	·0000,015	·0000,007	·0000,003	·0000,001	·0000,0004	·0000,0002	·0000,0001
1·2	·0000,033	·0000,015	·0000,007	·0000,003	·0000,001	·0000,0005	·0000,0002	·0000,0001	·0000,0000
1·3	·0000,015	·0000,006	·0000,003	·0000,001	·0000,0005	·0000,0002	·0000,0001	·0000,0000	
1·4	·0000,006	·0000,003	·0000,001	·0000,0005	·0000,0002	·0000,0001	·0000,0000		
1·5	·0000,003	·0000,001	·0000,0004	·0000,0002	·0000,0001	·0000,0000			
1·6	·0000,001	·0000,0004	·0000,0002	·0000,0001	·0000,0000				
1·7	·0000,0004	·0000,0002	·0000,0001	·0000,0000					
1·8	·0000,0002	·0000,0001	·0000,0000						
1·9	·0000,0001	·0000,0000							
2·0	·0000,0000								

	$h = 1\cdot 8$	$h = 1\cdot 9$	$h = 2\cdot 0$	$h = 2\cdot 1$	$h = 2\cdot 2$	$h = 2\cdot 3$	$h = 2\cdot 4$	$h = 2\cdot 5$	$h = 2\cdot 6$
$k =$ 0	·0000,196	·0000,092	·0000,0419	·0000,0184	·0000,0079	·0000,0032	·0000,0013	·0000,0005	·0000,0002
·1	·0000,102	·0000,046	·0000,0205	·0000,0088	·0000,0036	·0000,0015	·0000,0006	·0000,0002	·0000,0001
·2	·0000,051	·0000,023	·0000,0097	·0000,0040	·0000,0017	·0000,0006	·0000,0002	·0000,0001	·0000,0000
·3	·0000,025	·0000,011	·0000,0045	·0000,0018	·0000,0007	·0000,0003	·0000,0001	·0000,0000	
·4	·0000,012	·0000,005	·0000,0020	·0000,0008	·0000,0003	·0000,0001	·0000,0000		
·5	·0000,005	·0000,002	·0000,0009	·0000,0003	·0000,0001	·0000,0000			
·6	·0000,002	·0000,001	·0000,0004	·0000,0001	·0000,0000				
·7	·0000,001	·0000,0004	·0000,0001	·0000,0000					
·8	·0000,0004	·0000,0001	·0000,0001						
·9	·0000,0002	·0000,0001	·0000,0000						
1·0	·0000,0001	·0000,0000							
1·1	·0000,0000								

d/N for $r = -\cdot 90$.

	$h = 0$	$h = \cdot 1$	$h = \cdot 2$	$h = \cdot 3$	$h = \cdot 4$	$h = \cdot 5$	$h = \cdot 6$	$h = \cdot 7$	$h = \cdot 8$
$k =$ 0	·0717,832	·0535,024	·0385,698	·0268,454	·0180,108	·0116,302	·0072,185	·0043,011	·0024,576
·1	·0535,024	·0387,680	·0271,230	·0182,917	·0118,733	·0074,080	·0044,371	·0025,486	·0014,024
·2	·0385,698	·0271,230	·0183,863	·0119,967	·0075,240	·0045,302	·0026,157	·0014,469	·0007,661
·3	·0268,454	·0182,917	·0119,967	·0075,631	·0045,775	·0026,569	·0014,774	·0007,863	·0004,003
·4	·0180,108	·0118,733	·0075,240	·0045,775	·0026,707	·0014,928	·0007,987	·0004,087	·0001,999
·5	·0116,302	·0074,080	·0045,302	·0026,569	·0014,928	·0008,029	·0004,130	·0002,030	·0000,953
·6	·0072,185	·0044,371	·0026,157	·0014,774	·0007,987	·0004,130	·0002,041	·0000,963	·0000,434
·7	·0043,011	·0025,486	·0014,469	·0007,863	·0004,087	·0002,030	·0000,963	·0000,436	·0000,189
·8	·0024,576	·0014,024	·0007,661	·0004,003	·0001,999	·0000,953	·0000,434	·0000,189	·0000,078
·9	·0013,452	·0007,386	·0003,879	·0001,947	·0000,934	·0000,427	·0000,187	·0000,078	·0000,031
1·0	·0007,048	·0003,721	·0001,878	·0000,905	·0000,416	·0000,183	·0000,077	·0000,031	·0000,012
1·1	·0003,532	·0001,791	·0000,868	·0000,401	·0000,177	·0000,075	·0000,030	·0000,011	·0000,004
1·2	·0001,692	·0000,824	·0000,383	·0000,170	·0000,072	·0000,029	·0000,011	·0000,004	·0000,001
1·3	·0000,774	·0000,362	·0000,161	·0000,069	·0000,028	·0000,011	·0000,004	·0000,001	·0000,0005
1·4	·0000,338	·0000,151	·0000,065	·0000,026	·0000,010	·0000,004	·0000,001	·0000,0005	·0000,0001
1·5	·0000,141	·0000,060	·0000,025	·0000,010	·0000,004	·0000,001	·0000,0004	·0000,0001	·0000,0000
1·6	·0000,056	·0000,023	·0000,009	·0000,003	·0000,001	·0000,0004	·0000,0001	·0000,0000	
1·7	·0000,021	·0000,008	·0000,003	·0000,001	·0000,0004	·0000,0001	·0000,0000		
1·8	·0000,008	·0000,003	·0000,001	·0000,0004	·0000,0001	·0000,0000			
1·9	·0000,003	·0000,001	·0000,0003	·0000,0001	·0000,0000				
2·0	·0000,0009	·0000,0003	·0000,0001	·0000,0000					
2·1	·0000,0003	·0000,0001	·0000,0000						
2·2	·0000,0001	·0000,0000							
2·3	·0000,0000								

d/N for r = − ·90.

	h=·9	*h*=1·0	*h*=1·1	*h*=1·2	*h*=1·3	*h*=1·4	*h*=1·5	*h*=1·6	*h*=1·7
k= 0	·0013,452	·0007,048	·0003,532	·0001,692	·0000,774	·0000,338	·0000,141	·0000,056	·0000,021
·1	·0007,386	·0003,721	·0001,791	·0000,824	·0000,362	·0000,151	·0000,060	·0000,023	·0000,008
·2	·0003,879	·0001,878	·0000,868	·0000,383	·0000,161	·0000,065	·0000,025	·0000,009	·0000,003
·3	·0001·947	·0000,905	·0000,401	·0000,170	·0000,069	·0000,026	·0000,010	·0000,003	·0000,001
·4	·0000,934	·0000,416	·0000,177	·0000,072	·0000,028	·0000,010	·0000,004	·0000,001	·0000,0004
·5	·0000,427	·0000,183	·0000,075	·0000,029	·0000,011	·0000,004	·0000,001	·0000,0004	·0000,0001
·6	·0000,187	·0000,077	·0000,030	·0000,011	·0000,004	·0000,001	·0000,0004	·0000,0001	·0000,0000
·7	·0000,078	·0000,031	·0000,011	·0000,004	·0000,001	·0000,0005	·0000,0001	·0000,0000	
·8	·0000,031	·0000,012	·0000,004	·0000,001	·0000,0005	·0000,0001	·0000,0000		
·9	·0000,012	·0000,004	·0000,001	·0000,0005	·0000,0002	·0000,0000			
1·0	·0000,004	·0000,001	·0000,0005	·0000,0002	·0000,0000				
1·1	·0000,001	·0000,0005	·0000,0002	·0000,0000					
1·2	·0000,0005	·0000,0002	·0000,0000						
1·3	·0000,0002	·0000,0000							
1·4	·0000,0000								

	h=1·8	*h*=1·9	*h*=2·0	*h*=2·1	*h*=2·2	*h*=2·3
k= 0	·0000,008	·0000,003	·0000,0009	·0000,0003	·0000,0001	·0000,0000
·1	·0000,003	·0000,001	·0000,0003	·0000,0001	·0000,0000	
·2	·0000,001	·0000,0003	·0000,0001	·0000,0000		
·3	·0000,0004	·0000,0001	·0000,0000			
·4	·0000,0001	·0000,0000				
·5	·0000,0000					

d/N for r = − ·95.

	h=0	*h*=·1	*h*=·2	*h*=·3	*h*=·4	*h*=·5	*h*=·6	*h*=·7	*h*=·8
k= 0	·0505,416	·0330,242	·0202,154	·0115,376	·0061,136	·0029,965	·0013,543	·0005,629	·0002,146
·1	·0330,242	·0203,181	·0116,555	·0062,076	·0030,582	·0013,893	·0005,804	·0002,224	·0000,781
·2	·0202,154	·0116,555	·0062,393	·0030,895	·0014,107	·0005,923	·0002,282	·0000,805	·0000,260
·3	·0115,376	·0062,076	·0030,895	·0014,179	·0005,984	·0002,317	·0000,822	·0000,266	·0000,079
·4	·0061,136	·0030,582	·0014,107	·0005,984	·0002,329	·0000,830	·0000,270	·0000,080	·0000,022
·5	·0029,965	·0013,893	·0005,923	·0002,317	·0000,830	·0000,272	·0000,081	·0000,022	·0000,005
·6	·0013,543	·0005,804	·0002,282	·0000,822	·0000,270	·0000,081	·0000,022	·0000,006	·0000,001
·7	·0005,629	·0002,224	·0000,805	·0000,266	·0000,080	·0000,022	·0000,006	·0000,001	·0000,0003
·8	·0002,146	·0000,781	·0000,260	·0000,079	·0000,022	·0000,005	·0000,001	·0000,0003	·0000,0000
·9	·0000,749	·0000,250	·0000,076	·0000,021	·0000,005	·0000,001	·0000,0003	·0000,0000	
1·0	·0000,239	·0000,073	·0000,021	·0000,005	·0000,001	·0000,0003	·0000,0000		
1·1	·0000,070	·0000,020	·0000,005	·0000,001	·0000,0002	·0000,0000			
1·2	·0000,019	·0000,005	·0000,001	·0000,0002	·0000,0000				
1·3	·0000,004	·0000,001	·0000,0002	·0000,0000					
1·4	·0000,001	·0000,0002	·0000,0000						
1·5	·0000,0002	·0000,0000							
1·6	·0000,0000								

	h=·9	*h*=1·0	*h*=1·1	*h*=1·2	*h*=1·3	*h*=1·4	*h*=1·5
k= 0	·0000,749	·0000,239	·0000,070	·0000,019	·0000,004	·0000,001	·0000,0002
·1	·0000,250	·0000,073	·0000,020	·0000,005	·0000,001	·0000,0002	·0000,0000
·2	·0000,076	·0000,021	·0000,005	·0000,001	·0000,0002	·0000,0000	
·3	·0000,021	·0000,005	·0000,001	·0000,0002	·0000,0000		
·4	·0000,005	·0000,001	·0000,0002	·0000,0000			
·5	·0000,001	·0000,0003	·0000,0000				
·6	·0000,0003	·0000,0000					
·7	·0000,0000						

ON THE PROBABLE ERROR OF BISERIAL η.

By KARL PEARSON, F.R.S.

(1) In a paper in *Biometrika*, Vol. vii, pp. 248–257, entitled "On a New Method of determining Correlation where one Variable is given by alternative and the other by multiple Categories," I gave in 1910 the process of determining what is now generally termed "biserial η."

Let x be the alternative, y the multiple variate, $\bar{x}_y$ the distance from the division between the alternative categories of the mean of the array of x's corresponding to a given value of y, ${}_y\sigma_x$ its standard deviation and n_y its frequency. Let $\bar{x}$, σ_x and N be the corresponding quantities for the marginal total. Then, if

$$\gamma = \bar{x}/\sigma_x, \quad \gamma_y = \bar{x}_y/{}_y\sigma_x,$$

I showed that

$$\eta = \left[\frac{\dfrac{1}{N} S\left\{n_y\left(\dfrac{\bar{x}_y}{{}_y\sigma_x}\right)^2\right\} - \left(\dfrac{\bar{x}}{\sigma_x}\right)^2}{1 + \dfrac{1}{N} S\left\{n_y\left(\dfrac{\bar{x}_y}{{}_y\sigma_x}\right)^2\right\}}\right]^{\frac{1}{2}} \text{..........................(i)},$$

and explained that η might be found from the usual tables of the probability integral. I did not publish the probable error of this method of determining "biserial η" at the time, and it has remained one of the few outstanding cases where probable errors were still wanting. I now add the determination of the probable error, and owing to the kindness of my colleagues, Miss Ethel M. Elderton and Miss B. M. Cave, am able to give a table for its fairly easy computation.

(2) I take as my biserial table the following:

n_{11}	n_{21}	...	n_{1s}	......	ν_1
n_{12}	n_{22}	...	n_{2s}	......	ν_2
n_1	n_2	...	n_s	......	N

Here n_s stands for n_y, and the y variate is the horizontal or multiple category variate. We shall write

$$\kappa^2 = \frac{1}{N} S(n_s\gamma_s^2), \quad \gamma^2 = (\bar{x}/\sigma_x)^2 \text{..........................(ii)},$$

and thus

$$\eta^2 = \frac{\kappa^2 - \gamma^2}{1 + \kappa^2} = 1 - \frac{1 + \gamma^2}{1 + \kappa^2}.$$

Accordingly to a first approximation

$$\eta\delta\eta = \frac{1+\gamma^2}{(1+\kappa^2)^2}\kappa\delta\kappa - \frac{\gamma\delta\gamma}{1+\kappa^2},$$

where η, κ and γ are now mean values and $\delta\eta$, $\delta\kappa$ and $\delta\gamma$ are variations from those means. Squaring, summing for all values, and dividing by the number of cases:

$$\eta^2\sigma_\eta{}^2 = \frac{(1-\eta^2)^4}{(1+\gamma^2)^2}\kappa^2\sigma_\kappa{}^2 + \frac{\gamma^2}{(1+\kappa^2)^2}\sigma_\gamma{}^2 - \frac{2\gamma\kappa\,(1+\gamma^2)}{(1+\kappa^2)^3}\,\text{Mean}\,(\delta\kappa\delta\gamma) \quad \text{...(iii)}.$$

Hence to find σ_η we require to evaluate σ_κ, σ_γ and Mean $(\delta\gamma\delta\kappa)$. We shall now proceed to the consideration of these quantities.

We have $$\nu_1/N = \frac{1}{\sqrt{2\pi}}\int_\gamma^\infty e^{-\frac{1}{2}x^2}dx.$$

Hence $$\delta\nu_1/N = -\,\delta\gamma\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$$
$$= -\,\delta\gamma z,$$

where z is the ordinate of the frequency curve of the marginal total at the boundary of the alternative categories.

Thus $$\sigma^2_{\nu_1}/N^2 = \sigma_\gamma{}^2 z^2.$$

But if the sample has been taken from an indefinitely large population

$$\sigma^2_{\nu_1} = \nu_1\,(1-\nu_1/N).$$

Accordingly $$\sigma_\gamma{}^2 = \frac{\nu_1\nu_2}{N^3z^2} \quad \text{..(iv)}.$$

Again $$\kappa\delta\kappa = S\,(\kappa_s\delta\kappa_s),$$

where $$\kappa_s{}^2 = \frac{n_s}{N}\gamma_s{}^2,$$

$$\kappa^2\sigma_\kappa{}^2 = S\,(\kappa_s{}^2\sigma^2_{\kappa_s}) + 2S\,\{\kappa_s\kappa_{s'}\,\text{Mean}\,(\delta\kappa_s\delta\kappa_{s'})\} \quad \text{..............(v)}.$$

We require therefore to find $\sigma^2_{\kappa_s}$ and Mean $(\delta\kappa_s\delta\kappa_s{}')$. Now

$$2\kappa_s\delta\kappa_s = \frac{\delta n_s}{N}\gamma_s{}^2 + 2\frac{n_s}{N}\gamma_s\delta\gamma_s \quad \text{............................(vi)}.$$

But $$n_{1s} = n_s\frac{1}{\sqrt{2\pi}}\int_{\gamma_s}^\infty e^{-\frac{1}{2}x^2}dx,$$

$$\frac{\delta n_{1s}}{n_s} - \frac{n_{1s}\delta n_s}{n_s{}^2} = -\,\delta\gamma_s z_s \quad \text{..............................(vii)}.$$

And accordingly:

$$\frac{\sigma^2_{n_{1s}}}{n_s{}^2} - \frac{2n_{1s}}{n_s{}^3}\,\text{Mean}\,(\delta n_{1s}\delta n_s) + \frac{n^2_{1s}}{n_s{}^4}\sigma^2_{n_s} = z_s{}^2\sigma^2_{\gamma_s}.$$

But
$$\sigma^2_{n_{1s}} = n_{1s}\left(1 - \frac{n_{1s}}{N}\right), \quad \sigma^2_{n_s} = n_s\left(1 - \frac{n_s}{N}\right),$$

$$\begin{aligned}\text{Mean}\,(\delta n_{1s}\delta n_s) &= \text{Mean}\,(\delta n_{1s}\delta n_{2s}) + \text{Mean}\,(\delta n_{1s})^2\\ &= -\frac{n_{1s}n_{2s}}{N} + n_{1s}\left(1 - \frac{n_{1s}}{N}\right)\\ &= n_{1s}\left(1 - \frac{n_s}{N}\right).\end{aligned}$$

Thus
$$\sigma^2_{\gamma_s} = \frac{n_{1s}n_{2s}}{n_s{}^3 z_s{}^2} \quad \text{......(viii)}.$$

It will be seen that (viii) is of the same form as (iv), although the value of n_s is variable, while that of N is constant.

Now multiply (vii) by δn_s and sum and we have

$$\frac{n_{1s}}{n_s}\left(1 - \frac{n_s}{N}\right) - \frac{n_{1s}}{n_s{}^2}n_s\left(1 - \frac{n_s}{N}\right) = -z_s\,\text{Mean}\,(\delta\gamma_s\delta n_s).$$

Thus
$$\text{Mean}\,(\delta\gamma_s\delta n_s) = 0 \quad \text{......(ix)}.$$

Returning to (vi), squaring and summing, we find

$$4\kappa_s{}^2\sigma^2_{\kappa_s} = \frac{1}{N^2}\left\{\gamma_s{}^4 n_s\left(1 - \frac{n_s}{N}\right) + 4n_s{}^2\gamma_s{}^2\,\frac{n_{1s}n_{2s}}{n_s{}^3 z_s{}^2}\right\},$$

or,
$$\kappa_s{}^2\sigma^2_{\kappa_s} = \frac{1}{N^2}\left\{\tfrac{1}{4}\gamma_s{}^4 n_s\left(1 - \frac{n_s}{N}\right) + \frac{n_{1s}n_{2s}}{n_s}\left(\frac{\gamma_s}{z_s}\right)^2\right\} \quad \text{......(x)}.$$

Again returning to (vi), multiplying it by the corresponding values of $2\kappa_{s'}\delta\kappa_{s'}$ and summing, we reach

$$\begin{aligned}4\kappa_s\kappa_{s'}\,\text{Mean}\,(\delta\kappa_s\delta\kappa_{s'}) = &-\frac{n_s n_{s'}}{N^3}\gamma_s{}^2\gamma_{s'}{}^2 + 4\frac{n_s n_{s'}}{N^2}\gamma_s\gamma_{s'}\,\text{Mean}\,(\delta\gamma_s\delta\gamma_{s'})\\ &+ 2\frac{n_s}{N^2}\gamma_s\gamma_{s'}{}^2\,\text{Mean}\,(\delta n_{s'}\delta\gamma_s) + 2\frac{n_{s'}}{N^2}\gamma_{s'}\gamma_s{}^2\,\text{Mean}\,(\delta n_s\delta\gamma_{s'}).\end{aligned}$$

But
$$\text{Mean}\,(\delta\gamma_s\delta\gamma_{s'}) = \frac{1}{z_s z_{s'}}\left(-\frac{n_{1s}n_{1s'}}{n_s n_{s'}N} - \frac{n_{1s}n_{1s'}}{n_s{}^2 n_{s'}{}^2}\,\frac{n_s n_{s'}}{N} + \frac{n_{1s'}}{n_s n_{s'}{}^2}\,\frac{n_{1s}n_{s'}}{N} + \frac{n_{1s}}{n_s{}^2 n_{s'}}\,\frac{n_{1s'}n_s}{N}\right) = 0.$$

Further multiplying (vii) by $\delta n_{s'}$ we find

$$\text{Mean}\,(\delta n_{s'}\delta\gamma_s) = -\frac{1}{z_s}\left(-\frac{n_{1s}n_{s'}}{n_s N} + \frac{n_{1s}n_s n_{s'}}{n_s{}^2 N}\right) = 0, \text{ also.}$$

Accordingly
$$\kappa_s\kappa_{s'}\,\text{Mean}\,(\delta\kappa_s\delta\kappa_{s'}) = -\frac{1}{4}\frac{n_s n_{s'}}{N^3}\gamma_s{}^2\gamma_{s'}{}^2 \quad \text{......(xi)}.$$

Returning to (v) we can now evaluate it by aid of (x) and (xi); thus we have

$$\kappa^2\sigma_\kappa{}^2 = \frac{1}{N^2}\left\{\tfrac{1}{4}S\left\{\gamma_s{}^4 n_s\left(1 - \frac{n_s}{N}\right)\right\} + S\left\{\frac{n_{1s}n_{2s}}{n_s}\left(\frac{\gamma_s}{z_s}\right)^2\right\} - \tfrac{1}{2}S\,(n_s n_{s'}\gamma_s{}^2\gamma_{s'}{}^2)\right\} \quad \text{(xii)}.$$

It now remains to find Mean $(\delta\kappa\delta\gamma)$. We have:

$$\kappa \text{ Mean } (\delta\kappa\delta\gamma) = S\left(\kappa_s \left\{\text{Mean } (\delta\kappa_s\delta\gamma)\right\}\right)$$

$$= \tfrac{1}{2} S \left\{\frac{\gamma_s^2}{N} \text{ Mean } (\delta\gamma\delta n_s)\right\} + S\left\{\frac{n_s\gamma_s}{N} \text{ Mean } (\delta\gamma_s\delta\gamma)\right\}.$$

We must now find Mean $(\delta\gamma\delta n_s)$ and Mean $(\delta\gamma_s\delta\gamma)$.

$$\text{Mean } (\delta\gamma\delta n_s) = -\frac{\text{Mean } (\delta\nu_1\delta n_s)}{Nz}$$

$$= -\frac{1}{Nz}\left(n_{1s} - \frac{\nu_1 n_s}{N}\right).$$

$$\text{Mean } (\delta\gamma_s\delta\gamma) = \text{Mean } \left\{\frac{\delta\nu_1}{Nz}\left(\frac{\delta n_{1s}}{n_s z_s} - \frac{n_{1s}\delta n_s}{n_s^2 z_s}\right)\right\}$$

$$= \frac{1}{Nzn_s z_s}\left\{n_{1s}\left(1 - \frac{\nu_1}{N}\right) - \frac{n_{1s}}{n_s}\left(n_{1s} - \frac{\nu_1 n_s}{N}\right)\right\}$$

$$= \frac{n_{1s}}{Nzn_s z_s}\left(1 - \frac{n_{1s}}{n_s}\right).$$

Thus

$$\kappa \text{ Mean } (\delta\kappa\delta\gamma) = -\frac{1}{N^2 z}\left\{\tfrac{1}{2} S\,(n_{1s}\gamma_s^2) - \frac{\nu_1}{2N} S\,(n_s\gamma_s^2) - S\left(\frac{n_{1s}n_{2s}}{n_s z_s}\gamma_s\right)\right\} \quad \text{(xiii).}$$

We can now substitute in (iii) from (iv), (xii) and (xiii) and obtain σ_η^2. We have

$$\eta^2\sigma_\eta^2 = \frac{(1-\eta^2)^4}{(1+\gamma^2)^2}\left[\frac{1}{N^2}\left\{\tfrac{1}{4} S\left(\gamma_s^4 n_s\left(1 - \frac{n_s}{N}\right)\right) + S\left(\frac{n_{1s}n_{2s}}{n_s}\left(\frac{\gamma_s}{z_s}\right)^2\right) - \tfrac{1}{2} S\,(n_s n_{s'}\gamma_s^2\gamma_{s'}^2)\right\}\right]$$

$$+ \frac{(1-\eta^2)^2}{(1+\gamma^2)^2}\frac{\nu_1\nu_2}{N^3}\left(\frac{\gamma}{z}\right)^2$$

$$+ \frac{(1-\eta^2)^3}{(1+\gamma^2)^2}\frac{1}{N^2}\frac{\gamma}{z}\left\{S\,(n_{1s}\gamma_s^2) - \frac{\nu_1}{N} S\,(n_s\gamma_s^2) - 2S\left(\frac{n_{1s}n_{2s}}{n_s}\frac{\gamma_s}{z_s}\right)\right\}$$

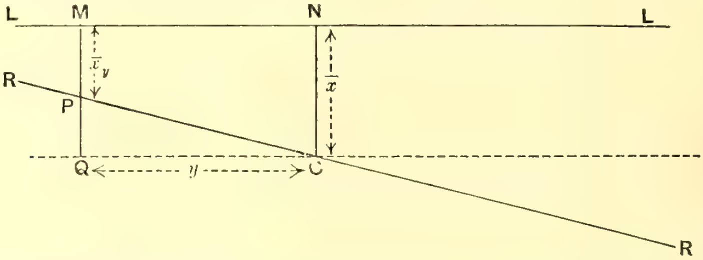
$$= \frac{(1-\eta^2)^2}{N(1+\gamma^2)^2}\left[\frac{\nu_1\nu_2}{N^2}\left(\frac{\gamma}{z}\right)^2 + \frac{(1-\eta^2)^2}{4}\left\{S\left(\frac{\gamma_s^4 n_s}{N}\right) - \left(S\left(\frac{n_s\gamma_s^2}{N}\right)\right)^2\right\}\right.$$

$$+ (1-\eta^2)^2 S\left\{\frac{n_{1s}n_{2s}}{n_s n_s}\frac{n_s}{N}\left(\frac{\gamma_s}{z_s}\right)^2\right\}$$

$$\left. + (1-\eta^2)\left(\frac{\gamma}{z}\right)\left\{S\left(\frac{n_{1s}\gamma_s^2}{N}\right) - \frac{\nu_1}{N} S\left(\frac{n_s}{N}\gamma_s^2\right) - 2S\left(\frac{n_{1s}n_{2s}}{n_s N}\left(\frac{\gamma_s}{z_s}\right)\right)\right\}\right]$$

.........(xiv).

Now we know all the quantities in the sums on the right of (xiv), because γ_s has had to be found for each array, and z_s can be at once determined from $\bar{x}_y/{}_y\sigma_x$ which is known. But the labour would be very considerable and hardly commensurate with the result desired, i.e. an approximate measure of the accuracy of determining the correlation by the biserial η method. We shall accordingly investigate the values of the above terms on the hypothesis of a large number of arrays with linear regression and homoscedasticity, or even with a complete Gaussian distribution of frequency.



In the first place let the division between the two series be the line *LL*. Let *C* be the centroid of the frequency surface and *RR* the regression line. Let $\bar{x} = CN$ and $\bar{x}_y = PM$. Then by the equation to the regression line (correlation $r = \eta$)

$$PQ/\sigma_x = rCQ/\sigma_y,$$

$$(\bar{x} - \bar{x}_y)/\sigma_x = ry/\sigma_y,$$

$$\frac{\bar{x}_y}{\sigma_x\sqrt{1-r^2}} = \frac{1}{\sqrt{1-r^2}}\left(\frac{\bar{x}}{\sigma_x} - r\frac{y}{\sigma_y}\right),$$

$$\gamma_s = \frac{1}{\sqrt{1-r^2}}(\gamma - ry') \text{ ....(xv).}$$

Hence

$$\gamma_s^2 = \frac{1}{1-r^2}(\gamma^2 - 2ry'\gamma + r^2y'^2),$$

$$\gamma_s^4 = \frac{1}{(1-r^2)^2}(\gamma^4 - 4ry'\gamma^3 + 6r^2y'^2\gamma^2 - 4r^3y'^3\gamma + r^4y'^4).$$

Thus, since we may suppose $S(n_sy') = S(n_sy'^3) = 0$ approximately:

$$S(n_s\gamma_s^2)/N = \frac{1}{1-r^2}(\gamma^2 + r^2),$$

$$S(n_s\gamma_s^4)/N = \frac{1}{(1-r^2)^2}(\gamma^4 + 6r^2\gamma^2 + r^4\beta_2'),$$

where β_2' is the second statistical β-constant for the y'-array, and since to our approximation $\eta = r$:

$$\frac{(1-\eta^2)^2}{4}\left[\frac{S(n_s\gamma_s^4)}{N} - \left\{\frac{S(n_s\gamma_s^2)}{N}\right\}^2\right] = \eta^2\left\{\gamma^2 + \tfrac{1}{4}\eta^2(\beta_2' - 1)\right\} \text{......(xvi).}$$

Again
$$S\left(\frac{n_{1s}\gamma_s^2}{N}\right) = \frac{1}{(1-r^2)} S\left\{\frac{n_{1s}}{N}(\gamma^2 - 2r\gamma y' + r^2y'^2)\right\}$$

$$= \frac{1}{(1-r^2)}\left\{\frac{\nu_1\gamma^2}{N} - 2r\gamma\frac{S(n_{1s}y')}{N} + r^2\frac{S(n_{1s}y'^2)}{N}\right\}.$$

But as in the theory of biserial r*

$$r = \frac{S(n_{1s}y')}{N}\Big/\left(z\frac{\nu_1}{N}\right),$$

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* *Biometrika*, Vol. VII. p. 97.

and thus $$S\left(\frac{n_{1s}y'}{N}\right) = rz\,\frac{\nu_1}{N}.$$

Again $$\frac{S\,(n_{1s}y'^2)}{N} = \frac{1}{N}\,S'\left\{n_x\,(1-r^2) + r^2\,\frac{x^2}{\sigma_x^{\,2}}\right\},$$

where the summation S' is from $x = \bar{x}$ to ∞; thus

$$\begin{aligned}\frac{S\,(n_{1s}y'^2)}{N} &= \frac{\nu_1}{N}\,(1-r^2) + r^2\int_\gamma^\infty \frac{1}{\sqrt{2\pi}}\,x^2 e^{-\frac{1}{2}x^2}dx\\ &= \frac{\nu_1}{N}\,(1-r^2) + r^2\gamma z + r^2\,\frac{\nu_1}{N}\\ &= \frac{\nu_1}{N} + r^2\gamma z.\end{aligned}$$

Accordingly we have

$$S\left(\frac{n_{1s}\gamma_s^{\,2}}{N}\right) = \frac{1}{1-\eta^2}\left\{\frac{\nu_1}{N}\,(\gamma^2 - 2\eta^2\gamma z + \eta^2) + \eta^4\gamma z\right\},$$

and accordingly

$$\begin{aligned}(1-\eta^2)\,\frac{\gamma}{z}\left\{\frac{S\,(n_{1s}\gamma_s^{\,2})}{N} - \frac{\nu_1}{N}\,S\left(\frac{n_s\gamma_s^{\,2}}{N}\right)\right\} &= \frac{\gamma}{z}\left\{\frac{\nu_1}{N}\,(\gamma^2 - 2\eta^2\gamma z + \eta^2 - \gamma^2 - \eta^2) + \eta^4\gamma z\right\}\\ &= \eta^2\gamma^2\left(\eta^2 - 2\,\frac{\nu_1}{N}\right)\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(xvii)}.\end{aligned}$$

Now substitute (xvi) and (xvii) in (xiv) and we find

$$\begin{aligned}\eta^2\sigma_\eta^{\,2} = \frac{(1-\eta^2)^2}{N\,(1+\gamma^2)^2}&\left[\frac{\nu_1\nu_2}{N^2}\left(\frac{\gamma}{z}\right)^2 + \eta^2\,(\gamma^2 + \tfrac{1}{2}\eta^2) + \eta^2\gamma^2\left(\eta^2 - 2\,\frac{\nu_1}{N}\right)\right.\\ &\left.+ (1-\eta^2)^2\,S\left\{\frac{n_{1s}n_{2s}}{n_s n_s}\left(\frac{\gamma_s}{z_s}\right)^2\frac{n_s}{N}\right\} - 2\,(1-\eta^2)\,\frac{\gamma}{z}\,S\left\{\frac{n_{1s}n_{2s}}{n_s n_s}\,\frac{\gamma_s}{z_s}\,\frac{n_s}{N}\right\}\right].\end{aligned}$$

Or,

$$\begin{aligned}\sigma_\eta^{\,2} = \frac{(1-\eta^2)^2}{N\,(1+\gamma^2)^2}&\left[\tfrac{1}{2}\eta^2 + \gamma^2\left(\eta^2 + \frac{\nu_2-\nu_1}{N}\right) + \frac{1}{\eta^2}\left(\frac{\gamma}{z}\right)^2\left\{\frac{\nu_1\nu_2}{N^2} - S\left(\frac{n_{1s}n_{2s}}{n_s N}\right)\right\}\right.\\ &\left.+ \frac{1}{\eta^2}\,S\left\{\frac{n_{1s}n_{2s}}{n_s N}\left((1-\eta^2)\,\frac{\gamma_s}{z_s} - \frac{\gamma}{z}\right)^2\right\}\right]\quad\ldots\ldots\ldots\ldots\text{(xviii)}.\end{aligned}$$

This form for the value of $\sigma_\eta^{\,2}$ shows that the probable error of η does not become infinite when $\eta = 0$; for in that case, we shall have $\dfrac{n_{1s}}{n_s} = \dfrac{\nu_1}{N}$ for all values of s, and $\dfrac{\gamma_s}{z_s} = \dfrac{\gamma}{z}$, so that the last two terms in $1/\eta^2$ take indeterminate forms which need evaluating. For calculation it is best to use the form

$$\begin{aligned}\sigma_\eta^{\,2} = \frac{(1-\eta^2)^2}{N\,(1+\gamma^2)^2}&\left[\tfrac{1}{2}\eta^2 + \gamma^2\left(\eta^2 + \frac{\nu_2-\nu_1}{N}\right)\right.\\ &\left.+ \frac{1}{\eta^2}\left\{\frac{\nu_1\nu_2}{N^2}\left(\frac{\gamma}{z}\right)^2 + (1-\eta^2)^2 S\left(\frac{n_{1s}n_{2s}}{n_s n_s}\left(\frac{\gamma_s}{z_s}\right)^2\frac{n_s}{N}\right) - 2\,(1-\eta^2)\,\frac{\gamma}{z}\,S\left(\frac{n_{1s}n_{2s}}{n_s n_s}\left(\frac{\gamma_s}{z_s}\right)\frac{n_s}{N}\right)\right\}\right]\\ &\ldots\ldots\ldots\text{(xix)}.\end{aligned}$$

The functions $$\psi_s'' = \frac{n_{1s}\,.\,n_{2s}}{n_s\,.\,n_s}\left(\frac{\gamma_s}{z_s}\right)^2 = \tfrac{1}{2}(1-\alpha^2)\left(\frac{x}{z}\right)^2.$$

$$\psi_s' = \frac{n_{1s}\,.\,n_{2s}}{n_s\,.\,n_s}\,\frac{\gamma_s}{z_s} = \tfrac{1}{2}(1-\alpha^2)\,\frac{x}{z}.$$

where x, $\frac{1}{2}(1+\alpha)$ and z are the values given in the tables of the probability function, have been calculated by my colleagues Miss Ethel M. Elderton and Miss B. M. Cave in the table given on p. 302 below. The labour was considerable as owing to the smallness of z for high values of x, the quantities involved had to be calculated to many places of decimals. If this table be used we can put (xix) in the form:

$$\sigma_\eta^{\,2} = \frac{(1-\eta^2)^2}{N(1+\gamma^2)^2}\left[\tfrac{1}{2}\eta^2 + \gamma^2\left(\eta^2 + \frac{\nu_2-\nu_1}{N}\right) + \frac{1}{\eta^2}\left\{\frac{\nu_1\nu_2}{N^2}\left(\frac{\gamma}{z}\right)^2 + (1-\eta^2)^2\, S\left(\psi_s''\,\frac{n_s}{N}\right) - 2(1-\eta^2)\frac{\gamma}{z}\, S\left(\psi_s'\,\frac{n_s}{N}\right)\right\}\right] \quad \text{.........(xx).}$$

I propose to illustrate this on the following example correlating nature of vaccination and severity of attack in the case of smallpox.

Severity of Attack.

Nature of Vaccination	(1) Haemorrhagic	(2) Confluent	(3) Abundant	(4) Sparse	(5) Very sparse	Totals
0 to 25 yrs. since	5	38	120	176	148	487
Over 25 yrs. or not at all	44	251	388	308	211	1202
Totals	49	289	508	484	359	1689
Positions of means	$\bar{x}_1/\sigma_1 = \gamma_1 = +1{\cdot}2700$	$\bar{x}_2/\sigma_2 = \gamma_2 = +1{\cdot}1194$	$\bar{x}_3/\sigma_3 = \gamma_3 = +{\cdot}7185$	$\bar{x}_4/\sigma_4 = \gamma_4 = +{\cdot}3488$	$\bar{x}_5/\sigma_5 = \gamma_5 = {\cdot}2218$	$\bar{x}/\sigma_x = \gamma = {\cdot}5583$
Ordinates at dichotomic line	$z_1 = {\cdot}17810$	$z_2 = {\cdot}21321$	$z_3 = {\cdot}30818$	$z_4 = {\cdot}37540$	$z_5 = {\cdot}38925$	$z = {\cdot}34137$
n_{1s}/n_s	·10204	·13149	·23622	·36364	·41226	$\frac{\nu_1}{N} = {\cdot}28834$
n_{2s}/n_s	·89796	·86851	·76378	·63636	·58774	$\frac{\nu_2}{N} = {\cdot}71166$
n_s/N	·02901	·17111	·30077	·28656	·21255	Sum = 1·00000
n_{1s}/N	·00296	·02250	·07105	·10420	·08763	Sum = $\cdot 28834 = \nu_1/N$
γ_s^2	1·61290	1·25306	·51624	·12166	·04920	$\gamma^2 = {\cdot}31170$
γ_s/z_s	7·13083	5·25022	2·33143	·92914	·56981	$\gamma/z = 1{\cdot}63547$
$(\gamma_s/z_s)^2$	50·84867	27·56489	5·43556	·86331	·32469	$(\gamma/z)^2 = 2{\cdot}67476$
$n_s\gamma_s^2/N$	·04679	·21441	·15527	·03486	·01046	$\kappa^2 = {\cdot}46179$
$n_s\gamma_s^4/N$	·07547	·26867	·06812	·00424	·00051	$\gamma^4 = {\cdot}09716$
$n_{1s}\gamma_s^2/N$	·00477	·02819	·03668	·01268	·00431	Sum = ·08663
$\frac{n_{1s}}{n_s}\,\frac{n_{2s}}{n_s}\,\frac{n_s}{N}\,\frac{\gamma_s}{z_s}$	·018955	·102594	·126515	·061613	·029346	Sum = ·339023
$\frac{n_{1s}}{n_s}\,\frac{n_{2s}}{n_s}\,\frac{n_s}{N}\left(\frac{\gamma_s}{z_s}\right)^2$	·135162	·538639	·294961	·057247	·016722	Sum = 1·042731

Now $\eta^2 = (\kappa^2 - \gamma^2)/(1 + \kappa^2) = \cdot 102{,}6755$ and $\eta = \cdot 3204$.

Again, $S(n_s \gamma_s^4)N = \cdot 41701$ actually; but if it be calculated from the value of p. 296, i.e.

$$S(n_s \gamma_s^4)N = \frac{1}{(1-\eta^2)^2}(\gamma^4 + 6\eta^2\gamma^2 + \eta^4\beta_2'),$$

taking $\beta_2' = 3$, its value $= \cdot 39843$, which is approximate enough for most practical purposes,

$$\frac{(1-\eta^2)^2}{4}\left\{S\left(\frac{\gamma_s^4 n_s}{N}\right) - \left(\frac{S(n_s\gamma_s^2)}{N}\right)^2\right\},$$

taken actually, $= \cdot 201{,}298\,\{\cdot 41701 - (\cdot 46179)^2\} = \cdot 041{,}0165$,

while the approximate value in (xvi) gives $\cdot 037{,}2751$.

In the next place

$$(1-\eta^2)\frac{\gamma}{z}\left\{S\left(\frac{n_{1s}\gamma_s^2}{N}\right) - \frac{\nu_1}{N}S\left(\frac{n_s}{N}\gamma_s^2\right)\right\},$$

taken actually, $= - \cdot 068{,}270$, while the approximate value in (xvii) gives $- \cdot 01517$.

Further*,

$$(1-\eta^2)\frac{\gamma}{z} \times 2S\left(\frac{n_{1s}n_{2s}}{n_s n_s}\frac{n_s}{N}\frac{\gamma_s}{z_s}\right) = 1\cdot 794{,}649 \times 1\cdot 63547 \times \cdot 339{,}023\dagger$$
$$= \cdot 995{,}062,$$

and will be seen to be of wholly different order to the terms already computed.

Again, $$\frac{\nu_1\nu_2}{N^2}\left(\frac{\gamma}{z}\right)^2 = \cdot 28834 \times \cdot 71166 \times 2\cdot 67476$$
$$= \cdot 548{,}861.$$

Lastly we have the term

$$(1-\eta^2)^2 S\left\{\frac{n_{1s}n_{2s}}{n_s n_s}\frac{n_s}{N}\left(\frac{\gamma_s}{z_s}\right)^2\right\} = \cdot 805{,}191 \times 1\cdot 042{,}731\ddagger$$
$$= \cdot 839{,}598.$$

Thus substituting in (xiv) we have

$$\sigma_\eta^2 = \frac{(1-\eta^2)^2}{N\eta^2(1+\gamma^2)^2}\{\cdot 548{,}861 + \cdot 041{,}0165 + \cdot 839{,}598 - \cdot 068{,}270 - \cdot 995{,}062\}$$
$$= \frac{\cdot 805{,}191 \times \cdot 366{,}144}{1689 \times \cdot 102{,}6755 \times (1\cdot 31170)^2} = \cdot 000{,}988{,}065.$$
$$\therefore\ \sigma_\eta = \cdot 03143,$$

and $$\text{Probable Error} = \cdot 67449\sigma_\eta = \cdot 0212.$$

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* In this case γ_s is of the same sign throughout, but caution must be taken to see from the data whether for certain arrays it changes sign.

† Found from the table of ψ', this is ·338,961.

‡ Found from the table of ψ'', this is 1·044,911. The differences in this and the previous value for ψ' depend on cutting off at 5 or 6 decimal places and in using in the different processes only first differences.

For comparison we will investigate the value as given by the less exact formula (xix):

$$\sigma_\eta{}^2 = \frac{{\cdot}805{,}191}{1689 \times (1{\cdot}31170)^2} \Big[{\cdot}051{,}338 + {\cdot}31170\,({\cdot}102{,}676 + {\cdot}423{,}327) + 9{\cdot}739{,}422\,({\cdot}548{,}861 + {\cdot}839{,}598 - {\cdot}995{,}062)\Big]$$

$$= {\cdot}000{,}277{,}077 \times 4{\cdot}046{,}752 = {\cdot}001{,}121{,}26.$$

Hence $$\sigma_\eta = {\cdot}03349,$$

and $$\text{Probable Error} = {\cdot}0226.$$

Thus for all practical purposes (xix) gives as good a result as the more lengthy formula (xiv). Had the value $r = {\cdot}3204$ been computed by the product moment method for a population of 1689 the probable error would have been ·0147. Thus the accuracies of the product-moment and the biserial η methods appear to be about roughly in the ratio of 2 to 3 for this case.

We will now turn back to the formula (xviii) and endeavour to express the quantities in curled brackets in powers of r or η. We have

$$z_s = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\gamma_s{}^2} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(\gamma - ry')^2}{1-r^2}}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\gamma^2} \{1 + r\gamma y' + \tfrac{1}{2} r^2 (\gamma^2 y'^2 - y'^2 - \gamma^2)\}.$$

Hence

$$\frac{\gamma_s}{z_s} = \frac{\gamma}{z}\left(1 - r\frac{y'}{\gamma} + \tfrac{1}{2}r^2\right)\{1 - r\gamma y' + \tfrac{1}{2}r^2(\gamma^2 y'^2 + y'^2 + \gamma^2)\}$$

$$= \frac{\gamma}{z}\left\{1 - ry'\frac{1+\gamma^2}{\gamma} + r^2(y'^2) + \tfrac{1}{2}r^2(1+\gamma^2)(1+y'^2)\right\} \quad \text{.........(xxi).}$$

Again $$\frac{n_{1s}}{n_s} = \frac{1}{\sqrt{2\pi}} \int_{\gamma_s}^{\infty} e^{-\frac{1}{2}x^2}\, dx,$$

and $$\gamma_s = \gamma - ry' + \tfrac{1}{2}r^2\gamma.$$

Thus $$\frac{n_{1s}}{n_s} = \frac{\nu_1}{N} + \frac{1}{\sqrt{2\pi}} \int_{\gamma - ry' + \frac{1}{2}r^2\gamma}^{\gamma} e^{-\frac{1}{2}x^2}\, dx.$$

Take $x = \gamma - x'$, so that x' will be small, thus

$$\frac{n_{1s}}{n_s} = \frac{\nu_1}{N} + z \int_0^{ry' - \frac{1}{2}r^2\gamma} (1 + \gamma x' + \tfrac{1}{2}\gamma^2 x'^2)(1 - \tfrac{1}{2}x'^2)\, dx'$$

$$= \frac{\nu_1}{N} + z\,(ry' - \tfrac{1}{2}\gamma r^2 + \tfrac{1}{2}r^2\gamma y'^2) + \text{etc.}$$

Similarly:

$$\frac{n_{2s}}{n_s} = \frac{\nu_2}{N} - z\,(ry' - \tfrac{1}{2}\gamma r^2 + \tfrac{1}{2}r^2\gamma y'^2) - \text{etc.},$$

$$\frac{n_{1s}n_{2s}}{n_s n_s} = \frac{\nu_1\nu_2}{N^2} - z\,\frac{\nu_2 - \nu_1}{N}\,ry' - z^2 r^2 y'^2 + Z\,\frac{\nu_2 - \nu_1}{N}\,\tfrac{1}{2}\gamma r^2(1 - y'^2) + \text{etc.} \ldots\text{(xxii).}$$

But $$S\left(\frac{n_s}{N}\right) = 1, \quad \frac{S\,(n_s y')}{N} = 0, \quad S\left(\frac{n_s y'^2}{N}\right) = 1,$$

and accordingly: $$S\left(\frac{n_{1s}n_{2s}}{n_s n_s}\frac{n_s}{N}\right) = \frac{\nu_1\nu_2}{N^2} - z^2\nu^2 + \text{etc.}$$

We have then
$$\frac{1}{\eta^2}\left(\frac{\gamma}{z}\right)^2\left\{\frac{\nu_1\nu_2}{N^2} - S\left(\frac{n_{1s}n_{2s}}{n_s N}\right)\right\} = \gamma^2 + \text{terms in } \eta^2 \text{ ....(xxiii).}$$

Again turning back to (xxi):
$$(1-\eta^2)\frac{\gamma_s}{z_s} - \frac{\gamma}{z} = \frac{\gamma}{z}\left(-ry'\frac{1+\gamma^2}{\gamma} + r^2(y'^2-1)\right) + \tfrac{1}{2}r^2(1+\gamma^2)(1+y'^2).$$

Square this, multiply by (xxii) and n_s/N, sum and note that sum of odd powers of y' go out. We find
$$\frac{1}{\eta^2}S\left\{\frac{n_{1s}n_{2s}}{n_s N}\left((1-\eta^2)\frac{\gamma_s}{z_s} - \frac{\gamma}{z}\right)^2\right\} = \frac{\nu_1\nu_2}{N^2}\frac{(1+\gamma^2)^2}{z^2} + \text{terms in } \eta^2\text{...(xxiv).}$$

Substituting (xxiii) and (xxiv) in (xviii) we deduce
$$\sigma_\eta{}^2 = \frac{(1-\eta^2)^2}{N}\left[\frac{\gamma^2}{(1+\gamma^2)^2}\frac{\nu_2-\nu_1}{N} + \frac{\gamma^2}{(1+\gamma^2)^2} + \frac{\nu_1\nu_2}{N^2z^2} + \text{terms in } \eta^2\right]$$
$$= \frac{(1-\eta^2)^2}{N}\left[\frac{\nu_1\nu_2}{N^2z^2} + \frac{2\nu_2}{N}\frac{\gamma^2}{(1+\gamma^2)^2} + \text{terms in } \eta^2\right].$$

$$\text{Probable Error of } \eta = \frac{\cdot 67449\,(1-\eta^2)}{\sqrt{N}}\left\{\frac{\nu_1\nu_2}{N^2z^2} + \frac{2\nu_2}{N}\frac{\gamma^2}{(1+\gamma^2)^2}\right\}^{\frac{1}{2}}$$
$$\times\ (1 + \text{terms in } \eta^2 \text{ and higher powers}) \text{......(xxv).}$$

This expression is of value for two reasons. First it indicates that the probable error of η found for non-associated variates is
$$= \frac{\cdot 67449}{\sqrt{N}}\left\{\frac{\nu_1\nu_2}{N^2z^2} + \frac{2\nu_2}{N}\frac{\gamma^2}{(1+\gamma^2)^2}\right\}^{\frac{1}{2}} \text{ ....(xxvi).}$$

And secondly (xxv) may give a good approximate value of the probable error of η, when we neglect the last factor altogether, say for values of η under ·4 or ·5. Thus in the illustration given above
$$\left\{\frac{\nu_1\nu_2}{N^2z^2} + \frac{2\nu_2}{N}\frac{\gamma^2}{(1+\gamma^2)^2}\right\}^{\frac{1}{2}} = \left\{\frac{\cdot 548{,}861}{\cdot 31170} + 1\cdot 42332\,\frac{\cdot 31170}{(1\cdot 31170)^2}\right\}^{\frac{1}{2}}$$
$$= 1\cdot 4208.$$

We have therefore to multiply the probable error as given by the product-moment method, i.e. in our case ·0147 by 1·4208, giving ·0209, in fair accordance with the more accurate value ·0226, it being remembered that a probable error is rarely worth more than one or two significant figures.

Table of Values of the Biserial η *Functions,* ψ' *and* ψ''.

x	ψ'	ψ''	x	ψ'	ψ''
00	0	0	2·00	·82357	30·5075
·05	·03132	·00393	2·05	·83084	34·9079
·10	·06258	·01577	2·10	·83770	39·9960
·15	·09371	·03563	2·15	·84416	45·8899
·20	·12465	·06375	2·20	·85025	52·7294
·25	·15534	·10043	2·25	·85599	60·6805
·30	·18571	·14608	2·30	·86140	69·9410
·35	·21572	·20121	2·35	·86650	80·7466
·40	·24530	·26644	2·40	·87132	93·3785
·45	·27441	·34251	2·45	·87587	108·1734
·50	·30299	·43030	2·50	·88016	125·535
·55	·33099	·53083	2·55	·88422	145·946
·60	·35839	·64531	2·60	·88807	169·991
·65	·38513	·77509	2·65	·89171	198·369
·70	·41118	·92177	2·70	·89516	231·930
·75	·43651	1·08716	2·75	·89843	271·696
·80	·46110	1·27337	2·80	·90154	318·909
·85	·48493	1·48278	2·85	·90449	375·075
·90	·50797	1·71815	2·90	·90731	442·028
·95	·53022	1·98263	2·95	·90998	521·999
1·00	·55165	2·27983	3·00	·91254	617·713
1·05	·57228	2·61391	3·05	·91498	732·506
1·10	·59208	2·98961	3·10	·91730	870·463
1·15	·61108	3·41242	3·15	·91953	1036·599
1·20	·62926	3·88863	3·20	·92166	1237·088
1·25	·64665	4·42549	3·25	·92371	1479·538
1·30	·66324	5·03136	3·30	·92567	1773·344
1·35	·67906	5·71590	3·35	·92755	2130·136
1·40	·69412	6·49026	3·40	·92936	2564·332
1·45	·70844	7·36736	3·45	·93110	3093·849
1·50	·72203	8·36218	3·50	·93277	3740·988
1·55	·73493	9·49211	3·55	·93438	4533·572
1·60	·74715	10·77737	3·60	·93593	5506·376
1·65	·75871	12·24149	3·65	·93743	6702·955
1·70	·76965	13·91195	3·70	·93888	8177·972
1·75	·77999	15·82082	3·75	·94027	10,000·169
1·80	·78975	18·00564	3·80	·94162	12,256·169
1·85	·79896	20·51038	3·85	·94292	15,055·358
1·90	·80765	23·38668	3·90	·94418	18,536·141
1·95	·81584	26·69522	3·95	·94540	22,874·013
2·00	·82357	30·50747	4·00	·94658	28,291·951

A PRELIMINARY REPORT ON SOME BREEDING EXPERIMENTS WITH FOXGLOVES.

By ERNEST WARREN, D.Sc. (Lond.).

In 1914 in the garden of the writer there occurred among some ordinary foxgloves (*Digitalis gloxiniaeflora*) a specimen of the variety known as *monstrosa* in which there is a large, more or less regular, peloric or crown flower arising by the fusion of a variable number of ordinary flowers at the top of the axis. The idea arose that it would be of interest to ascertain whether this peloric character would blend in the offspring when the two forms of foxgloves were crossed, or whether, as would appear more likely, the character would be transmitted in its entirety or not at all.

In working at this particular character it was found that the shape, the spotting, and the general intensity of colouration of the ordinary zygomorphic flowers varied greatly, and accordingly a record has been made of these different characters in the first generation. The second generation is now being raised; but, as it will be some time before flowering takes place, a preliminary report on the results of the observation on the first generation will be of interest, since they indicate the general trend of the experiment.

I am much indebted to Dr Conrad Akerman for making most of the measurements and for ever-ready assistance in the experiment; to Mr A. W. Cullingworth, Mr W. G. Rump and Miss R. Meanwell for general help in the cultivation of the plants, mounting the pressed flowers, etc., and to Mr George Carter, Mr W. Bainbridge, Mr C. R. C. McDonald and Mr H. C. Burnup for growing some of the plants in their gardens, also to Mr J. F. Nicolson for allowing me to examine and take specimens of plants from his private grounds.

I. *Material.*

The parent plants were the following:

(1) Medium-purple crowned foxglove raised from seed from Erfurt, Germany = *A*.

(2) Very-pale-purple crowned foxglove from Mr George Carter's Nursery, Pietermaritzburg, raised from Erfurt seed = *C* (1).

(3) White ordinary foxglove (Carter) = *C* (2).

(4) Light-purple ordinary foxglove (Carter) = *C* (3).

(5) Medium-purple ordinary foxglove from Mr P. T. Badock's garden, Hilton Rd = *B* (1).

(6) Pale-purple ordinary foxglove (Badock) = B (2).

(7) Medium-purple ordinary foxglove (Badock) = B (3).

(8) Dark-purple ordinary foxglove (Badock) = B (4).

(9) Dark-purple crowned foxglove from Mr J. F. Nicolson's garden, Boshoff Rd; seed from Messrs Dobbie, Edinburgh = N (1).

(10) Dark-purple creased lip ordinary foxglove (Nicolson) = N (2).

These plants were variously crossed or self-fertilized so that seed-capsules of some 36 different pairings were obtained. Unfortunately many of the families were completely killed soon after germination owing to scorching by the semi-tropical sun of Natal. There were ultimately obtained records of the flowering of 188 plants, the product of 19 families. In every case the contents of one capsule only were sown. Sowing was done in the middle of December, 1914, and flowering occurred in October and November of 1915. Most of the plants grew very luxuriantly and the flower-axes reached a height of 4 to 5 ft.

The following is a list of the families showing the parentage (seed-plant placed first) and the number of plants which flowered. Except where indicated the pollen was taken from the ordinary zygomorphic flowers:

I A	A (self-fertilized) ...	35
I	C (1) (self-fertilized)	7
II	C (1) × B (4) ...	8
III	B (4) × C (1) ...	5
IV	B (4) (self-fertilized)	5
V	C (1) × B (3) ...	23
VIII	C (1) × C (2) ...	10
IX	C (2) × C (1) ...	10
IX A	C (1) × N (2) ...	4
X	C (2) (self-fertilized)	11
XI	A × C (3) ...	17
XII	C (3) × A* ...	1
XIV	A × C (2) ...	14
XV	C (2) × A* ...	8
XV A	A × C (2) ...	7
XV B	C (2) × A† ...	1
XXIV	A × B (4) ...	13
XXV	B (4) × A† ...	6
XXIX	A × N (1) ...	3

All the families except I A were grown in beds on a piece of land close to the Natal Museum. I A plants were raised in different private gardens of Pietermaritzburg.

II. *The Inheritance of certain Characters.*

The characters dealt with are:

(1) The possession of a crown or a terminal peloric flower at the top of the flower-axis.

(2) The presence of brown spots in place of dark-purple spots on the lower inside surface of the ordinary zygomorphic flower.

(3) The amount of spotting on the lower inside surface of the ordinary flower, expressed as the percentage of area of this surface covered by the spots.

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* Pollen from crown flower.

† Pollen from terminal crown flower of side flower-axis.

(4) The intensity of the purple colouration over the general outer surface of the ordinary flower.

(5) The ratio of the maximum width of the corolla at the mouth to the total length of the corolla, expressed $\frac{\text{width}}{\text{length}} \times 1000$.

With reference to the spotting on the inside of the flower the percentage area covered by the spots was estimated by the help of a diagram consisting of a series of equal sized rectangles each of which was spotted to a known extent: 4 %, 8 %, 12 %, 16 %,......64 %.

For estimating the intensity of the purple colouration over the general outer surface a colour scale was obtained in the following way. A very weak solution* of a mixture of aniline dyes was prepared which resembled the purple colour of the flowers. This solution was placed in upright rectangular glass jars of varying internal dimensions. With transmitted light the intensity of colour, of course, varied according to the depth of fluid. Jars of internal dimensions, measured from back to front, of 20 mm., 30 mm., 40 mm., 50 mm., 60 mm., 70 mm., 80 mm., 90 mm., 100 mm., 140 mm., 280 mm., were employed, and the colour of the solution, as viewed from the front of the jar with light coming from behind, was compared with the general colour of the flowers. Thus, if the colour of the flower was nearest, say, to the colour of the solution in a jar with a thickness of fluid of 70 mm. the colour of the plant was said to be 70. For a comparatively small range, as from 20 to 100 mm., the method appeared to work satisfactorily, but less reliance can be placed on the greater depths. In any case a scale of some definite physical significance is obtained.

The colour of the flowers is due to the presence of a purple cell-sap. In the case of the general colouration of the whole surface both the outside and the inside epidermal cells bear this coloured sap. In the spots, where the colouration is much more intense, the coloured cells are confined to the epidermis on the inside of the flower. In flowers which are white, with the exception of the deeply coloured spots, there may be some exceedingly faint traces of isolated streaks of purple on the inside of the flower, these are due to a slight colouration of the inside epidermal cells.

The flowers were measured with compasses and scale. The width was the maximum distance between the reflexed margins of the mouth, measured from right to left, and the length, measured along the adcauline surface, was the distance from the middle of the straightened-out margin of the mouth to the middle of the insertion-edge of the corolla.

1. *The peloric Character.*

The peloric flower at the top of the axis would appear to arise by the fusion of a varying number of the uppermost flowers and the concurrent entire suppression of the internodes between such flowers. Usually the apex of the main axis is unaffected, and after the peloric flower has withered it continues to grow through the centre of the crown and may produce a considerable number of zygomorphic flowers. On this out-growing axis a second and subsequently even a third peloric or crown flower may be produced. These are always less perfect than the first. In other plants the peloric flower is permanently the terminal flower, and the main axis ends in ovary and pistil.

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* Water 100 c.c., rubin 2·375 mg., gentian violet 0·125 mg. (Grübler).

In plant *A* the peloric flower consisted of four fused flowers, in plant *C* (1) of about eight flowers, and in both cases the main axis continued to grow on the withering of the crown. In *N* (1) the peloric flower was a regular deep cup consisting of only three fused flowers and the main axis terminated in an ovary.

In the case of side flower-shoots the axis seldom continues to grow, but generally terminates in a pistil. In plant *A* the terminal flower of the side shoot consisted of only two flowers which had fused laterally and the resulting structure was bilaterally symmetrical. In plant *C* (1) the side shoots terminated in a beautifully regular shallow cup consisting of three fused flowers.

In connection with the formation of the crown flower it is clear from the arrangement of the stamens, and the lobing and spotting of the corolla, that the structure arises as if the flowers had split along the middle line of the upper surface, spread out and fused with their neighbours along the split edges.

As an abnormality there occasionally occurred among the plants zygomorphic flowers completely split along the upper surface: they were otherwise quite normal flowers. Sometimes where split the edges gaped apart, and thus there is a tendency for the flowers to arise in a flat, leaf-like condition.

It may be supposed in fact that the complete suppression of certain internodes causes a definite number of flower-buds to fuse together in such a manner that when growth occurs the resulting structure has the appearance of being produced by the joining together of several zygomorphic flowers which have been cut open along the middle line of the upper surface and have been connected together to form a cup or shallow saucer.

It was soon found that imperfect crowns of different degrees of incompleteness could occur, and this fact raised the presumption that in the cross of a crowned plant and an ordinary plant the offspring would exhibit more or less intermediate characters.

It seemed quite hopeless to attempt to find a natural scale by means of which the intensity of the peloric character in different plants could be measured, and the following empirical grading was adopted.

(1) Plants in which the main axis carried a perfect crown of a variable number of flowers (3, 4, 8) was placed in the 100 % grade. On some few plants the corolla of the crown flowers was quite small and green, or even entirely absent, but the arrangement of the sepals indicated clearly that the peloric character was fully developed; such plants were also placed under the 100 % grade.

(2) Sometimes the main axis had its terminal portion spirally twisted in 1–2 turns, and in this region the flowers were excessively crowded owing to the great reduction in the length of the internodes. The flowers were very abnormal, and were usually split open and formed large, flat, coloured, petaloid structures which matured long before the lower normal flowers of the axis. The side shoots of such plants usually carried more or less perfect crowns. These plants were placed in the 75 % grade.

(3) Plants in which the main axis was ordinary and carried no crown, but the side flower-axes terminated in crowns, consisting of two or more fused flowers, were placed in the 50 % grade.

(4) Plants in which the main axis was markedly dwarfed, lumpy and curved, owing to the shortening of the internodes, but the flowers more or less normal, were placed in the 50 % grade.

(5) Plants in which the terminal portion of the main axis was somewhat curved, in which the upper internodes were reduced and of which the side shoots also showed such a tendency, were placed in the 25 % grade.

Spiral bending of the axis always occurs with the peloric condition when the crown consists of more than three flowers and the axis continues to grow through the crown. It is doubtless caused by the suppression of the internodes.

In the following table the results obtained in the first generation are summarized.

Parentage	Offspring						
	No.	Crowned or tendency for same					Not crowned
		100 %	75 %	50 %	25 %	Total crowned	
IA. *A* (crowned 100 %) ♀ × ♂ (self) ...	33	33	0	0	0	33	0
I. *C* (1) (crowned 100 %) ♀ × ♂ (self) ...	7	3	0	4	0	7	0
X. *C* (2) (ordinary) ♀ × ♂ (self)	11	1	1	1	0	3	8
IV. *B* (4) (ordinary) ♀ × ♂ (self)	5	0	0	0	0	0	5
VIII. *C* (1) (crowned) ♀ × *C* (2) (ordinary) ♂	10	2	0	1	0	3	7
IX. *C* (2) (ordinary) ♀ × *C* (1) (crowned) ♂	10	5	0	1	0	6	4
II. *C* (1) (crowned) ♀ × *B* (4) (ordinary) ♂	8	0	0	0	0	0	8
III. *B* (4) (ordinary) ♀ × *C* (1) (crowned) ♂	5	0	0	0	0	0	5
V. *C* (1) (crowned) ♀ × *B* (3) (ordinary) ♂	23	0	0	0	0	0	23
XI. *A* (crowned) ♀ × *C* (3) (ordinary) ♂ ...	17	2	4	3	1	10	7
XII. *C* (3) (ordinary) ♀ × *A* (crowned) ♂ ...	1	0	1	0	0	1	0
XIV. *A* (crowned) ♀ × *C* (2) (ordinary) ♂ ...	14	7	2	0	0	9	5
XV. *C* (2) (ordinary) ♀ × *A* (crowned) ♂ ...	8	2	0	1	0	3	5
XVA. *A* (crowned) ♀ × *C* (2) (ordinary) ♂ ...	7	2	0	0	0	2	5
XVB. *C* (2) (ordinary) ♀ × *A* (crowned) ♂ ...	1	0	0	0	1	1	0
IXA. *C* (1) (crowned) ♀ × *N* (2) (ordinary) ♂	4	1	0	0	0	1	3
XXIX. *A* (crowned) ♀ × *N* (1) (crowned) ♂ ...	4	4	0	0	0	4	0
XXIV. *A* (crowned) ♀ × *B* (4) (ordinary) ♂	13	0	0	0	0	0	13
XXV. *B* (4) (ordinary) ♀ × *A* (crowned) ♂	6	0	0	0	0	0	6
Totals	...	...	...	...	...	83	104

It will be of interest to compare these figures with those that would be expected if the character behaved as a Mendelian unit. In the experimental results it will be noticed that all plants showing any tendency for a crown are grouped together as opposed to the plants exhibiting no such tendency.

To obtain the theoretical figures, and speaking in Mendelian terminology, it will be necessary to make certain assumptions as to the homozygous or heterozygous nature of the parent plants with regard to the presence or absence of a crown. The absence is dominant over the presence of a crown.

In the cases of parents *A*, *C* (1), *C* (2) and *B* (4) we have experimental evidence of the gametic nature of the plants. Reference to the above table will show that *A* and *C* (1) are pure recessives (families I A and I), *C* (2) is heterozygous (family X), and *B* (4) almost certainly homozygous dominant (family IV). For the other parents we assume that *B* (3) and *B* (4) are homozygous dominant, *N* (1) pure recessive, and *C* (3) and *N* (2) heterozygous.

With these assumptions, which are warranted by the behaviour of the parents in the various crossings, we can calculate the theoretical expectation for each family. The totals, to the nearest whole figure, are 83 plants crowned and 104 plants not crowned. The experimental result as shown above gives the same figures. In the light of other investigations this identity between the theoretical expectation and experimental result is to be regarded, of course, as unusual and accidental.

Of the 83 plants with a crown or a tendency for a crown the number in each grade is given below:

100 %	75 %	50 %	25 %	Total
62	8	11	2	83

In the various crossings of crowned parents and non-crowned parents there is thus a marked tendency for the character to be transmitted in its entirety or not at all. Notwithstanding this fact it nevertheless appears that the character can be diluted; for example in cross no. XI, *A* (crowned) ♀ × *C* (3) (ordinary) ♂, we have:

Crowned				Not crowned
100 %	75 %	50 %	25 %	
2	4	3	1	7

Thus, although the character appears to be capable of blending yet it is usually transmitted as a simple Mendelian unit.

2. *The Presence of Brown Spots.*

Among the various families there occurred plants with white corolla which possessed brown or greenish spots in place of the usual dark-purple spots. Such brown-spotted flowers only occurred when the corolla was white. If there was the faintest trace of purple on any part of the general surface of the corolla the spots were almost invariably purple. On the other hand the intensity of the colouration of the spots appeared to be in no way related to the intensity of the general colouration of the corolla, since some of the darkest coloured spots were found in flowers with an otherwise white corolla. In the terminology of the Mendelians it may be said that distinct factors control the general colouration of the corolla and the colour of the spots.

In this connection it is interesting to remember that the general colouration of the corolla is due to coloured sap in the cells of both the outer and inner epidermis of the flower, while the colouration of the spots arises through the colour in the inner epidermal cells only.

A microscopical examination of the brown spots revealed the presence of a granular brownish substance and of a great accumulation of typical starch grains. Such starch grains were few or absent in the surrounding epidermal cells; also they did not occur in the normal dark purple or crimson spots. The brown substance is almost certainly a decomposition product of anthocyanin, the normal colouring matter. Dr F. W. Bews, Professor of Botany, Natal University College, has suggested that possibly the starch also is to be regarded as a decomposition product of the anthocyanin: but the great abundance of the starch somewhat militates against this view.

The brown substance is insoluble in alcohol, ether, HCl (50 %) and potash (1 %).

With the naked eye there is a sharp discontinuity between white flowers with purple spots and white flowers with brown spots. Microscopical examination, however, shows that the discontinuity is not really so sharply defined. In many of the coloured cells of the purple spots traces of the brown substance may be detected, and at first it was thought possible that the removal of the purple colouring substance with alcohol would leave brown spots. This, however, was found not to be the case. Only rarely is the brown substance in sufficient quantity to mark out the spot on the removal of the anthocyanin. The point to notice is that brownish purple spots intermediate between the normal purple spots and the brown spots are only rarely found. Traces of an intermediate condition can be found microscopically, but there appears to be normally a gap between a small amount of decomposition product and total decomposition.

It will be remembered that there occurs a similar gap between ordinary foxgloves and crowned foxgloves; intermediate conditions are possible but only rarely occur.

If we assume that absence of decomposition (purple spots) is dominant and total decomposition (brown spots) recessive, and that the parents which produced white offspring were heterozygous dominants with respect to this character, we can compare the experimental result with the theoretical Mendelian expectation.

Parentage (Heterozygous dominants)		Offspring			
		Observation		Theoretical expectation	
		Purple spots	Brown spots	Purple spots	Brown spots
X.	*C* (2) (self-fertilized)	9	2	8·25	2·75
IA.	*A* (self-fertilized) ...	26	7	24·75	8·25
XVA.	*A* ♀ × *C* (2) ♂ ...	6	1	5·25	1·75
XVB.	*C* (2) ♀ × *A* ♂ ...	0	1	0·75	0·25
XI.	*A* ♀ × *C* (3) ♂ ...	12	5	12·75	4·25
XII.	*C* (3) ♀ × *A* ♂ ...	1	0	0·75	0·25
XIV.	*A* ♀ × *C* (2) ♂ ...	9	5	10·50	3·50
XV.	*C* (2) ♀ × *A* ♂ ...	6	2	6·00	2·00
Totals ...		69	23	69·00	23·00

The assumption as to the heterozygous nature of the parents is warranted by the consistency which is shown when a supposed heterozygous dominant is crossed with an undoubted homozygous dominant. Thus we have:

Parentage	Offspring			
	Observation		Theoretical expectation	
	Purple	Brown	Purple	Brown
C (2) Heterozygous dominant ♀ × *C* (1) homozygous dominant ♂	10	0	10	0
C (1) Homozygous dominant ♀ × *C* (2) heterozygous dominant ♂	10	0	10	0
Totals	20	0	20	0

The fact that the observed result and the theoretical expectation in the above two tables should be identical may be regarded as somewhat accidental, but nevertheless the agreement is rather striking.

3. *The Amount of Spotting on the inner Surface of the Flower.*

By means of the diagram already referred to the percentage area, with respect to the lower portion of the inside surface of the flower, covered by the spots was estimated in six ordinary zygomorphic flowers in each plant. This character appeared to exhibit no discontinuity in its inheritance. All intermediate conditions seemed to occur in the offspring; for example a parent with much spotting crossed with a parent with few spots tended to have offspring with a medium amount of spotting. It was quite impossible to sort out the parents or offspring into two or more groups of "much spotting," "little spotting," "medium spotting," etc. As far as can be judged from the material at hand the units of spotting, if such occur, are so small that the series is practically a continuous one.

The only satisfactory way of dealing with such a character which exhibits no discontinuity is the statistical method of Galton and Pearson. From the nature of the crossings it may not be always possible to deal statistically with the relationship of both male and female parent to the offspring. For example, if a single plant be crossed with the pollen of a dozen different plants, and families raised, there will be one female parent only, but 12 different male parents and twelve different mid-parents. Such a method of procedure is not free from objection, but it is the only course that is possible with the material at present available. In each plant the amount of spotting was determined in the bottom six flowers of the main axis, that is in the first six flowers which opened.

Another point to note is that some of the families were raised from seed obtained by self-fertilization. Now, the differences, if any, in the strength of inheritance in parthenogenesis, self-fertilization and cross-fertilization have not yet been fully investigated from an experimental point of view, and accordingly the heredity constants have been calculated both with the inclusion and the exclusion of the self-fertilized families.

Ordinary correlation tables have been made, utilizing the six flowers of each plant, and Prof. Pearson's constants have been calculated.

As an example the following table is given:

Spotting: Mid-Parent and Offspring (omitting self-fertilized Plants).

Mid-parent, percentage of spotting	54	96	189	144	105	91	45	31	12	6	Totals
8—11	34	36	36	2	—	—	—	—	—	—	108
12—15	20	43	72	39	7	—	—	—	—	—	181
16—19	—	3	53	55	47	42	28	24	4	2	258
20—23	—	—	—	4	5	2	1	—	—	—	12
24—27	—	9	27	39	18	13	7	—	—	—	113
28—31	—	5	1	5	28	29	2	2	4	2	78
32—35	—	—	—	—	—	5	7	5	4	2	23
Offspring, percentage of spotting	*0—3*	*4—7*	*8—11*	*12—15*	*16—19*	*20—23*	*24—27*	*28—31*	*32—35*	*36—39*	773

The general results may be summarized in the form of a table:

Relationship	No. of mid-parents	No. of off-spring	No. of flowers	Standard deviation of mid-parent	Standard deviation of offspring	Coefficient of correlation
Mid-parent and offspring (including self-fertilized families)	15	176	1027	10·509	8·500	·563
Mid-parent and offspring (excluding self-fertilized families)	11	127	773	6·610	7·790	·545
Male parent and offspring (including self-fertilized families)	15	176	1027	16·733	8·595	·431
Male parent and offspring (excluding self-fertilized families)	11	127	773	15·249	7·790	·320

With the small number of parents available the constants are bound to be erratic, and the results can only be regarded as giving a rough indication of the strength of inheritance. The general tendency is for a low parental correlation: the mean of the two paternal correlations is ·38. The average of some 52 correlation coefficients of parental heredity obtained from a variety of different animal species with various characters has been given by Prof. Karl Pearson as ·45, while the mean of the parental coefficients for a series of characters in the Shirley Poppy was ·33.

4. *General Colouration of Corolla.*

We have seen that the general colouration of the corolla is due to coloured sap in the epidermal cells of both the inside and outside of the corolla. The intensity of this colouration was estimated, as already described, by a colour-scale obtained by viewing by transmitted light varying depths of a standard tinted solution. There is discontinuity in the colour of the flowers of the plants. The flowers can be divided into (1) purple and (2) not purple (white). For this subdivision a purple flower must be defined as one in which the general surface of the corolla is coloured. The colour may be quite pale or very dark. A white flower is one in which the general surface is white, but there may be a few extremely faint blushes of purple on the corolla, especially on the inner surface.

Such definitions may seem empirical and unsatisfactory, but nevertheless it was found in practice to work quite easily owing to the fact that exceedingly faint *general* colouration of the whole corolla did not occur. There was a clear gap in the series between faintly "purple flowers" and the "white flowers." In the whole series the palest purple flower was represented by a depth of 15 mm. of coloured fluid and there was only one such plant.

In the accompanying table the families are arranged in the order of the mid-parental colour—the mean of the colours of the male and female parents.

Parentage		Offspring																														
	R = Purple W = White	Mid-parental colour	Purple (Uniform colouration of outer surface)																													Not Purple
			250—254	*245—249*	*240—244*	*235—239*	*230—234*	*225—229*	*220—224*	....	*100—104*	*95—99*	*90—94*	*85—89*	*80—84*	*75—79*	*70—74*	*65—69*	*60—64*	*55—59*	*50—54*	*45—49*	*40—44*	*35—39*	*30—34*	*25—29*	*20—24*	*15—19*	*10—14*	*5—9*	*0—4*	Total Purple	
X.	*C* (2), W ♀ ♂ ...	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	11
IX.	*C* (2), W × *C* (1), R	17	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	2	—	—	—	—	—	5	5
VIII.	*C* (1), R × *C* (2), W	17	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	4	—	—	—	—	—	7	3
I.	*C* (1), R ♀ ♂ ...	34	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	0	1	1	2	1	—	—	—	—	—	—	6	1
XVA.	*A*, R × *C* (2), W ...	35	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	0	0	0	0	0	1	1	1	1	—	—	—	—	6	1
XVB.	*C* (2), W × *A*, R ...	35	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	1
XV.	*C* (2), W × *A*, R ...	35	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	0	0	0	1	0	0	1	1	1	—	—	—	6	2
XIV.	*A*, R × *C* (2), W ...	35	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4	1	0	1	0	0	2	0	1	—	—	—	—	—	9	5
V.	*C* (1), R × *B* (3), R	54	—	—	—	—	—	—	—	—	—	—	—	—	—	1	4	2	4	4	4	2	2	—	—	—	—	—	—	—	—	23	0
IXA.	*C* (1), R × *N* (2), R	54	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	0	3	—	—	—	—	—	—	—	—	—	—	—	—	4	0
II.	*C* (1), R × *B* (4), R	57	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	0	1	3	0	0	2	—	—	—	—	—	—	—	8	0
III.	*B* (4), R × *C* (1), R	57	—	—	—	—	—	—	—	—	—	—	—	1	0	2	0	0	0	1	0	0	0	1	—	—	—	—	—	—	—	5	0
XII.	*C* (3), R × *A*, R ...	62	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	0
XI.	*A*, R × *C* (3), R ...	62	—	—	—	—	—	—	—	—	—	—	—	2	0	0	0	3	1	0	1	0	0	0	4	0	1	—	—	—	—	12	5
IA.	*A*, R ♀ ♂	70	—	—	—	—	—	—	—	—	—	—	—	—	4	2	7	3	2	0	2	1	—	—	—	—	—	—	—	—	—	21	7
XXIX.	*A*, R × *N* (1), R ...	72	1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	0
XXV.	*B* (4), R × *A*, R ...	75	1	—	—	—	—	—	—	—	1	0	0	0	2	0	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	6	0
XXIV.	*A*, R × *B* (4), R ...	75	2	—	—	—	—	—	—	—	1	0	2	1	1	2	1	1	1	1	—	—	—	—	—	—	—	—	—	—	—	13	0
IV.	*B* (4), R ♀ ♂ ...	80	—	—	—	—	—	—	2	—	2	0	0	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5	0
	Totals	—	4	0	0	0	0	0	2	..	4	0	2	6	7	7	16	17	14	8	11	4	4	11	9	9	3	1	0	0	0	139	41

By reference to the horizontal line of totals at the bottom of the table it will be seen that among the purple offspring there are none under the first three units of the colour scale, 0–4, 5–9, 10–14. There would also appear to be discontinuity at the other end of the scale, but it is very doubtful as to how far reliance can be placed on the colour-scale where the depth of the fluid is considerable.

Taking the families altogether there were 139 purple plants and 41 white. The Mendelian expectation may be calculated for each family on making the following assumptions: purple colour dominant to white; *C* (2) pure recessive; *A*, *C* (1) and *C* (3) heterozygous dominants; *B* (3), *B* (4), *N* (1) and *N* (2) homozygous dominants.

Justification for these assumptions is given by the consistency with which the different parents act in different pairings. The sum totals of the calculated expectations for all the families are 131 purple and 49 white. It will be seen that these figures are in very fair agreement with the experimental result which is 139 purple and 41 white. This may be regarded as a typical Mendelian result.

A point of interest is to ascertain as to how far the various intensities of the colouration among the purple plants are inherited. It is possible that some observers would regard the different intensities of the purple colouration in a family as representing mere fluctuating variations which could not be inherited. According to the pure-line theory the different intensities of purple colouration occurring within the limits of a family resulting from self-fertilization cannot be inherited. It will be possible to test this theory in the generation of plants now being raised, but the present material cannot give a definite reply to the question.

An inspection of the above table (p. 313) will show that as we pass to the higher mid-parental colours there is a *gradual* rise in the colouration of the families. This implies that the various shades of purple are not distinct types, but that they can blend perfectly with one another.

The fact that in closely similar matings the families raised show a regular progressive rise in the intensity of colouration with the rise in the mid-parental colouration does not favour the expectation that any support for the pure-line theory will be forthcoming from the present investigation.

It is desired to find the correlation with respect to the intensity of colour between the purple parents and the purple offspring. For this purpose, in order to have strictly homogeneous material, the parents should be homozygous dominants. The material will not permit such a calculation. The purple plants of the different families include both homozygous dominants and heterozygous dominants. Now the homozygous dominants are found to be darker in colour than the heterozygous dominants, and so in any family consisting of an equal number of heterozygous and homozygous dominants we may approximately divide the individuals into the two groups by regarding the darker half as homozygous and the lighter half as heterozygous dominant. In this way we can form a correlation table between the parents and the heterozygous or the homozygous offspring.

Using the symbols introduced by Prof. Bateson, namely RR homozygous recessive, DR heterozygous dominant, DD homozygous dominant, we have, on crossing individuals of the different gametic constitutions, the following theoretical results:

$RR \times DR = 1\ DR$ (light purple) + $1\ RR$ (white),

$DR \times DR = 1\ DD$ (dark purple) + $2\ DR$ (light purple) + RR (white),

$DR \times DD = 1\ DD$ (dark purple) + $1\ DR$ (light purple).

In calculating the correlation it is advisable to use only parents and offspring of similar gametic constitution; thus, DR parents with DR offspring, or DD parents and DD offspring. RR parents and RR offspring being white are excluded.

To accomplish this it is necessary in any given family, obtained by crossing together RR, DR and DD parents, to sort out the purple offspring into DR and DD individuals. To do this with complete certainty it would be necessary to self-fertilize each individual and obtain families. The labour involved is too great to be undertaken lightly. Owing to the fact that DR offspring are paler than DD offspring we may approximately divide the families into the two groups of individuals by taking the darker half (in the case of DR and DD parentage) or the darker one-third (in the case of DR and DR parentage) of the plants as being DD individuals in constitution and the remainder as being DR individuals.

Thus for example:

Parentage	Family under Colour-Scale (see preceding table)
VIII. $C\,(1) \times C\,(2) = DR \times RR$	1, 2, 4 (DR), RR (excluded from table)
I. $C\,(1) \times C\,(1) = DR \times DR$	1, 0, 1, 1, 2, 1 ($\frac{1}{3}\ DD$ and $\frac{2}{3}\ DR$) = 1, 0, 1 DD + 1, 2, 1 DR
IXA. $C\,(1) \times N\,(2) = DR \times DD$	1, 0, 3 ($\frac{1}{2}\ DD$ and $\frac{1}{2}\ DR$) = 1, 0, 1 DD + 2 DR

In this way a correlation table can be made with fair accuracy between DR parent and DR offspring. The paucity of the material, however, prevents much reliance being placed on the actual figures obtained. At most the figures can give only a rough indication of the strength of inheritance.

When a heterozygous purple plant is crossed with a white plant (recessive) both purple and white offspring are produced. The coloured offspring, however, tend to be paler than the coloured parent. Thus the colour of the coloured

DR Female Parent and DR Offspring.

Female parent, general colouration	2	6	8	8	3	3	12	5	5	12	8	2	1	1	Totals
30—34	—	4	3	5	3	2	8	4	—	—	—	—	—	—	29
35—39	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
40—44	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
45—49	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
50—54	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
55—59	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
60—64	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
65—69	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
70—74	2	2	5	3	0	1	4	1	5	11	8	2	1	1	46
Offspring, general colouration	*20—24*	*25—29*	*30—34*	*35—39*	*40—44*	*45—49*	*50—54*	*55—59*	*60—64*	*65—69*	*70—74*	*75—79*	*80—84*	*85—89*	76

DR Male Parent and DR Offspring.

Male parent, general colouration	1	2	3	6	5	2	1	3	1	5	6	7	1	1	Totals
30—34	—	—	2	2	5	1	0	0	1	0	0	0	1	—	12
35—39	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
40—44	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
45—49	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
50—54	—	1	0	4	0	0	0	1	0	1	1	—	—	—	8
55—59	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
60—64	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
65—69	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
70—74	1	1	1	0	0	1	1	2	0	4	5	7	0	1	24
Offspring, general colouration	*15—19*	*20—24*	*25—29*	*30—34*	*35—39*	*40—44*	*45—49*	*50—54*	*55—59*	*60—64*	*65—69*	*70—74*	*75—79*	*80—84*	44

parent has been diluted by crossing with the colourless parent. Since such blending occurs it will be of interest to find the correlation between the mid-parental colour and the colour of the *DR* offspring. The *DD* and *DR* offspring were separated from one another by the method above described.

Mid-Parent and DR Offspring (including self-fertilized Families).

Mid-parent, general colouration	1	3	9	9	11	4	3	10	6	9	12	9	3	2	1	Totals
15—19	—	—	6	3	3	—	—	—	—	—	—	—	—	—	—	12
20—24	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
25—29	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
30—34	—	—	—	1	2	1	—	—	—	—	—	—	—	—	—	4
35—39	1	2	3	1	3	1	0	1	0	3	5	1	—	—	—	21
40—44	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
45—49	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
50—54	—	—	—	—	—	2	2	4	4	2	—	—	—	—	—	14
55—59	—	—	—	—	3	0	0	2	1	0	0	0	1	—	—	7
60—64	—	1	0	4	0	0	0	1	0	1	2	—	—	—	—	9
65—69	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
70—74	—	—	—	—	—	—	1	2	0	2	3	6	0	0	1	15
75—79	—	—	—	—	—	—	—	—	1	1	2	2	2	2	—	10
Offspring, general colouration	*15—19*	*20—24*	*25—29*	*30—34*	*35—39*	*40—44*	*45—49*	*50—54*	*55—59*	*60—64*	*65—69*	*70—74*	*75—79*	*80—84*	*85—89*	92

The general results may be summarized as follows:

Relationship	No. of mid-parents or parents	No. of *DR* off-spring	Standard deviation of parents	Standard deviation of *DR* offspring	Co-efficient of corre-lation
Mid-parent colour and colour of *DR* offspring (including self-fertilized families)	17	92	19·280	17·520	·675
Mid-parent colour and colour of *DR* offspring (excluding self-fertilized families)	15	74	18·514	17·800	·587
Male parent (*RD*) colour and colour of *DR* off-spring (including self-fertilized families)	8	44	17·249	18·336	·471
Female parent (*DR*) colour and colour of *DR* offspring (including self-fertilized families)	12	76	19·351	16·563	·415

We see from the above that the strength of inheritance, with respect to the intensity of colouration, between individuals of similar gametic constitution follows a similar statistical law to that exhibited by characters where discontinuity cannot be detected, where complete blending occurs and no trace of segregation can be found.

It might have been anticipated by some that the varying intensities of the general colouration among individuals of similar gametic constitution were casual, fluctuating variations which could not be inherited. The above results point to the view that small variations of this nature are inheritable, and consequently they can be of importance in the evolution of species.

5. *The Ratio of the Breadth to the Length of the Corolla.*

The parent plants differed markedly in the relative width of the corolla. *C* (1), *C* (2) and *N* (1) were wide, *B* (1), *B* (3) and *B* (4) were narrow, and the remainder were of medium width. As in the case of the amount of spotting of the corolla, no distinct discontinuity in the variation of the character could be noticed either in the parents or offspring. As far as could be judged the variation was a continuous one, or one with such small steps as practically to be equivalent to the same.

The following tables of correlation are given as records:

Ratio, Mid-parent and Offspring (excluding self-fertilized Families).

Mid-parent. $\frac{\text{Breadth}}{\text{Length}}1000$	3	4	11	28	39	47	77	80	102	103	80	61	45	30	31	9	7	6	7	3	1	Totals
503—516	3	0	3	14	17	11	16	14	15	11	7	3	—	—	—	—	—	—	—	—	—	114
517—530	—	2	3	3	0	1	0	2	1	—	—	—	—	—	—	—	—	—	—	—	—	12
531—544	—	—	—	1	3	5	10	10	23	24	11	14	2	3	2	—	—	—	—	—	—	108
545—558	—	2	5	10	19	29	44	47	54	53	51	30	25	10	9	4	1	2	1	—	—	396
559—572	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
573—586	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
587—600	—	—	—	—	—	1	7	5	5	10	10	9	16	15	18	4	6	4	6	3	1	120
601—614	—	—	—	—	—	—	—	2	4	5	1	5	2	2	2	1	—	—	—	—	—	24
Offspring. $\frac{\text{Breadth}}{\text{Length}}1000$	*419—432*	*433—446*	*447—460*	*461—474*	*475—488*	*489—502*	*503—516*	*517—530*	*531—544*	*545—558*	*559—572*	*573—586*	*587—600*	*601—614*	*615—628*	*629—642*	*643—656*	*657—670*	*671—684*	*685—698*	*699—712*	774

Ratio, Male parent and Offspring (including self-fertilized Families).

Male parent. $\frac{\text{Breadth}}{\text{Length}}1000$	1	9	27	58	86	121	145	164	147	109	74	28	9	16	5	1	Totals
480—499	—	3	7	20	32	18	31	19	15	11	—	—	—	—	—	—	156
500—519	—	—	1	2	14	24	17	34	24	12	8	1	0	1	—	—	138
520—539	1	4	15	25	23	30	48	53	48	29	13	5	2	2	—	—	298
540—559	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
560—579	—	2	4	8	13	36	34	31	35	25	28	18	6	10	2	—	252
580—599	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
600—619	—	—	—	3	4	13	15	27	25	32	25	4	1	3	3	1	156
Offspring. $\frac{\text{Breadth}}{\text{Length}}1000$	*400—419*	*420—439*	*440—459*	*460—479*	*480—499*	*500—519*	*520—539*	*540—559*	*560—579*	*580—599*	*600—619*	*620—639*	*640—659*	*660—679*	*680—699*	*700—719*	1000

Relationship	No. of parents	No. of off-spring	No. of flowers	Standard deviation of parents	Standard deviation of offspring	Coefficient of correlation
Mid-parent and offspring (including self-fertilized families)	15	176	1000	29·626	49·220	·554
Mid-parent and offspring (excluding self-fertilized families)	11	127	774	25·886	47·164	·509
Male parent and offspring (including self-fertilized families)	15	176	1000	38·783	49·236	·341
Male parent and offspring (excluding self-fertilized families)	11	127	774	38·898	47·358	·300

The parental correlation is thus low as in the case of the area covered by spots. Such a condition increases the probable error of the numerical results, and many more parents would be necessary in order to obtain more reliable figures. Nevertheless it may be noted that the statistical results obtained with the various characters are very fairly consistent with one another and the parental inheritance of the characters investigated tends to be somewhat low.

III. *The Variability of the Race and of the Families.*

According to the pure-line theory the variability within a family consisting of individuals of similar gametic constitution and arising by self-fertilization of the parent should be small, supposing that no new mutation should arise, and it is of such a nature that it is incapable of being transmitted by inheritance. Selection within such a family would not be effective. In other words, families obtained by self-fertilizing individuals at the two ends of the range of variation would exhibit the same mean. The material at present to hand will not throw direct light on the question, but indirectly it lends no support to the theory.

The comparative variabilities of the race and of the individual families are given in the table on p. 320. The colour of the different flowers on an axis was found to be essentially uniform; but in the two other characters there was some variability among the flowers on the same axis. Therefore in these two characters (the area covered by spots, and the ratio of the width to the length of the corolla) the mean of the six basal flowers of the axis was taken as expressive of the nature of the plant.

Thus the mean variability of a family is 67 % of the variability of the whole race. Owing to the fact that the majority of these families arose through cross-fertilization this large family variability is not a proof against the pure-line theory. It may be stated here that with the four self-fertilized families raised there was some tendency for a reduction in the variability, but nevertheless it remained very considerable.

The average percentage variability of the self-fertilized families is 50 % of the variability of the race. This is to be compared with 67 % for cross-fertilized families. The number of the families is, however, too few for drawing any absolute conclusions. With such a high variability in these families it would appear improbable that the families obtained by self-fertilizing the individuals at the extremes of the range would both exhibit the same mean. It may be added that in the parthenogenetic aphis (*Hyalopterus*) it has been shown that the variability of the family amounted to 60 % of that of the race.

Character	No. of individuals in generation	Standard deviation (S.D.) of generation	No. of families with six or more individuals	No. of individuals	Mean standard deviation (S.D.) of the families	$\frac{\text{S.D. fam.}}{\text{S.D. gen.}} \times 100$
Intensity of general colouration (*DR* individuals)	90	3·490	8	69	2·349	67
Ratio $\frac{\text{Breadth}}{\text{Length}}$ corolla	172	3·324	13	154	2·414	73
Spotting	178	1·816	13	160	1·000	61
					Mean of percentages ...	67

It must be remembered that Johanessen's experiments have been shown by Karl Pearson to be inconclusive with reference to the pure-line theory. The size of the different seeds produced by any given individual plant varies greatly, and there is no inherent reason to suppose that the offspring obtained from the small seeds and large seeds taken from the same plant would exhibit any difference in the size of their seeds. It might reasonably be supposed that a small seed and a large seed from the same plant would both inherit the average character of the parent irrespective of their actual size. One might as well expect that the plants obtained by budding from a small begonia leaf would have smaller leaves than those obtained from a larger leaf of the same plant.

IV. *Fraternal Correlation.*

The likeness of brothers to one another has been investigated, and the relationship was studied after the method adopted by Prof. Pearson. He takes each individual of a family and compares its deviation from the mean with that of every other member of the family. These combinations can be plotted out in a correlation table in a similar manner to the relationship of offspring and parent. The coefficient of correlation for all possible pairs of brethren will be a measure of their resemblance, just as the standard deviation is a measure of their dissimilarity.

The fraternal correlation was calculated for (1) the intensity of the general colouration of the purple flowers, (2) the area of the spots, (3) the ratio of the width to the length of the corolla.

For the intensity of the general colouration only offspring having a similar gametic constitution were taken. Heterozygous individuals, or those which are supposed to produce two kinds of gametes, were used. With regard to the ratio and the area covered by the spots the nature of the individual was determined by taking the mean of the varying values obtained from six different flowers from the flowering axis. This procedure was unnecessary in the case of the colour, as the individual flowers on the same axis did not differ from one another perceptibly.

As a record the fraternal correlation table for spotting is herewith given:

Percentage spotting, 1st brother	281	378	425	596	300	168	147	21	16	4	Totals
0—3	64	98	70	47	2	—	—	—	—	—	281
4—7	98	138	73	65	4	—	—	—	—	—	378
8—11	70	73	124	91	34	20	8	1	4	—	425
12—15	47	65	91	178	107	53	48	5	2	—	596
16—19	2	4	34	107	64	49	30	8	2	—	300
20—23	—	—	20	53	49	20	19	4	2	1	168
24—27	—	—	8	48	30	19	36	1	4	1	147
28—31	—	—	1	5	8	4	1	0	1	1	21
32—35	—	—	4	2	2	2	4	1	0	1	16
36—39	—	—	—	—	—	1	1	1	1	—	4
Percentage of spotting, 2nd brother	*0—3*	*4—7*	*8—11*	*12—15*	*16—19*	*20—23*	*24—27*	*28—31*	*32—35*	*36—39*	2336

The comparative results obtained for parental correlation and fraternal correlation are shown in the following table:

Character	Parental			Fraternal	
	No. of families	No. of offspring	Correlation	No. of pairs of brothers	Correlation
Intensity of purple colouration of corolla	14	98	·47	318	·52
Area of spots	19	178	·53	1168	·61
Ratio $\frac{\text{Width}}{\text{Length}}$ corolla ...	19	170	·37	1032	·29
Mean of correlations ...			·46	—	·47

The parental correlations given above differ somewhat from those given in an earlier part of the paper owing to the fact that the mean of the character, as it occurred in the six flowers, was taken instead of using the individual flowers of the plant.

The means of the coefficients of the three characters are ·46 and ·47 for the parental and fraternal correlation respectively. These results are not very divergent from those obtained by Prof. Pearson from very various sources; he gives the average parental correlation as about ·48.

V. *Summary.*

The data at present available were derived from crossing in various ways 10 cultivated foxglove plants with different characteristics, namely: crowned or peloric, not crowned, purple of different intensities, white, relatively wide and relatively narrow corolla, etc. There were 19 families; each family being raised from the seed of a single capsule. Altogether 188 offspring which flowered were obtained.

The nature of the inheritance of the following characters was studied: (1) peloric or crown character, (2) the colour of the spots, (3) the relative area covered by the spots, (4) the colour of the corolla and the degree of intensity of the purple colouration of the general surface, (5) the ratio of the width to the length of the corolla.

The numerical results obtained are given below:

Character	Mendelian Inheritance (parent and offspring)				Statistical Relationship (parent and offspring)		
	Observed		Theoretical Expectation		Coefficient of Correlation		
					Foxgloves		Mean of coefficients from various sources
	No. with character present	No. with character absent	No. with character present	No. with character absent	Including self-fertilized families	Excluding self-fertilized families	
Crown	83	104	83	104	—	—	·45
Brown spots	23	69	23	69	—	—	
Percentage area covered by spots	—	—	—	—	·43	·32	
General purple colouration	41	139	49	131	·47	·41	
Ratio of breadth to length of corolla	—	—	—	—	·34	·30	
Totals	147	312	155	304	·41	·34	
					·38		

(1) The peloric character may be regarded as arising by the suppression of certain internodes of the flower-axis and the concurrent fusion of the flower-buds resulting in a more or less regular cup or saucer-shaped flower at the top of the

axis, which matures before the ordinary zygomorphic flowers. This variety is known as *monstrosa*.

When a peloric plant is crossed with a non-peloric plant the offspring are generally either completely peloric or completely non-peloric. There is no marked tendency for the offspring to be intermediate in character. Notwithstanding this fact the character is capable of dilution, since in the case of one family arising from such a cross nearly all of the offspring were more or less intermediate in nature. Certain of the internodes were much reduced, but not entirely suppressed, and the flowers involved arose in the form of petaloid blades which matured before the lower normal flowers, as in the case of a typical crowned plant.

The gametic nature of the parents, as to whether they were pure or impure in the Mendelian sense with reference to the peloric character, could be judged either by raising families obtained by self-fertilization or by their action in the various crosses that were made. Families derived from self-fertilization from four parents were successfully raised, and in the case of the remaining parents the marked consistency in their behaviour in the different crosses indicated clearly as to whether they were homozygous or heterozygous.

The non-peloric condition was dominant.

On dividing the offspring into peloric plants (including those which showed any tendency towards a peloric condition) and non-peloric plants, the numbers were 83 peloric and 104 non-peloric. Calculating the Mendelian expectation for each family and adding the results the figures obtained were identically the same.

(2) The results of reciprocal crosses appeared to be the same; that is the families obtained from ♀ *A* × ♂ *B* did not differ in any constant manner from those derived from ♀ *B* × ♂ *A*.

(3) The results obtained by using pollen from the crown flower, the ordinary zygomorphic flower and of flowers with abnormally split open corolla did not appear to differ from one another in the least.

(4) The colour of the seeds is very definitely related to the colour of the flowers of the seed-plant. Plants with white flowers had very pale buff-coloured seeds, those with light purple corollas had light brown seeds, while those with dark purple corollas had very dark brown seeds. As might be expected, a plant with white flowers crossed with pollen from a plant with dark purple flowers produced pale coloured seeds, since the coloured testa of the seed is formed by the seed-plant and is not a result of fertilization.

This observation emphasizes the fact that great caution is necessary in regarding characters as being independent of one another. It is often stated that an organism may be pure bred in one character and not in another; but probably this is true only in a very limited sense. De Vries holds that a change in any one character involves some change in the constitution of the organism as a whole.

(5) In plants in which the general internal and external surfaces of the corolla are white, the spots which occur may be purple or brown. Sometimes spots were nearly absent and the corolla appeared to be practically pure white, but when such was the case some sign of brown spots could be found on some few of the flowers. An obviously intermediate condition between purple spots and brown spots was rare, and judged by ordinary inspection the spots were either purple *or* brown, seldom brownish purple. The brown colouring matter may be regarded as a decomposition product of the purple anthocyanin. In plants in which the corolla was coloured purple over the general surface the spots were never entirely brown. Thus it would appear from the present observations that the causes which lead to the total decomposition of the anthocyanin in the purple spots of otherwise white flowers are never wholly effective in flowers with purple corolla. Accordingly although there is some independence between the two characters, the colouration of the spots and the colouration of the general surface of the corolla, yet nevertheless there seems to be a certain relationship between the two, since the spots were never found to be completely brown unless the general surface of the corolla was white. A feature of unknown significance may be mentioned, the cells of the spots containing the brown colouring matter were found to be densely crowded with typical starch grains, while the cells of the purple spots containing the anthocyanin in solution in the cell-sap contained little or no starch.

Purple spots were dominant and brown spots recessive.

The observed results in the different families totalled 69 plants with purple spots and 23 with brown spots. The Mendelian expectation calculated for each family and added together yielded exactly the same figures. Since completely brown spots were never found in coloured flowers, only those families in which plants with white flowers occurred were included in this calculation.

(6) With regard to the general colouration of the corolla it was found that some of the parents were heterozygous and others homozygous. The results of the crossings were distinctly Mendelian in character. Purple was dominant and white recessive. There were 41 white and 139 purple offspring; the theoretical expectation, calculated for each family and added together, was 49 white and 131 purple.

A point of considerable interest was noticed, namely that in dealing with parents and offspring of similar gametic constitution (i.e. heterozygous and homozygous dominant) the inheritance of the intensity of colouration followed the usual statistical laws. A light purple plant crossed with a dark purple plant might give some perfectly white offspring, but those offspring that were coloured, if of the same gametic character as the parent, tended to be intermediate in shade between the two parents. Thus the intensity of colouration is not a mere fluctuating uninheritable character, but its inheritance follows the usual laws found by statisticians to hold in the case of characters where a Mendelian relationship cannot be detected. The parental coefficient of correlation between heterozygous dominant parents and heterozygous dominant offspring including self-fertilized families was ·47 for male parent and ·41 for female parent. These figures are not far removed from Prof. Pearson's mean coefficient ·45.

(7) The remaining two characters dealt with, viz. the percentage area covered by the spots and the relative width of the corolla, exhibited no discontinuity in their inheritance, and consequently a Mendelian relationship could not be detected. The parental coefficients of correlation for the spotting, including and excluding the self-fertilized families, were ·43 and ·32 respectively, and for the width ·34 and ·30. The mean of all four coefficients is ·35 which has to be compared with Prof. Karl Pearson's mean coefficient of ·45 derived from various sources. Owing to the fewness in the number of parents the probable errors of the foxglove coefficients of correlation are large, but nevertheless there appears to be a distinct tendency for the parental correlation to be low. Similarly Prof. Karl Pearson found parental inheritance to be low in the Shirley Poppy: the mean value being ·33.

(8) The mean of the seven mid-parental correlations, calculated for the crown character, the spots, the general colouration and width of the corolla, is ·57 and the mean of the six parental correlations is ·38.

(9) The experiment at the present stage cannot throw any direct light on the pure-line theory. The variability which occurred among cross-fertilized families amounted to 67 % of the variability of the race, and among several self-fertilized families it was 50 %. With such a high variability it would appear to be unlikely that the families, raised by self-fertilizing the two brethren at the two ends of the range of variation of the families, would exhibit the same mean.

(10) The resemblance of the individuals of a family to one another with reference to several characters was calculated, and the mean of these fraternal correlations was found to be ·47. Prof. Karl Pearson gives a mean value of ·5 or a little more for the fraternal correlation of a series of diverse characters.

VI. *Theoretical Conclusions.*

We thus see that among the characters examined we find typical Mendelian and typical non-Mendelian results, and the relationship between the two types of inheritance is by no means clear. It has been said that Mendelian inheritance occurs in the case of characters which do not mix, but this view is not warranted by the present results. For example, the peloric character can be diluted by crossing with the ordinary type, but such dilution does not usually take place; also, the general purple colouration of the corolla is diluted when a dark parent is crossed with a pale parent; but, nevertheless, a very pale general colouration is exceedingly rare, and the flower is as a rule either distinctly purple (although perhaps rather pale) or white. Thus we have discontinuity even when characters are capable of dilution.

From the experimental data which have been accumulated during the last decade there is no doubt that segregation of characters may occur in crossing hybrids, and by defining the unit-character in a suitable manner the familiar ratios so well known to Mendelians do appear in the case of certain characters. The above expression "defining the unit-character in a suitable manner" is well illustrated in the case of the general colouration of the foxglove flowers. Provided we define a "purple flower" as meaning a flower in which the corolla is more or less uniformly tinted all over we obtain a simple Mendelian relationship; but many of the white flowers, on being closely scrutinized, were seen to possess an excessively faint tinge of purple over small isolated areas, the remainder of the corolla being dead white. If these plants had been included under the category of "purple flowers" no Mendelian relationship would be apparent. There is, however, a certain justification for excluding such from the category of "purple flowers," since there is a distinct gap in the series between the flowers which are tinted faintly purple all over and the white flowers with an extremely slight blush of purple over small irregularly placed areas.

A similar relation occurs in connection with the brown spots. Provided the spots are examined with the naked eye they are very generally either brown or purple; brownish purple spots were seldom observed, and a simple Mendelian relationship occurs, brown spots being recessive. If, however, the purple spots are examined under the microscope the distinction between the two kinds of spots is seen to be not so sharp, since a certain amount of decomposition product may be present, although not as a general rule in sufficient quantity to give a spot a brownish purple appearance to the naked eye. Here, again, we have a distinct gap in the series; it occurs between a small amount of decomposition and total decomposition. A strict Mendelian might hold that in the first case the slight tinge of purple occurring in the white flowers was a character distinct from the purple colouration of the general surface of the corolla, and was controlled by a different determinant or group of determinants; but such a supposition could scarcely be applied to explain the occurrence of the gap between the small amount of decomposition and the total decomposition of the anthocyanin of the spots. By presupposing the existence of many factors controlling a character and the linkage of factors it may be possible to express in Mendelian terms almost any experimental result that may be obtained, but it must be confessed that there is an artificiality about the process which does not inspire confidence in the interpretation.

With regard to the peloric character, in order to obtain a simple Mendelian relationship, it is necessary to include in the peloric category all plants which exhibited any distinct tendency towards the peloric condition. There is a gap in the series between flower-axes which exhibit some shortening of the internodes of the upper flowers and a marked shortening resulting in a distinct abnormality; and it is this gap in the series which gives the clue as to where the line of division should be drawn between peloric plants and non-peloric plants.

The question arises as to how far a so-called unit-character is capable of change. In the course of evolution any given character must of necessity have arisen in some way from some other character, and according to de Vries it arose by a sudden mutation. It is clear, however, that the somatic nature of an individual tends to reflect its gametic constitution, and obversely any change in the somatic nature of an individual tends to imply a change in the determinants of the germ-cells; and therefore the factor or determinant, say, of the general colouration of the corolla in a pale purple heterozygous dominant foxglove differs to some extent from that in a darker purple heterozygous plant: and the offspring of these two plants, if crossed with the same pollen, and bred under the same environment (in its widest sense) would not be the same. I have deduced some evidence in this investigation to show that the relationship of parents and offspring with respect to the fluctuations in the intensity of expression of a Mendelian character is similar to that of a non-Mendelian character provided the parents and offspring are of the same class, i.e. all heterozygous or all homozygous. It follows from this that a slight variation in the nature of the factor in the parent is accompanied by a slight somatic variation in the offspring and will be subjected to natural selection.

It appears to be premature to attempt to visualize these complicated phenomena in the nuclear behaviour of the sexual cells, although Morgan and others* have made ingenious attempts at so doing. In any such attempt it seems necessary to assume the existence of factors, determinants, plasms or something, no matter what it is called, which bears the heredity influence. Speaking in these mechanical terms it may be suggested that characters which exhibit no discontinuity in their inheritance, i.e. show no Mendelian relationships, are transmitted by a determinant or a system of determinants uniformly distributed through the chromosomes, and perhaps also through the cytoplasm of the germinal cells; while those characters which do exhibit Mendelian relationships depend on determinants which are localized in the chromosomes only, and possibly in definite places in the chromosomes. Morgan has recently referred to this last conception as the linear arrangement of factors in the chromosome.

In both cases, whether localized or not, the determinants controlling a particular character are variable in nature and are therefore subject to gradual modification by the natural selection of the individuals which bear the genital cells.

Mendelian inheritance in its original simple form, as it sometimes appears in the crossing of varieties, would seem to be, so to speak, an incidental phenomenon arising through the nuclear changes and the localization of the determinants, and it is open to question whether it has had any marked influence in the evolution of species. It is conceivable that evolution has taken place in spite of, rather than by the aid of, such Mendelian inheritance. The hereditary influence of the parent varies slightly through changes from unknown causes in the nature of the determinants, and the small resulting somatic variations in the offspring would appear to constitute the chief means by which evolution may slowly proceed and adaptation to the environment can be effected.

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* Morgan, T. H. and others, *The Mechanism of Mendelian Heredity*, New York, 1915.

ON THE DISTRIBUTION OF THE CORRELATION CO-EFFICIENT IN SMALL SAMPLES. APPENDIX II TO THE PAPERS OF "STUDENT" AND R. A. FISHER.

A COOPERATIVE STUDY

By H. E. SOPER, A. W. YOUNG, B. M. CAVE, A. LEE
AND K. PEARSON

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(1) *Introductory.* In a paper of 1908* "Student" dealt experimentally with the distribution of the correlation coefficient of small samples, and gave empirical curves—in particular for the case of zero correlation in the sampled population—which have proved remarkably exact. The problem was next considered in 1913 by H. E. Soper† who obtained the mean correlation and the standard deviation of the distribution of correlations to second approximations. Of the

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* *Biometrika*, Vol. VI. p. 302 *et seq.*

† *Biometrika*, Vol. IX. p. 91 *et seq.*

formulae he gives for $\bar{r}$ and σ_r of the distribution of the correlation r in samples of n from a population of correlation ρ, we have found in practice the most exact are*

$$\bar{r} = \rho \left\{1 - \frac{1}{2n}(1-\rho^2) - \frac{3}{8n^2}(1-\rho^2)(1+3\rho^2)\right\} \quad \text{..................(i)}$$

and
$$\sigma_r^{\,2} = \frac{(1-\rho^2)^2}{n}\left(1 + \frac{1 + 5{\cdot}5\rho^2}{n}\right) \quad \text{...(ii).}$$

Soper also by assuming a Pearson curve of limited range + 1 to − 1 of type

$$y = y_0\left(1 - \frac{x}{a_1}\right)^{m_1}\left(1 + \frac{x}{a_2}\right)^{m_2}$$

deduces the modal value $\check{r}$ of r as approximately

$$\check{r} = \bar{r}\,\frac{\{1 - (\sigma_r^{\,2} + \bar{r}^2)\}}{1 + 2\bar{r}^2 - 3(\sigma_r^{\,2} + \bar{r}^2)} \quad \text{............................(iii),}$$

so that $\check{r}$ would be determined from a knowledge of $\bar{r}$ and $\sigma_r^{\,2} + \bar{r}^2$.

The next step was taken by R. A. Fisher who gave in 1915[†] the actual frequency distribution of r, namely the curve

$$y_n = f_n(r) = \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi(n-3)!}(1-r^2)^{\frac{n-4}{2}}\frac{d^{n-2}}{d(r\rho)^{n-2}}\left(\frac{\cos^{-1}(-\rho r)}{\sqrt{1-\rho^2 r^2}}\right) \quad \text{......(iv).}$$

Except for very low values of n this expression for y_n does not provide a formula from which the ordinates of the frequency curve for r can be readily determined, and as the problem was left by Fisher there were no rapid means of numerically determining either $\bar{r}$ or $\check{r}$ or again $\sigma_r^{\,2}$.

Clearly in order to determine the approach to Soper's approximations, and ultimately to the normal curve as n increases we require expressions for the moment coefficients of (iv), and further for practical purposes we require to table the ordinates of (iv) in the region for which n is too small for Soper's formulae to provide adequate approximations. These are the aims of the present paper. It is only fair to state that the arithmetic involved has been of the most strenuous kind and has needed months of hard work on the part of the computers engaged[‡]. On the other hand the algebra has often been of a most interesting and suggestive character.

(2) *On Properties of the Function* $U = \cos^{-1}(-x)/\sqrt{1-x^2}$.

We have
$$\frac{dU}{dx} = \frac{1}{1-x^2} + \frac{x\cos^{-1}(-x)}{(1-x^2)^{\frac{3}{2}}},$$

or
$$(1-x^2)\frac{dU}{dx} = 1 + xU.$$

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* See *loc. cit.* pp. 105 and 107.

[†] *Biometrika*, Vol. x. p. 507 *et seq.*

[‡] Besides those whose names are given under the title, we have to thank I. Horwitz for some calculating aid, Ethel M. Elderton and D. Heron for occasional assistance, especially in the experimental part of the work, and lastly but very far from least we have to acknowledge the untiring work of H. Gertrude Jones and Adelaide G. Davin in the construction of the models the beauty and accuracy of which are not more than suggested in the plates.

Apply Leibnitz's Theorem and we have

$$(1-x^2)\frac{d^nU}{dx^n}-2x(n-1)\frac{d^{n-1}U}{dx^{n-1}}-2\frac{(n-1)(n-2)}{1\,.\,2}\frac{d^{n-2}U}{dx^{n-2}}=x\frac{d^{n-1}U}{dx^{n-1}}+(n-1)\frac{d^{n-2}U}{dx^{n-2}},$$

or, $$(1-x^2)\frac{d^nU}{dx^n}-x(2n-1)\frac{d^{n-1}U}{dx^{n-1}}-(n-1)^2\frac{d^{n-2}U}{dx^{n-2}}=0 \quad \text{.........(v).}$$

Put $x=0$ and we have

$$\left(\frac{d^nU}{dx^n}\right)_0=(n-1)^2\left(\frac{d^{n-2}U}{dx^{n-2}}\right)_0,$$

but clearly $U_0=\frac{1}{2}\pi$ and $(dU/dx)_0=1$.

Hence by Maclaurin's Theorem

$$\frac{\cos^{-1}(-x)}{\sqrt{1-x^2}}=\frac{\pi}{2}\left(1+\frac{1^2}{2\,!}x^2+\frac{3^2\,.\,1^2}{4\,!}x^4+\ldots+\frac{(2s-1)^2(2s-3)^2\ldots1^2}{(2s)\,!}x^{2s}+\ldots\right)$$
$$+\left(x+\frac{2^2}{3\,!}x^3+\frac{4^2\,.\,2^2}{5\,!}x^5+\ldots+\frac{(2s)^2(2s-2)^2\ldots2^2}{(2s+1)\,!}x^{2s+1}+\ldots\right)\ldots\text{(vi).}$$

We are now in a position to give the successive differentials of U which may be either even or odd. We have for the two cases

$$\frac{d^{2s}}{dx^{2s}}\left\{\frac{\cos^{-1}(-x)}{\sqrt{1-x^2}}\right\}$$
$$=\frac{\pi}{2}(2s-1)^2(2s-3)^2\ldots1^2\left\{1+\frac{(2s+1)^2}{2\,!}x^2+\frac{(2s+1)^2(2s+3)^2}{4\,!}x^4+\ldots\right\}$$
$$+(2s)^2(2s-2)^2\ldots2^2\left\{x+\frac{(2s+2)^2}{3\,!}x^3+\frac{(2s+2)^2(2s+4)^2}{5\,!}x^5+\ldots\right\}\ldots\text{(vii),}$$

$$\frac{d^{2s-1}}{dx^{2s-1}}\left\{\frac{\cos^{-1}(-x)}{\sqrt{1-x^2}}\right\}$$
$$=\frac{\pi}{2}(2s-1)^2(2s-3)^2\ldots1^2\left\{x+\frac{(2s+1)^2}{3\,!}x^3+\frac{(2s+1)^2(2s+3)^2}{5\,!}x^5+\ldots\right\}$$
$$+(2s-2)^2(2s-4)^2\ldots2^2\left\{1+\frac{(2s)^2}{2\,!}x^2+\frac{(2s)^2(2s+2)^2}{4\,!}x^4+\ldots\right\}\ldots\text{(vii)}^{\text{bis}},$$

the development of the several series being clear.

For calculation of y_{2s} or y_{2s-1} the above series are idle, just as they are when substituted in the equation for $dy_n/dr=0$ which gives $\check{r}$. They converge far too slowly to be of use for numerical evaluations. But as we shortly shall show, they are, after certain transformations, most valuable in determining the moment coefficients.

Now $$y_n=\frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi(n-3)\,!}(1-r^2)^{\frac{n-4}{2}}\frac{d^{n-2}U}{dx^{n-2}};$$

multiply (v) by $$(1-\rho^2)^{\frac{n+1}{2}}(1-r^2)^{\frac{n-2}{2}}\Big/(\pi(n-1)\,!),$$

and we have

$$(1-\rho^2r^2)\,y_{n+2}-\frac{\rho r\,(2n-1)\sqrt{1-\rho^2}\sqrt{1-r^2}}{n-1}\,y_{n+1}-\frac{(1-\rho^2)(1-r^2)}{(n-1)(n-2)}(n-1)^2\,y_n=0,$$

thus

$$y_{n+2}=\frac{2n-1}{n-1}\,\kappa_1 y_{n+1}+\frac{n-1}{n-2}\,\kappa_2 y_n \quad\text{.....................(viii).}$$

Here

$$\kappa_1=\frac{\rho r\sqrt{1-\rho^2}\sqrt{1-r^2}}{1-\rho^2r^2},\qquad \kappa_2=\frac{(1-\rho^2)(1-r^2)}{1-\rho^2r^2}$$

are constant for ρ and r given and thus (viii) enables us to deduce y_{n+2} from y_{n+1} and y_n for a given ρ and r. But by simple differentiation

$$\left.\begin{aligned} y_3&=\frac{1-\rho^2}{\pi\sqrt{1-r^2}}\left(\frac{1}{1-\rho^2r^2}+\frac{\rho r\cos^{-1}(-\rho r)}{(1-\rho^2r^2)^{\frac{3}{2}}}\right)\\ y_4&=\frac{(1-\rho^2)^{\frac{3}{2}}}{\pi}\left(\frac{3\rho r}{(1-\rho^2r^2)^2}+\frac{(1+2\rho^2r^2)\cos^{-1}(-\rho r)}{(1-\rho^2r^2)^{\frac{5}{2}}}\right)\end{aligned}\right\}\quad\text{.........(ix).}$$

Hence if y_3 and y_4 be calculated for a series of values of r and ρ all higher values may be reached by a repeated use of (viii). The values chosen were: ρ proceeding by ·1 from 0 to 1 and r proceeding by ·05 from -1 to $+1$.

The disadvantage of this method of calculating y_n is that, except by independent computing, there is no means of checking accuracy until all the ordinates have been deduced, and any mistake in y_n for a low value of n is perpetuated throughout the series. When all the ordinates have been found, say for $n = 25$, then the smoothness of these ordinates and the fact that they give the correct total area with a suitable graduation-formula will be checks on the accuracy of the whole system of ordinates. In this manner Table A, p. 379, was calculated.

Another method of approaching the value of y_n is of some advantage. We may take

$$y_n=\frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi\,(n-3)!}\,(1-r^2)^{\frac{n-4}{2}}\left(\frac{v_n}{(1-\rho^2r^2)^{n-2}}+\frac{u_n\cos^{-1}(-\rho r)}{(1-\rho^2r^2)^{\frac{2n-3}{2}}}\right)\quad\text{......(x),}$$

where v_n and u_n are functions of ρr in integer positive powers, and if we substitute in (viii) we obtain

$$\left.\begin{aligned} v_{n+2}&=(2n-1)\,\rho r v_{n+1}+(n-1)^2(1-\rho^2r^2)\,v_n\\ u_{n+2}&=(2n-1)\,\rho r u_{n+1}+(n-1)^2(1-\rho^2r^2)\,u_n\end{aligned}\right\}\quad\text{...............(xi).}$$

We may write y_n in the form

$$y_n=\frac{n-2}{(n-2)!}\,\frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi}\,(1-r^2)^{\frac{n-4}{2}}\,\frac{d^{n-2}}{d(\rho r)^{n-2}}\left\{\frac{\cos^{-1}(-\rho r)}{\sqrt{1-\rho^2r^2}}\right\},$$

or

$$y_2=(n-2)_{n=2}\,\frac{\sqrt{1-\rho^2}}{\pi\,(1-r^2)}\left(\frac{0}{(1-\rho^2r^2)^0}+\frac{\cos^{-1}(-\rho r)}{(1-\rho^2r^2)^{\frac{1}{3}}}\right).$$

Hence $v_2 = 0$ and $u_2 = 1$, while y_2 will vanish for all values of r, except $r = \pm 1$ owing to the factor $(n-2)_{n=2}$. Thus (ix) gives us

$$v_3 = 1, \qquad u_3 = \rho r,$$

whence by (xi)

$$v_4 = 3\rho r, \qquad u_4 = 1 + 2\rho^2 r^2,$$

$$v_5 = 4 + 11\rho^2 r^2, \qquad u_5 = \rho r\,(9 + 6\rho^2 r^2),$$

$$v_6 = \rho r\,(55 + 50\rho^2 r^2), \qquad u_6 = 9 + 72\rho^2 r^2 + 24\rho^4 r^4,$$

and the successive values can be rapidly calculated, much faster than by actually differentiating out (iv). It is, however, shortest to insert the numerical values of ρ and r in (xi) and deduce the v_n's and u_n's numerically in succession. (Table A was, however, in the present case deduced from (viii). We did this by direct calculation of the values of

$$\left(\frac{y_n}{n-2}\right)_{n=2} = \frac{\sqrt{1-\rho^2}}{\pi(1-r^2)}\,\frac{\cos^{-1}(-\rho r)}{(1-\rho^2 r^2)^{\frac{1}{2}}},$$

and y_3 in equation (ix). Equation (viii) then gave us the numerical values of y_4, y_5, etc. in succession.)

We may write (x) in the form

$$y_n = \frac{(1-\rho^2)^{\frac{3}{2}}}{\pi} V_n\,(v_n + u_n U) \text{..........................(xii)},$$

where
$$V_n = \frac{\{(1-\rho^2)(1-r^2)\}^{\frac{n-4}{2}}}{(n-3)!\,(1-\rho^2 r^2)^{n-2}} \quad \text{and} \quad U = \frac{\cos^{-1}(-\rho r)}{\sqrt{1-\rho^2 r^2}}.$$

Here V_n, v_n, u_n and U are symmetrical in ρ and r and accordingly ρ and r can be interchanged. The problem approached this way involves:

(*a*) calculating $(1-\rho^2)^{\frac{3}{2}}/\pi$ for various values of ρ;

(*b*) U for various values of ρr;

(*c*) V_n for various values of ρ, r and n;

(*d*) determining u_n and v_n in succession from (xi) for various values of ρr and n.

Lastly we may use the series for y_n to be given later (see Eqn. (xliii)) which develops y_n in inverse powers of $(n-1)$. Actually we have adopted (viii) for tabling the ordinates of the first 25 curve-series, and the last expansion for verification and higher cases.

(3) *On the Determination of the Moment Coefficients.* We shall next determine the value of the moment coefficients about $r = 0$, as origin, and shall deal with the even and odd coefficients independently. Let them be μ'_{2p} and μ'_{2p+1}. Clearly, the total area having been taken as unity:

$$\mu'_{2p} = \int_{-1}^{+1} y_n r^{2p}\, dr$$

$$= \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi\,(n-3)\,!}\int_{-1}^{+1}(1-r^2)^{\frac{n-4}{2}}\, r^{2p}\,\frac{d^{n-2}U}{d\,(\rho r)^{n-2}}\,dr.$$

Now

$$r^{2p} = r^{2p} - (r^2-1)^p + (-1)^p\,(1-r^2)^p$$

$$= pr^{2p-2} - \frac{p\,(p-1)}{2\,!}\,r^{2p-4} + \frac{p\,(p-1)\,(p-2)}{3\,!}\,r^{2p-6} - \dots + (-1)^p\,(1-r^2)^p.$$

Hence

$$\mu'_{2p} = p\mu'_{2p-2} - \frac{p(p-1)}{2\,!}\,\mu'_{2p-4} + \frac{p\,(p-1)\,(p-2)}{3\,!}\,\mu'_{2p-6} - \dots + (-1)^p\,\chi_{2p} \quad \text{(xiii)},$$

where $$\chi_{2p} = \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi\,(n-3)\,!}\int_{-1}^{+1}(1-r^2)^{\frac{n-4+2p}{2}}\,\frac{d^{n-2}U}{d\,(\rho r)^{n-2}}\,dr.$$

Thus using (vii)$^{\text{bis}}$ on the assumption that n is odd and remembering that odd powers of r will now disappear we reach:

$$\chi_{2p} = \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi\,(n-3)\,!}\,(n-3)^2\,(n-5)^2 \dots 2^2\left(i_0 + \frac{(n-1)^2}{2\,!}\,\rho^2 i_2 + \frac{(n+1)^2\,(n-1)^2}{4\,!}\,\rho^4 i_4 + \dots\right),$$

where $$i_{2m} = \int_{-1}^{+1}(1-r^2)^{\frac{n-4+2p}{2}}\,r^{2m}\,dr.$$

Now we may write $r = \cos\phi$, so that

$$i_{2m} = 2\int_0^{\frac{\pi}{2}} \sin^{n+2p-3}\phi\,\cos^{2m}\phi\,d\phi,$$

and we have $$i_{2m} = \frac{2m-1}{n+2p-3+2m}\,i_{2m-2} \quad \dots\dots\dots\dots\dots\dots\dots\dots\text{(xiv)}.$$

Thus

$$\chi_{2p} = \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi\,(n-3)\,!}\,(n-3)^2\,(n-5)^2 \dots 2^2 i_0\left(1 + \frac{(n-1)^2}{2\,!}\,\rho^2\,\frac{1}{n-1+2p}\right.$$
$$\left. + \frac{(n+1)^2\,(n-1)^2}{4\,!}\,\rho^4\,\frac{1}{n-1+2p}\,\frac{3}{n+1+2p} + \dots\right)$$

$$= \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi\,(n-3)\,!}\,(n-3)^2\,(n-5)^2 \dots 2^2 i_0 F\left(\frac{n-1}{2},\ \frac{n-1}{2},\ \frac{n-1}{2}+p,\ \rho^2\right),$$

where F as usual denotes the hypergeometrical series. But by a well-known transformation due to Euler

$$F\,(\alpha,\ \beta,\ \gamma,\ x) = (1-x)^{\gamma-\alpha-\beta}\,F\,(\gamma-\alpha,\ \gamma-\beta,\ \gamma,\ x) \quad \dots\dots\dots\text{(xv)},$$

and accordingly

$$F\left(\frac{n-1}{2},\ \frac{n-1}{2},\ \frac{n-1}{2}+p,\ \rho^2\right)=(1-\rho^2)^{p-\frac{n-1}{2}}F\left(p,\ p,\ \frac{n-1}{2}+p,\ \rho^2\right)$$

or
$$\chi_{2p}=\frac{(1-\rho^2)^p}{\pi\,(n-3)\,!}\,(n-3)^2\,(n-5)^2\,\dots\,2^2 i_0\,.\,F\left(p,\ p,\ \frac{n-1}{2}+p,\ \rho^2\right).$$

Now
$$i_0=2\int_0^{\frac{\pi}{2}}\sin^{n+2p-3}\phi\,d\phi=2q_{n+2p-2},\ \text{say},$$

and
$$2q_n=2\int_0^{\frac{\pi}{2}}\sin^{n-1}\phi\,d\phi$$

is known to be
$$=\frac{(n-2)\,(n-4)\,\dots\,1}{(n-1)\,(n-3)\,\dots\,2}\,\pi\ \ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\text{(xvi)},$$

if n be odd as supposed above. Thus finally we have

$$\chi_{2p}=(1-\rho^2)^p\,\frac{q_{n+2p-2}}{q_{n-2}}\,F\left(p,\ p,\ \frac{n-1}{2}+p,\ \rho^2\right)\ \dots\dots\dots\dots\text{(xvii)}.$$

A Table of $q_n=\int_0^{\frac{\pi}{2}}\sin^{n-1}\phi\,d\phi$ from $n=1$ to $n=105$ is given on p. 377 below.

Now (xvii) has only been proved for n odd. If n be even we must take the first series of (vii) and this gives

$$\chi_{2p}=\frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi\,(n-3)\,!}\,\frac{\pi}{2}\,(n-3)^2\,(n-5)^2\,\dots\dots\,1^2 i_0'\left(1+\frac{(n-1)^2}{2\,!}\rho^2\,\frac{1}{n+2p-1}\right.$$
$$\left.+\frac{(n+1)^2\,(n-1)^2}{4\,!}\rho^4\,\frac{1}{n+2p-1}\cdot\frac{3}{n+2p+1}+\dots\right),$$

where
$$i_0'=2\int_0^{\frac{\pi}{2}}\sin^{n+2p-3}\phi\,d\phi=2q_{n+2p-2}$$

and
$$2q_n=\frac{(n-2)\,(n-4)\,\dots\,2}{(n-1)\,(n-3)\,\dots\,3}\,.\,2\dots\dots\dots\dots\dots\dots\dots\dots\text{(xviii)},$$

since n is even. Thus

$$\chi_{2p}=(1-\rho^2)^{\frac{n-1}{2}}\,\frac{q_{n+2p-2}}{q_{n-2}}\,F\left(\frac{n-1}{2},\ \frac{n-1}{2},\ \frac{n-1}{2}+p,\ \rho^2\right)$$
$$=(1-\rho^2)^p\,\frac{q_{n+2p-2}}{q_{n-2}}\,F\left(p,\ p,\ \frac{n-1}{2}+p,\ \rho^2\right)\ \dots\dots\dots\dots\dots\dots\dots\text{(xix)},$$

or (xvii) holds whether n be even or odd.

As particular cases we have for $p=1$

$$\mu_2'=\bar{r}^2+\sigma_r^2=1-\chi_2$$
$$=1-\frac{q_n}{q_{n-2}}\,(1-\rho^2)\left(1+\frac{2^2}{n+1}\,\frac{\rho^2}{2}+\frac{2^2\,.\,4^2}{1\,.\,2\,(n+1)\,(n+3)}\,\frac{\rho^4}{4}\right.$$
$$\left.+\frac{2^2\,.\,4^2\,.\,6^2}{1\,.\,2\,.\,3\,.\,(n+1)\,(n+3)\,(n+5)}\,\frac{\rho^6}{8}+\dots\right)$$
$$\dots\dots\dots\dots\dots\text{(xx)},$$

or $$\sigma_r^2 = 1 - \bar{r}^2 - \frac{n-2}{n-1}(1-\rho^2)\left(1 + \frac{2^2}{n+1}\frac{\rho^2}{2} + \frac{2^2 . 4^2}{1 . 2 . (n+1)(n+3)}\frac{\rho^4}{4} + \frac{2^2 . 4^2 . 6^2}{1 . 2 . 3 . (n+1)(n+3)(n+5)}\frac{\rho^6}{8} + \dots\right) \quad \text{..................(xx)}^{\text{bis}},$$

and again for $p = 2$

$$\mu_4' = \mu_4 + 4\mu_3'\mu_1' - 6\mu_2'\mu_1'^2 + 3\mu_1'^4$$

$$= 2\mu_2' - 1 + \frac{n(n-2)}{(n+1)(n-1)}(1-\rho^2)^2\left\{1 + \frac{4^2}{n+3}\frac{\rho^2}{2} + \frac{4^2 . 6^2}{1 . 2 (n+3)(n+5)}\frac{\rho^4}{4} + \frac{4^2 . 6^2 . 8^2}{1 . 2 . 3 (n+3)(n+5)(n+7)}\frac{\rho^6}{8} + \dots\right\} \quad \text{...............(xxi)}.$$

The series in (xx) and (xxi) for $n = 25$ and upwards converge with sufficient rapidity to determine μ_2' and μ_4' rapidly and therefore with accurate values for μ_1' and μ_3' will give μ_2 or σ_r^2 and μ_4, and thus provide the determination of β_2.

We will now determine μ'_{2p+1} in like manner. We have

$$\mu'_{2p+1} = \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi(n-3)!}\int_{-1}^{+1}(1-r^2)^{\frac{n-4}{2}} r^{2p+1}\frac{d^{n-2}U}{d(\rho r)^{n-2}}dr,$$

but

$$r^{2p+1} = r\{r^{2p} - (r^2-1)^p + (-1)^p(1-r^2)^p\}$$

$$= r\left\{pr^{2p-2} - \frac{p(p-1)}{2!}r^{2p-4} + \frac{p(p-1)(p-2)}{3!}r^{2p-6} - \dots + (-1)^p(1-r^2)^p\right\}.$$

Hence $$\mu'_{2p+1} = p\mu'_{2p-1} - \frac{p(p-1)}{2!}\mu'_{2p-3} + \frac{p(p-1)(p-2)}{3!}\mu'_{2p-5} - \dots\dots + (-1)^p\chi_{2p+1} \quad \text{................................(xxii)},$$

where

$$\chi_{2p+1} = \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi(n-3)!}\int_{-1}^{+1}(1-r^2)^{\frac{n-4+2p}{2}} r \frac{d^{n-2}U}{d(r\rho)^{n-2}}dr$$

$$= \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi(n-3)!}\frac{1}{n-2+2p}\int_{-1}^{+1}\frac{d^{n-2}U}{d(r\rho)^{n-2}}d\left(-(1-r^2)^{\frac{n-2+2p}{2}}\right),$$

or integrating by parts

$$= \frac{\rho(1-\rho^2)^{\frac{n-1}{2}}}{\pi(n-3)!(n-2+2p)}\int_{-1}^{+1}(1-r^2)^{\frac{n-2+2p}{2}}\frac{d^{n-1}U}{d(r\rho)^{n-1}}dr.$$

Assuming n odd we must keep the first series in the value of $\dfrac{d^{n-1}U}{d\,(r\rho)^{n-1}}$, and we reach

$$\chi_{2p+1} = \frac{\rho\,(1-\rho^2)^{\frac{n-1}{2}}}{\pi\,(n-3)!\;(n-2+2p)}\,\frac{\pi}{2}\,(n-2)^2\,(n-4)^2\,\dots\,1^2\,.\,2q_{n+2p}$$

$$\times\left(1+\frac{n^2}{2!}\rho^2\,\frac{1}{n+1+2p}+\frac{n^2\,(n+2)^2}{4!}\,\frac{1}{n+1+2p}\,\frac{3}{n+3+2p}\,\rho^4+\text{etc.}\,\dots\right)$$

$$= \frac{\rho\,(1-\rho^2)^{\frac{n-1}{2}}}{2\,(n-3)!\;(n-2+2p)}\,2q_{n+2p}\,(n-2)^2\,(n-4)^2\,\dots\,1^2$$

$$\times\,F\left(\frac{n}{2},\,\frac{n}{2},\,\frac{n+1}{2}+p,\,\rho^2\right)$$

$$= \frac{\rho\,(1-\rho^2)^p\,(n-2)}{n-2+2p}\,\frac{q_{n+2p}}{q_{n-1}}\,F\left(p+\frac{1}{2},\,p+\frac{1}{2},\,\frac{n+1}{2}+p,\,\rho^2\right)\quad\text{......(xxiii)},$$

if we use Euler's reduction formula, and note that for n odd, or $n-1$ even

$$2q_{n-1} = \frac{(n-3)\,(n-5)\,\dots\,2}{(n-2)\,(n-4)\,\dots\,1}\,.\,2.$$

If we start with n even we reach an absolutely identical formula by a different route. Thus we have

$$\mu'_{2p+1} = p\mu'_{2p-1} - \frac{p\,(p-1)}{2!}\,\mu'_{2p-3} + \frac{p\,(p-1)\,(p-2)}{3!}\,\mu'_{2p-5} - \dots$$

$$+\,(-1)^p\,\frac{\rho\,(1-\rho^2)^p\,(n-2)}{n-2+2p}\,\frac{q_{n+2p}}{q_{n-1}}\,F\left(p+\frac{1}{2},\,p+\frac{1}{2},\,\frac{n+1}{2}+p,\,\rho^2\right)$$

$$\text{...............(xxiv)}.$$

Taking p in succession equal to zero and to unity we find

$$\mu_1' = \bar{r} = \rho\,\frac{q_n}{q_{n-1}}\left(1+\frac{1^2}{n+1}\,\frac{\rho^2}{2}+\frac{1^2\,.\,3^2}{1\,.\,2\,.\,(n+1)\,(n+3)}\,\frac{\rho^4}{4}\right.$$

$$\left.+\,\frac{1^2\,.\,3^2\,.\,5^2}{3!\,(n+1)\,(n+3)\,(n+5)}\,\frac{\rho^6}{8}+\dots\right)\text{......(xxv)},$$

$$\mu_3' = \mu_3 + 3\mu_2'\bar{r} - 2\bar{r}^3 = \bar{r} - \rho\,(1-\rho^2)\,\frac{q_{n+2}}{q_{n-1}}\,\frac{n-2}{n}$$

$$\times\left(1+\frac{3^2}{n+3}\,\frac{\rho^2}{2}+\frac{3^2\,.\,5^2}{1\,.\,2\,(n+3)\,(n+5)}\,\frac{\rho^4}{4}+\frac{3^2\,.\,5^2\,.\,7^2}{3!\,(n+3)\,(n+5)\,(n+7)}\,\frac{\rho^6}{8}+\dots\right)$$

$$\text{...............(xxvi)}.$$

Equations (xxv) and (xxvi) provide the values of the odd moment coefficients about zero and this in fairly rapidly converging series. From them we can deduce the value about the mean μ_3 and thus find the fundamental β_1. Table X, p. 377, again gives the requisite values of q_n for the range $n = 1$ to 105.

Illustration. Samples of 25 are taken out of a population in which two variates have the correlation $\rho = \cdot 6$. Determination of the nature of the distribution of r in these samples.

Here $n = 25$, and with $\rho = \cdot 6$ we find from (xx), (xxi), (xxv) and (xxvi) the values*

$$\mu_1' = \bar{r} = \cdot 591{,}825, \qquad \mu_2' = \cdot 368{,}739,$$

$$\mu_3' = \cdot 238{,}293, \qquad \mu_4' = \cdot 158{,}510.$$

Further $$\mu_2 = \cdot 018{,}482, \qquad \sigma_r = \cdot 135{,}950,$$

$$\mu_3 = - \cdot 001{,}812{,}380, \qquad \mu_4 = \cdot 001{,}279{,}141,$$

giving $$\beta_1 = \cdot 520{,}265, \qquad \beta_2 = 3\cdot 744{,}573.$$

The distribution is thus very far from normal.

Hence by the formula†:

$$\text{Distance from mean to mode} = \frac{\sigma_r \sqrt{\beta_1}\,(\beta_2 + 3)}{2\,(5\beta_2 - 6\beta_1 - 9)} \quad \text{.........(xxvii)},$$

we find $$\check{r} - \bar{r} = \cdot 050{,}094,$$

$$\check{r} = \cdot 64192.$$

We shall see later that the actual value is

$$\check{r} = \cdot 64194,$$

or the approximation is very close.

The skewness is given by

$$Sk. = (\check{r} - \bar{r})/\sigma_r = \cdot 36847,$$

thus indicating that there is but little approach to normality.

Fig. 1, p. 338, shows the excellent fit of a Pearson curve of Type II to the distribution. The equation is

$$y = \cdot 31004 \left(1 - \frac{x}{\cdot 31075}\right)^{5\cdot 7536} \left(1 + \frac{x}{9\cdot 64157}\right)^{178\cdot 5135}.$$

We see that when $n = 25$, Pearson's curves—fitted by moments not by range—adequately describe the frequencies, but there is still no real approach to a Gaussian distribution.

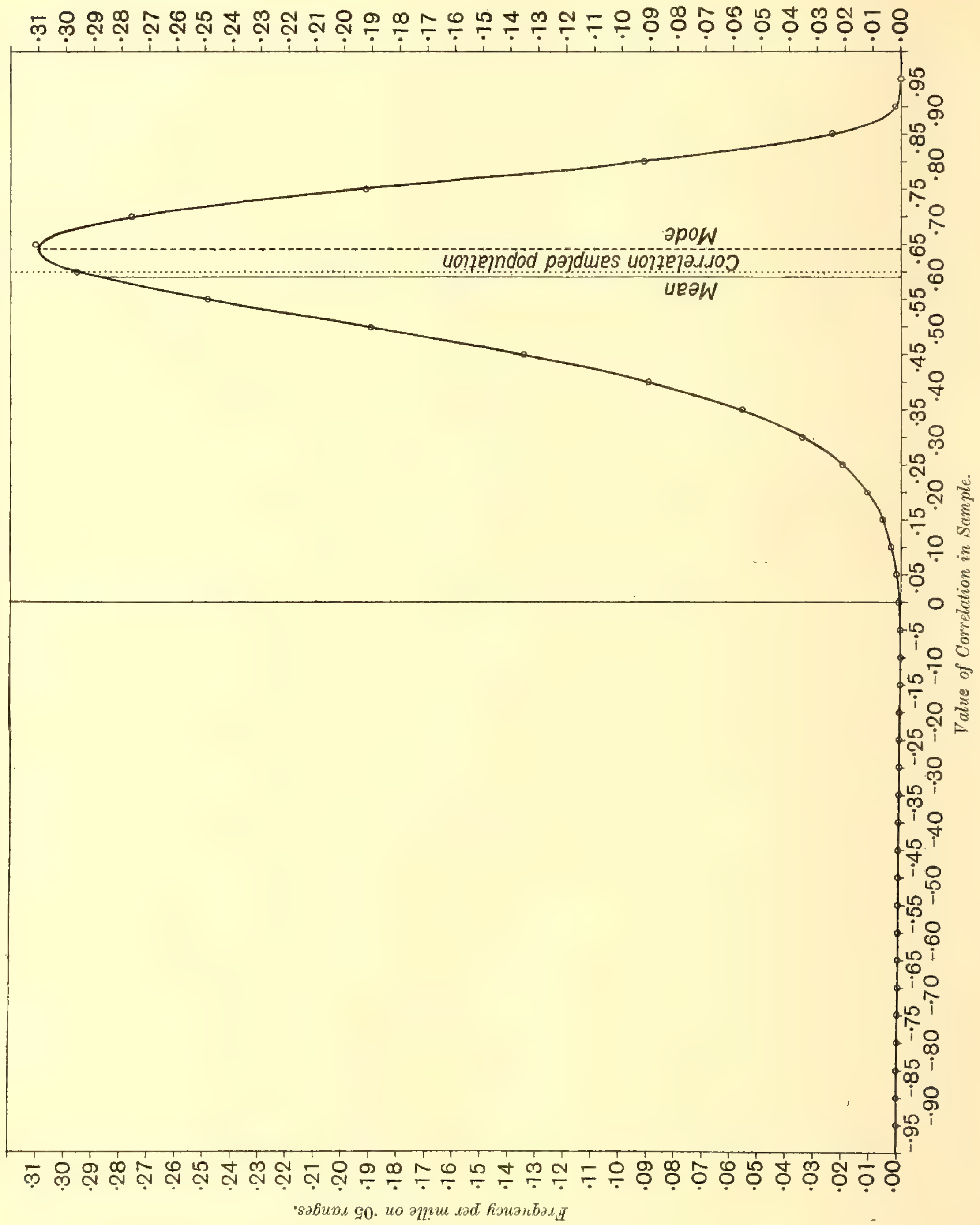
The series-expansions which have been given for the determination of the moments are of very little service when n is less than 25. We have therefore to consider formulae for deducing in succession the moments about $r = 0$ for $n = 5$ to $n = 25$.

---

* The values were in every case worked out to nine places of decimals.

† Pearson: Mathematical Contributions to the Theory of Evolution, XII, p. 7. *Drapers' Company Research Memoirs, Biometric Series*, Cambridge University Press.

Fig. 1. Comparison of Values of Frequency Ordinates for $n=25$, $\rho=0{\cdot}6$ as given by complete theory and by a Pearson Skew Curve of Frequency. The dots mark true ordinates.



By (xxv)

$$\bar{r}_{n+2} = \rho \frac{q_{n+2}}{q_{n+1}} \left(1 + \frac{1^2}{n+3} \frac{\rho^2}{2} + \dots + \frac{1 \,.\, 3^2 \dots (2s-1)^2}{s!\,(n+3)\,(n+5) - (n+2s+1)} \frac{\rho^{2s}}{2^s} + \dots\right),$$

$$\bar{r}_n = \rho \frac{q_n}{q_{n-1}} \left(1 + \frac{1^2}{n+1} \frac{\rho^2}{2} + \dots + \frac{1^2 \,.\, 3^2 \dots (2s-1)^2}{s!\,(n+1)\,(n+3) \dots (n+2s-1)} \frac{\rho^{2s}}{2^s} + \dots\right),$$

$$\bar{r}_{n+2} - \bar{r}_n = \rho \frac{q_{n+2}}{q_{n-1}} \left\{\frac{n}{n-1} \left(1 + \frac{1^2}{n+3} \frac{\rho^2}{2} + \dots + \frac{1^2 \,.\, 3^2 \dots (2s-1)^2}{s!\,(n+3)\,(n+5) \dots (n+2s+1)} \frac{\rho^{2s}}{2^s} + \dots\right) - \frac{n+1}{n} \left(1 + \frac{1^2}{n+1} \frac{\rho^2}{2} + \dots + \frac{1^2 \,.\, 3^2 \dots (2s-1)^2}{s!\,(n+1)\,(n+3) \dots (n+2s-1)} \frac{\rho^{2s}}{2^s} + \dots\right)\right\},$$

since

$$\frac{q_{n-1}}{q_{n+1}} = \frac{n}{n-1}.$$

The general term is therefore

$$\frac{1}{(n-1)\,(n-2)\,\rho} \frac{q_{n+2}}{q_{n-1}} \frac{n-2}{n} \left(\frac{n^2\,(n+1)}{n+2s+1} - (n^2-1)\right) \frac{1^2 \,.\, 3^2 \dots (2s-1)^2}{s!\,(n+1)\,(n+3) \dots (n+2s-1)} \frac{\rho^{2s}}{2^s}.$$

Now in (xxvi) we have seen that

$$\bar{r}_n - \mu'_{3,n} = \rho\,(1-\rho^2) \frac{q_{n+2}}{q_{n-1}} \frac{n-2}{n} \left\{1 + \frac{3^2}{n+3} \frac{\rho^2}{2} + \dots + \frac{3^2 \,.\, 5^2 \dots (2s+1)^2}{s!\,(n+3)\,(n+5) \dots (n+2s+1)} \frac{\rho^{2s}}{2^s} + \dots\right\},$$

and the general term is

$$\rho \frac{q_{n+2}}{q_{n-1}} \frac{n-2}{n} \left(\frac{3^2 \,.\, 5^2 \dots (2s+1)^2}{s!\,(n+3)\,(n+5) \dots (n+2s+1)} - \frac{2 \,.\, 3^2 \,.\, 5^2 \dots (2s-1)^2}{(s-1)!\,(n+3)\,(n+5) \dots (n+2s-1)}\right) \frac{\rho^{2s}}{2^s}$$

$$= \rho \frac{q_{n+2}}{q_{n-1}} \frac{n-2}{n} \left(\frac{(2s+1)^2}{n+2s+1} - 2s\right) \frac{1^2 \,.\, 3^2 \,.\, 5^2 \dots (2s-1)^2}{s!\,(n+3)(n+5) \dots (n+2s-1)} \frac{\rho^{2s}}{2^s}$$

$$= \rho \frac{q_{n+2}}{q_{n-1}} \frac{n-2}{n} \frac{(n+1)\,(1-2s\,(n-1))}{n+2s+1} \frac{1^2 \,.\, 3^2 \,.\, 5^2 \dots (2s-1)^2}{s!\,(n+1)\,(n+3) \dots (n+2s-1)} \frac{\rho^{2s}}{2^s}$$

$$= \rho \frac{q_{n+2}}{q_{n-1}} \frac{n-2}{n} \left(\frac{n^2\,(n+1)}{n+2s+1} - (n^2-1)\right) \frac{1^2 \,.\, 3^2 \,.\, 5^2 \dots (2s-1)^2}{s!\,(n+1)\,(n+3)\,(n+5) \dots (n+2s+1)} \frac{\rho^{2s}}{2^s}$$

$$= (n-1)\,(n-2)\,(\bar{r}_{n+2} - \bar{r}_n).$$

Hence

$$\bar{r}_{n+2} - \bar{r}_n = \frac{\bar{r}_n - \mu'_{3,n}}{(n-1)\,(n-2)} \qquad \text{(xxviii)}.$$

This result expresses the mean for samples of $n + 2$ in terms of the mean for samples of n and the third moment of samples of n.

Next let

$$\chi_{2p,n} = \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi\,(n-3)\,!}\int_{-1}^{+1}(1-r^2)^{\frac{n-4+2p}{2}}\,\frac{d^{n-2}U}{d\,(\rho r)^{n-2}}\,dr$$

$$= \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi\,(n-3)\,!}\,f_{p,n}.$$

Now integrate $f_{p,n}$ by parts *twice*:

$$f_{p,n} = \frac{n-4+2p}{\rho}\int_{-1}^{+1} r\,(1-r^2)^{\frac{n-6+2p}{2}}\,\frac{d^{n-3}U}{d\,(\rho r)^{n-3}}\,dr$$

$$= \frac{n-4+2p}{\rho}\int_{-1}^{+1}\{(n-6+2p)\,(1-r^2)^{\frac{n-8+2p}{2}}$$

$$- (n-6+2p+1)\,(1-r^2)\}^{\frac{n-6+2p}{2}} \times \frac{d^{n-4}U}{d\,(\rho r)^{n-4}}\,dr$$

$$= \frac{n-4+2p}{\rho^2}\,\{(n-6+2p)\,f_{p-1,n-2} - (n-6+2p+1)\,f_{p,n-2}\}.$$

Or returning to the $\chi_{2p,n}$ notation

$$\chi_{2p,n} = \frac{1-\rho^2}{\rho^2}\,\frac{n-4+2p}{(n-3)\,(n-4)}\,\{(n-6+2p)\,\chi_{2p-2,n-2} - (n-6+2p+1)\,\chi_{2p,n-2}\}.$$

As special cases put $p = 1$ and 2, and change n to $n + 2$. We have*

$$\chi_{2,n+2} = \frac{1-\rho^2}{\rho^2}\cdot\frac{n}{n-1}\cdot\left\{\chi_{0,n} - \frac{n-1}{n-2}\,\chi_{2,n}\right\} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(xxix)},$$

$$\chi_{4,n+2} = \frac{1-\rho^2}{\rho^2}\cdot\frac{n+2}{n-1}\cdot\left\{\frac{n}{n-2}\,\chi_{2,n} - \frac{n+1}{n-2}\,\chi_{4,n}\right\}\ldots\ldots\ldots\ldots\ldots\text{(xxx)}.$$

But $\chi_{0,n} = 1$ and $\chi_{2,n} = 1 - \mu'_{2,n}$,

$$\chi_{4,n} = 1 - 2\mu'_{2,n} + \mu'_{4,n}.$$

Accordingly

$$\mu'_{2,n+2} = 1 - \frac{1-\rho^2}{\rho^2}\,\frac{n}{n-2}\left(\mu'_{2,n} - \frac{1}{n-1}\right)\ldots\ldots\ldots\ldots\ldots\ldots\text{(xxxi)},$$

which can be verified directly from (xx) or (xx)$^{\text{bis}}$. Again instead of working with the series for $\chi_{4,n+2}$ above (xxx), we can replace it by one involving the moments about $r = 0$, directly:

$$\mu'_{4,n+2} = 1 - \frac{1-\rho^2}{\rho^2}\left\{\frac{(n+1)\,(n+2)}{(n-1)\,(n-2)}\,\mu'_{4,n} + \frac{n^2-6n-4}{(n-1)\,(n-2)}\,\mu'_{2,n} - \frac{1}{n-1}\right\}$$

$$\ldots\ldots\ldots\text{(xxxii)}.$$

---

* The process of integrating by parts shows that we must have $n > 2$.

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The diagram referred to on p. 341 will appear with Part II of the paper.

It remains to determine the formula for $\mu'_{3, n+2}$. We have

$$\chi_{2p+1, n} = \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi\,(n-3)\,!} \int_{-1}^{+1} (1-r^2)^{\frac{n-4+2p}{2}}\, r\, \frac{d^{n-2}U}{d\,(r\rho)^{n-2}}\, dr,$$

whence by double integration by parts to reduce the U differential coefficient we obtain

$$\chi_{2p+1, n} = \frac{1-\rho^2}{\rho^2} \frac{n-4+2p}{(n-3)\,(n-4)} \left\{(n-6+2p)\,\chi_{2p-1, n-2} - \frac{n-4+2p+1}{n-4}\,\chi_{2p+1, n-2}\right\}.$$

Putting $p=1$ and changing n to $n+2$ we have

$$\chi_{3, n+2} = \frac{1-\rho^2}{\rho^2} \frac{n}{n-1} \left\{\chi_{1, n} - \frac{n+1}{n-2}\,\chi_{3, n}\right\} \quad \text{..............(xxxiii).}$$

This may again be read as a formula for $\mu'_{3, n+2}$:

$$\mu'_{3, n+2} = \bar{r}_n \left(1 - \frac{n}{n-1} \frac{1-\rho^2}{\rho^2}\right) + \frac{\bar{r}_n - \mu'_{3, n}}{(n-1)\,(n-2)} \left(1 + n\,(n+1)\,\frac{1-\rho^2}{\rho^2}\right) \quad \text{(xxxiv).}$$

Starting with the values of the μ's for $n = 3$, 4, 25 and 26, the moment coefficients about $r = 0$ have been determined for $n = 5$ to 25 in succession. As controls the values for $n = 20$ had already been determined and those for $n = 10$ were also obtained at a very considerable expenditure of labour from the very slowly converging series of Formulae (xx), (xxi), (xxv) and (xxvi). The initial values of the moment coefficients (i.e. those for $n = 3$, 4, 25 and 26) had to be calculated generally to 15 and sometimes to 20 significant figures, owing to the numerical factors in (xxviii), (xxxi), (xxxii) and (xxxiv) being frequently greater than unity, and thus errors in the last figure being repeatedly multiplied. According to the special value of ρ, it was found best sometimes to deduce moment coefficients of $n + 2$ from those for n, and sometimes those of n from those for $n + 2$, i.e. to work up from 3 and 4, or down from 25 and 26. It seems unnecessary to enter at length here into the many difficulties that arose in the course of these calculations. We think they have all been successfully surmounted and that our final values may be trusted to the figures actually recorded in the tables. We thus found the moment coefficients and from them the values of β_1 and β_2 for the ten values of ρ from 0 to ·9, and for the values of n, 2 to 25, 50, 100 and 400. Diagram I shows that our 270 frequency curves are fairly well distributed over the most frequently occurring portion of the β_1, β_2 plane. Now our view is that the constants β_1, β_2 describe adequately for statistical purposes the bulk of the usual frequencies distributions. But we have provided tables of the values of the ordinates for the above 270 curves. Hence *by interpolation* it will now be possible to determine rapidly ordinates which will graduate with reasonable accuracy any frequency distribution whatever quite apart from the idea of sampling normally correlated variates*.

---

* Francis Galton frequently insisted on the importance of forming Tables of frequency ordinates, which would graduate any frequency distribution in the β_1, β_2 plane. A scheme for covering this plane

In order to make use of our ordinates for graduating frequency curves we must express the distance from our origin to our mean (i.e. from $r = 0$ to $r = \bar{r}_n$) in terms of the standard deviation, and further the unit of argument of the abscissae, i.e. ·05 in r, also in terms of the standard deviation. Our interpolated frequency ordinates (reduced of course, to the size of the actual population) will then have to be plotted to intervals of $\cdot 05\sigma_g/\sigma_r$, the origin being $\bar{r}_n\sigma_g/\sigma_r$ from the mean of the graduated data, where σ_g is the standard deviation of the graduated data. Care must be taken to so choose the axis of abscissae of the graduated data that the sign of μ_3 is the same in the graduated material and the graduating frequencies. Table C gives the distance from the mean to the origin of coordinates in each case and also the abscissal unit for plotting both in terms of the standard deviation.

(4) *On the Determination of the Mode.* Differentiating (iv) we have

$$\frac{dy_n}{dr} = \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi\,(n-3)\,!}\left\{(1-r^2)^{\frac{n-4}{2}}\,\rho\,\frac{d^{n-1}U}{d\,(r\rho)^{n-1}} - r\,(n-4)\,(1-r^2)^{\frac{n-6}{2}}\,\frac{d^{n-2}U}{d\,(r\rho)^{n-2}}\right\}.$$

Hence the mode $\breve{r}$ is given by

$$0 = (1-\breve{r}^2)\,\rho^2\left(\frac{d^{n-1}U}{d\,(r\rho)^{n-1}}\right) - \breve{r}\rho\,\frac{d^{n-2}U}{d\,(r\rho)^{n-2}}\,(n-4),$$

or writing $\breve{\rho}^2 = \breve{r}\rho$, we have

$$(\rho^2 - \breve{\rho}^4)\,\frac{d^{n-1}\breve{U}}{d\,(\breve{\rho}^2)^{n-2}} = (n-4)\,\breve{\rho}^2\,\frac{d^{n-2}\breve{U}}{d\,(\breve{\rho}^2)^{n-2}} \quad \text{..............(xxxv),}$$

where $\breve{U}$ is U with $\breve{\rho}$ put for $r\rho$.

Now (xxxv) is by no means easy to solve adequately, for if we solve it by approximation, $r = \rho$ and $\breve{\rho} = \rho^2$ is not sufficiently close for an effective first approximation, especially when ρ differs considerably from zero. We have indeed from (v) the relation

$$(1-\breve{\rho}^4)\,\frac{d^{n-1}\breve{U}}{d\,(\breve{\rho}^2)^{n-1}} - \breve{\rho}^2\,(2n-3)\,\frac{d^{n-2}\breve{U}}{d\,(\breve{\rho}^2)^{n-2}} - (n-2)^2\,\frac{d^{n-3}\breve{U}}{d\,(\breve{\rho}^2)^{n-3}} = 0 \quad \text{(xxxvi),}$$

and this might be combined with (xxxv) to deduce in succession relations between lower pairs of differential coefficients, till we ultimately reach a relation between $d\breve{U}/d\breve{\rho}^2$ and $\breve{U}$, but the process is too laborious except for very low values of n.

Fisher has outlined another method of approaching the mode*. It is easy to see that

$$\begin{aligned}\frac{\cos^{-1}(-x)}{\sqrt{1-x^2}} &= \frac{2}{\sqrt{1-x^2}}\left(\tan^{-1}\frac{1-x}{\sqrt{1-x^2}} - \tan^{-1}\frac{-x}{\sqrt{1-x^2}}\right)\\ &= \frac{2}{\sqrt{1-x^2}}\left[\tan^{-1}\left(\frac{\xi - x}{\sqrt{1-x^2}}\right)\right]_0^1\\ &= 2\int_0^1 \frac{d\xi}{(\xi - x)^2 + 1 - x^2} = 2\int_0^1 \frac{d\xi}{\xi^2 - 2x\xi + 1},\end{aligned}$$

---

with a series of Pearson-curves has been long under consideration, but the immense labour of calculating the ordinates of 400 to 500 curves has so far prevented the actualisation of this idea. The present ordinate-tables go some way to supply the need Galton pointed out.

* *Biometrika*, Vol. x. p. 520.

or, if $\xi = e^{-z}$,

$$= \int_0^\infty \frac{dz}{\cosh z - x} = \int_0^\infty \frac{dz}{\cosh z - \rho r}, \quad \text{if } x = \rho r.$$

But

$$y_n = \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi (n-3)!} (1-r^2)^{\frac{n-4}{2}} \frac{d^{n-2}}{d(\rho r)^{n-2}} \left(\frac{\cos^{-1}(-\rho r)}{\sqrt{1-\rho^2 r^2}} \right)$$

$$= \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi (n-3)!} (1-r^2)^{\frac{n-4}{2}} \frac{d^{n-2}}{d(\rho r)^{n-2}} \int_0 \frac{dz}{\cosh z - \rho r}$$

$$= (n-2) \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi} (1-r^2)^{\frac{n-4}{2}} \int_0^\infty \frac{dz}{(\cosh z - \rho r)^{n-1}} \quad \text{...(xxxvii)},$$

$$= \frac{(n-2)(1-\rho^2)^{\frac{n-1}{2}}}{\pi} (1-r^2)^{\frac{n-4}{2}} I_{n-1}, \text{ say} \text{............(xxxvii)}^{\text{bis}}.$$

Substituting in Eqn. (viii) we find

$$n(1-\rho^2 r^2) I_{n+1} = (2n-1) \rho r I_n + (n-1) I_{n-1} \quad \text{......(xxxviii)},$$

as the reduction formula for the I_n's.

Similarly, if

$$I'_{n-1} = \int_0^\infty \frac{dz}{(\cosh z - \rho_0^2)^{n-1}},$$

then

$$n(1-\rho_0^4) I'_{n+1} = (2n-1) \rho_0^2 I'_n + (n-1) I'_{n-1} \quad \text{...(xxxviii)}^{\text{bis}}.$$

Now using value (xxxvii)$^{\text{bis}}$ for y_n, the equation for the mode is

$$(n-4) \breve{r} \breve{I}_{n-1} = \rho (1-\breve{r}^2)(n-1) \breve{I}_n,$$

or, if as before, $\breve{\rho}^2 = \rho \breve{r}$, we have:

$$(n-4) \breve{\rho}^2 \breve{I}_{n-1} = (\rho^2 - \breve{\rho}^4)(n-1) \breve{I}_n \text{.................(xxxix)}.$$

This combined with

$$n(1-\breve{\rho}^4) \breve{I}_{n+1} = (2n-1) \breve{\rho}^2 \breve{I}_n + (n-1) \breve{I}_{n-1} \text{ ....(xl)},$$

should determine the mode.

Now assume $\breve{\rho}^2 = \rho_0^2 + \epsilon$, where ρ_0^2 is some first approximation to $\breve{\rho}^2$, then we find

$$\epsilon = -\frac{(n-4)\rho_0^2 I'_{n-1} - (n-1)(\rho^2 - \rho_0^4) I_n'}{(n-4) I'_{n-1} + (n-1)(n-2)\rho_0^2 I_n' - (n-1)(\rho^2 - \rho_0^4) I'_{n+1}} \quad \text{.........(xli)}.$$

If we had obtained an approximation ρ_0^2 to $\breve{\rho}^2$, we could start with

$$I_1' = \frac{\cos^{-1}(-\rho_0^2)}{\sqrt{1-\rho_0^4}} \quad \text{and} \quad I_2' = \frac{1}{1-\rho_0^4} + \frac{\rho_0^2 \cos^{-1}(-\rho_0^2)}{(1-\rho_0^4)^{\frac{3}{2}}} \quad \text{......(xlii)},$$

and by aid of (xxxviii)$^{\text{bis}}$ determine the I''s in succession. If $E_n = I_n / I_{n-1}$ we can put our results in the forms (xliii) and (xliv) below, and calculate successive E's:

$$\epsilon = -\frac{\left\{ \frac{(n-4)\rho_0^2}{E_n} - (n-1)(\rho^2 - \rho_0^4) \right\}}{\frac{n-4}{E_n} + (n-1)(n-2)\rho_0^2 - n(n-1)(\rho^2 - \rho_0^4) E_{n+1}} \quad \text{...(xliii)},$$

and $$(n-1)(1-\rho_0{}^4)E_n = (2n-3)\rho_0{}^2 + \frac{n-2}{E_{n-1}} \quad \text{...............(xliv).}$$

But even this would be laborious had we to find successive values of E_n from (xliv). Actually, if n be moderately large, E_n and E_{n-1} tend to equality fairly rapidly. For example the following are the values of E_n for $\rho_0 = {\cdot}6$:

$$I_1 = 2{\cdot}078{,}4173, \; I_2 = 1{\cdot}873{,}8688 \text{ and therefore } E_2 = {\cdot}901{,}5845.$$

E_2	·901,5845	E_9	1·470,8511	E_{16}	1·511,0031	E_{23}	1·527,2571
E_3	1·257,5588	E_{10}	1·475,5703	E_{17}	1·514,1874	E_{24}	1·528,7778
E_4	1·183,5106	E_{11}	1·486,5966	E_{18}	1·516,9985	E_{25}	1·530,1728
E_5	1·451,8703	E_{12}	1·492,1848	E_{19}	1·519,5018	E_{26}	1·531,4570
E_6	1·377,5430	E_{13}	1·498,5199	E_{20}	1·521,7436	E'	1·529,7263
E_7	1·453,2879	E_{14}	1·503,1022	E_{21}	1·523,7636	E''	1·531,0459
E_8	1·445,7342	E_{15}	1·507,3770	E_{22}	1·525,5928	E'''	1·530,1488

Clearly E_n and E_{n-1} approach equality. Now put $E_{25} = E_{24}$ for $n = 25$ in (xliv) and we have for $\rho_0 = {\cdot}6$

$$20{\cdot}8896E'^2 - 16{\cdot}92E' - 23 = 0,$$

which gives for the root required

$$E' = 1{\cdot}529{,}7263.$$

But we might also have made $E_{25} = E_{26}$ and so reached

$$21{\cdot}7600E''^2 - 17{\cdot}64E'' - 24 = 0,$$

which gives $$E'' = 1{\cdot}531{,}0459.$$

It is better therefore in finding E_n to equate E_n and E_{n-1} than E_n and E_{n+1}.

A still closer approximation may be found by noting that

$$E_n - E' = \epsilon = E_{n+1} - E'', \text{ nearly,}$$

where ϵ is very small. Hence since

$$n(1-\rho_0{}^4)E_{n+1}E_n - (2n-1)\rho_0{}^2E_n - (n-1) = 0,$$

we have

$$\epsilon = \frac{(n-1) + (2n-1)\rho_0{}^2E' - n(1-\rho_0{}^4)E'E''}{n(1-\rho_0{}^4)(E'+E'') - (2n-1)\rho_0{}^2},$$

or, $$E''' = E' + \epsilon = \frac{(n-1) + n(1-\rho_0{}^4)E'^2}{n(1-\rho_0{}^4)(E'+E'') - (2n-1)\rho_0{}^2} \quad \text{............(xlv).}$$

For the case of $\rho_0 = {\cdot}6$ and $n = 25$ we find

$$E''' = 1{\cdot}530{,}1488,$$

and $E_{25} - E''' = {\cdot}000{,}0240$, a close agreement. As a matter of fact as we only use E_n in a small term the approximation E' is generally quite sufficient.

In the above method all turns on finding a good value of $\rho_0{}^2$, i.e. a first approximation to the value of the product of ρ and $\check{r}$. This may be obtained in either of the following ways:

First, choose the values of ρ_0^2 and E' to satisfy the simultaneous equations

$$\frac{(n-4)\rho_0^2}{E'} - (n-1)(\rho^2 - \rho_0^4) = 0,$$

and

$$(n-1)(1-\rho_0^4)E' = (2n-3)\rho_0^2 + \frac{n-2}{E'}.$$

Or, we have for ρ_0 the equation

$$\frac{(n-1)(n-4)(1-\rho_0^4)\rho_0^2}{(n-1)(\rho^2-\rho_0^4)} = (2n-3)\rho_0^2 + \frac{(n-2)(n-1)(\rho^2-\rho_0^4)}{(n-4)\rho_0^2},$$

which writing $\rho_0^4 = z$ gives us

$$(n-4)^2(1-z)z = (2n-3)(n-4)z(\rho^2-z) + (n-2)(n-1)(\rho^2-z)^2,$$

or

$$6z^2 - z\{(n-4)^2 + \rho^2(5n-8)\} + (n-2)(n-1)\rho^4 = 0 \quad \text{......(xlvi)}.$$

As illustration if $n = 25$ and $\rho = \cdot 6$

$$6z^2 - 483\cdot 12z + 71\cdot 5392 = 0,$$

giving $z = \cdot 148,351,$

or, $\rho_0^2 = \breve{r}\rho = \cdot 385,164,$

and $\breve{r} = \cdot 64194,$

a value* in excellent agreement with the results on p. 337, and needing no further approximation.

Again suppose $n = 5$, and $\rho = \cdot 6$, we have

$$6z^2 - 7\cdot 12z + 1\cdot 5552 = 0.$$

Hence $z = \cdot 288,6295$ and $\rho_0^2 = \cdot 537,2425$ leading to $\breve{r} = \cdot 895,404$ as our approximation. We shall now use this value of ρ_0^2 to determine the true system of E's corresponding to this value.

We have

$$n(1-\rho_0^4)E_{n+1} = (2n-1)\rho_0^2 + \frac{n-1}{E_n},$$

while

$$E_2 = \rho_0^2 + \frac{1}{\sqrt{1-\rho_0^4}\cos^{-1}(-\rho_0^2)} \quad \text{.....................(xlvii)}$$

$$= 1\cdot 091,8073.$$

Substituting in

$$n \times \cdot 711,3705 E_{n+1} = (2n-1) \times \cdot 537,2425 + \frac{n-1}{E_n},$$

we obtain the series

$$E_2 = 1\cdot 091,8073, \qquad E_3 = 1\cdot 776,5988,$$

$$E_4 = 1\cdot 786,2042, \qquad E_5 = 1\cdot 911,8858, \qquad E_6 = 1\cdot 947,6088.$$

The values show us that $E_5 = E_4$ was naturally much rougher in this case than that of $n = 25$. However we find $\epsilon = + \cdot 001,1177$, $\rho_0^2 = \cdot 538,3602$ and $\breve{r} = \cdot 897,267$, as our next approximation, involving no very great change.

---

* Repeated use of Eqn. (xli) only modified this result to $\breve{r} = \cdot 641,939$.

To confirm this value of $\check{r}$ we take as the first approximation to $\check{r}$ the value given in the method of the following section, i.e. $\check{r} = \cdot 91344$ giving $\rho_0{}^2 = \cdot 548{,}064$ and

$$n \times \cdot 699{,}6259 E_{n+1} = (2n - 1) \times \cdot 548{,}0640 + \frac{n-1}{E_n}.$$

Using (xlvii) we find $E_2 = 1 \cdot 103{,}9149$ and hence

$$E_3 = 1\cdot 822{,}4446, \qquad E_4 = 1\cdot 828{,}4768,$$
$$E_5 = 1\cdot 957{,}1738, \qquad E_6 = 1\cdot 994{,}3056,$$

leading to $\qquad \epsilon = - \cdot 008{,}8172,$

and $\qquad \rho_0{}^2 = \cdot 539{,}2468,$

or $\qquad \check{r} = \cdot 89875.$

It will be seen that our two methods of approaching the true value of $\check{r}$ still differ to some extent, although probably serviceable enough for practical purposes. Accordingly we will now make a further approximation starting from $\check{r} = \cdot 8980$ or $\rho_0{}^2 = \cdot 5388$, and we have

$$E_2 = 1\cdot 093{,}5399, \qquad E_3 = 1\cdot 783{,}0638, \qquad E_4 = 1\cdot 792{,}1629,$$
$$E_5 = 1\cdot 918{,}2742, \qquad E_6 = 1\cdot 954{,}1949.$$

These give $\qquad \epsilon = - \cdot 000{,}4924,$

and consequently $\qquad \rho_0{}^2 = \cdot 538{,}3036,$

with $\qquad \check{r} = \cdot 89717,$

a value no doubt correct to four figures.

It is clear that the process of finding the mode for n small is much more laborious than for $n = 25$ or over, because E_n is not nearly E_{n+1}. Actually the value given for E' by the simultaneous equation process from which we started is

$$E' = 1\cdot 881{,}8787,$$

which is only a rough approximation to the value $E_5 = 1\cdot 911{,}8858$. That method must therefore be followed by further approximations when n is much smaller than 25.

(5) *Determination of Ordinates and Mode by Expansions.*

Approximate Expression for the Ordinates. We may proceed to expand the Eqn. (xxxvii) in powers of $1/n$ or $1/(n-1)$. This will involve a knowledge of the expansion of

$$I_n = \int_0^\infty \frac{dz}{(\cosh z - \rho_0{}^2)^n}, \text{ where } \rho_0{}^2 = \rho \check{r},$$

and can be achieved by a process to which Pearson drew attention in 1902*.

Let
$$\frac{1}{\cosh z - \rho_0{}^2} = \frac{1}{1 - \rho_0{}^2} e^{-(a_2' z^2 + a_4' z^4 + a_6' z^6 + a_8' z^8 + \dots)} \quad \text{.........(xlviii).}$$

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* *Biometrika*, Vol. I. p. 393.

Then, if

$$v = \log(\cosh z - \rho_0^2) = \log(1 - \rho_0^2) + a_2' z^2 + a_4' z^4 + a_6' z^6 + \ldots,$$

it follows that

$$a_2' = \frac{1}{2!}\left(\frac{d^2 v}{dz^2}\right)_0, \qquad a_4' = \frac{1}{4!}\left(\frac{d^4 v}{dz^4}\right)_0,$$

and so on.

Now $$\frac{dv}{dz} = \frac{\sinh z}{\cosh z - \rho_0^2}, \quad \text{or} \quad (\cosh z - \rho_0^2)\frac{dv}{dz} = \sinh z.$$

Apply Leibnitz's Theorem, differentiating $(2s - 1)$ times, and we have

$$\sinh z \frac{dv}{dz} + (2s-1)\cosh z \frac{d^2 v}{dz^2} + \frac{(2s-1)(2s-2)}{2!}\sinh z \frac{d^3 v}{dz^3} + \ldots$$
$$+ (\cosh z - \rho_0^2)\frac{d^{2s} v}{dz^{2s}} = \cosh z.$$

Hence when $z = 0$

$$(2s-1)\left(\frac{d^2 v}{dz^2}\right)_0 + \frac{(2s-1)(2s-2)(2s-3)}{3!}\left(\frac{d^4 v}{dz^4}\right)_0 + \ldots + (1-\rho_0^2)\left(\frac{d^{2s} v}{dz^{2s}}\right)_0 = 1.$$

Now put s in succession 1, 2, 3, etc. and there results

$$(1-\rho_0^2)\left(\frac{d^2 v}{dz^2}\right)_0 = 1, \qquad 3\left(\frac{d^2 v}{dz^2}\right)_0 + (1-\rho_0^2)\left(\frac{d^4 v}{dz^4}\right)_0 = 1,$$

$$5\left(\frac{d^2 v}{dz^2}\right)_0 + 10\left(\frac{d^4 v}{dz^4}\right)_0 + (1-\rho_0^2)\left(\frac{d^6 v}{dz^6}\right)_0 = 1,$$

$$7\left(\frac{d^2 v}{dz^2}\right)_0 + 35\left(\frac{d^4 v}{dz^4}\right)_0 + 21\left(\frac{d^6 v}{dz^6}\right)_0 + (1-\rho_0^2)\left(\frac{d^8 v}{dz^8}\right)_0 = 1,$$

$$9\left(\frac{d^2 v}{dz^2}\right)_0 + 84\left(\frac{d^4 v}{dz^4}\right)_0 + 126\left(\frac{d^6 v}{dz^6}\right)_0 + 36\left(\frac{d^8 v}{dz^8}\right)_0 + (1-\rho_0^2)\left(\frac{d^{10} v}{dz^{10}}\right)_0 = 1,$$

etc., etc.

These lead to

$$a_2' = \frac{1}{2}\frac{1}{1-\rho_0^2}, \qquad a_4' = -\frac{2+\rho_0^2}{24(1-\rho_0^2)^2},$$

$$a_6' = \frac{16 + 13\rho_0^2 + \rho_0^4}{720(1-\rho_0^2)^3}, \qquad a_8' = -\frac{(272 + 297\rho_0^2 + 60\rho_0^4 + \rho_0^6)}{40320(1-\rho_0^2)^4},$$

$$a_{10}' = \frac{7936 + 10841\rho_0^2 + 3651\rho_0^4 + 251\rho_0^6 + \rho_0^8}{3{,}628{,}800(1-\rho_0^2)^5}, \text{ etc.}$$

Accordingly we have, raising (xlviii) to the nth power and expanding the exponential after the term in $a_2' z^2$,

$$\frac{1}{(\cosh z - \rho_0^2)^n} = \frac{1}{(1-\rho_0^2)^n} e^{-n a_2' z^2}\{1 - n a_4' z^4 - n a_6' z^6 - n(a_8' - \tfrac{1}{2} n a_4'^2) z^8$$
$$- n(a_{10}' - n a_4' a_6') z^{10} - n(a_{12}' - n a_4' a_8' - \tfrac{1}{2} n a_6'^2 + \tfrac{1}{6} n^2 a_4'^3) z^{12} + \ldots\}.$$

Remembering that

$$\int_{-\infty}^{+\infty} e^{-\frac{nz^2}{2(1-\rho_0^2)}} z^{2s} dz = \sqrt{2\pi} \sqrt{\frac{1-\rho_0^2}{n}} (2s-1)(2s-3)\dots 1 \times \frac{(1-\rho_0^2)^s}{n^s},$$

we find

$$I_n = \int_0^\infty \frac{dz}{(\cosh z - \rho_0^2)^n} = \frac{1}{2} \frac{\sqrt{2\pi}}{(1-\rho_0^2)^n} \sqrt{\frac{1-\rho_0^2}{n}} \left(1 + \frac{1}{8} \frac{\rho_0^2 + 2}{n} + \frac{1}{128} \frac{9\rho_0^4 + 12\rho_0^2 + 4}{n^2} \right.$$
$$+ \frac{75\rho_0^6 + 90\rho_0^4 - 20\rho_0^2 - 40}{1024n^3}$$
$$\left. + \frac{3675\rho_0^8 + 4200\rho_0^6 - 2520\rho_0^4 - 3360\rho_0^2 - 336}{32768n^4} + \text{etc.}\right) \quad \text{...(xlix).}$$

But
$$y_n = \frac{(n-2)(1-\rho^2)^{\frac{n-1}{2}} (1-r^2)^{\frac{n-4}{2}}}{\pi} I_{n-1}, \quad \text{if} \quad \rho_0^2 = \rho r,$$

and thus we have

$$y_n = \frac{1}{\sqrt{2\pi}} \frac{n-2}{\sqrt{n-1}} (1-\rho^2)^{\frac{3}{2}} \chi_0(\rho, r)$$
$$\times \left(1 + \frac{\phi_1(\rho r)}{(n-1)} + \frac{\phi_2(\rho r)}{(n-1)^2} + \frac{\phi_3(\rho r)}{(n-1)^3} + \frac{\phi_4(\rho r)}{(n-1)^4} + \dots\right) \quad \text{...(l),}$$

where
$$\chi_0(\rho, r) = \frac{(1-\rho^2)^{\frac{n-4}{2}} (1-r^2)^{\frac{n-4}{2}}}{(1-\rho r)^{n-\frac{3}{2}}}$$

is symmetrical in ρ and r and

$$\left.\begin{aligned} \phi_1(\rho r) &= \frac{r\rho + 2}{8}, \qquad \phi_2(\rho r) = \frac{(3r\rho + 2)^2}{128}, \\ \phi_3(\rho r) &= \frac{5\{15(r\rho)^3 + 18(r\rho)^2 - 4(r\rho) - 8\}}{1024}, \\ \phi_4(\rho r) &= \frac{3675(r\rho)^4 + 4200(r\rho)^3 - 2520(r\rho)^2 - 3360(r\rho) - 336}{32768} \end{aligned}\right\} \quad \text{...(li),}$$

thus depend only on the product of ρ and r.

We may write

$$y_n = \frac{n-2}{\sqrt{n-1}} (1-\rho^2)^{\frac{3}{2}} \chi(\rho, r) \left\{1 + \frac{\phi_1(\rho r)}{(n-1)} + \frac{\phi_2(\rho r)}{(n-1)^2} + \frac{\phi_3(\rho r)}{(n-1)^3} + \frac{\phi_4(\rho r)}{(n-1)^4} + \dots\right\} \quad \text{.........(lii),}$$

where
$$\log \chi(\rho, r) = -(n-1) \log \chi_1 - \log \chi_2,$$

and
$$\chi_1 = \frac{1-\rho r}{\{(1-\rho^2)(1-r^2)\}^{\frac{1}{2}}},$$

$$\chi_2 = \frac{\sqrt{2\pi}\{(1-\rho^2)(1-r^2)\}^{\frac{3}{2}}}{(1-\rho r)^{\frac{1}{2}}},$$

both being symmetrical in r and ρ

Table C in Appendix gives the values of $\log \dfrac{n-2}{\sqrt{n-1}}$, $\log(1-\rho^2)^{\frac{3}{2}}$, $\log \chi_1$, $\log \chi_2$, ϕ_1, ϕ_2, ϕ_3, ϕ_4, and enables the ordinates of the frequency curves to be calculated with considerable rapidity for $n = 25$ and upwards*.

Approximate Expressions for the Mode.

Writing
$$I_n' = \int_0^\infty \frac{dz}{(\cosh z - \rho\breve{r})^n},$$
we find
$$I_n' = \frac{1}{2} \frac{\sqrt{2\pi}}{(1-\rho\breve{r})^n} \sqrt{\frac{1-\rho\breve{r}}{n}} \left(1 + \frac{\phi_1'}{n} + \frac{\phi_2'}{n^2} + \frac{\phi_3'}{n^3} + \dots\right),$$
where ϕ_s' stands for $\phi_s(\rho\breve{r})$. We now use Eqn. (xxxix) and find
$$\frac{\breve{r} - \rho\breve{r}^2}{\rho - \rho\breve{r}^2} = \frac{n-1}{n-4}\sqrt{\frac{n-1}{n}}\, R,$$
where
$$R = \left(1 + \frac{\phi_1'}{n} + \frac{\phi_2'}{n^2} + \frac{\phi_3'}{n^3} + \dots\right) \Big/ \left(1 + \frac{\phi_1'}{(n-1)} + \frac{\phi_2'}{(n-1)^2} + \frac{\phi_3'}{(n-1)^3} + \dots\right) \quad \dots\dots\dots\text{(liii)}.$$

If we expand this in inverse powers of $\dfrac{1}{n-1}$, we deduce
$$R = 1 - \frac{\phi_1'}{(n-1)^2} + \frac{\phi_1'^2 - 2\phi_2' + \phi_1'}{(n-1)^3} - \frac{\phi_1'^3 + \phi_1'^2 + \phi_1' - 3\phi_1'\phi_2' - 3\phi_2' + 3\phi_3'}{(n-1)^4} + \dots.$$
Again
$$\frac{n-1}{n-4}\sqrt{\frac{n-1}{n}} = 1 + \frac{5}{2(n-1)} + \frac{63}{8(n-1)^2} + \frac{373}{16(n-1)^3} + \frac{8987}{128(n-1)^4} + \dots.$$
Thus we have
$$\frac{\breve{r} - \rho\breve{r}^2}{\rho - \rho\breve{r}^2} = 1 + \frac{5}{2(n-1)} + \frac{63 - \phi_1'}{8(n-1)^2} + \frac{373 - 24\phi_1' + 16\phi_1'^2 - 32\phi_2'}{16(n-1)^3}$$
$$+ \frac{8987 - 816\phi_1' + 192\phi_1'^2 - 128\phi_1'^3 + 384\phi_1'\phi_2' - 256\phi_2' - 384\phi_3'}{128(n-1)^4} + \dots.$$

Bringing the first term on the right to the left we reach after substituting for the ϕ''s
$$\breve{r} = \rho\left(1 + \frac{5(1-\breve{r}^2)}{2(n-1)} + \frac{(61-\rho\breve{r})(1-\breve{r}^2)}{8(n-1)^2} + \frac{(367 - 5\rho\breve{r} - 2\rho^2\breve{r}^2)(1-\breve{r}^2)}{16(n-1)^3}\right.$$
$$\left. + \frac{(17606 - 195\rho\breve{r} - 81\rho^2\breve{r}^2 - 50\rho^3\breve{r}^3)(1-\breve{r}^2)}{256(n-1)^4} + \dots\right) \quad \dots\dots\text{(liv)}.$$

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* The ordinates calculated by the rising difference formula were tested in this manner. For $n = 25$ the accordance was excellent, and quite good enough for practical purposes at $n = 10$. Below this (lii) becomes less reliable and needs more terms.

This series has now to be inverted and leads after considerable algebra to

$$\check{r} = \rho\left(1 + \frac{5}{2}\frac{(1-\rho^2)}{(n-1)} + \frac{(61 - 101\rho^2)(1-\rho^2)}{8(n-1)^2} + \frac{(367 - 1480\rho^2 + 1273\rho^4)(1-\rho^2)}{16(n-1)^3}\right.$$
$$\left. + \frac{(17606 - 125727\rho^2 + 246783\rho^4 - 143782\rho^6)(1-\rho^2)}{256(n-1)^4} + \dots\right) \dots\dots\text{(lv)}.$$

The above series is of very considerable interest from more than one standpoint*. In the first place it appears that Soper's approximation (*Biometrika,* Vol. IX. p. 108) was not valid. He obtained

$$\check{r} = \rho\left\{1 + \frac{3}{2}\frac{(1-\rho^2)}{(n-1)} + \frac{(41 + 23\rho^2)(1-\rho^2)}{8(n-1)^2} + \dots\right\}.$$

Thus for $n = 25$, $\rho = \cdot 6$, Soper's formula gives ·62811, and (lv) gives ·64205, while the exact value is ·64194. It is clear that the coefficient $\frac{3}{2}$ in Soper's second term of the series can never approach the $\frac{5}{2}$ of the more exact expression. At first the difference was found very perplexing, especially when the algebra had been verified; but the solution appears to lie in the consideration that the best fitting Pearson curve to the frequency is not one tied down to the range -1 to $+1$. That curve is fitted by two moments only, but if we fit a curve by the first four moments and use the general expression

$$\check{r} = \bar{r} + \frac{\sigma_r\sqrt{\beta_1}(\beta_2 + 3)}{2(5\beta_2 - 6\beta_1 - 9)} \dots\dots\dots\dots\dots\dots\dots\dots\dots\text{(lvii)},$$

we obtain

$$\check{r} = \cdot 64192,$$

a value close to the true value. In other words the use of the third and fourth moments to find the mode is far more important than fixing down the range to the theoretically possible values; that process determines much more quickly the form of the frequency curve, but it does not give nearly such a good fit as allowing the Pearson curve freedom to adjust itself by means solely of the first four moments†. On the other hand a Pearson curve determined by the first four moments does describe fairly accurately the frequency distributions of r for $n = 25$ and upwards: see p. 337.

(6) *Equation for Modes and Antimodes* ($n = 3$).

Still another method of approaching the modal value has been found occasionally of service‡.

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* We have used the expansion in terms of $1/(n-1)$ rather than $1/n$ as $(n-1)$ appears to arise more simply in all the formulae. The form in $1/n$ is

$$\check{r} = \left(1 + \frac{5(1-\rho^2)}{2n} + \frac{(81 - 101\rho^2)(1-\rho^2)}{8n^2} + \frac{(651 - 1884\rho^2 + 1273\rho^4)(1-\rho^2)}{16n^3} + \dots\right) \dots\text{(lvi)}.$$

† The Pearson curve determined from the range does not give good values of the frequency for $n = 25$, even when we use the true values of $\bar{r}$ and σ_r and not Soper's approximations to these constants.

‡ It was used successfully in calculating the antimode in the case of samples of three, when the correlation in the sampled population was low. It gave a fairly good "jumping off point" even for higher values of the correlation.

Starting from the equation (xxxix)

$$(n-4)\,\breve{\rho}^2 I_{n-1} = (\rho^2 - \breve{\rho}^4)\,(n-1)\,I_n,$$

where $I_n = \int_0^\infty \frac{dz}{(\cosh z - \breve{\rho}^2)^{n-1}}$ and $\breve{\rho}^2 = \rho\breve{r}$, $\breve{r}$ being the modal value of r, we may expand I_n and I_{n-1} in terms of powers of $\breve{\rho}^2$, the coefficients involving

$$\int_0^\infty \frac{dz}{(\cosh z)^{m-1}} = \int_0^{\frac{\pi}{2}} \sin^{m-1}\theta\, d\theta = q_m.$$

We find at once

$$(n-4)\,\breve{\rho}^2 \left(q_{n-1} + (n-1)\,\breve{\rho}^2 q_n + \frac{(n-1)\,n}{1\,.\,2}\,\breve{\rho}^4 q_{n+1} + \frac{(n-1)\,n\,(n+1)}{1\,.\,2\,.\,3}\,\breve{\rho}^6 q_{n+2} + \ldots\ldots\right)$$

$$= (\rho^2 - \breve{\rho}^4)\,(n-1)\left(q_n + n\breve{\rho}^2 q_{n+1} + \frac{n\,(n+1)}{1\,.\,2}\,\breve{\rho}^4 q_{n+2} + \frac{n\,(n+1)\,(n+2)}{1\,.\,2\,.\,3}\,\breve{\rho}^6 q_{n+3} + \ldots\ldots\right).$$

Rearranging and substituting $\rho\breve{r}$ for $\breve{\rho}^2$ and noting that

$$q_{m+1} = \frac{m-1}{m}\, q_{m-1},$$

we have

$$\rho\,(n-1)\,q_n = \breve{r} q_{n-1}\,(n - 4 - \rho^2\,(n-1)^2) + \tfrac{1}{2}(n-1)\,\breve{r}^2 \rho q_n\,(2\,(n-3) - \rho^2 n^2) + \frac{(n-1)\,n}{6}\,\breve{r}^3 \rho^2 q_{n+1}\,(3\,(n-2) - \rho^2\,(n+1)^2) + \text{etc.} \ldots\ldots(\text{lviii}),$$

where the form of the successive terms is sufficiently obvious, and the series converges rapidly if ρ be small.

For the particular case in which we have chiefly used this equation to determine $\breve{r}$, namely samples of three, $\breve{r}$ corresponds to an antimode and the equation is for $n = 3$:

$$\begin{aligned} 2\rho q_3 = {} & -\breve{r} q_2\,(1 + 4\rho^2) - \breve{r}^2 q_3 9\rho^3 + \breve{r}^3 q_4 \rho^2\,(3 - 16\rho^2) \\ & + \breve{r}^4 q_5 \rho^3\,(8 - 25\rho^2) + \breve{r}^5 q_6 \rho^4\,(15 - 36\rho^2) \\ & + \breve{r}^6 q_7 \rho^5\,(24 - 49\rho^2) + \breve{r}^7 q_8 \rho^6\,(35 - 64\rho^2) \\ & + \text{etc.} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{lix}). \end{aligned}$$

An equation which led to $\breve{r}$ with singular accuracy and comparative ease for small values of ρ by aid of Table X for q_n.

(7) *Tables and Models.*

Table A (p. 379) gives the values of the mean, of the mode, of the standard deviation, of β_1 and β_2 and thus of the skewness of the frequency distributions of r. It will be seen that long after we have reached the limit of what are usually treated as small samples, the skewness of the distribution of r is very considerable. The approach to the normal curve is very slow, and the "probable error of the correlation coefficient," i.e. $\cdot 67449\,(1-r^2)/\sqrt{n}$ as usually recorded, has very little worth. Models have been prepared to illustrate these points as follows:

Model A gives for $n=2$ to $n=25$, the distribution of r for $\rho=\cdot 6$.

Model B gives for $n=2$ to $n=25$, the distribution of r for $\rho=\cdot 8$.

Model C gives for $n=3$, the distribution of r for $\rho=0$ to $\cdot 9$.

Model D gives for $n=4$, the distribution of r for $\rho=0$ to $\cdot 9$.

Model E gives for $n=25$, the distribution of r for $\rho=0$ to $\cdot 9$.

Model F gives for $n=50$, the distribution of r for $\rho=0$ to $\cdot 9$.

(Further models are in process of construction for low values of n.)

Even the photographs of such models form a striking warning of the dangers which arise (i) from small samples, and (ii) from judging results from even repeated small samples; the modal value of the frequency distribution for the correlation of these will be very sensibly higher than the correlation of the sampled population.

(8) *On the Determination of the "most likely" Value of the Correlation in the Sampled Population*, i.e. $\hat{\rho}$.

We now turn to another point. Suppose we have found the value of the correlation in a small sample to be r, what is the most reasonable value $\hat{\rho}$ to give to the correlation ρ of the sampled population?

Now we know that

$$y_n=(n-2)\frac{(1-\rho^2)^{\frac{n-1}{2}}}{\pi}(1-r^2)^{\frac{n-4}{2}}\int_0^\infty\frac{dz}{(\cosh z-\rho r)^{n-1}},$$

and if $\phi(\rho)\,d\rho$ were the law of distribution of ρ's, we ought to make

$$\frac{n-2}{\pi}(1-\rho^2)^{\frac{n-1}{2}}\phi(\rho)(1-r^2)^{\frac{n-4}{2}}\int_0^\infty\frac{dz}{(\cosh z-\rho r)^{n-1}}\,d\rho$$

a maximum with ρ, or in other words deduce the value of ρ for a given r from

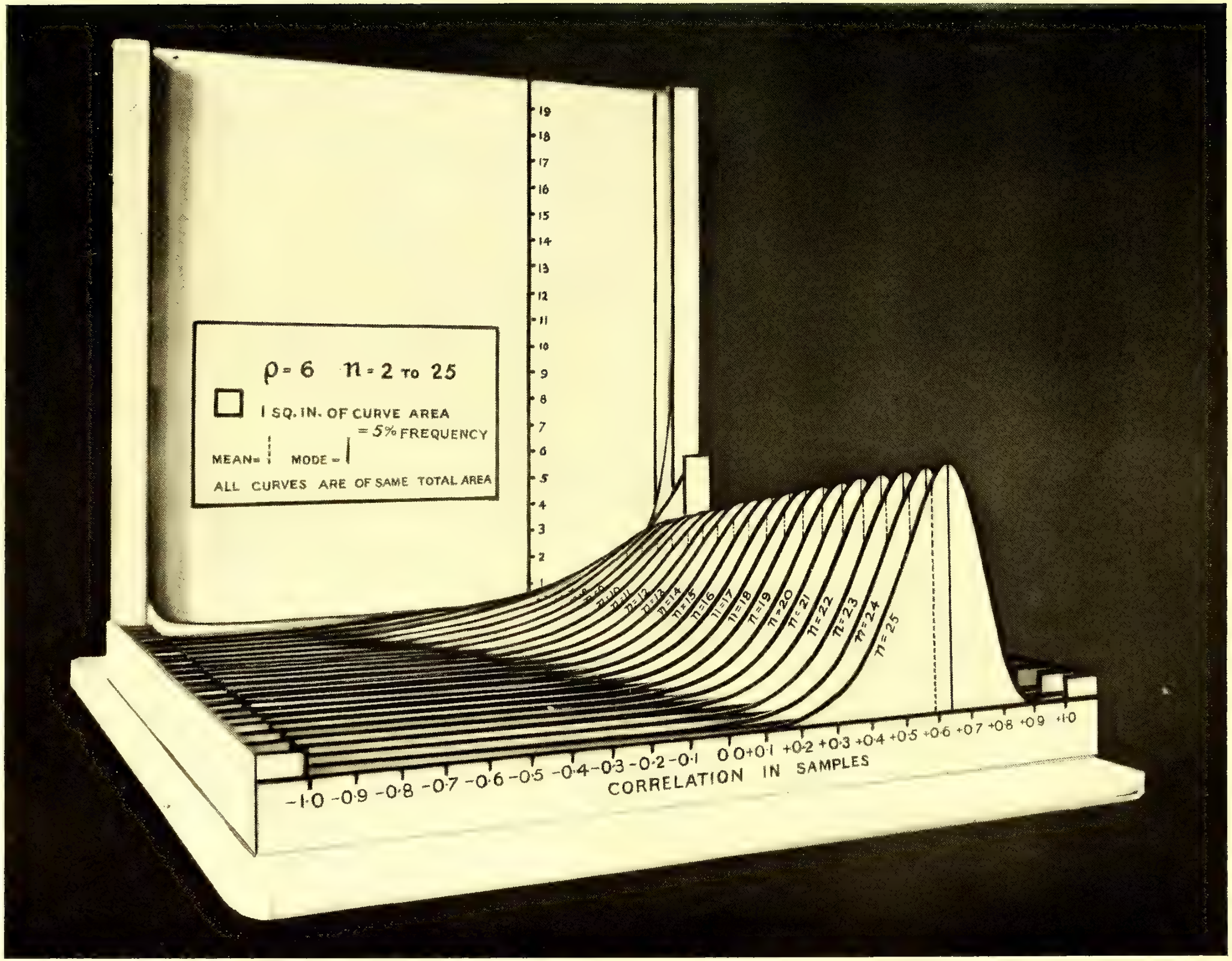
$$\frac{d}{d\rho}\left\{\int_0^\infty\frac{(1-\rho^2)^{\frac{n-1}{2}}\phi(\rho)\,dz}{(\cosh z-\rho r)^{n-1}}\right\}=0 \quad\text{.......................(lx).}$$

Fisher puts $\phi(\rho)$ equal to a constant and then differentiating out reaches the equation

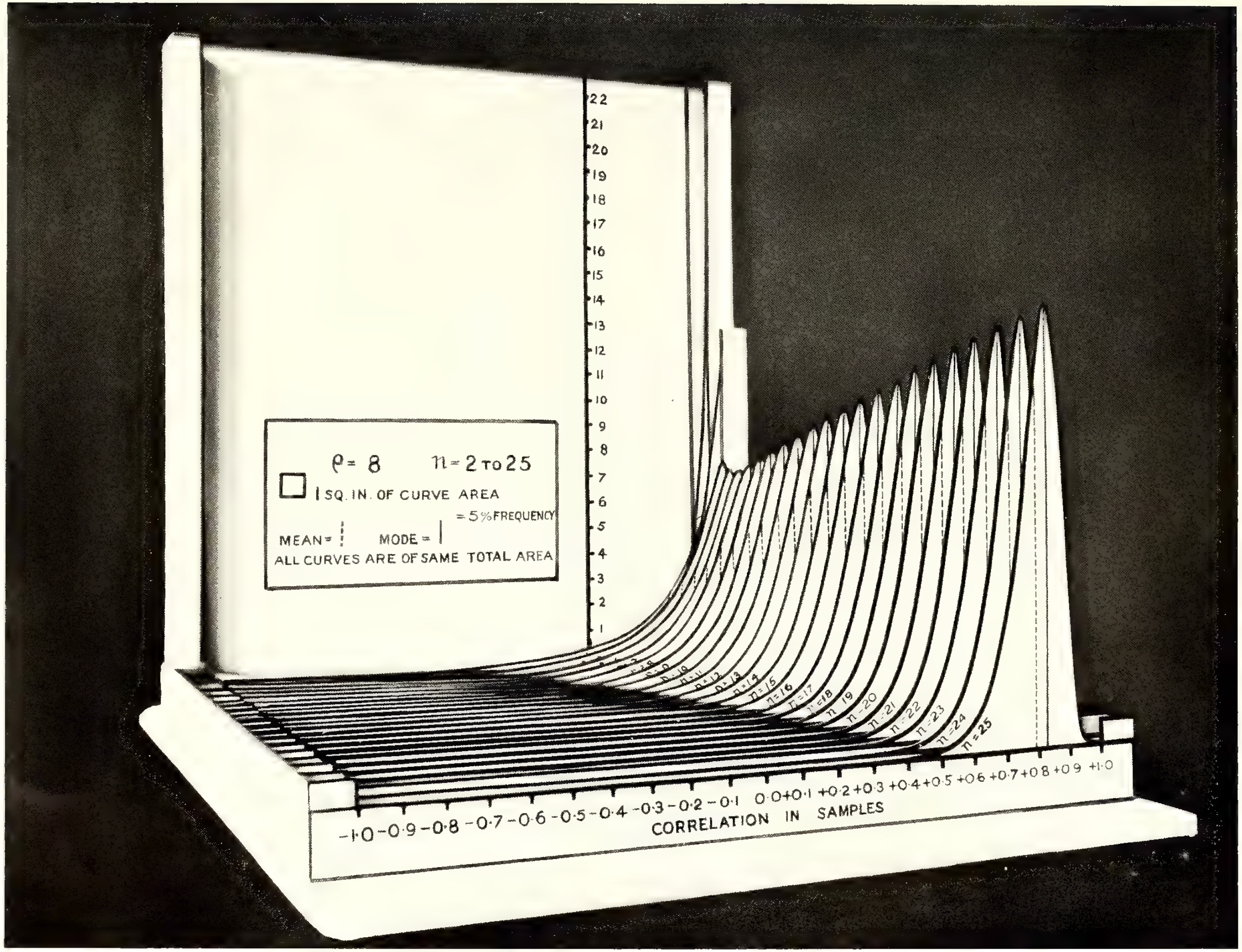
$$\int_0^\infty\frac{(r-\rho\cosh z)\,dz}{(\cosh z-\rho r)^n}=0 \quad\text{..............................(lxi),}$$

which should provide the value of ρ in terms of r. He solves this only to a first approximation, obtaining,

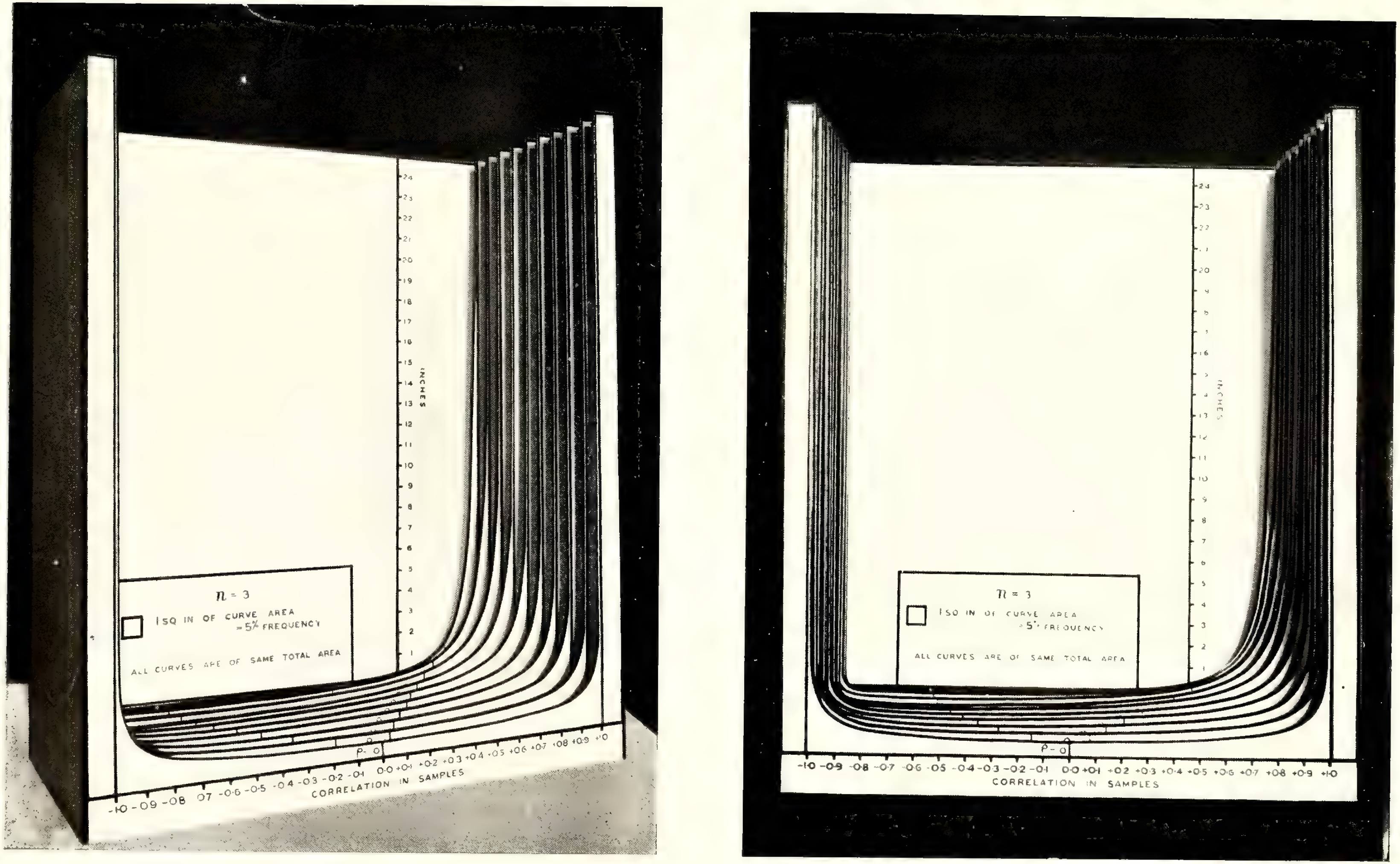
$$\rho=r\left(1-\frac{1-r^2}{2n}\right) \quad\text{..............................(lxii).}$$



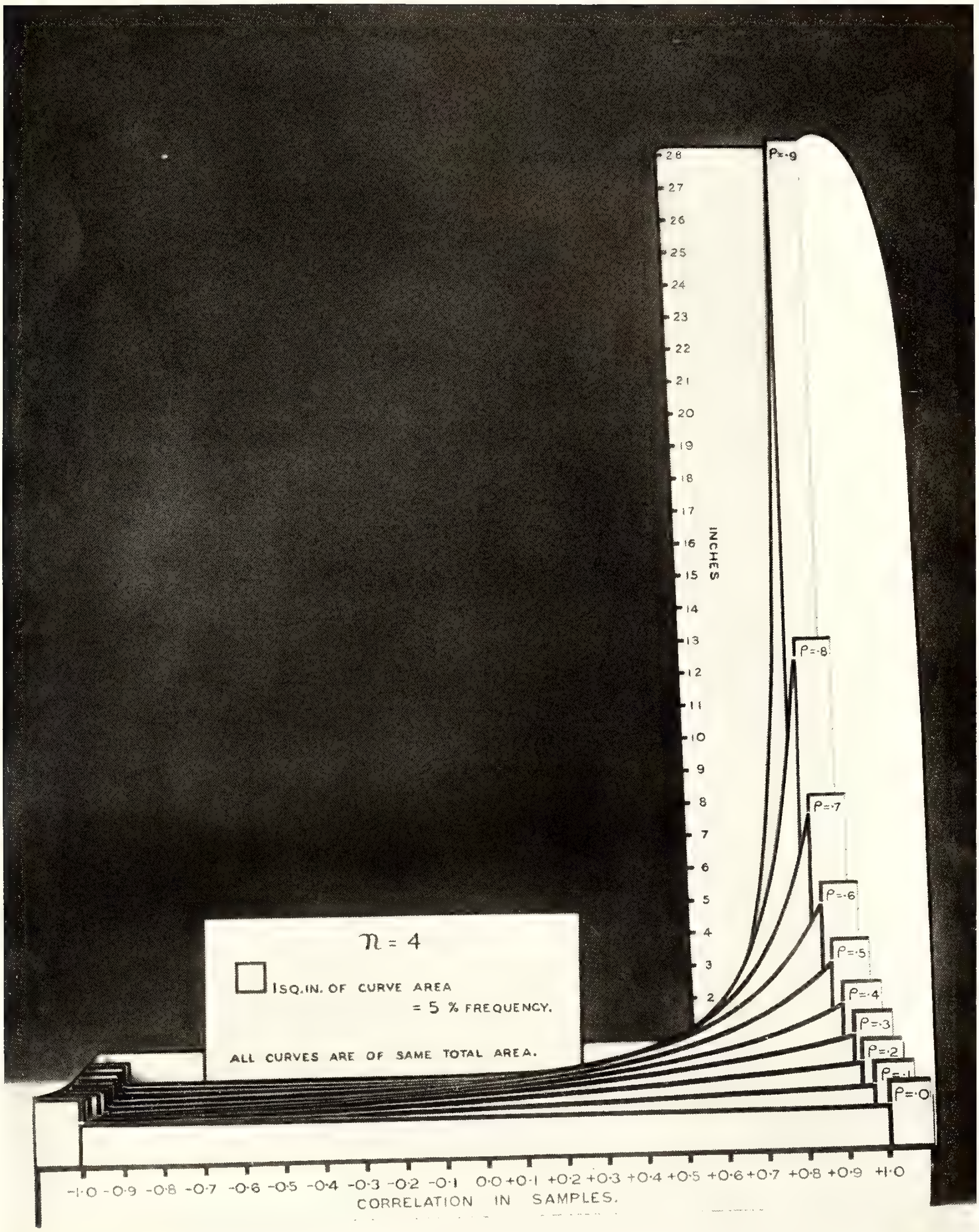
Correlation in Small Samples. $\rho = 0{\cdot}6$. Frequency curves for samples of sizes two to twenty-five, showing the changes in type from a skew "cocked hat" to **J**- and **U**-forms. Model A.


ρ = 8   n = 2 TO 25
1 SQ. IN. OF CURVE AREA = 5% FREQUENCY
MEAN =   MODE =
ALL CURVES ARE OF SAME TOTAL AREA
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
n = 9
n = 10
n = 11
n = 12
n = 13
n = 14
n = 15
n = 16
n = 17
n = 18
n 19
n = 20
n = 21
n = 22
n = 23
n = 24
n = 25
-1·0 -0·9 -0·8 -0·7 -0·6 -0·5 -0·4 -0·3 -0·2 -0·1 0·0 +0·1 +0·2 +0·3 +0·4 +0·5 +0·6 +0·7 +0·8 +0·9 +1·0
CORRELATION IN SAMPLES

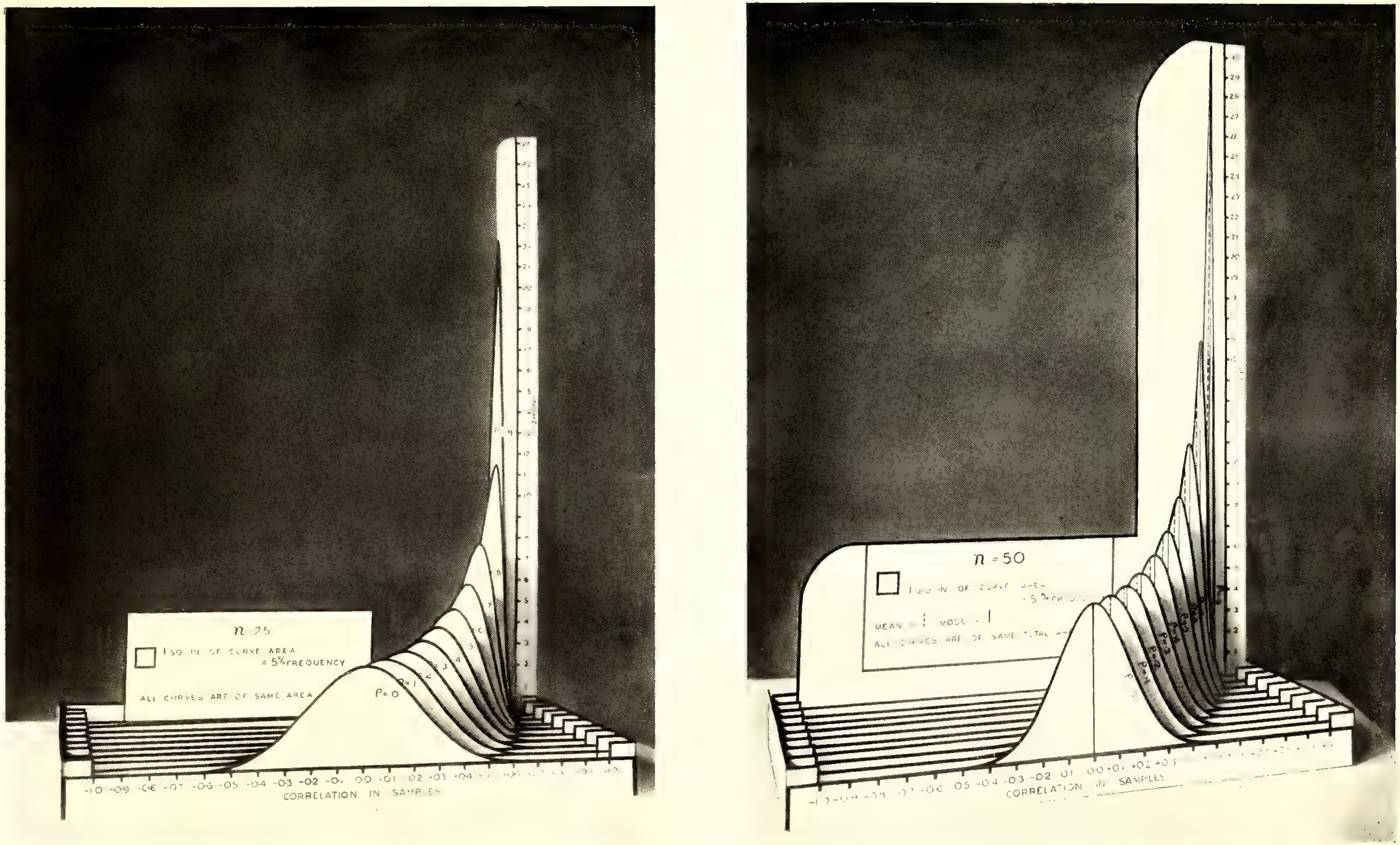
Correlation in Small Samples. $\rho = 0{\cdot}8$. Frequency curves for samples of sizes two to twenty-five, showing the changes in type from a skew "cocked hat" to J- and U-forms. Model B.


$n = 3$
1 SQ IN OF CURVE AREA = 5% FREQUENCY
ALL CURVES ARE OF SAME TOTAL AREA
INCHES
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
ρ = 0
-1·0 -0·9 -0·8 -0·7 -0·6 -0·5 -0·4 -0·3 -0·2 -0·1 0·0 +0·1 +0·2 +0·3 +0·4 +0·5 +0·6 +0·7 +0·8 +0·9 +1·0
CORRELATION IN SAMPLES

Correlation in Small Samples, $\rho=0{\cdot}0$ to $\rho=0{\cdot}9$. Frequency curves for samples of size three. Model C in two aspects, illustrating the various **U**-forms of curves, which occur in this case.



Correlation in Small Samples, $\rho=0{\cdot}0$ to $\rho=0{\cdot}9$ for samples of four. Model D, illustrating forms of frequency passing from the rectangle to marked J-forms of curves, which occur in this case.


n = 25
1·50 in. of curve area = 5% frequency
All curves are of same area
ρ = 0
CORRELATION IN SAMPLES
n = 50
Mean
Mode
All curves are of same total area
CORRELATION IN SAMPLES

Correlation in Samples of 25 and 50 for $\rho = 0{\cdot}0$ to $\rho = 0{\cdot}9$. Models E and F, illustrating forms still deviating very considerably from normality and increasing in skewness with increase of ρ.

This is a lower approximation than Soper's second approximation, in similar cases, and we know that even Soper's values are not sufficiently accurate when n is as large as 25. Hence no very great confidence can be put in (lxii).

But there is another point about (lx) which is of great importance. Fisher's equation, our (lxi), is deduced on the assumption that $\phi(\rho)$ is constant. In other words he assumes a horizontal frequency curve for ρ, or holds that *à priori* all values of ρ are equally likely to occur. This raises a number of philosophical points and difficulties. We ask:

When we are in absolute ignorance as to ρ, is it according to our experience that all values of the correlation are equally likely to occur? We think this question must probably if not certainly be answered in the negative. Very high correlations are relatively rare, and most biometricians would find it difficult to cite straight away a couple of cases of the correlation equal to $-\cdot 95$ although they could cite a score in which the correlation was sensibly zero, or again about $\cdot 5$. Every biometrician is seeking high correlations, for these are for him the all important data, but he knows how difficult and rare they are to find*. The equal distribution of ignorance which applies so well to many statistical ratios, does not seem valid in the case of correlations†. We generally know quite approximately

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* We have recently had occasion to table (*a*) nearly 400 correlations between characters of the human femur, and (*b*) over 300 for characters of the human skull. The distributions were very far indeed from horizontal straight lines, and to suppose *à priori* such distributions horizontal could only lead to grave errors.

† A similar problem arises in the case of standard deviations. If Σ be the S.D. of the sample and σ of the sampled population, then the frequency curve for S.D.'s is (*Biometrika*, Vol. X. p. 523)

$$y = y_0 \frac{\Sigma^{n-2}}{\sigma^{n-1}} e^{-\frac{1}{2}\frac{n\Sigma^2}{\sigma^2}}.$$

Now, if we make this a maximum for variation of σ, we obtain

$$\sigma = \sqrt{\frac{n}{n-1}}\,\Sigma \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\alpha)$$

as the "best value" of σ.

This was pointed out to the Editor by "Student," and was a desirable criticism of the statement made (Vol. X. pp. 528–9) that the most reasonable value to give to Σ was the mode of the sampled population, i.e. to take the observed $\Sigma = \breve{\Sigma} = \sqrt{\frac{n-2}{n}}\,\sigma$ or suppose

$$\sigma = \sqrt{\frac{n}{n-2}}\,\Sigma \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\beta).$$

Equations (α) and (β) are not identical. But again (α) is based on the assumption that *all* values of the S.D. are *à priori* equally likely to occur. But surely this is not a result in accordance with our experience! Values of σ from 0 to ∞ are not equally within our experience, and there is almost an absurdity in talking about a standard deviation varying from 0 to ∞; are we to include all possible scales in this distribution? The S.D. of stature might certainly be anything from practically zero to infinity if we measured it first in "light years" and then in microns. Or, are we to measure our S.D.'s all in the same units, when we suppose the distribution of S.D.'s to be of equal probability from zero to infinity? How is this to be done in the case of an absolute length and an index? Given a definite problem, there is certainly no *à priori* likelihood that the S.D. will have every value from 0 to ∞, if we confine ourselves to one scale. It must practically be less than the mean value, and in most actual the correlation of the characters in the population samples, and desire to ascertain whether a small sample of some population *similar to a known population* confirms our experience.

For example, we may have twenty pairs of brothers recorded for some special character. Our *à priori* knowledge is certainly not that all correlations between pairs of brothers from − 1 to + 1 are equally likely to occur! On the contrary we anticipate a value which will not be very far from 0·5. And this *à priori* conviction is so great, that if the small sample did not give a value which considering the size of the sample was compatible with the correlation in the sampled population being near 0·5, we should suspect errors in the measurement or some form of disturbing selection. In such cases, and something like them appears to us most frequent in biometric practice, it is we think erroneous to apply Bayes' Theorem. All it seems possible to do is to assume that we have drawn a value near the mode of our distribution, for our sampled population is much more likely to have a single value, that of our *à priori* experience, than every possible value from − 1 to + 1. If Bayes' Theorem confirms this value—so much the better; if it does not, its fundamental hypothesis is usually so unjustified that it seems most unreasonable to assert that it must give the most likely value of the correlation in the sampled population.

The fuller solution of Eqn. (lxi) thus appears to have academic rather than practical value. Still certain points of theoretical interest arise in the discussion of both (lx) and (lxi). Let us suppose that our *à priori* knowledge consists in the distribution of ρ about a mean $\bar{\rho}$ with a standard deviation κ. It is convenient to take $\kappa^2 = m\,(1 - \bar{\rho}^2)$, where m is an arbitrary constant. Probably $\kappa = 0$, whenever $\bar{\rho} = 1$, and this suggested this form; but since m is quite arbitrary we lay no stress on this point. The equation to determine the most likely value of ρ now becomes

$$\frac{d}{d\rho}\int_0^\infty \frac{(1-\rho^2)^{\frac{n-1}{2}}\, e^{-\frac{(\rho-\bar{\rho})^2}{2m\,(1-\bar{\rho})^2}}}{(\cosh z - \rho r)^{n-1}}\, dz = 0,$$

or

$$\frac{(\rho-\bar{\rho})\,(1-\rho^2)}{m\,(n-1)\,(1-\bar{\rho}^2)}\, I_{n-1} + \rho I_{n-1} = (1-\rho^2)\, r I_n \quad \text{............(lxiii)}.$$

Now this equation cannot in general be solved unless we know the order of the product $m\,(n-1)$. Certain cases, however, can be considered. If m be very large, i.e. if there be very considerable scatter in our past experience of ρ, then

$$\rho I_{n-1} = (1-\rho^2)\, r I_n \text{.................................(lxiv)},$$

problems is very narrowly limited. For example we measure twenty individuals of a population for stature, and seek the best value of the variability of the sampled population from the result. Would it not be unreasonable to suppose that *à priori* this variability may be equally likely to have any value from 0 to ∞ ? Our *à priori* knowledge is that it is somewhere between 2″·5 and 3″·0 and very far from equally likely even between these values. To justify the equal distribution of our ignorance, we should have to assume that we neither knew the exact character measured, nor the unit in which it was measured, and such ignorance can only be very exceptional in the present state of our knowledge.

an equation identical with what we obtain by the "equal distribution of our ignorance." The same result is also reached if m be only moderately large and n very big. In other words "the equal distribution of our ignorance," even if we really have some knowledge of the frequency distribution of ρ, will not lead us badly astray in the case of big samples. The matter is quite otherwise, however, in the case of *small* samples; unless our knowledge is very limited (m very large) we have no right whatever to take (lxiv) as applying to such small samples. Indeed when m is fairly small ρ will not differ substantially from $\bar{\rho}$, and the solution of (lxiii) will differ widely from that of (lxiv). We may consider these cases in succession.

Case (i). *Very slight knowledge of ρ, or on the other hand a large sample.*

Here we are justified in using (lxiv). We can attempt its solution in two different ways as in the case of the mode.

Let $\hat{\rho}$ be the most likely value of ρ and let us write $\hat{\rho}r = \rho_1^2$, then

$$(1 - \hat{\rho}^2)\, r\hat{I}_n = \hat{\rho}\hat{I}_{n-1},$$

or

$$(r^2 - \rho_1^4)\, \hat{I}_n = \rho_1^2\hat{I}_{n-1}.$$

Now let ρ_0^2 be a first approximation to ρ_1^2 and suppose $\rho_1^2 = \rho_0^2 + \epsilon$, where ϵ is small. Then

$$(r^2 - \rho_0^4 - 2\rho_0^2\epsilon)\,(I_n' + n\epsilon I'_{n+1}) = (\rho_0^2 + \epsilon)\,(I'_{n-1} + \overline{n-1}\,\epsilon I_n'),$$

where

$$I_n' = \int_0^\infty \frac{dz}{(\cosh z - \rho_0^2)^n}.$$

Hence remembering that

$$n\,(1 - \rho_0^4)\, I'_{n+1} = (2n - 1)\,\rho_0^2 I_n' + (n - 1)\, I'_{n-1},$$

we find

$$\epsilon = \frac{I_n'\,(r^2 - \rho_0^4) - \rho_0^2 I'_{n-1}}{I'_{n-1} + \rho_0^2\,(n + 1)\, I_n' - n\,(r^2 - \rho_0^4)\, I'_{n+1}},$$

or

$$\epsilon = (1 - \rho_0^4)\frac{(r^2 - \rho_0^4)\, E_n - \rho_0^2}{(1 - \rho_0^4) - (n - 1)(r^2 - \rho_0^4) + \rho_0^2\{(n + 1)(1 - \rho_0^4) - (2n - 1)(r^2 - \rho_0^4)\}\, E_n} \quad \text{..................(lxv)},$$

where

$$(1 - \rho_0^4)\,(n - 1)\, E_n = (2n - 3)\,\rho_0^2 + \frac{n - 2}{E_{n-1}} \quad \text{...............(lxvi)},$$

and

$$E_n = I_n'/I'_{n-1}.$$

Now (lxv) and (lxvi) may be treated exactly like the corresponding equations for the determination of the mode. If n be moderately large, we may put

$$E_n = E_{n-1} = E'$$

in (lxv), and if we know ρ_0^2 obtain the value of E', which value substituted for E_n in (lxvi) gives us ϵ and thus a new approximation. If we cannot guess a good value for ρ_0^2 (although $\rho_0^2 = \rho^2$ is in this case usually sufficient) we can treat the numerator of (lxv) equated to zero, and (lxvi) with E' for E_n and E_{n-1} as simultaneous equations to find E' and ρ_0^2, and so obtain a good approximation straight off, when n is of the order 25, or a fair first approximation when n is smaller.

Applying this we have from (lxv)

$$E_n = E_{n-1} = \rho_0^2/(r^2 - \rho_0^4).$$

Hence

$$(n-1)(1-\rho_0^4)\rho_0^4 - (2n-3)\rho_0^4(r^2-\rho_0^4) - (n-2)(r^2-\rho_0^4)^2 = 0,$$

or

$$\rho_0^2 = r^2\sqrt{\frac{n-2}{n-1-r^2}},$$

and therefore

$$\hat{\rho} = r \times \sqrt{\frac{n-2}{n-1-r^2}} \quad \text{.........................(lxvii)}.$$

Thus it will be seen that on the hypothesis of the equal distribution of ignorance for $n = 100$, the ratio of $\hat{\rho}$ to r will differ less than ·99402 from unity. On the other hand if n be 5, the ratio of $\hat{\rho}$ to r may differ from unity by as much as ·8660 does. For example, if $n = 5$, then $\hat{\rho} = {\cdot}26278$ if $r = {\cdot}3$. But for a sample of five the standard deviation of a value of ρ between ·2 and ·3 is of the order ·18 to ·20, so that there is little to be gained by treating the observed ·30 as corresponding to a sampled population of ·26.

We shall now proceed to determine an expansion for $\hat{\rho}$. If R be the ratio of Eqn. (liii), we find from (lxiv)

$$\hat{\rho} = r(1-\hat{\rho}^2)\,\hat{I}_n/\hat{I}_{n-1},$$

or

$$\hat{\rho} = \frac{r(1-\hat{\rho}^2)}{1-\hat{\rho}r}\sqrt{\frac{n-1}{n}}\,R$$

$$= \frac{r(1-\hat{\rho}^2)}{1-\hat{\rho}r}\left(1+\frac{1}{n-1}\right)^{-\frac{1}{2}}\left(1-\frac{\phi_1}{(n-1)^2}+\frac{\phi_1^2-2\phi_2+\phi_1}{(n-1)^3}+\dots\right),$$

whence on substituting

$$\hat{\rho} = r\left(1-\frac{1}{2}\frac{1-\hat{\rho}^2}{n-1}+\frac{1}{8}\frac{(1-r\hat{\rho})(1-\hat{\rho}^2)}{(n-1)^2}+\frac{1}{16}\frac{(1+r\hat{\rho}-2r^2\hat{\rho}^2)(1-\hat{\rho}^2)}{(n-1)^3}+\dots\right),$$

and after inversion

$$\hat{\rho} = r\left(1-\frac{1}{2}\frac{1-r^2}{n-1}+\frac{(1-5r^2)(1-r^2)}{8(n-1)^2}+\frac{(1+8r^2-17r^4)(1-r^2)}{16(n-1)^3}+\dots\right) \quad \text{.........(lxviii)}.$$

This result gives us a measure of the correctness of (lxvii), for that equation may be written

$$\hat{\rho} = r\left(\frac{1-\dfrac{1}{n-1}}{1-\dfrac{r^2}{n-1}}\right)^{\frac{1}{2}}$$

$$= r\left\{1-\frac{(1-r^2)}{2(n-1)}-\frac{(1+3r^2)(1-r^2)}{8(n-1)^2}-\frac{(1+2r^2+5r^4)(1-r^2)}{16(n-1)^3}+\dots\right\} \text{(lxix)}.$$

Thus the divergence begins as early as the term in $1/(n-1)^2$ and (lxvii) can only be trusted for rough approximations to $\hat{\rho}$.

Illustration. Suppose $r = \cdot 6$, what is the "most likely" value of ρ, on the assumption of equal distribution of ignorance? Let $n = 25$, then we find from (lxviii)

$$\hat{\rho} = \cdot 59194,$$

while (lxvii) gives ·59182, an agreement adequate for most statistical purposes.

If $n = 5$, and $r = \cdot 6$, we find

$$\begin{aligned}\hat{\rho} &= \cdot 55058 \text{ from (lxviii)},\\ &= \cdot 56695 \text{ from (lxvii)}.\end{aligned}$$

There is now considerable divergence in the two methods and another approximation is desirable. Let us take $\rho_0{}^2 = \hat{\rho}r = \cdot 33035$, then to find E_n we have

$$(n-1) \times \cdot 890{,}8689 E_n = (2n-3) \times \cdot 33035 + \frac{n-2}{E_{n-1}}$$

E_2 will be given by (xlvii) and equals ·885,5939, whence we determine

$$E_3 = 1\cdot 189{,}9819, \quad E_4 = 1\cdot 246{,}8946, \quad E_5 = 1\cdot 324{,}1082,$$

and accordingly from (lxv)

$$\epsilon = \cdot 001{,}3081,$$

$$r\hat{\rho} = \rho_0{}^2 + \epsilon = \cdot 331{,}8581,$$

and $\hat{\rho} = \cdot 553{,}097$, a value not far removed from that found by the first approximation. We conclude that even when n is small, quite good results will be found from (lxviii) and that it is probably better to use this rather than (lxvii) in such cases as the starting point for a second approximation.

Case (ii). *Close à priori Knowledge of* ρ.

We will now suppose m small, so that the first approximation to ρ may be taken as $\bar{\rho}$. We substitute in (lxiii) $\rho = \bar{\rho} + \psi$ and we find, neglecting squares of ψ,

$$\left(\frac{\psi}{m(n-1)} + \bar{\rho} + \psi\right)(\bar{I}_{n-1} + (n-1)\, r\psi \bar{I}_n) = (1 - \bar{\rho}^2 - 2\bar{\rho}\psi)(r\bar{I}_n + nr\psi \bar{I}_{n+1}),$$

where
$$\bar{I}_n = \int_0^\infty \frac{dz}{(\cosh z - \bar{\rho}r)^{n-1}}.$$

This leads us to

$$\psi = \frac{(1-\bar{\rho}^2)\, r\bar{I}_n - \bar{\rho}\bar{I}_{n-1}}{\bar{I}_{n-1}\left(1 + \frac{1}{m(n-1)}\right) + \bar{\rho}\,(n+1)\, r\bar{I}_n - (1-\bar{\rho}^2)\, nr^2 \bar{I}_{n+1}},$$

whence remembering that

$$n\,(1 - (r\bar{\rho})^2)\, \bar{I}_{n+1} = (2n-1)\, r\bar{\rho}\bar{I}_n + (n-1)\, \bar{I}_{n-1},$$

and writing $\bar{E}_n = \bar{I}_n/\bar{I}_{n-1}$, we have after some transformations

$$\begin{aligned}\hat{\rho} &= \bar{\rho} + \psi \\ &= \bar{\rho} + \frac{(1 - r^2\bar{\rho}^2)\,((1 - \bar{\rho}^2)\,r\bar{E}_n - \bar{\rho})}{(1 - r^2\bar{\rho}^2)\left(1 + \dfrac{1}{m\,(n-1)}\right) - (n-1)\,r^2\,(1 - \bar{\rho}^2) + r\bar{\rho}\,\{(n+1)(1 - r^2\bar{\rho}^2) - (2n-1)\,r^2\,(1 - \bar{\rho}^2)\}\,\bar{E}_n}\end{aligned} \qquad \text{............(lxx)},$$

where $$(n-1)\,(1 - r^2\bar{\rho}^2)\,\bar{E}_n = (rn - 3)\,r\bar{\rho} + \frac{n-2}{\bar{E}_{n-1}} \qquad \text{..............(lxxi)}.$$

The method is now straightforward, at least for n moderately large. We put $\bar{E}_n = \bar{E}_{n-1} = \bar{E}'$ in (lxxi) and substitute the resulting value of $\bar{E}'$ for $\bar{E}_n$ in (lxx), and thus reach the small correction on $\bar{\rho}$.

Illustration. In a sample of 25 pairs only of parent and child the correlation for a certain character was found to be ·6. What is the most reasonable value to give to ρ in the sampled population?

If we distributed our ignorance equally the result would be that stated on p. 357, i.e.

$$\hat{\rho} = \cdot 59194.$$

But, in applying Bayes' Theorem to this case, to what result of experience do we appeal? Clearly the only result of experience by which we could justify this "equal distribution of ignorance" would be the accumulative experience that in past series the correlation of parent and child had taken with equal frequency of occurrence every value from -1 to $+1$. To appeal to such a result is absurd; Bayes' Theorem ought only to be used where we have in past experience, as for example in the case of probabilities and other statistical ratios, met with every admissible value with roughly equal frequency. There is no such experience in this case. On the contrary the mean value of ρ for very long series of frequencies of 1000 and upwards is known to be $+\cdot 46$ and the range is hardly more than ·40 to ·52. We may accordingly take $\bar{\rho} = \cdot 46$ and $m\,(1 - \bar{\rho}^2) = \kappa^2 =$ about ·0004, whence

$$m = \cdot 0004/\cdot 7884 = \cdot 000{,}507 \text{ say}.$$

Thus $\dfrac{1}{m\,(n-1)} = 82\cdot 1828$ and the term containing it is the dominating term in Equation (lxiii). Thus $\hat{\rho}$ will differ little from $\bar{\rho}$. We find

$$\hat{\rho} = \bar{\rho} + \frac{\cdot 437{,}006\bar{E}_n - \cdot 424{,}959}{70\cdot 034{,}491 + 2\cdot 790{,}925\bar{E}_n},$$

from (lxx).

We next determine $\bar{E}_n = \bar{E}'$ from (lxxi), i.e.

$$24 \times \cdot 923{,}824\bar{E}'^2 - 47 \times \cdot 276\bar{E}' - 23 = 0,$$

which gives us $\bar{E}' = 1\cdot 352{,}2185$, thus $\psi = \cdot 00225$ and

$$\hat{\rho} = \cdot 46225,$$

a totally different "most likely value" from that obtained by "equally distributing our ignorance."

Statistical workers cannot be too often reminded that there is no validity in a mathematical theory pure and simple. Bayes' Theorem must be based on experience, the experience that where we are *à priori* in ignorance all values are equally likely to occur. This is not the case in the present illustration, and we must use our past experience in the same way as we should use our past experience of equal frequency; the appeal to this experience has here absolutely the same validity as in Bayes' case and cannot be for a moment neglected. We see that our new experience scarcely modifies the old and this is what we should naturally conjecture would be the case. If we increase the size of the new sample, then ultimately $1/m\,(n-1)$ becomes very small, and we approach nearer the value ·59194 given by Bayes' Theorem. But past experience will bias the value obtained from the new material for a long time, and we see that according to the value of the past experience $\hat{\rho}$ may vary from ·46225 to ·59194. It will thus be evident that in problems like the present the indiscriminate use of Bayes' Theorem is to be deprecated. It has unfortunately been made into a fetish by certain purely mathematical writers on the theory of probability, who have not adequately appreciated the limits of Edgeworth's justification of the theorem by appeal to *general* experience.

Case (iii). *Past Experience a Factor, but not the Dominating Factor of Judgment.*

Cases can arise in which $\rho = \bar{\rho}$ is not a very close approximation, i.e. when we have some past experience, but not a very concentrated one of like correlations. In this case we must return to Equation (lxiii), and we shall assume $\hat{\rho}r = {\rho_0}^2 + \epsilon$, where ${\rho_0}^2$ is some fairly close approximation to $\hat{\rho}r$. We shall write $\bar{\rho}r = {\bar{\rho}_0}^2$. We find

$$\epsilon = \frac{(r^2 - {\rho_0}^4)\, I_n - \left\{{\rho_0}^2 + \dfrac{({\rho_0}^2 - {\bar{\rho}_0}^2)\,(r^2 - {\rho_0}^4)}{m\,(n-1)\,(r^2 - {\bar{\rho}_0}^4)}\right\} I_{n-1}}{\left\{1 + \dfrac{r^2 + 2{\rho_0}^2{\bar{\rho}_0}^2 - 3{\rho_0}^4}{m(n-1)(r^2 - {\bar{\rho}_0}^4)}\right\} I_{n-1} + \left\{(n+1)\,{\rho_0}^2 + \dfrac{({\rho_0}^2 - {\bar{\rho}_0}^2)\,(r^2 - {\rho_0}^4)}{m\,(r^2 - {\bar{\rho}_0}^4)}\right\} I_n - n\,(r^2 - {\rho_0}^4)\, I_{n+1}} \quad \text{............(lxxii)},$$

where $$n\,(1 - {\rho_0}^4)\, I_{n+1} = (2n-1)\,{\rho_0}^2 I_n + (n-1)\, I_{n-1} \quad \text{..............(lxxiii)},$$

equations which can be readily expressed in terms of E's.

Unfortunately the approximation obtained by equating the numerator of ϵ to zero and using (lxxiii) as simultaneous equations is not very rapidly obtained as the resulting equation is now of the eighth order. It is better from the data themselves to guess a reasonable value for ${\rho_0}^2$ and start the approximation from this.

Illustration. The correlation between the maximum length and breadth of crania is not very definitely known. Its mean is about ·30, but the values determined for it range from nearly zero to ·6. Assuming the standard deviation to be ·1, what is the "most likely value" to give to this correlation in the case of a sample of 25 skulls showing a correlation of ·50?

Here $\bar{\rho} = \cdot 30$, $m(1-\bar{\rho}^2) = \cdot 01$, and $n = 25$. $\bar{\rho}_0{}^2 = r\bar{\rho} = \cdot 15$. We will assume as a first approximation to $\hat{\rho}$, $\hat{\rho} = \cdot 40$, hence $\rho_0{}^2 = \cdot 20$. Equation (lxxiii) for $n = n - 1$ gives

$$(n-1) \times \cdot 96 E_n = (2n-3) \times \cdot 20 + (n-2)/E_{n-1}.$$

Put $n = 25$, and $E' = E_n = E_{n-1}$, and we have to find E',

$$23 \cdot 04 E'^2 - 9 \cdot 4 E' - 23 = 0,$$

which gives $E' = 1 \cdot 223{,}7367$; from this we deduce $E_{n+1} = 1 \cdot 225{,}5025$, and

$$\epsilon = - \cdot 029{,}9238,$$

leading to $r\hat{\rho} = \cdot 170{,}0762$, or $\hat{\rho} = \cdot 34015$.

Starting again with $\rho_0{}^2 = \cdot 17008$, we find

$$23 \cdot 305{,}7472 E'^2 - 7 \cdot 99376 E' - 23 = 0,$$

giving $E' = 1 \cdot 179{,}6109$ and $E_{n+1} = 1 \cdot 181{,}3579$.

Whence we deduce $\epsilon = + \cdot 003{,}999$,

and accordingly $r\hat{\rho} = \cdot 174{,}079$ and $\hat{\rho} = \cdot 34816$, a close enough approximation.

But if we had "equally distributed our ignorance" we should have found*

$$\hat{\rho} = \cdot 49217.$$

These results seem extremely suggestive. If we were to observe the correlation of length and breadth of skull in a new sample of 25 skulls, then an observed value of ·50 would give a "most likely value" on the equal distribution of ignorance of ·4922.

But no biometrician would admit absolute ignorance in such a case; the correlation has been determined rather vaguely and not very adequately so that results range from something like zero to ·6. But this *à priori* knowledge leads on precisely the same basis as Bayes' Theorem to the value $\hat{\rho} = \cdot 3482$—a result very much closer to previous experience of the mean value, than to the observed result. And there are relatively few cases in which some such, if only vague, *à priori* experience does not exist.

In the light of the above illustrations we consider it justifiable to assert that the results deduced from the principle of the "equal distribution of ignorance" have academic rather than practical value, and we hold that to apply it without consideration of its basis to the problem of finding the most likely values of the statistical constants of a sampled population from the values observed in a small sample may lead to results very wide from the truth.

(9) *Special Cases of Frequency for n small.*

We shall now discuss individually the lowest sample sizes.

(i) *Samples of Two, $n = 2$.* Here

$$\bar{r} = \frac{\sin^{-1}\rho}{\frac{1}{2}\pi},$$

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* Equation (lxvii) would give $\hat{\rho} = \cdot 49204$, nearly as good practically as (lxviii)

and the distribution consists of $\dfrac{\cos^{-1}(-\rho)}{\pi}$ at $r = +1$ and $\dfrac{\cos^{-1}\rho}{\pi}$ at $r = -1$*, or at what is the same thing $\frac{1}{2}(1 + \bar{r})$ and $\frac{1}{2}(1 - \bar{r})$.

The moments are

$$\mu_1' = \mu_3' = \bar{r}, \qquad \mu_2' = \mu_4' = 1,$$

and accordingly

$$\mu_2 = 1 - \bar{r}^2, \qquad \mu_3 = -2\bar{r}(1 - \bar{r}^2), \qquad \mu_4 = (1 - \bar{r}^2)(1 + 3\bar{r}^2) \ldots \text{(lxxiv)}.$$

Hence $\beta_1 = 4\bar{r}^2/(1 - \bar{r}^2), \qquad \beta_2 = (1 + 3\bar{r}^2)/(1 - \bar{r}^2),$

and accordingly

$$\beta_2 - \beta_1 - 1 = 0.$$

TABLE I. *Samples of Two.*

ρ, value of correlation in sampled population	$\bar{r}$, mean correlation of samples	σ_r	β_1	β_2	Number of positive correlations per 1000 samples	Number of negative correlations per 1000 samples
0·0	0	1	0	1	500·000	500·000
0·1	·063,7686	·997,965	·016,332	1·016,332	531·884	468·116
0·2	·128,1884	·991,750	·066,827	1·066,827	564·094	435·906
0·3	·193,9734	·981,007	·156,387	1·156,387	596·987	403·013
0·4	·261,9798	·965,007	·294,764	1·294,764	630·990	369·010
0·5	·333,3333	·942,809	·500,000	1·500,000	666·667	333·333
0·6	·409,6655	·912,236	·806,686	1·806,686	704·833	295·167
0·7	·493,6334	·869,670	1·288,724	2·288,724	746·817	253·183
0·8	·590,3345	·807,164	2·139,534	3·139,534	795·167	204·833
0·9	·712,8674	·701,299	4·133,056	5·133,056	856·434	143·566
1·0	1·000,0000	·000,000	∞	∞	1,000·000	·000,000

The distributions are two lumps given by the last two columns, and are accurately given by Pearson's skew frequency distributions for the relation $\beta_2 - \beta_1 - 1 = 0$ (see *Phil. Trans.* Vol. 216 A, p. 433).

(ii) *Samples of Three*, $n = 3$.

$$y_3 = N \frac{1 - \rho^2}{\pi} \frac{1}{\sqrt{1 - r^2}} \frac{dU}{dx},$$

where

$$x = r\rho \quad \text{and} \quad U = \cos^{-1}(-x)/\sqrt{1 - x^2}$$

as before. Hence

$$\mu_p' = \int_{-1}^{+1} r^p y_3 dr = \frac{1}{\rho^{p+1}} \int_{-\rho}^{+\rho} x^p y_3 dx$$

$$= \frac{\rho(1 - \rho^2)}{\pi \rho^{p+1}} \int_{-\rho}^{+\rho} \frac{x^p}{\sqrt{\rho^2 - x^2}} \frac{dU}{dx} dx.$$

---

* Since we can determine at sight whether any pair is positively or negatively correlated, this gives a method of determining ρ by simply counting the number of $+1$ and -1 correlations in the arrays, say m_p and m_n, then $\rho = \cos \pi \left(\dfrac{m_n}{m_n + m_p}\right)$ and the probable error of the determination is

$$\frac{\cdot 67449}{\sqrt{N}} \sqrt{1 - \rho^2} \sqrt{\frac{m_n}{N}\left(1 - \frac{m_n}{N}\right)},$$

N being the number of pairs used. Cf. "Student," *Biometrika*, Vol. VI. p. 304.

Now let $x = \rho \sin \phi$, then

$$\mu_p' = \frac{1-\rho^2}{\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \sin^p \phi \frac{dU}{dx} \, d\phi.$$

Now
$$\sin \phi \frac{dU}{dx} = \frac{dU}{d\rho},$$

hence
$$\mu_p' = \frac{1-\rho^2}{\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \sin^{p-1} \phi \frac{dU}{d\rho} \, d\phi$$

$$= \frac{1-\rho^2}{\pi} \frac{d}{d\rho} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \sin^{p-1} \phi U \, d\phi.$$

But
$$U = \int_0^\infty \frac{dz}{\cosh z - \rho \sin \phi},$$

thus
$$\mu_p' = \frac{1-\rho^2}{\pi} \frac{d}{d\rho} \int_0^\infty dz \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{\sin^{p-1}\phi}{\cosh z - \rho \sin \phi} \, d\phi \ldots .$$

Write
$$\psi = \frac{\pi}{2} + \phi \quad \text{and} \quad \cosh z = \eta,$$

we have
$$\mu_p' = \frac{1-\rho^2}{\pi} \frac{d}{d\rho} \int_0^\infty dz \int_0^\pi \frac{(-1)^{p-1} \cos^{p-1} \psi \, d\psi}{\eta + \rho \cos \psi} \quad \ldots\ldots\ldots\ldots \text{(lxxv)}.$$

Let $p = 1$, then we find

$$\mu_1' = \frac{2(1-\rho^2)}{\pi} \frac{d}{d\rho} \int_0^\infty dz \int_0^\pi \frac{d(\tan \frac{1}{2}\psi)}{(\eta+\rho) + (\eta-\rho) \tan^2 \frac{1}{2}\psi}.$$

But

$$\int_0^\pi \frac{d(\tan \frac{1}{2}\psi)}{(\eta+\rho) + (\eta-\rho)\tan^2 \frac{1}{2}\psi} = \left[\frac{1}{\sqrt{\eta^2-\rho^2}} \tan^{-1} \left(\frac{\tan \frac{1}{2}\psi}{\sqrt{\frac{\eta+\rho}{\eta-\rho}}} \right) \right]_0^\pi = \frac{\pi}{2} \frac{1}{\sqrt{\eta^2-\rho^2}}.$$

Thus
$$\mu_1' = (1-\rho^2) \frac{d}{d\rho} \int_0^\infty \frac{dz}{\sqrt{\eta^2-\rho^2}}.$$

Now take
$$\eta = \frac{1}{\sin \phi'} = \cosh z,$$

hence
$$-\frac{\cos \phi'}{\sin^2 \phi'} d\phi' = \sinh z dz,$$
$$= \sqrt{\eta^2 - 1} dz,$$

thus
$$dz = -\operatorname{cosec} \phi' d\phi'.$$

It follows that
$$\mu_1' = (1-\rho^2) \frac{d}{d\rho} \int_0^{\frac{\pi}{2}} \frac{d\phi'}{\sqrt{1-\rho^2 \sin^2 \phi'}}.$$

Let as usual
$$F\left(k,\ \frac{\pi}{2}\right) = \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}},$$

and
$$E\left(k,\ \frac{\pi}{2}\right) = \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 \sin^2 \phi}\, d\phi.$$

Then we have
$$\mu_1' = (1 - \rho^2) \frac{dF\left(\rho,\ \frac{\pi}{2}\right)}{d\rho}.$$

But
$$\frac{dF_1}{dk} = \frac{1}{kk'^2}(E_1 - k'^2 F_1),$$

where
$$k' = \sqrt{1 - k^2},$$

E_1 and F_1 denoting the complete elliptic integrals*.

Thus finally
$$\bar{r} = \frac{1}{\rho}\{E_1 - (1 - \rho^2) F_1\} \quad \text{........................(lxxvi)},$$

and $\bar{r}$ is known, as soon as ρ is given, from tables of the complete elliptic integrals.

Returning to Equation (xx) and putting $n = 3$ we have
$$\mu_2' = 1 - \frac{1}{2}(1 - \rho^2)\left(1 + \frac{\rho^2}{2} + \frac{\rho^4}{3} + \frac{\rho^6}{4} + \frac{\rho^8}{5} + \dots\right)$$
$$= 1 + \frac{1}{2}\frac{1 - \rho^2}{\rho^2}\left\{- \rho^2 - \frac{1}{2}(\rho^2)^2 - \frac{1}{3}(\rho^2)^3 - \frac{1}{4}(\rho^2)^4 - \dots\right\}$$
$$= 1 + \frac{1}{2}\frac{1 - \rho^2}{\rho^2}\log_e(1 - \rho^2) \quad \text{..(lxxvii)},$$

and further
$$\sigma_r^2 = \mu_2 = 1 - \bar{r}^2 + \frac{1}{2}\frac{1 - \rho^2}{\rho^2}\log_e(1 - \rho^2) \quad \text{...............(lxxviii)}.$$

We now turn to the third moment and may anticipate a recurrence of the elliptic integrals. We shall obtain our result on the whole most briefly by appealing to Equations (xxii) and (xxiii) on pp. 335 and 336. We have
$$\mu_3' = \chi_1 - \chi_3 = \frac{\rho(1 - \rho^2)}{2}\left\{I_3 \,.\, F\left(\frac{3}{2},\ \frac{3}{2},\ 2,\ \rho^2\right) - \frac{1}{3} I_5 \,.\, F\left(\frac{3}{2},\ \frac{3}{2},\ 3,\ \rho^2\right)\right\},$$

or since
$$I_3 = \frac{\pi}{2}, \quad I_5 = \frac{3}{4}\frac{\pi}{2},$$

$$\mu_3' = \frac{\rho(1 - \rho^2)}{2}\frac{\pi}{2}\left\{F\left(\frac{3}{2},\ \frac{3}{2},\ 2,\ \rho^2\right) - \frac{1}{4} F\left(\frac{3}{2},\ \frac{3}{2},\ 3,\ \rho^2\right)\right\}$$
$$= \frac{\rho(1 - \rho^2)}{2}\frac{\pi}{2}\left\{S\left[\frac{(3\,.\,5 \dots 2s + 1)^2}{2^{2s}(s)\,!\,(s + 1)\,!}\rho^{2s}\right] - \frac{1}{2} S\left[\frac{(3\,.\,5 \dots 2s + 1)^2}{2^{2s}(s)\,!\,(s + 2)\,!}\rho^{2s}\right]\right\}$$
$$= \frac{\rho(1 - \rho^2)}{2}\frac{\pi}{2}\left\{S\left[\frac{(3\,.\,5 \dots 2s + 1)^2}{2^{2s}(s)\,!\,(s + 1)\,!}\left(1 - \frac{1}{2s + 4}\right)\rho^{2s}\right]\right\}$$

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* Cayley, *Elementary Treatise on Elliptic Functions*, p. 48.

$$= \frac{\rho(1-\rho^2)}{2}\frac{\pi}{2}\left\{S\left[\frac{(3\,.\,5\,\ldots\,2s+3)^2}{2\,.\,2^{2s}(s+2)!\,(s+2)!}\,\frac{(s+1)(s+2)}{2s+3}\,\rho^{2s}\right]\right\}$$

$$= \frac{(1-\rho^2)}{2\rho^2}\frac{\pi}{2}\frac{d}{d\rho}\left\{S\left[\frac{(3\,.\,5\,\ldots\,2s+3)^2}{2^{2s}(s+2)!\,(s+2)!}\,\frac{s+1}{2s+3}\,\frac{\rho^{2s+4}}{2^2}\right]\right\}$$

$$= \frac{1-\rho^2}{\rho^2}\frac{d}{d\rho}\left\{\frac{\pi}{2}S\left[\frac{(3\,.\,5\,\ldots\,2s+3)^2}{2^4\,.\,2^{2s}(s+2)!\,(s+2)!}\left(1-\frac{1}{2s+3}\right)\rho^{2s+4}\right]\right\}$$

$$= \frac{1-\rho^2}{\rho^2}\frac{d}{d\rho}\left\{\frac{\pi}{2}F\left(\frac{1}{2},\ \frac{1}{2},\ 1,\ \rho^2\right)+\frac{\pi}{2}F\left(-\frac{1}{2},\ \frac{1}{2},\ 1,\ \rho^2\right)\right\}$$

$$= \frac{1-\rho^2}{\rho^2}\frac{d}{d\rho}\{F_1(\rho)+E_1(\rho)\}$$

$$= \frac{1-\rho^2}{\rho^2}\left\{\frac{E_1-(1-\rho^2)F_1}{\rho(1-\rho^2)}+\frac{E_1-F_1}{\rho}\right\},$$

or

$$\mu_3' = \frac{2-\rho^2}{\rho^3}E_1 - \frac{2(1-\rho^2)}{\rho^3}F_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(lxxix)},$$

where E_1 and F_1 are as before the complete elliptic integrals.

In order to obtain the fourth moment coefficient about zero we will return to formulae (xx) and (xxi) of pp. 334–5 and write

$$\mu_2' = 1 - \frac{n-2}{n-1}(1-\rho^2)f_2$$

and

$$\mu_4' = 2\mu_2' - 1 + \frac{n(n-2)}{(n+1)(n-1)}(1-\rho^2)^2 f_4,$$

where f_2 and f_4 are the hypergeometrical series.

Now the general term of f_2 is

$$\frac{(2\,.\,4\,.\,6\,.\,8\,\ldots\,2s+2)^2\,\rho^{2s+2}}{(s+1)!\,(n+1)(n+3)\ldots(n+2s+1)\,2^{s+1}},$$

and the general term of f_4 is

$$\frac{(4\,.\,6\,.\,8\,\ldots\,2s+2)^2\,\rho^{2s}}{(s)!\,(n+3)(n+5)\ldots(n+2s+1)\,2^{s}}.$$

Hence it follows that

$$\frac{n+1}{4\rho}\frac{df_2}{d\rho} = f_4,$$

or we have

$$\mu_4' = 1 - \frac{2(n-2)}{n-1}(1-\rho^2)f_2 + \frac{n(n-2)}{4(n-1)}(1-\rho^2)^2\frac{df_2}{\rho\,d\rho} \quad \ldots\ldots\ldots\text{(lxxx)}.$$

Thus if we are able to sum f_2 algebraically, we can determine μ_4' algebraically*

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* The corresponding formula for μ_3' and $\mu_1' = \bar{r}$ is

$$\mu_3' = \bar{r} - (1-\rho^2)(n-2)\frac{d}{d\rho}\left(\frac{\bar{r}}{\rho}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(lxxxi)}.$$

If we put $n=3$ and $\bar{r} = \frac{1}{\rho}(E_1-(1-\rho^2)F_1)$ we find

$$\mu_3' = \frac{2-\rho^2}{\rho^3}E_1 - \frac{2(1-\rho^2)}{\rho^3}F_1$$

confirming the result in Eqn. (lxxix).

Putting $n=3$ and writing $f_2 = -\frac{1}{\rho^2}\log_e(1-\rho^2)$, we have

$$\mu_4' = 1 + \frac{1-\rho^2}{\rho^2}\log_e(1-\rho^2) - \frac{3}{8}\frac{(1-\rho^2)^2}{\rho}\frac{d}{d\rho}\left\{\frac{\log_e(1-\rho^2)}{\rho^2}\right\}$$

$$= 1 + \frac{1-\rho^2}{4\rho^4}\{3\rho^2 + (3+\rho^2)\log_e(1-\rho^2)\} \text{ ....(lxxxii).}$$

This completes the moment coefficients for samples of three.

As ρ may be determined by considering the ratio of negative to positive correlations in samples of two, so it may be determined by considering the ratio of positive to negative correlations in samples of three. Let m_p be the number of positive and m_n the number of negative correlations, then since

$$y_3 = N\frac{1-\rho^2}{\pi}\frac{1}{\sqrt{1-r^2}}\frac{1}{r}\frac{dU}{d\rho},$$

$$m_p = \frac{1-\rho^2}{\pi}\frac{d}{d\rho}\int_0^{+1}\frac{U\,dr}{r\sqrt{1-r^2}}$$

$$= \frac{1-\rho^2}{\pi}\frac{d}{d\rho}\int_0^{+1}\frac{\cos^{-1}(-\rho r)\,dr}{r\sqrt{(1-r^2)(1-r^2\rho^2)}}$$

and

$$m_n = \frac{1-\rho^2}{\pi}\frac{d}{d\rho}\int_{-1}^{0}\frac{\cos^{-1}(-\rho r)\,dr}{r\sqrt{(1-r^2)(1-r^2\rho^2)}}.$$

Now put in the latter integral $r = -r$, then

$$m_n = \frac{1-\rho^2}{\pi}\frac{d}{d\rho}\int_0^{+1}\frac{-\cos^{-1}(\rho r)\,dr}{r\sqrt{(1-r^2)(1-r^2\rho^2)}}.$$

But $\cos^{-1}(-\rho r) + \cos^{-1}(\rho r) = \pi$, hence

$$\frac{m_p - m_n}{m_p + m_n} = (1-\rho^2)\frac{d}{d\rho}\int_0^{+1}\frac{dr}{r\sqrt{(1-r^2)(1-r^2\rho^2)}}$$

$$= (1-\rho^2)\int_0^1\frac{r^2\rho\,dr}{\sqrt{1-r^2}\,(1-r^2\rho^2)^{\frac{3}{2}}},$$

or if $r = \sin\phi$,

$$= (1-\rho^2)\int_0^{\frac{\pi}{2}}\frac{\rho\sin\phi\,d\phi}{(1-\rho^2\sin^2\phi)^{\frac{3}{2}}}$$

$$= -(1-\rho^2)\int_0^{\frac{\pi}{2}}\frac{d(\rho\cos\phi)}{(1-\rho^2+\rho^2\cos^2\phi)^{\frac{3}{2}}}.$$

Or again if $\rho\cos\phi = \sqrt{1-\rho^2}\tan\theta$,

$$\frac{m_p - m_n}{m_p + m_n} = +(1-\rho^2)\int_0^{\sin^{-1}\rho}\frac{\sqrt{1-\rho^2}\cos^3\theta}{(1-\rho^2)^{\frac{3}{2}}}\frac{d\theta}{\cos^2\theta}$$

$$= \int_0^{\sin^{-1}\rho}\cos\theta\,d\theta = \rho,$$

or
$$\rho = (m_p - m_n)/(m_p + m_n) \text{ ....(lxxxiii)},$$
a very simple formula if m_p or m_n has been found.

Clearly
$$\rho = (2m_p - N)\,N,$$
and
$$\delta\rho = 2\delta m_p/N,$$
$$\sigma_\rho{}^2 = 4\sigma^2{}_{m_p} = 4\,\frac{m_p}{N^2}\left(1 - \frac{m_p}{N}\right).$$

Thus the probable error of ρ found in this manner is
$$\frac{\cdot67449}{\sqrt{N}}\,2\sqrt{\frac{m_p}{N}\left(1 - \frac{m_p}{N}\right)} \text{ ....(lxxxiv)},$$
and can easily be evaluated, for it gives:
$$\text{Probable Error of } \rho = \frac{\cdot67449}{\sqrt{N}}\sqrt{1 - \rho^2} \text{ ....(lxxxv)}.$$

We see it is larger by the factor $\dfrac{1}{\sqrt{1 - \rho^2}}$, which is greater than unity, than the usual value for the product moment process of the correlation. But a new point arises: N is the number of triplets in the present process, and N the number of individuals in the product moment process. If we take M triplets and N individuals, we have to compare
$$\cdot67449\sqrt{1 - \rho^2}/\sqrt{M} \quad \text{with} \quad \cdot67449\,(1 - \rho^2)/\sqrt{N},$$
and these probable errors will be equal if
$$M = N/(1 - \rho^2).$$

If the number of triplets be $> N/(1 - \rho^2)$ the triplet process will be more accurate than the product moment method. The number of triplets required for equality of probable errors are for the various values of ρ:

$\rho = 0 \quad M = N, \qquad \rho = \cdot5 \quad M = 1\cdot333N,$

$\rho = \cdot1 \quad M = 1\cdot010N, \qquad \rho = \cdot6 \quad M = 1\cdot563N,$

$\rho = \cdot2 \quad M = 1\cdot042N, \qquad \rho = \cdot7 \quad M = 1\cdot923N,$

$\rho = \cdot3 \quad M = 1\cdot099N, \qquad \rho = \cdot8 \quad M = 2\cdot778N,$

$\rho = \cdot4 \quad M = 1\cdot190N, \qquad \rho = \cdot9 \quad M = 5\cdot263N.$

This series would seem to suggest, since a triplet contains three individuals, that to use the triplet process with equal exactness with the product moment process, in the case, say, of $\rho = \cdot5$, we should need a population of $4N$. But this assumes that each triplet is based upon three independent individuals. Actually a population of N provides $\frac{1}{6}N\,(N - 1)\,(N - 2)$ triplets and if these could be considered as an *independent* sample of M triplets, we should have a less value of the probable error of ρ by the triplet process using all possible sets than by the product moment process, provided
$$\rho^2 < 1 - \frac{6}{(N - 1)\,(N - 2)}.$$

For example if $N = 10$, for all values of ρ between $+ \cdot 957$ and $- \cdot 957$, the 120 triplets will give a better result than the 10 individuals. Even 50 triplets would be better than 10 individuals for all values of ρ between $+ \cdot 894$ and $- \cdot 894$. But the question arises whether we can consider the 120 triplets from a sample of 10 individuals as much a random sample as 120 triplets from an indefinitely large population, and this can hardly be the case. It may be, however, that 50 triplets out of the 120 would be sufficiently independent to give a better result than 10 individuals. It is very desirable that a full study should be made of such restricted sampling, for without such study it is not possible to assert how far the probable errors of doublet or triplet procedure are greater than those of the product moment method.

Of course in such a case as that referred to, the labour of the triplet process will be considerably greater, for we have to determine the *sign* of the correlation in 50 or 120 cases, instead of applying the product moment process to 10 individuals, and the labour rapidly increases with increase in the size of the set (doublet, triplet, etc.) and the size of the sample. Still the labour may be worth while in the case of small populations, where the best result is of considerable importance. We have not endeavoured to extend the theory to quadruplets or quintettes, because the labour of determining the sign of the correlation in these cases is very considerable.

In the case of triplets, we require the sign of the product moment

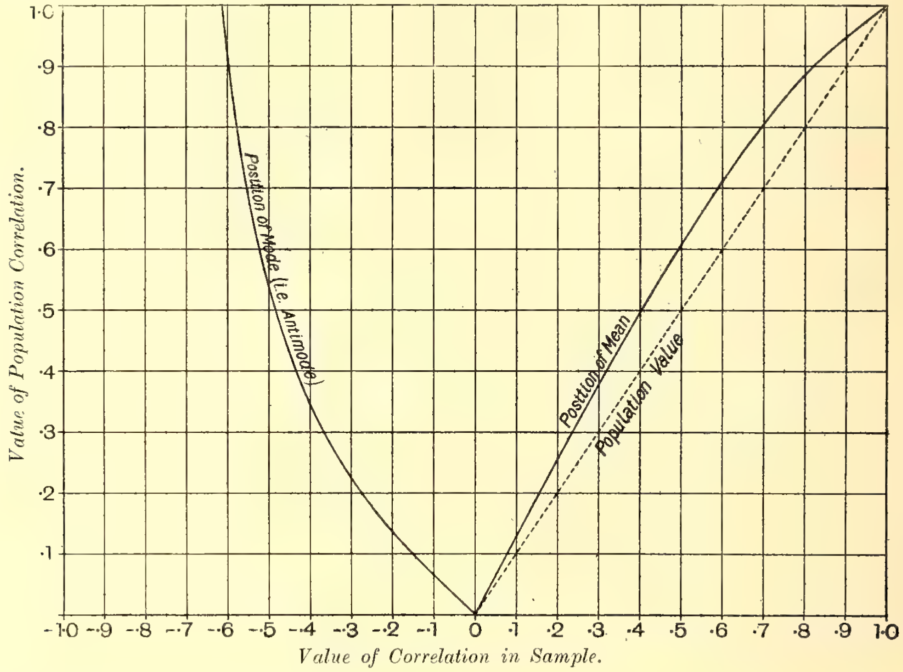
$$\frac{x_1 y_1 + x_2 y_2 + x_3 y_3}{3} - \frac{(x_1 + x_2 + x_3)(y_1 + y_2 + y_3)}{9},$$

or the sign of $$(x_2 - x_1)(y_2 - \bar{y}) + (x_3 - x_1)(y_3 - \bar{y}).$$

Now suppose the triplet arranged according to the character x in ascending order x_1, x_2, x_3, then $x_2 - x_1$ and $x_3 - x_1$ will always be positive, and accordingly if either both $y_2 - \bar{y}$ and $y_3 - \bar{y}$ are positive, or both negative the sign of the correlation is obvious. On the other hand if $y_2 - \bar{y}$ and $y_3 - \bar{y}$ are of *opposite* sign, the matter has got to be a little more carefully considered. But if $\bar{y}$ has been found and the above differences determined, in most cases it is not needful to actually multiply out, in order to realise the sign. A graphic process depending on the plotting of the triplet triangle seemed on the whole more laborious than the above.

Of course in samples of three the U-shaped distributions give a minimum where $dy/dr = 0$, and we have therefore an *antimode*, not a mode. The values of this antimode are recorded in Table II, p. 368. It will be seen that all the antimodes are negative for positive correlations in the sampled population. The antimode asymptotes to the value $- \cdot 613{,}9616$, which it reaches when $\rho = +1$. In this case the value actually fails at $\rho = +1$, for the equation $dy/dr = 0$ is satisfied then for all values of r, owing to the presence of the factor $(1 - \rho^2)^{\frac{n-1}{2}}$: see Equation (xxxvii). Nevertheless the antimode curve goes right up to the point indicated above and this value must be used for the purpose of interpolation

FIG. 2. Mode (Antimode) and Mean Curves for $n=3$, and for values of ρ from 0 to $+1$.



between $\rho = \cdot95$ and $\rho = 1\cdot00$. The curve is shown in Fig. 2, and both antimodal and mean lines on the photograph of the model of the frequency surface.

The distributions are after $\rho = \cdot2$ very skew U-shaped frequency curves, whose β_1, β_2 lie in the U-area of Pearson's skew curves, which, however, do not reproduce the antimode very closely. The ordinates are given in Table A (p. 379).

TABLE II. *Samples of Three.*

ρ, value of correlation in sampled population	$\bar{r}$, mean correlation of samples	$\check{r}$, antimode of samples	$\sigma_{\bar{r}}$	β_1	β_2	Number of positive correlations per 1000 samples	Number of negative correlations per 1000 samples
0·0	·0000,0000	·000,0000	·707,1068	·000,0000	1·500,0000	500	500
0·1	·0786,3836	−·151,2541	·704,5029	·028,0136	1·532,3082	550	450
0·2	·1578,7706	−·275,5141	·696,5708	·115,2406	1·632,9424	600	400
0·3	·2383,6407	−·367,9037	·682,9273	·272,2374	1·814,1763	650	350
0·4	·3208,5431	−·435,4082	·662,8536	·520,5707	2·101,1480	700	300
0·5	·4062,9889	−·485,5321	·635,1363	·901,7817	2·542,3242	750	250
0·6	·4960,0160	−·523,8811	·597,7313	1·500,1840	3·236,2938	800	200
0·7	·5919,3885	−·553,8751	·546,9866	2·510,5375	4·411,4204	850	150
0·8	·6975,5118	−·577,9216	·475,4818	4·503,0667	6·738,8238	900	100
0·9	·8204,3635	−·597,5916	·363,4654	10·222,6204	13·467,2160	950	50
0·95	·8742,5455	−·606,1358	·268,4676	21·078,0376	26·340,9890	975	25
1·00	1·0000,0000	−·613,9616	·000,0000	∞	∞	1,000	0

(iii) *Samples of Four*, $n = 4$.

Here, if $x = r\rho$,

$$y_4 = \frac{N(1-\rho^2)^{\frac{3}{2}}}{\pi} \frac{d^2}{dx^2}\left(\frac{\cos^{-1}(-x)}{\sqrt{1-x^2}}\right);$$

and if
$$U = \cos^{-1}(-x)/\sqrt{1-x^2},$$

then
$$\mu_q' = \frac{(1-\rho^2)^{\frac{3}{2}}}{\pi\rho^{q+1}}\left\{\left[x^q\frac{dU}{dx} - qx^{q-1}U\right]_{-\rho}^{+\rho} + q(q-1)\int_{-\rho}^{+\rho} x^{q-2}U\,dx\right\}.$$

To find the first four moments about $r = 0$, we have to determine

$$\int_{-\rho}^{+\rho} x^2 U dx, \qquad \int_{-\rho}^{+\rho} xU\,dx, \qquad \int_{-\rho}^{+\rho} U\,dx,$$

$$x^4\frac{dU}{dx} - 4x^3U, \quad x^3\frac{dU}{dx} - 3x^2U, \quad x^2\frac{dU}{dx} - 2xU \quad \text{and} \quad x\frac{dU}{dx} - U.$$

The results are most briefly expressed by using Fisher's notation $\rho = \sin\alpha$ and remembering that

$$\cos^{-1}(-\rho) + \cos^{-1}(\rho) = \pi, \quad \text{and} \quad \alpha = \cos^{-1}(-\rho) - \frac{\pi}{2}.$$

For the purposes of integration we put $x = -\cos\theta$ and integrate in terms of θ. We find

$$\int_{-\rho}^{+\rho} x^2 U\,dx = \tfrac{1}{2}\pi\{\alpha - \cos\alpha\sin\alpha\},$$

$$\int_{-\rho}^{+\rho} xU\,dx = 2\{\sin\alpha - \alpha\cos\alpha\},$$

$$\int_{-\rho}^{+\rho} U\,dx = \pi\alpha,$$

$$x^4\frac{dU}{dx} - 4x^3U = \pi\left\{\frac{\sin^5\alpha}{\cos^3\alpha} - 4\frac{\sin^3\alpha}{\cos\alpha}\right\},$$

$$x^3\frac{dU}{dx} - 3x^2U = 2\left\{\frac{\sin^3\alpha}{\cos^2\alpha} + \alpha\frac{\sin^4\alpha}{\cos^3\alpha} - 3\alpha\frac{\sin^2\alpha}{\cos\alpha}\right\},$$

$$x^2\frac{dU}{dx} - 2xU = \pi\left\{\frac{\sin^3\alpha}{\cos^3\alpha} - 2\frac{\sin\alpha}{\cos\alpha}\right\},$$

$$x\frac{dU}{dx} - U = 2\left\{\frac{\sin\alpha}{\cos^2\alpha} + \alpha\frac{\sin^2\alpha}{\cos^3\alpha} - \alpha\frac{1}{\cos\alpha}\right\}.$$

Hence substituting in the several values of μ_q', we obtain*

$$\bar{r} = \mu_1' = \frac{2}{\pi}\{\cot\alpha + \alpha(1 - \cot^2\alpha)\} \quad \text{..................(lxxxvi)},$$

---

* Tested by formulae (lxxx) and (lxxxi), p. 364, which can be put in the forms

$$\mu_3' = \mu_1'\{(n-2)/\rho^2 - (n-3)\} - \frac{(n-2)(1-\rho^2)}{\rho}\frac{d\mu_1'}{d\rho} \quad \text{.....................(xc)},$$

and
$$\mu_4' = \tfrac{1}{2}(n-2) - \tfrac{1}{2}(n-4)\mu_2' - \tfrac{1}{4}n\frac{1-\rho^2}{\rho}\frac{d\mu_2'}{d\rho} \quad \text{..............................(xci)}.$$

$$\sigma_r^2 + \bar{r}^2 = \mu_2' = 1 - 2\cot^2 a + 2a\cot^3 a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(lxxxvii)},$$

$$\mu_3' = \frac{2}{\pi}\{\cot a + 6\cot^3 a + a\,(1 - 3\cot^2 a - 6\cot^4 a)\} \quad \ldots\ldots\ldots\text{(lxxxviii)},$$

$$\mu_4' = 1 - 4\cot^2 a - 6\cot^4 a + a\,(6\cot^3 a + 6\cot^5 a)^* \quad \ldots\ldots\ldots\text{(lxxxix)}.$$

From these formulae were calculated the values given in the following table.

TABLE III. *Samples of Four.*

ρ, value of correlation of sampled population	$\bar{r}$, mean value of correlation in samples	μ_3, 3rd moment coefficient	σ_r	Usual value assumed for σ_r, i.e. $\frac{1-\rho^2}{\sqrt{n-1}}$	β_1	β_2	β_2/β_1
0·0	0	0	·577,3503	·577,3503	0	1·800,000	∞
0·1	·084,9678	– ·033,6268	·574,5653	·571,5768	·031,429	1·839,929	58·5418
0·2	·170,4532	– ·065,2863	·566,0965	·554,2563	·129,510	1·964,665	15·1700
0·3	·257,0089	– ·092,9620	·551,5835	·525,3887	·306,862	2·190,708	7·1391
0·4	·345,2652	– ·114,5383	·530,3576	·484,9742	·589,510	2·552,205	4·3294
0·5	·435,9911	– ·127,7520	·501,3081	·433,0127	1·028,270	3·116,256	3·0306
0·6	·530,1976	– ·130,1567	·462,6087	·369,5042	1·728,423	4·022,982	2·3275
0·7	·629,3378	– ·119,1407	·411,1087	·294,4486	2·940,226	5·609,288	1·9078
0·8	·735,7362	– ·092,1708	·340,7311	·207,8461	5·428,946	8·922,221	1·6435
0·9	·853,9806	– ·048,1281	·236,6586	·109,6966	13·184,043	19·571,006	1·4844
0·95	·920,8889	– ·021,4678	·157,7942	·056,2917	29·8558	43·4082	1·4539
0·98	·965,7599	– ·006,4363	·088,2666	·022,8631	87·5994	130·1935	1·4862
0·99	·982,1321	– ·002,4358	·055,4859	·011,4893	203·3250	311·7316	1·5332
1·00	1	0	0	0	∞	∞	1·8305 †

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* These expressions cannot be applied to the case of $\rho = 0$. We must return to Equation (ix) and put $\rho = 0$, finding $y_4 = \frac{1}{2}N$ or a horizontal line.

† The ratio β_2/β_1 equals in the limit $(27\pi^2 - 256)(3\pi^2 - 16)/(9\pi^2 - 80)^2 = 1{\cdot}8305$. This may be shown by putting $a = \frac{1}{2}\pi - \epsilon$, or $\rho = 1 - \frac{1}{2}\epsilon^2$, or $\epsilon = \sqrt{1 - \rho^2}$, where ϵ is small. We then find for values of ρ near unity

$$\bar{r} = \rho^2 - \frac{8}{3\pi}(1 - \rho^2)^{\frac{3}{2}}, \qquad \mu_2 = \left(\pi - \frac{16}{3\pi}\right)(1 - \rho^2)^{\frac{3}{2}},$$

$$\mu_3 = -\left(3\pi - \frac{80}{3\pi}\right)(1 - \rho^2)^{\frac{3}{2}}, \quad \mu_4 = \left(9\pi - \frac{256}{3\pi}\right)(1 - \rho^2)^{\frac{3}{2}}.$$

These lead to

$$\beta_1 = \frac{\left(3\pi - \frac{80}{3\pi}\right)^2}{\left(\pi - \frac{16}{3\pi}\right)^3 (1 - \rho^2)^{\frac{3}{2}}}, \qquad \beta_2 = \frac{\left(9\pi - \frac{256}{3\pi}\right)}{\left(\pi - \frac{16}{3\pi}\right)^2 (1 - \rho^2)^{\frac{3}{2}}},$$

and give the above result.

(iv) *General Case of small Samples, $n > 4$.*

Equations (xxviii), (xxxi), (xxxii), and (xxxiv) enable us to express the moment-coefficients of a sample of $n + 2$ in terms of those of a sample of n. But we have found algebraic expressions for the moment-coefficients for $n = 3$ and $n = 4$ in terms of (*a*) the complete elliptic integrals and logarithmic functions, (*b*) trigonometrical functions of α and $\cot \alpha$. Hence all even samples can have their moment-coefficients expressed in terms of α and $\cot \alpha$, and all odd samples can have their odd moment-coefficients expressed in terms of the complete elliptic integrals and their even moments in terms of logarithmic functions. The former result has been already noticed by Fisher[*]. The arithmetical calculation of the successive moment-coefficients after $n = 4$ by the difference formulae is, however, shorter than obtaining the algebraical expressions and then substituting arithmetical values, and has been followed in our calculations.

(10) *Approach of the Distribution as n increases to a Normal Character.*

It is well known that for the "probable error" to have meaning the distribution must approach the Gaussian for which $\beta_1 = 0$, $\beta_2 = 3$. It is clear that these conditions are by no means fulfilled for samples of 25 or 50, whatever be the value of ρ. There is nearer approach in the *low* values of ρ in samples of 100, but there is considerable deviation for $\rho = 5$ and upwards.

TABLE IV. *Values of the Frequency Constants for the Correlation in Samples of 25.*

ρ	$\bar{r}$ mean	Actual $\check{r}$[†] mode	$\check{r}$ from Pearson's formula[‡]	Actual σ	$\frac{1-\rho^2}{\sqrt{n-1}}$	β_1	β_2
0·0	0	0	0	·2041,241	·2041,241	0	2·769,2305
0·1	·0979,577	·11173	·11127	·2022,954	·2020,829	·012,3106	2·791,6002
0·2	·1960,288	·22258	·22177	·1967,883	·1959,592	·049,8655	2·860,0511
0·3	·2943,287	·33172	·33090	·1875,386	·1857,530	·114,6242	2·978,8302
0·4	·3929,765	·43840	·43758	·1744,356	·1714,643	·210,1771	3·155,8537
0·5	·4920,974	·54197	·54149	·1573,152	·1530,931	·342,3386	3·404,2283
0·6	·5918,251	·64194	·64190	·1359,499	·1306,395	·520,2635	3·745,3432
0·7	·6923,054	·73792	·73826	·1100,322	·1041,033	·758,5549	4·214,8982
0·8	·7937,001	·82966	·83025	·0791,481	·0734,847	1·081,1286	4·869,2635
0·9	·8961,933	·91703	·91736	·0427,345	·0387,836	1·533,4124	5·858,3872
1·0	1	1	1	0	0	∞	∞

It will be realised that while the ordinary value for the standard deviation of r and the distribution of r by a normal curve is fairly close for samples of 400, there is still a quite sensible deviation from normality in the case of $\rho = \cdot 8$ or over. In fact it may be said that for the size of ordinary samples, there is always a sensible

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[*] Fisher, *Biometrika*, Vol. x. p. 516.

[†] See Sections (4) and (5) above.

[‡] $\check{r} = \bar{r} + \mu_3 (\beta_2 + 3)/\{\mu_2 (10\beta_2 - 12\beta_1 - 18)\}$.

TABLE V. *Values of the Frequency Constants for the Correlation in Samples of* 50.

ρ	$\bar{r}$ mean	Actual $\breve{r}$ mode	$\breve{r}$ from Pearson's formula*	Actual σ	$\frac{1-\rho^2}{\sqrt{n-1}}$	β_1	β_2
0·0	0	0	0	·142,857	·142,857	0	2·88236
0·1	·098,995	·1054	·1053	·141,505	·141,429	·00666	2·89184
0·2	·198,047	·2104	·2102	·137,439	·137,143	·02683	2·93350
0·3	·297,218	·3147	·3144	·130,634	·130,000	·06107	2·99909
0·4	·396,565	·4180	·4177	·121,049	·120,000	·11041	3·09417
0·5	·496,150	·5198	·5196	·108,620	·107,143	·17635	3·22240
0·6	·596,038	·6201	·6199	·093,260	·091,429	·26110	3·38912
0·7	·696,295	·7184	·7183	·074,878	·072,857	·36774	3·60222
0·8	·796,989	·8146	·8146	·053,324	·051,429	·50037	3·87312
0·9	·898,198	·9085	·9085	·028,434	·027,143	·66608	4·22186
1·0	1	1	1	0	0	∞	∞

TABLE VI. *Values of the Frequency Constants for the Correlation in Samples of* 100.

ρ	$\bar{r}$ mean	Actual $\breve{r}$ mode	$\breve{r}$ from Pearson's formula*	Actual σ	$\frac{1-\rho^2}{\sqrt{n-1}}$	β_1	β_2
0·0	0	0	0	·100,5038	·100,5038	0	2·94060
0·1	·099,5016	·10258	·10255	·099,5260	·099,4987	·00346	2·94736
0·2	·199,0319	·20499	·20494	·096,5887	·096,4836	·01390	2·96774
0·3	·298,6219	·30708	·30701	·091,6832	·091,4584	·03147	3·00213
0·4	·398,3013	·40868	·40860	·084,7934	·084,4232	·05644	3·05118
0·5	·498,1002	·50964	·50957	·075,8968	·075,3778	·08919	3·11583
0·6	·598,0498	·60982	·60976	·064,9640	·064,3224	·13025	3·19739
0·7	·698,1815	·70907	·70903	·051,9577	·051,2569	·18031	3·29767
0·8	·798,5279	·80726	·80724	·036,8329	·036,1814	·24027	3·41896
0·9	·899,1225	·90427	·90423	·019,5352	·019,0957	·31148	3·57898
1·0	1	1	1	0	0	∞	∞

TABLE VII. *Values of the Frequency Constants for the Correlation in Samples of* 400.

ρ	$\bar{r}$ mean	Actual $\breve{r}$ mode	$\breve{r}$ from Pearson's formula*	Actual σ	$\frac{1-\rho^2}{\sqrt{n-1}}$	β_1	β_2
0·0	0	0	0	·0500,626	·0500,626	0	2·9850
0·1	·0998,760	·1006,250	·1006,232	·0495,654	·0495,620	·00089	2·9868
0·2	·1997,595	·2012,116	·2012,082	·0480,733	·0480,601	·00357	2·9921
0·3	·2996,579	·3017,217	·3017,171	·0455,851	·0455,570	·00804	3·0010
0·4	·3995,788	·4021,171	·4021,115	·0420,988	·0420,526	·01433	3·0138
0·5	·4995,297	·5023,602	·5023,584	·0376,115	·0375,470	·02250	3·0297
0·6	·5995,181	·6024,134	·6024,089	·0321,195	·0320,401	·03245	3·0498
0·7	·6995,517	·7022,401	·7022,386	·0256,183	·0255,319	·04435	3·0725
0·8	·7996,380	·8018,037	·8018,013	·0181,023	·0180,255	·05820	3·1017
0·9	·8997,849	·9010,725	·9010,668	·0095,653	·0095,119	·07402	3·1342
1·0	1	1	1	0	0	∞	∞

* $\breve{r} = \bar{r} + \mu_3 (\beta_2 + 3)/\{\mu_2 (10\beta_2 - 12\beta_1 - 18)\}$.

deviation from normality for high values of ρ in the sampled population, and the usual "probable error of r" must be treated with caution in these cases. Pearson's curves would give better results, as is indicated by the agreement of $\breve{r}$ to five figures.

(11) *Table for determining the Mode* $\breve{r}$ *of the Frequency Distribution of considerable Size* n *when the Correlation in the sampled Population is known to be* ρ.

The required value of the mode is

$$\breve{r} = \rho + \frac{\nu_1(\rho)}{n-1} + \frac{\nu_2(\rho)}{(n-1)^2} + \frac{\nu_3(\rho)}{(n-1)^3} + \frac{\nu_4(\rho)}{(n-1)^4},$$

when ρ is positive; if ρ be negative $\breve{r}$ has the same value as for ρ positive, but with opposite sign.

TABLE VIII. *Functions required in determining the Mode of a large or fairly large Sample.*

ρ	$\nu_1(\rho)$	$\nu_2(\rho)$	$\nu_3(\rho)$	$\nu_4(\rho)$
·00	0	0	0	0
·05	·12468,75000	+ ·37872,26953	+1·13249,90199	+3·36913,85919
·10	·24750,00000	+ ·74237,62500	+2·18002,51688	+6·33154,83878
·15	·36656,25000	+1·07636,49609	+3·06395,34936	+8·53428,44284
·20	·48000,00000	+1·36704,00000	+3·71804,16000	+9·72192,80640
·25	·58593,75000	+1·60217,28516	+4·09383,77380	+9·77504,47690
·30	·68250,00000	+1·77142,87500	+4·16514,90563	+8·72821,01391
·35	·76781,25000	+1·86684,01172	+3·93125,67461	+6·77045,11170
·40	·84000,00000	+1·88328,00000	+3·41856,48000	+4·22414,17680
·45	·89718,75000	+1·81893,55078	+2·68036,91046	+1·50278,32653
·50	·93750,00000	+1·67578,12500	+1·79443,35938	− ·94981,38428
·55	·95906,25000	+1·46005,27734	+ ·85806,01815	−2·73364,05345
·60	·96000,00000	+1·18272,00000	− ·01966,08000	−3·57140,42880
·65	·93843,75000	+ ·85996,06641	− ·72873,29366	−3·38017,64160
·70	·89250,00000	+ ·51363,37500	−1·17258,21188	−2·31971,93665
·75	·82031,25000	+ ·17175,29297	−1·28614,42566	− ·79706,28440
·80	·72000,00000	− ·13104,00000	−1·05802,56000	+ ·59787,92160
·85	·58968,75000	− ·35300,16797	− ·55704,89418	+1·25603,36730
·90	·42750,00000	− ·44481,37500	+ ·03650,10188	+ ·84819,10191
·95	·23156,25000	− ·34910,94141	+ ·39461,98756	− ·19874,71356
1·00	0	0	0	0

The above Table will give the value of $\breve{r}$ correctly to about the sixth figure if $n = 100$ or more, to about the fourth figure if $n = 25$ or more. Below 25 it can only serve as a "taking off point" for more accurate approximations, and these are fairly troublesome if n be very low. It will be found best to interpolate for the expression to be added to ρ.

Illustration. To find the modal $\breve{r}$ for samples of 9 when $\rho = {\cdot}2852$.

$$\breve{r} = \rho + {\cdot}106{,}272, \text{ for } \rho = {\cdot}25,$$

$$\breve{r} = \rho + {\cdot}121{,}126, \text{ for } \rho = {\cdot}30.$$

Hence
$$\begin{aligned}\check{r} &= \rho + \cdot 116{,}729, \text{ for } \rho = \cdot 2852\\ &= \cdot 4019, \text{ say.}\end{aligned}$$

We cannot, however, be certain that this is correct to more than two figures.

Equation (xlvi) gives us $\check{r} = \cdot 4038$.

We will therefore start with $\check{r} = \cdot 4030$, say, as the basis of a more elaborate approximation, or $\rho_0{}^2 = \cdot 1149372$ say.

Hence calculating I_1 and I_2 and using the difference formula we find

$$\begin{array}{ll} I_1 = 1\cdot 6972{,}3599, & I_6 = 1\cdot 0465{,}6745,\\ I_2 = 1\cdot 2110{,}7453, & I_7 = 1\cdot 0879{,}1327,\\ I_3 = 1\cdot 0715{,}7031, & I_8 = 1\cdot 1443{,}9624,\\ I_4 = 1\cdot 0262{,}1201, & I_9 = 1\cdot 2145{,}9528,\\ I_5 = 1\cdot 0236{,}1262, & I_{10} = 1\cdot 2980{,}8251. \end{array}$$

Thus the equality of I_9 and I_{10} has not been reached, so that we could hardly anticipate (xlvi) giving a very good result. Using (xli) we find

$$\epsilon = + \cdot 0006{,}0228,$$

a sufficiently small correction, leading to $\rho_0{}^2 = \cdot 1155{,}3948$, and $\check{r} = \cdot 40512$, correct to the fourth figure. Table VIII for $n = 9$ gives $\check{r}$ in error by about 0·8 %.

(12) *Table for determining the "most probable" value $\hat{\rho}$ of the correlation in a sampled population from the knowledge of the correlation r in a sample of size n, when n is considerable and it is legitimate to distribute ignorance equally.*

The required value is

$$\hat{\rho} = r - \frac{\lambda_1(r)}{n-1} - \frac{\lambda_2(r)}{(n-1)^2} - \frac{\lambda_3(r)}{(n-1)^3},$$

when r is positive; if r be negative, $\hat{\rho}$ has the same value as for r positive, but with opposite sign.

The above formula using Table IX will give $\hat{\rho}$ correct to five figures if $n = 25$ or over, and correct to four figures if $n = 10$ or over.

It appears best to interpolate not for the separate λ-functions, but for the total value to be subtracted from r to find $\hat{\rho}$. Thus, suppose we require to find $\hat{\rho}$ for $r = \cdot 6781$ and for $n = 16$. We have for $r = \cdot 65$

$$\hat{\rho} = \cdot 65 - \cdot 0127{,}3515,$$

and for $r = \cdot 70$
$$\hat{\rho} = \cdot 70 - \cdot 0121{,}8204.$$

Therefore a difference of $- \cdot 0005{,}5311$ corresponds to a rise of $\cdot 05$ and accordingly one of $- \cdot 0003{,}1085$ to a rise of $\cdot 0281$. Thus

$$\begin{aligned}\hat{\rho} &= \cdot 6781 - \cdot 0124{,}2430\\ &= \cdot 6657, \text{ accurately.}\end{aligned}$$

TABLE IX. *Functions required in determining the "most probable" value* $\hat{\rho}$ *of the Correlation.*

r	$\lambda_1(r)$	$\lambda_2(r)$	$\lambda_3(r)$
·00	0	0	0
·05	·02493,75000	− ·00615,64453	− ·00317,92000
·10	·04950,00000	− ·01175,62500	− ·00667,19813
·15	·07331,25000	− ·01626,62109	− ·01073,47255
·20	·09600,00000	− ·01920,00000	− ·01551,36000
·25	·11718,75000	− ·02014,16016	− ·02099,99084
·30	·13650,00000	− ·01876,87500	− ·02699,79938
·35	·15356,25000	− ·01487,63672	− ·03310,98746
·40	·16800,00000	− ·00840,00000	− ·03874,08000
·45	·17943,75000	+ ·00056,07422	− ·04312,99059
·50	·18750,00000	+ ·01171,87500	− ·04541,01563
·55	·19181,25000	+ ·02457,59766	− ·04470,17533
·60	·19200,00000	+ ·03840,00000	− ·04024,32000
·65	·18768,75000	+ ·05220,05859	− ·03156,41987
·70	·17850,00000	+ ·06470,62500	− ·01870,45688
·75	·16406,25000	+ ·07434,08203	− ·00248,33679
·80	·14400,00000	+ ·07920,00000	+ ·01517,76000
·85	·11793,75000	+ ·07702,79297	+ ·03087,17070
·90	·08550,00000	+ ·06519,37500	+ ·03926,26688
·95	·04631,25000	+ ·04066,81641	+ ·03257,27752
1·00	0	0	0

(13) *On the Table for* $q_n = \int_0^{\frac{\pi}{2}} \sin^{n-1}\phi\, d\phi$.

While Table X, p. 377, gives the value of $q_n = \int_0^{\frac{\pi}{2}} \sin^{n-1}\phi\, d\phi$ to ten figures, and therefore will be of use in calculating the values of the moments, the reader may be compelled to deal with values of n greater than those tabled, or even may need more than ten significant figures. The present values were calculated to twelve figures, by means of the simple relations

$$q_{2p+2} = q_{2p} - \frac{1}{2p+1} q_{2p},$$

$$q_{2p+1} = q_{2p-1} - \frac{1}{2p} q_{2p-1},$$

with the control relation $q_{2p} \times q_{2p-1} = \frac{\pi}{2} \frac{1}{2p-1}$, and the occasional direct calculation of individual q_n's to control the accuracy by use of Degen's Tables* and Briggs's *Arithmetica Logarithmica*†.

Since
$$q_{2p+1} = \frac{(2p)!}{2^{2p}\{(p)!\}^2} \frac{\pi}{2} \quad \text{..............................(xcii)},$$

and
$$q_{2p} = \frac{2^{2p}\{(p)!\}^2}{2p\,(2p)!} \quad \text{..............................(xciii)},$$

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* Tabularum ad faciliorem et breviorem probabilitatis computationem utilium Enneas. C. F. Degen, Havniae, 1824.

† Londini, W. Jones, 1624.

and Degen gives the logarithms of the factorials up to 1200! to eighteen mantissa figures, there is no difficulty in getting the logarithms of the q_n's to 18 figures. All we need is the value of either $\log \frac{\pi}{2}$ or $\frac{\pi}{2}$ to an adequate number of places*. But for modern methods of machine calculation the logarithm is of small service and we need in this case to find also the antilogarithm. Let us illustrate the process in the determination of q_{104}:

$$\log q_{104} = \left.\begin{array}{l} 104 \log 2 \\ + 2 \log (52!) \\ - \log 104 \\ - \log (104!) \end{array}\right) \begin{array}{rr} = & 31{\cdot}307119{,}549054{,}044280 \\ + & 135{\cdot}813296{,}784409{,}541764 \\ - & 2{\cdot}017033{,}339298{,}780355 \\ - & 166{\cdot}012795{,}764264{,}301069 \end{array}$$

$$= \bar{1}{\cdot}090587{,}229{,}900{,}504{,}620.$$

Thus far the work is very straightforward. But to obtain the antilogarithm to twelve figures is another matter. Tables like the original Vega (to 10 figures) or Mendizabel (to 8 figures) are not of service. We are thus compelled to use Briggs's 14 figure Table of Logarithms, but the fundamental defect of that magnificent piece of work is the largeness of the differences. The nearest logarithm to the above is log (·12319) = 1·090575,455222,21 with the remainder

$$r = {\cdot}000011{,}774678{,}29,$$

and the difference ·000035,252606,20. Mere linear interpolation gives

$${\cdot}1231{,}9334{,}0087{,}32,$$

which is wrong in the tenth figure.

We have therefore used the method of inverse interpolation given in the *Tables for Statisticians*† as (vii)$^{\text{bis}}$ on p. xiv. Unfortunately there are two misprints in the value given there (corrected in the *Errata*); it should run

$$\theta^2 \tfrac{1}{4} (- u_0 - u_1 + u_{-1} + u_2) + \theta \tfrac{1}{4} (5u_1 - 3u_0 - u_{-1} - u_2) + u_0 - u_0 (\theta) = 0 \qquad \text{.........(xciv)}.$$

But from Briggs's tables,

$$\begin{aligned} u_{-1} &= {\cdot}0905{,}4019{,}9754{,}24, \\ u_0 &= {\cdot}0905{,}7545{,}5222{,}21, \\ u_{+1} &= {\cdot}0906{,}1070{,}7828{,}41, \\ u_{+2} &= {\cdot}0906{,}4595{,}7573{,}32, \end{aligned}$$

whence

$${\cdot}0001{,}4101{,}614786\, \theta = {\cdot}0000{,}4709{,}8713{,}16 + \theta^2 \times {\cdot}0000{,}0005723{,}06,$$

or,

$$\theta = \frac{{\cdot}4709{,}8713{,}16}{1{\cdot}4101{,}6147{,}86} + \theta^2 \frac{{\cdot}0000{,}5723{,}06}{1{\cdot}4101{,}6147{,}86}.$$

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* $\frac{\pi}{2} = 1{\cdot}570796{,}326794{,}8966$; $\log \frac{\pi}{2} = {\cdot}196119{,}877030{,}1527$.

† Cambridge University Press, 1914.

To a first approximation

$$\theta_1 = \frac{\cdot4709{,}8713{,}16}{1\cdot4101{,}6147}\left(1 - \frac{\cdot0000{,}0000{,}86}{1\cdot4101{,}6147}\right),$$

arranging it thus as most machines cannot divide by more than *nine* figures. Hence

$$\theta_1 = \cdot3339{,}9517{,}625.$$

Substitute in the θ^2 term and we find

$$\theta_2 = \cdot3339{,}9970{,}356,$$

or the value of $q_{104} = \cdot1231{,}9333{,}9997$ to twelve figures. Found by the continuous process it was ·1231,9333,9996, so that the value tabled is correct to the tenth figure.

TABLE X. *Values of the Integral* $q_n = \int_0^{\frac{\pi}{2}} \sin^{n-1}\phi\, d\phi = \int_0^{\frac{\pi}{2}} \cos^{n-1}\phi\, d\phi$.

n	q_n	n	I_n	n	I_n
1	1·5707,9632,68	36	·2103,4114,55	71	·1492,6566,48
2	1·0000,0000,00	37	·2074,4030,47	72	·1482,1822,53
3	·7853,9816,34	38	·2046,5624,97	73	·1471,9253,06
4	·6666,6666,67	39	·2019,8134,93	74	·1461,8783,86
5	·5890,4862,25	40	·1994,0865,35	75	·1452,0344,23
6	·5333,3333,33	41	·1969,3181,56	76	·1442,3866,74
7	·4908,7385,21	42	·1945,4502,78	77	·1432,9287,07
8	·4571,4285,71	43	·1922,4296,29	78	·1423,6543,80
9	·4295,1462,06	44	·1900,2072,49	79	·1414,5578,26
10	·4063,4920,63	45	·1878,7380,46	80	·1405,6334,38
11	·3865,6315,85	46	·1857,9804,21	81	·1396,8758,53
12	·3694,0836,94	47	·1837,8959,15	82	·1388,2799,39
13	·3543,4956,20	48	·1818,4489,23	83	·1379,8407,82
14	·3409,9234,10	49	·1799,6064,16	84	·1371,5536,75
15	·3290,3887,90	50	·1781,3377,19	85	·1363,4141,06
16	·3182,5951,83	51	·1763,6142,88	86	·1355,4177,49
17	·3084,7394,91	52	·1746,4095,29	87	·1347,5604,54
18	·2995,3837,01	53	·1729,6986,29	88	·1339,8382,35
19	·2913,3650,74	54	·1713,4584,06	89	·1332,2472,67
20	·2837,7319,28	55	·1697,6671,73	90	·1324,7838,73
21	·2767,6968,21	56	·1682,3046,16	91	·1317,4445,19
22	·2702,6018,36	57	·1667,3516,87	92	·1310,2258,08
23	·2641,8924,20	58	·1652,7905,00	93	·1303,1244,70
24	·2585,0974,08	59	·1638,6042,44	94	·1296,1373,58
25	·2531,8135,69	60	·1624,7771,02	95	·1289,2614,44
26	·2481,6935,12	61	·1611,2941,74	96	·1282,4938,07
27	·2434,4361,24	62	·1598,1414,12	97	·1275,8316,37
28	·2389,7789,37	63	·1585,3055,58	98	·1269,2722,22
29	·2347,4919,77	64	·1572,7740,88	99	·1262,8129,47
30	·2307,3727,67	65	·1560,5351,59	100	·1256,4512,90
31	·2269,2422,44	66	·1548,5775,63	101	·1250,1848,17
32	·2232,9413,87	67	·1536,8906,87	102	·1244,0111,78
33	·2198,3284,24	68	·1525,4644,65	103	·1237,9281,04
34	·2165,2764,98	69	·1514,2893,53	104	·1231,9334,00
35	·2133,6717,06	70	·1503,3562,85	105	·1226,0249,49

Something corresponding to the above process must be used for values of q_n with $n > 105$, if we require the function correct to ten or twelve figures. The usual approximate formulae for the factorial, or for the Γ-function, do not converge rapidly enough and generally give the logarithm, so that they really involve the use of the logarithm tables to 14 places and the antilogarithm process.

For values of 100 and upwards the following formula will be found good:

For n even we deduce by Stirling's Theorem from Eqn. (xcii)

$$q_n = \frac{1{\cdot}2533{,}1413{,}7315}{\sqrt{n}}\left(1 + \frac{{\cdot}25}{n} + \frac{{\cdot}03125}{n^2} - \frac{{\cdot}039{,}0625}{n^3} - \frac{{\cdot}0102{,}5390{,}625}{n^4}\right) \quad \text{(xcv)}.$$

For example, $n = 100$

$$\begin{aligned} q_{100} &= 1{\cdot}2533{,}1413{,}7315 \times {\cdot}1002{,}5031{,}64165, \\ &= {\cdot}1256{,}4512{,}9017{,}89, \end{aligned}$$

which is correct to twelve places.

For n odd we deduce by Stirling's Theorem from Eqn. (xcii) by somewhat more lengthy algebra precisely the same value.

[Owing to the growth of this memoir far beyond its original limits, it has been found impossible to include in this first portion the experimental work which accompanied the algebraical investigations, nor to give illustrations of the various uses which the tables serve. These matters are therefore reserved for a continuation of this memoir, which will appear later.]

APPENDIX.

CORRELATION IN SMALL SAMPLES.

TABLE A. *Ordinates and Constants of Frequency Curves**.

$n = 3$. ρ variate (correlation in population sampled).

r variate (correlation in sample).	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
− *·95*	1019·41	874·99	744·21	624·87	515·25	413·97	319·92	232·19	150·04	72·82
− *·90*	730·25	631·33	540 26	456·01	377·73	304·70	236·31	172·05	111·49	54·25
− *·85*	604·25	526·19	453·08	384·48	319·97	259·17	201·74	147·37	95·78	46·73
− *·80*	530·52	465·35	403·21	344·03	287·69	234·02	182·85	134·03	87·38	42·75
− *·75*	481·24	425·22	370·79	318·13	267·34	218·43	171·35	126·04	82·44	40·45
− *·70*	445·72	396·74	348·18	300·44	253·76	208·27	164·05	121·12	79·49	39·12
− *·65*	418·87	375·59	331·78	287·95	244·48	201·60	159·47	118·20	77·84	38·44
− *·60*	397·89	359·43	319·60	279·03	238·18	197·36	156·81	116·70	77·14	38·22
− *·55*	381·13	346·87	310·50	272·73	234·08	194·94	155·60	116·29	77·18	38·38
− *·50*	367·55	337·02	303·74	268·44	231·71	193·97	155·57	116·79	77·82	38·85
− *·45*	356·44	329·29	298·82	265·77	230·74	194·20	156·54	118·06	79·02	39·61
− *·40*	347·30	323·28	295·41	264·44	230·97	195·48	158·40	120·05	80·72	40·63
− *·35*	339·80	318·70	293·29	264·28	232·26	197·72	161·09	122·71	82·91	41·93
− *·30*	333·68	315·35	292·30	265·17	234·53	200·85	164·58	126·05	85·60	43·51
− *·25*	328·75	313·08	292·30	267·01	237·71	204·86	168·86	130·08	88·82	45·38
− *·20*	324·87	311·77	293·24	269·76	241·79	209·74	173·97	134·82	92·59	47·57
− *·15*	321·95	311·36	295·05	273·39	246·77	215·51	179·94	140·33	96·98	50·12
− *·10*	319·91	311·80	297·71	277·91	252·66	222·22	186·82	146·69	102·04	53·08
− *·05*	318·71	313·06	301·21	283·32	259·51	229·93	194·72	153·99	107·88	56·51
·00	318·31	315·13	305·58	289·66	267·38	238·73	203·72	162·34	114·59	60·48
+ *·05*	318·71	318·02	310·83	296·99	276·35	248·73	213·97	171·89	122·33	65·09
+ *·10*	319·91	321·75	317·02	305·38	286·51	260·05	225·63	182·84	131·27	70·48
+ *·15*	321·95	326·39	324·22	314·94	298·02	272·89	238·91	195·41	141·63	76·79
+ *·20*	324·87	331·99	332·52	325·79	311·04	287·45	254·08	209·89	153·71	84·24
+ *·25*	328·75	338·66	342·06	338·10	325·78	304·00	271·45	226·66	167·88	93·11
+ *·30*	333·68	346·53	353·00	352·08	342·52	322·88	291·45	246·18	184·60	103·75
+ *·35*	339·80	355·76	365·56	368·00	361·58	344·53	314·60	269·08	204·53	116·66
+ *·40*	347·30	366·59	380·02	386·21	383·43	369·48	341·59	296·15	228·52	132·54
+ *·45*	356·44	379·33	396·75	407·18	408·63	398·48	373·30	328·48	257·73	152·34
+ *·50*	367·55	394·39	416·27	431·53	437·95	432·48	410·96	367·53	293·80	177·48
+ *·55*	381·13	412·36	439·27	460·12	472·45	472·82	456·23	415·36	339·09	210·05
+ *·60*	397·89	434·08	466·77	494·15	513·63	521·35	511·47	474·92	397·07	253·32
+ *·65*	418·87	460·81	500·25	535·44	563·68	580·81	580·15	550·63	473·12	312·61
+ *·70*	445·72	494·50	542·05	586·77	626·01	655·44	667·63	649·38	575·90	396·99
+ *·75*	481·24	538·43	596·05	652·77	706·23	752·17	782·72	782·63	720·25	523·20
+ *·80*	530·52	598·63	669·27	741·91	814·50	883·48	941·21	971·06	933·85	724·94
+ *·85*	604·25	687·68	776·82	871·73	971·83	1074·99	1175·29	1256·88	1274·71	1079·64
+ *·90*	730·25	838·25	956·77	1087·46	1232·06	1391·88	1566·09	1745·85	1890·70	1804·95
+ *·95*	1019·41	1180·31	1361·48	1568·00	1806·93	2088·24	2426·12	2839·55	3341·78	3802·31
+ *1·00*	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
Mean	·0000	·0786	·1579	·2384	·3209	·4063	·4960	·5919	·6976	·8204
Antimode	− ·0000	− ·1513	− ·2755	− ·3679	− ·4354	− ·4855	− ·5239	− ·5539	− ·5779	− ·5976
σ	·7071	·7045	·6966	·6829	·6629	·6351	·5977	·5470	·4755	·3635
$(1-\rho^2)/\sqrt{n-1}$	·7071	·7000	·6788	·6435	·5940	·5303	·4525	·3606	·2546	·1344
β_1	·0000	·0280	·1152	·2722	·5206	·9018	1·5002	2·5105	4·5031	10·2226
β_2	1·5000	1·5323	1·6329	1·8142	2·1011	2·5423	3·2363	4·4114	6·7388	13·4672

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* In all cases the total frequency of the curve is taken as 1000.

TABLE A. *Ordinates and Constants of Frequency Curves.*

$n = 4$. ρ variate (correlation in population sampled).

r variate (correlation in sample).	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	500·00	386·33	295·47	222·28	163·11	115·34	77·06	46·93	24·03	8·02
− *·95*	500·00	390·84	301·85	228·97	169·22	120·41	80·89	49·50	25·46	8·53
− *·90*	500·00	395·43	308·42	235·94	175·66	125·79	84·99	52·27	27·01	9·09
− *·85*	500·00	400·09	315·19	243·21	182·44	131·52	89·38	55·26	28·69	9·69
− *·80*	500·00	404·83	322·17	250·79	189·59	137·61	94·10	58·49	30·52	10·36
− *·75*	500·00	409·64	329·36	258·71	197·13	144·10	99·16	61·99	32·51	11·08
− *·70*	500·00	414·54	336·78	266·97	205·09	151·02	104·61	65·79	34·69	11·89
− *·65*	500·00	419·52	344·44	275·61	213·51	158·41	110·48	69·92	37·08	12·77
− *·60*	500·00	424·58	352·34	284·64	222·42	166·31	116·82	74·41	39·70	13·75
− *·55*	500·00	429·73	360·49	294·09	231·85	174·77	123·68	79·32	42·58	14·83
− *·50*	500·00	434·96	368·90	303·97	241·84	183·84	131·10	84·68	45·76	16·04
− *·45*	500·00	440·28	377·59	314·32	252·44	193·57	139·16	90·56	49·28	17·39
− *·40*	500·00	445·69	386·57	325·17	263·69	204·03	147·92	97·03	53·19	18·90
− *·35*	500·00	451·20	395·84	336·54	275·65	215·29	157·46	104·15	57·54	20·59
− *·30*	500·00	456·80	405·43	348·48	288·38	227·44	167·88	112·01	62·40	22·51
− *·25*	500·00	462·50	415·34	361·01	301·94	240·55	179·28	120·72	67·85	24·68
− *·20*	500·00	468·30	425·59	374·17	316·40	254·74	191·79	130·41	73·98	27·16
− *·15*	500·00	474·19	436·20	388·01	331·84	270·11	205·53	141·20	80·91	29·99
− *·10*	500·00	480·20	447·17	402·57	348·34	286·81	220·69	153·27	88·77	33·25
− *·05*	500·00	486·30	458·53	417·89	366·00	304·96	237·43	166·83	97·73	37·02
·00	500·00	492·52	470·30	434·04	384·94	324·76	256·00	182·11	108·00	41·41
+ *·05*	500·00	498·85	482·49	451·07	405·26	346·38	276·64	199·40	119·83	46·56
+ *·10*	500·00	505·29	495·13	469·04	427·09	370·06	299·67	219·05	133·53	52·64
+ *·15*	500·00	511·84	508·23	488·02	450·60	396·04	325·45	241·50	149·52	59·89
+ *·20*	500·00	518·52	521·81	508·08	475·94	424·63	354·41	267·28	168·29	68·60
+ *·25*	500·00	525·31	535·90	529·31	503·31	456·16	387·08	297·05	190·49	79·17
+ *·30*	500·00	532·23	550·53	551·79	532·91	491·05	424·09	331·63	216·99	92·15
+ *·35*	500·00	539·28	565·72	575·62	564·99	529·75	466·20	372·07	248·90	108·29
+ *·40*	500·00	546·46	581·49	600·89	599·81	572·82	514·35	419·70	287·74	128·65
+ *·45*	500·00	553·77	597·89	627·74	637·68	620·92	569·71	476·24	335·53	154·74
+ *·50*	500·00	561·22	614·93	656·28	678·96	674·81	633·70	543·96	395·13	188·83
+ *·55*	500·00	568·81	632·65	686·66	724·05	735·44	708·15	625·86	470·54	234·34
+ *·60*	500·00	576·54	651·09	719·03	773·41	803·91	795·33	725·97	567·57	296·69
+ *·65*	500·00	584·41	670·29	753·55	827·57	881·57	898·20	849·83	694·88	384·78
+ *·70*	500·00	592·44	690·28	790·43	887·15	970·08	1020·55	1005·20	865·74	514·05
+ *·75*	500·00	600·62	711·11	829·86	952·87	1071·44	1167·38	1203·12	1101·28	712·86
+ *·80*	500·00	608·96	732·82	872·07	1025·55	1188·16	1345·36	1459·79	1436·61	1037·65
+ *·85*	500·00	617·46	755·47	917·33	1106·18	1323·35	1563·48	1799·62	1933·28	1612·67
+ *·90*	500·00	626·13	779·10	965·92	1195·91	1480·93	1834·16	2260·62	2706·28	2751·97
+ *·95*	500·00	634·97	803·76	1018·17	1296·08	1665·90	2174·80	2904·38	3989·04	5425·82
+ *1·00*	500·00	643·98	829·53	1074·43	1408·32	1884·66	2610·44	3835·42	6309·30	13781·45
Mean	·0000	·0850	·1705	·2570	·3453	·4360	·5302	·6293	·7357	·8540
Mode	Non-existent									
σ	·5774	·5746	·5661	·5516	·5304	·5013	·4626	·4111	·3407	·2367
$(1-\rho^2)/\sqrt{n-1}$	·5774	·5716	·5543	·5254	·4850	·4330	·3695	·2944	·2078	·1097
β_1	·0000	·0314	·1295	·3069	·5895	1·0283	1·7284	2·9402	5·4289	13·1840
β_2	1·8000	1·8399	1·9647	2·1907	2·5522	3·1163	4·0230	5·6093	8·9222	19·5710

TABLE A—(*continued*).

$n = 5$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	198·78	141·35	99·03	67·80	44·87	28·25	16·49	8·50	3·48	·80
− *·90*	277·50	200·54	142·43	98·66	65·96	41·90	24·65	12·79	5·27	1·22
− *·85*	335·36	246·33	177·39	124·35	84·01	53·86	31·94	16·70	6·92	1·62
− *·80*	381·97	285·19	208·27	147·80	100·93	65·32	39·06	20·58	8·59	2·02
− *·75*	421·08	319·60	236·74	170·10	117·44	76·75	46·30	24·59	10·33	2·45
− *·70*	454·64	350·80	263·63	191·85	133·96	88·43	53·84	28·82	12·20	2·91
− *·65*	483·79	379·53	289·41	213·36	150·72	100·55	61·79	33·37	14·24	3·42
− *·60*	509·30	406·24	314·40	234·88	167·91	113·24	70·28	38·30	16·48	3·99
− *·55*	531·68	431·25	338·80	256·56	185·68	126·63	79·40	43·68	18·96	4·62
− *·50*	551·33	454·76	362·75	278·52	204·14	140·84	89·26	49·59	21·72	5·34
− *·45*	568·52	476·92	386·34	300·86	223·41	156·00	99·98	56·12	24·82	6·16
− *·40*	583·47	497·83	409·64	323·65	243·58	172·23	111·67	63·37	28·32	7·10
− *·35*	596·35	517·56	432·68	346·96	264·76	189·65	124·47	71·45	32·27	8·17
− *·30*	607·30	536·16	455·50	370·82	287·04	208·40	138·53	80·49	36·78	9·41
− *·25*	616·40	553·64	478·09	395·29	310·51	228·62	154·02	90·65	41·92	10·86
− *·20*	623·76	570·02	500·44	420·38	335·28	250·49	171·14	102·08	47·83	12·54
− *·15*	629·42	585·27	522·52	446·12	361·44	274·17	190·10	115·02	54·64	14·52
− *·10*	633·43	599·37	544·31	472·50	389·09	299·86	211·15	129·71	62·54	16·86
− *·05*	635·82	612·28	565·73	499·53	418·31	327·76	234·60	146·44	71·74	19·65
·00	636·62	623·95	586·71	527·18	449·20	358·10	260·76	165·58	82·51	22·98
+ *·05*	635·82	634·31	607·16	555·41	481·84	391·13	290·03	187·57	95·19	27·01
+ *·10*	633·43	643·27	626·97	584·15	516·30	427·11	322·86	212·91	110·22	31·91
+ *·15*	629·42	650·74	645·99	613·31	552·65	466·33	359·76	242·26	128·14	37·93
+ *·20*	623·76	656·59	664·06	642·76	590·91	509·09	401·33	276·40	149·65	45·39
+ *·25*	616·40	660·68	680·97	672·33	631·08	555·72	448·27	316·29	175·68	54·74
+ *·30*	607·30	662·86	696·46	701·77	673·11	606·51	501·37	363·13	207·42	66·60
+ *·35*	596·35	662·91	710·24	730·79	716·87	661·77	561·55	418·41	246·48	81·84
+ *·40*	583·47	660·60	721·95	758·98	762·14	721·75	629·83	483·98	295·00	101·75
+ *·45*	568·52	655·66	731·14	785·81	808·54	786·62	707·37	562·17	355·92	128·18
+ *·50*	551·33	647·74	737·25	810·60	855·49	856·39	795·39	655·88	433·30	163·98
+ *·55*	531·68	636·41	739·59	832·42	902·10	930·77	895·14	768·74	532·81	213·57
+ *·60*	509·30	621·15	737·29	850·06	947·05	1009·03	1007·71	905·20	662·54	284·13
+ *·65*	483·79	601·26	729·21	861·85	988·36	1089·62	1133·68	1070·61	834·18	387·69
+ *·70*	454·64	575·83	713·82	865·50	1023·03	1169·66	1272·51	1270·96	1064·70	545·45
+ *·75*	421·08	543·59	689·01	857·73	1046·50	1243·92	1421·02	1511·86	1378·85	796·89
+ *·80*	381·97	502·63	651·67	833·60	1051·45	1302·93	1570·47	1795·27	1811·55	1220·21
+ *·85*	335·36	449·87	596·86	785·20	1025·58	1328·96	1699·62	2109·71	2406·01	1981·91
+ *·90*	277·50	379·52	515·46	698·04	945·84	1286·02	1756·53	2398·74	3182·44	3457·80
+ *·95*	198·78	277·21	385·59	538·03	757·76	1085·21	1595·22	2436·67	3917·02	6392·40
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0884	·1773	·2671	·3584	·4517	·5480	·6482	·7541	·8687
Mode	·0000	·3264	·5520	·6936	·7863	·8500	·8965	·9318	·9595	·9817
σ	·5000	·4972	·4886	·4740	·4528	·4239	·3858	·3358	·2691	·1748
$(1-\rho^2)/\sqrt{n-1}$	·5000	·4950	·4800	·4550	·4200	·3750	·3200	·2550	·1800	·0950
β_1	·0000	·0315	·1299	·3077	·5909	1·0315	1·7297	2·9374	5·4065	13·0290
β_2	2·0000	2·0429	2·1769	2·4201	2·8097	3·4191	4·4027	6·1333	9·7830	21·7579

TABLE A. *Ordinates and Constants of Frequency Curves.*

$n = 6$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
– *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
– *·95*	73·125	47·28	30·03	18·55	10·99	6·12	3·10	1·35	·44	·07
– *·90*	142·500	94·07	60·81	38·13	22·88	12·89	6·60	2·89	·95	·15
– *·85*	208·125	140·28	92·30	58·76	35·74	20·37	10·54	4·66	1·54	·25
– *·80*	270·000	185·83	124·49	80·50	49·64	28·64	14·97	6·68	2·23	·36
– *·75*	328·125	230·63	157·34	103·38	64·65	37·76	19·96	9·00	3·03	·50
– *·70*	382·500	274·58	190·82	127·43	80·85	47·84	25·59	11·66	3·96	·66
– *·65*	433·125	317·59	224·87	152·69	98·33	58·96	31·92	14·71	5·05	·85
– *·60*	480·000	359·54	259·44	179·18	117·16	71·24	39·06	18·20	6·32	1·07
– *·55*	523·125	400·33	294·48	206·94	137·45	84·78	47·09	22·22	7·79	1·33
– *·50*	562·500	439·82	329·89	235·97	159·29	99·73	56·16	26·83	9·52	1·64
– *·45*	598·125	477·89	365·60	266·29	182·79	116·21	66·38	32·13	11·55	2·02
– *·40*	630·000	514·41	401·49	297·90	208·03	134·39	77·92	38·25	13·93	2·46
– *·35*	658·125	549·23	437·46	330·80	235·13	154·44	90·95	45·31	16·73	3·00
– *·30*	682·500	582·20	473·37	364·95	264·20	176·55	105·68	53·47	20·03	3·64
– *·25*	703·125	613·16	509·07	400·33	295·32	200·93	122·35	62·92	23·94	4·41
– *·20*	720·000	641·93	544·37	436·88	328·62	227·80	141·22	73·89	28·59	5·35
– *·15*	733·125	668·33	579·08	474·49	364·17	257·41	162·61	86·65	34·12	6·50
– *·10*	742·500	692·18	612·97	513·07	402·05	290·01	186·88	101·53	40·75	7·91
– *·05*	748·125	713·27	645·79	552·46	442·32	325·89	214·44	118·92	48·71	9·65
·00	750·000	731·39	677·23	592·47	485·02	365·35	245·76	139·31	58·32	11·80
+ *·05*	748·125	746·31	706·99	632·84	530·14	408·70	281·39	163·28	69·98	14·50
+ *·10*	742·500	757·78	734·67	673·26	577·63	456·23	321·94	191·55	84·21	17·90
+ *·15*	733·125	765·56	759·87	713·34	627·35	508·26	368·14	224·98	101·67	22·24
+ *·20*	720·000	769·38	782·10	752·61	679·10	565·04	420·76	264·66	123·24	27·81
+ *·25*	703·125	768·94	800·84	790·47	732·53	626·81	480·72	311·90	150·07	35·06
+ *·30*	682·500	763·96	815·48	826·21	787·15	693·69	548·97	368·33	183·69	44·60
+ *·35*	658·125	754·11	825·36	858·93	842·24	765·62	626·56	435·94	226·18	57·33
+ *·40*	630·000	739·06	829·70	887·57	896·80	842·35	714·54	517·20	280·35	74·61
+ *·45*	598·125	718·45	827·66	910·82	949·48	923·20	813·87	615·10	350·06	98·48
+ *·50*	562·500	691·90	818·27	927·11	998·43	1006·98	925·29	733·19	440·67	132·11
+ *·55*	523·125	659·02	800·46	934·53	1041·18	1091·61	1048·92	875·62	559·69	180·65
+ *·60*	480·000	619·39	772·99	930·76	1074·42	1173·81	1183·82	1046·92	717·69	252·62
+ *·65*	433·125	572·56	734·52	913·01	1093·74	1248·41	1326·99	1251·36	929·54	362·77
+ *·70*	382·500	518·05	683·50	877·89	1093·27	1307·53	1471·73	1491·35	1215·82	537·74
+ *·75*	328·125	455·38	618·19	821·29	1065·22	1339·14	1604·80	1763·58	1603·56	828·01
+ *·80*	270·000	384·02	536·66	738·25	999·24	1325·11	1701·20	2050·08	2122·58	1334·30
+ *·85*	208·125	303·41	436·70	622·74	881·50	1237·97	1714·92	2297·17	2783·31	2265·98
+ *·90*	142·500	212·95	315·86	467·45	693·60	1036·09	1561·73	2364·83	3479·97	4043·96
+ *·95*	73·125	112·04	171·33	263·48	410·83	655·98	1086·58	1899·93	3578·10	7013·72
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0906	·1816	·2734	·3665	·4614	·5587	·6594	·7646	·8766
Mode	·0000	·2197	·4101	·5599	·6747	·7630	·8321	·8870	·9319	·9689
σ	·4472	·4444	·4360	·4216	·4007	·3725	·3356	·2878	·2253	·1397
$(1-\rho^2)/\sqrt{n-1}$	·4472	·4427	·4293	·4070	·3757	·3354	·2862	·2281	·1610	·0850
β_1	·0000	·0304	·1251	·2959	·5667	·9838	1·6418	2·7599	4·9848	11·4554
β_2	2·1429	2·1863	2·3222	2·5682	2·9615	3·5746	4·5585	6·2752	9·8417	21·1653

TABLE A—(*continued*).

$n = 7$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	25·84	15·19	8·75	4·87	2·58	1·27	·56	·20	·05	·01
− *·90*	70·30	42·38	24·93	14·14	7·62	3·81	1·69	·63	·16	·02
− *·85*	124·08	76·72	46·12	26·66	14·60	7·39	3·34	1·25	·33	·04
− *·80*	183·35	116·30	71·46	42·10	23·44	12·05	5·51	2·08	·56	·06
− *·75*	245·63	159·85	100·42	60·33	34·17	17·84	8·26	3·16	·85	·10
− *·70*	309·15	206·44	132·64	81·28	46·85	24·84	11·67	4·53	1·24	·14
− *·65*	372·52	255·27	167·80	104·93	61·59	33·20	15·83	6·22	1·72	·20
− *·60*	434·60	305·66	205·61	131·26	78·49	43·03	20·83	8·30	2·32	·27
− *·55*	494·46	356·96	245·82	160·28	97·70	54·50	26·81	10·85	3·08	·37
− *·50*	551·33	408·59	288·14	191·98	119·36	67·80	33·92	13·93	4·01	·49
− *·45*	604·53	459·99	332·29	226·34	143·61	83·12	42·31	17·66	5·16	·63
− *·40*	653·49	510·59	377·96	263·34	170·62	100·70	52·20	22·16	6·58	·82
− *·35*	697·73	559·87	424·83	302·91	200·54	120·78	63·81	27·58	8·32	1·05
− *·30*	736·85	607·28	472·53	344·98	233·54	143·64	77·42	34·10	10·48	1·35
− *·25*	770·51	652·32	520·67	389·43	269·77	169·60	93·33	41·94	13·13	1·72
− *·20*	798·41	694·44	568·81	436·10	309·36	198·97	111·92	51·36	16·41	2·19
− *·15*	820·34	733·15	616·47	484·78	352·43	232·12	133·60	62·69	20·47	2·80
− *·10*	836·13	767·91	663·12	535·18	399·06	269·42	158·87	76·33	25·50	3·56
− *·05*	845·65	798·24	708·17	586·95	449·29	311·28	188·29	92·77	31·77	4·55
·00	848·83	823·62	750·99	639·65	503·10	358·10	222·51	112·60	39·60	5·82
+ *·05*	845·65	843·56	790·87	692·72	560·37	410·29	262 29	136·56	49·43	7·48
+ *·10*	836·13	857·59	827·06	745·49	620·88	468·23	308·46	165·58	61·82	9·65
+ *·15*	820·34	865·26	858·73	797·15	684·25	532·27	361·98	200·77	77·52	12·53
+ *·20*	798·41	866·13	884·99	846·70	749·91	602·63	423·92	243·54	97·54	16·38
+ *·25*	770·51	859·80	904·89	893·00	817·07	679·42	495·43	295·61	123·21	21·58
+ *·30*	736·85	845·93	917·44	934·67	884·59	762·49	577·72	359·10	156·38	28·71
+ *·35*	697·73	824·21	921·57	970·10	950·96	851·33	671·98	436·64	199 53	38·62
+ *·40*	653·49	794·40	916·22	997·43	1014·18	944·93	779·26	531·38	256·18	52·62
+ *·45*	604·53	756·38	900·29	1014·55	1071·64	1041·52	900·25	647·10	331·09	72·76
+ *·50*	551·33	710·11	872·71	1019·07	1120·03	1138·25	1034·91	788·14	431·03	102·38
+ *·55*	494·46	655·69	832·49	1008·34	1155·12	1230·82	1181·85	959·18	565·53	147·00
+ *·60*	434·60	593·44	778·80	979·51	1171·74	1312·87	1337·35	1164·60	747·90	216·12
+ *·65*	372·52	523·87	711·02	929·65	1163·57	1375·33	1493·80	1406·95	996·60	326·69
+ *·70*	309·15	447·82	628·96	855·90	1123·23	1405·52	1637·17	1683·53	1336·03	510·28
+ *·75*	245·63	366·56	533·05	755·92	1042·49	1386·41	1743·33	1979·38	1794·82	828·27
+ *·80*	183·35	281·92	424·74	628·49	913·07	1296·13	1772·80	2252·77	2393·94	1404·91
+ *·85*	124·08	196·62	307·09	474·80	728·55	1109·20	1664·80	2407·27	3099·80	2495·12
+ *·90*	70·30	114·82	186·02	300·94	489·11	802·95	1336·07	2244·07	3664·14	4555·85
+ *·95*	25·84	43·51	73·17	124·05	214·20	381·46	712·23	1426·14	3147·79	7414·56
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0921	·1845	·2777	·3720	·4678	·5658	·6667	·7713	·8814
Mode	·0000	·1813	·3484	·4919	·6106	·7087	·7894	·8563	·9122	·9595
σ	·4082	·4055	·3973	·3832	·3629	·3356	·3001	·2545	·1958	·1175
$(1-\rho^2)/\sqrt{n-1}$	·4082	·4042	·3919	·3715	·3429	·3062	·2613	·2082	·1470	·0776
β_1	·0000	·0288	·1186	·2798	·5340	·9222	1·5263	2·5318	4·4611	9·6408
β_2	2·2500	2·2929	2·4267	2·6686	3·0535	3·6503	4·5968	6·2238	9·5129	19·3424

TABLE A. *Ordinates and Constants of Frequency Curves.*

$n = 8$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	8·91	4·76	2·49	1·25	·59	·26	·10	·03	·01	·00
− *·90*	33·84	18·63	9·97	5·12	2·47	1·10	·42	·13	·03	·00
− *·85*	72·19	40·95	22·48	11·80	5·82	2·62	1·03	·33	·07	·01
− *·80*	121·50	71·02	40·02	21·48	10·80	4·95	1·98	·63	·13	·01
− *·75*	179·44	108·11	62·53	34·34	17·61	8·22	3·33	1·08	·23	·02
− *·70*	243·84	151·45	89·96	50·58	26·48	12·58	5·19	1·71	·38	·03
− *·65*	312·66	200·21	122·17	70·35	37·63	18·23	7·66	2·57	·57	·05
− *·60*	384·00	253·56	158·99	93·82	51·30	25·35	10·84	3·69	·83	·07
− *·55*	456·10	310·59	200·22	121·13	67·75	34·18	14·89	5·16	1·18	·10
− *·50*	527·34	370·40	245·57	152·40	87·25	44·96	19·98	7·06	1·64	·14
− *·45*	596·26	432·05	294·69	187·72	110·08	58·00	26·31	9·47	2·25	·19
− *·40*	661·50	494·55	347·19	227·13	136·53	73·61	34·12	12·53	3·03	·27
− *·35*	721·88	556·91	402·57	270·64	166·89	92·15	43·68	16·38	4·04	·36
− *·30*	776·34	618·14	460·28	318·19	201·43	114·03	55·33	21·22	5·34	·49
− *·25*	823·97	677·22	519·66	369·65	240·45	139·67	69·47	27·27	7·03	·66
− *·20*	864·00	733·11	579·99	424·79	284·17	169·57	86·55	34·83	9·19	·88
− *·15*	895·79	784·83	640·42	483·31	332·81	204·24	107·11	44·26	11·98	1·17
− *·10*	918·84	831·37	700·04	544·75	386·52	244·24	131·79	56·00	15·57	1·57
− *·05*	932·82	871·77	757·84	608·54	445·36	290·13	161·34	70·61	20·22	2·09
·00	937·50	905·10	812·68	673·93	509·27	342·52	196·61	88·81	26·24	2·80
+ *·05*	932·82	930·49	863·38	740·00	578·06	401·96	238·59	111·47	34·07	3·77
+ *·10*	918·84	947·15	908·63	805·60	651·31	468·99	288·43	139·69	44·29	5·08
+ *·15*	895·79	954·37	947·08	869·36	728·36	544·03	347·39	174·87	57·69	6·89
+ *·20*	864·00	951·55	977·31	929·66	808·23	627·31	416·87	218·74	75·35	9·42
+ *·25*	823·97	938·24	997·87	984·60	889·50	718·81	498·39	273·48	98·76	12·97
+ *·30*	776·34	914·13	1007·32	1031·99	970·27	818·08	593·47	341·77	129·97	18·05
+ *·35*	721·88	879·12	1004·28	1069·38	1048·02	924·04	703·52	426·94	171·88	25·40
+ *·40*	661·50	833·34	987·47	1094·05	1119·51	1034·75	829·63	532·99	228·55	36·23
+ *·45*	596·26	777·15	955·78	1103·06	1180·66	1147·03	972·16	664·66	305·76	52·50
+ *·50*	527·34	711·26	908·44	1093·37	1226·48	1256·06	1130·10	827·21	411·69	77·48
+ *·55*	456·10	636·70	845·06	1061·99	1251·03	1354·86	1300·15	1025·98	558·02	116·83
+ *·60*	384·00	554·91	765·85	1006·21	1247·49	1433·64	1475·18	1265·10	761·16	180·59
+ *·65*	312·66	467·80	671·79	924·01	1208·46	1479·32	1642·00	1544·83	1043·60	287·38
+ *·70*	243·84	377·81	564·92	814·59	1126·64	1475·21	1778·44	1856·10	1434·02	473·04
+ *·75*	179·44	287·97	448·65	679·20	996·09	1401·54	1849·45	2169·84	1962·40	809·47
+ *·80*	121·50	202·00	328·13	522·33	814·61	1237·97	1804·24	2418·01	2637·69	1445·38
+ *·85*	72·19	124·36	210·79	353·40	587·92	970·50	1578·45	2464·22	3372·91	2684·78
+ *·90*	33·84	60·42	106·94	189·15	336·77	607·69	1116·43	2080·31	3769·68	5016·00
+ *·95*	8·91	16·49	30·50	57·02	109·05	216·63	456·02	1045·85	2706·04	7661·16
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0932	·1867	·2808	·3759	·4725	·5709	·6718	·7760	·8847
Mode	·0000	·1613	·3135	·4505	·5698	·6722	·7594	·8338	·8975	·9524
σ	·3780	·3753	·3673	·3536	·3339	·3074	·2733	·2299	·1746	·1024
$(1-\rho^2)/\sqrt{n-1}$	·3780	·3742	·3628	·3439	·3175	·2835	·2419	·1928	·1361	·0718
β_1	·0000	·0272	·1117	·2630	·5000	·8586	1·4088	2·3044	3·9581	8·0434
β_2	2·3333	2·3751	2·5051	2·7393	3·1101	3·6800	4·5751	6·0839	9·0368	17·2512

TABLE A—(*continued*).

$n = 9$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
– *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
– *·95*	3·02	1·47	·69	·31	·13	·05	·02	·00	·00	—
– *·90*	16·03	8·06	3·92	1·82	·79	·31	·10	·03	·00	—
– *·85*	41·32	21·50	10·78	5·14	2·28	·91	·31	·08	·01	—
– *·80*	79·21	42·66	22·04	10·78	4·89	2·00	·70	·19	·03	—
– *·75*	128·96	71·93	38·30	19·23	8·93	3·72	1·32	·37	·06	·00
– *·70*	189·20	109·29	60·01	30·95	14·72	6·27	2·27	·64	·11	·01
– *·65*	258·15	154·47	87·49	46·39	22·61	9·85	3·64	1·04	·19	·01
– *·60*	333·77	206·90	120·93	65·95	32·98	14·69	5·55	1·62	·29	·02
– *·55*	413·87	265·84	160·41	90·03	46·21	21·08	8·14	2·42	·45	·03
– *·50*	496·20	330·31	205·86	118·99	62·73	29·33	11·58	3·52	·66	·04
– *·45*	578·53	399·18	257·08	153·13	83·00	39·81	16·09	4·99	·96	·06
– *·40*	658·72	471·20	313·71	192·70	107·46	52·93	21·93	6·97	1·37	·09
– *·35*	734·71	544·95	375·25	237·86	136·60	69·16	29·41	9·57	1·93	·12
– *·30*	804·64	618·94	441·03	288·69	170·90	89·03	38·90	12·99	2·68	·17
– *·25*	866·82	691·61	510·20	345·14	210·81	113·15	50·86	17·45	3·70	·25
– *·20*	919·77	761·33	581·74	407·03	256·77	142·16	65·83	23·24	5·06	·35
– *·15*	962·26	826·48	654·46	473·98	309·16	176·78	84·46	30·73	6·89	·48
– *·10*	993·32	885·42	726·99	545·46	368·27	217·80	107·54	40·41	9·36	·68
– *·05*	1012·24	936·57	797·78	620·65	434·27	266·02	135·99	52·88	12·66	·95
·00	1018·59	978·46	865·14	698·50	507·13	322·29	170·89	68·91	17·11	1·33
+ *·05*	1012·24	1009·68	927·20	777·65	586·61	387·41	213·52	89·51	23·11	1·86
+ *·10*	993·32	1029·05	982·02	856·41	672·14	462·13	265·33	115·94	31·22	2·63
+ *·15*	962·26	1035·54	1027·55	932·73	762·75	547·03	327·98	149·85	42·24	3·73
+ *·20*	919·77	1028·41	1061·74	1004·19	856·97	642·43	403·32	193·30	57·27	5·33
+ *·25*	866·82	1007·20	1082·54	1068·00	952·68	748·20	493·28	248·93	77·88	7·67
+ *·30*	804·64	971·79	1088·08	1121·01	1047·05	863·56	599·83	320·04	106·28	11·16
+ *·35*	734·71	922·48	1076·68	1159·76	1136·34	986·80	724·70	410·75	145·67	16·43
+ *·40*	658·72	859·99	1047·01	1180·62	1215·85	1114·86	869·09	526·05	200·65	24·55
+ *·45*	578·53	785·54	998·27	1179·91	1279·81	1242·93	1032·99	671·79	277·86	37·28
+ *·50*	496·20	700·85	930·34	1154·15	1321·44	1363·83	1214·32	854·38	386·97	57·71
+ *·55*	413·87	608·22	843·95	1100·46	1333·12	1467·50	1407·47	1079·97	541·89	91·38
+ *·60*	333·77	510·46	740·94	1016·99	1306·81	1540·47	1601·27	1352·44	762·41	148·52
+ *·65*	258·15	410·96	624·47	903·63	1234·97	1565·77	1776·21	1669·36	1075·58	248·83
+ *·70*	189·20	313·58	499·21	762·81	1111·97	1523·66	1901·22	2014·01	1515·00	431·66
+ *·75*	128·96	222·57	371·51	600·46	936·53	1394·27	1930·95	2341·13	2111·97	778·77
+ *·80*	79·21	142·39	249·41	427·14	715·16	1163·62	1807·20	2554·55	2860·84	1463·91
+ *·85*	41·32	77·38	142·36	258·83	466·86	835·66	1472·97	2482·96	3612·89	2844·14
+ *·90*	16·03	31·28	60·49	116·98	228·19	452·62	918·20	1898·32	3818·02	5437·47
+ *·95*	3·02	6·15	12·51	25·79	54·64	121·08	287·39	755·00	2290·26	7794·37
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0940	·1883	·2832	·3789	·4760	·5747	·6756	·7793	·8869
Mode	·0000	·1492	·2920	·4238	·5420	·6463	·7374	·8168	·8861	·9467
σ	·3536	·3510	·3431	·3298	·3107	·2852	·2524	·2109	·1586	·0915
$(1-\rho^2)/\sqrt{n-1}$	·3536	·3500	·3394	·3217	·2670	·2652	·2263	·1803	·1273	·0672
β_1	·0000	·0256	·1051	·2468	·4677	·7983	1·2989	2·0963	3·5152	6·7561
β_2	2·4000	2·4403	2·5657	2·7907	3·1456	3·6857	4·5239	5·9099	8·5326	15·3136

TABLE A. *Ordinates and Constants of Frequency Curves.*

$n = 10$. ρ variate (correlation in population sampled).

r variate (correlation in sample).	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	1·01	·45	·19	·08	·03	·01	·00	·00	—	—
− *·90*	7·50	3·44	1·53	·64	·25	·09	·03	·01	—	—
− *·85*	23·37	11·15	5·11	2·21	·88	·31	·09	·02	·00	—
− *·80*	51·03	25·33	12·00	5·34	2·19	·80	·24	·06	·01	—
− *·75*	91·59	47·29	23·18	10·64	4·47	1·67	·52	·12	·02	—
− *·70*	145·09	77·94	39·56	18·72	8·09	3·09	·98	·23	·03	—
− *·65*	210·66	117·77	61·92	30·23	13·43	5·25	1·71	·42	·06	—
− *·60*	286·72	166·85	90·90	45·82	20·95	8·41	2·81	·70	·10	·00
− *·55*	371·15	224·86	127·01	66·13	31·14	12·85	4·39	1·12	·17	·01
− *·50*	461·43	291·10	170·55	91·82	44·57	18·91	6·63	1·73	·26	·01
− *·45*	554·77	364·50	221·63	123·45	61·84	27·00	9·73	2·60	·41	·02
− *·40*	648·27	443·69	280·14	161·56	83·58	37·60	13·93	3·83	·61	·03
− *·35*	739·03	526·99	345·68	206·59	110·50	51·29	19·56	5·52	·91	·04
− *·30*	824·22	612·48	417·62	258·84	143·28	68·70	27·02	7·85	1·33	·06
− *·25*	901·22	698·04	495·03	318·47	182·65	90·58	36·79	11·03	1·92	·09
− *·20*	967·68	781·38	576·65	385·42	229·29	117·77	49·48	15·32	2·76	·13
− *·15*	1021·57	860·14	660·97	459·39	283·82	151·22	65·82	21·09	3·92	·20
− *·10*	1061·26	931·94	746·13	539·76	346·77	191·94	86·73	28·82	5·55	·29
− *·05*	1085·57	994·42	830·01	625·59	418·49	241·06	113·28	39·13	7·83	·42
·00	1093·75	1045·39	910·20	715·49	499·09	299·70	146·80	52·84	11·02	·62
+ *·05*	1085·57	1082·81	984·11	807·66	588·33	369·02	188·84	71·03	15·49	·91
+ *·10*	1061·26	1104·96	1048·94	899·79	685·53	450·06	241·23	95·11	21·75	1·35
+ *·15*	1021·57	1110·49	1101·85	989·04	789·44	543·65	306·06	126·91	30·57	1·99
+ *·20*	967·68	1098·49	1139·99	1072·04	898·05	650·25	385·66	168·83	43·03	2·98
+ *·25*	901·22	1068·59	1160·70	1144·96	1008·47	769·73	482·55	223·95	60·70	4·48
+ *·30*	824·22	1021·01	1161·60	1203·51	1116·77	900·98	599·22	296·23	85·91	6·82
+ *·35*	739·03	956·66	1140·83	1243·14	1217·79	1041·59	737·88	390·62	122·04	10·51
+ *·40*	648·27	877·13	1097·21	1259·23	1305·16	1187·27	899·90	513·21	174·12	16·44
+ *·45*	554·77	784·74	1030·51	1247·45	1371·20	1331·27	1084·96	671·18	249·62	26·17
+ *·50*	461·43	682·54	941·67	1204·17	1407·27	1463·74	1289·77	872·31	359·56	42·50
+ *·55*	371·15	574·23	833·02	1127·09	1404·16	1571·18	1506·11	1123·76	520·21	70·66
+ *·60*	286·72	464·10	708·50	1015·97	1353·14	1636·20	1718·18	1429·27	754·96	120·77
+ *·65*	210·66	356·82	573·73	873·46	1247·48	1638·20	1899·37	1783·34	1095·94	213·01
+ *·70*	145·09	257·23	436·02	706·04	1084·83	1555·63	2009·23	2160·46	1582·39	389·45
+ *·75*	91·59	170·01	304·07	524·71	870·38	1371·13	1993·01	2497·22	2247·23	740·79
+ *·80*	51·03	99·20	187·37	345·25	620·62	1081·22	1789·53	2668·18	3067·85	1466·04
+ *·85*	23·37	47·59	95·03	187·37	366·47	711·33	1358·89	2473·52	3826·37	2979·24
+ *·90*	7·50	16·01	33·82	71·51	152·84	333·27	746·59	1712·69	3823·56	5828·60
+ *·95*	1·01	2·27	5·07	11·53	27·06	66·90	179·06	538·90	1916·67	7841·74
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0946	·1896	·2850	·3813	·4787	·5776	·6785	·7819	·8887
Mode	·0000	·1411	·2774	·4050	·5219	·6270	·7206	·8036	·8771	·9422
σ	·3333	·3308	·3232	·3103	·2917	·2671	·2355	·1958	·1461	·0832
$(1-\rho^2)/\sqrt{n-1}$	·3333	·3300	·3200	·3033	·2800	·2500	·2133	·1700	·1200	·0633
β_1	·0000	·0242	·0989	·2317	·4374	·7431	1·2002	1·9122	3·1377	5·7475
β_2	2·4545	2·4933	2·6137	2·8292	3·1669	3·6774	4·4598	5·7290	8·0534	13·6667

TABLE A—(*continued*).

$n = 11$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	·34	·14	·05	·02	·01	·00	·00	—	—	—
− *·90*	3·48	1·46	·59	·22	·08	·02	·01	·00	—	—
− *·85*	13·10	5·73	2·40	·94	·34	·11	·03	·01	—	—
− *·80*	32·59	14·90	6·47	2·63	·97	·31	·08	·02	—	—
− *·75*	64·48	30·82	13·91	5·84	2·22	·74	·20	·04	·00	—
− *·70*	110·28	55·10	25·85	11·22	4·40	1·51	·42	·09	·01	—
− *·65*	170·38	89·00	43·43	19·52	7·90	2·78	·80	·17	·02	—
− *·60*	244·13	133·37	67·72	31·55	13·19	4·77	1·41	·30	·04	—
− *·55*	329·91	188·52	99·67	48·15	20·80	7·76	2·35	·51	·06	—
− *·50*	425·31	254·27	140·04	70·22	31·39	12·08	3·76	·84	·10	·00
− *·45*	527·29	329·89	189·38	98·64	45·66	18·15	5·83	1·34	·17	·01
− *·40*	632·37	414·10	247·94	134·25	64·43	26·48	8·77	2·08	·27	·01
− *·35*	736·81	505·13	315·62	177·84	88·59	37·70	12·90	3·16	·43	·01
− *·30*	836·83	600·74	391·97	230·03	119·07	52·54	18·61	4·71	·65	·02
− *·25*	928·73	698·31	476·06	291·27	156·85	71·87	26·38	6·91	·99	·03
− *·20*	1009·12	794·88	566·57	361·75	202·94	96·71	36·86	10·01	1·49	·05
− *·15*	1074·98	887·29	661·66	441·31	258·26	128·20	50·84	14·35	2·21	·08
− *·10*	1123·87	972·26	759·03	529·42	323·64	167·66	69·32	20·37	3·27	·12
− *·05*	1153·95	1046·54	855·92	625·00	399·73	216·51	93·53	28·70	4·80	·19
·00	1164·10	1107·05	949·18	726·44	486·84	276·25	124·99	40·16	7·04	·29
+ *·05*	1153·95	1150·99	1035·30	831·45	584·86	348·45	165·55	55·88	10·29	·44
+ *·10*	1123·87	1176·02	1110·55	937·05	693·05	434·44	217·39	77·34	15·02	·68
+ *·15*	1074·98	1180·37	1171·11	1039·52	809·89	535·54	283·09	106·54	21·93	1·06
+ *·20*	1009·12	1163·01	1213·24	1134·42	932·85	652·40	365·55	146·17	32·04	1·65
+ *·25*	928·73	1123·74	1233·55	1216·69	1058·17	784·94	467·92	199·73	46·90	2·60
+ *·30*	836·83	1063·29	1229·19	1280·75	1180·68	931·80	593·39	271·79	68·83	4·13
+ *·35*	736·81	983·38	1198·19	1320·82	1293·66	1089·82	744·74	368·23	101·35	6·67
+ *·40*	632·37	886·74	1139·72	1331·30	1388·77	1253·34	923·68	496·34	149·79	10·92
+ *·45*	527·29	777·05	1054·45	1307·30	1456·28	1413·46	1129·63	664·75	222·30	18·21
+ *·50*	425·31	658·86	944·78	1245·35	1485·58	1557·29	1358·02	882·89	331·21	31·02
+ *·55*	329·91	537·38	815·03	1144·27	1466·08	1667·54	1597·69	1159·22	495·09	54·17
+ *·60*	244·13	418·24	671·55	1006·07	1388·90	1722·77	1827·66	1497·43	741·14	97·35
+ *·65*	170·38	307·09	522·51	836·93	1249·15	1699·11	2013·49	1888·67	1107·09	180·79
+ *·70*	110·28	209·16	377·49	647·80	1049·15	1574·50	2105·04	2297·63	1638·62	348·37
+ *·75*	64·48	128·73	246·69	454·52	801·88	1336·70	2039·33	2640·85	2370·71	698·67
+ *·80*	32·59	68·50	139·53	276·64	533·90	995·96	1756·78	2762·99	3261·77	1455·70
+ *·85*	13·10	29·01	62·88	134·46	285·17	600·28	1242·87	2443·04	4017·99	3094·34
+ *·90*	3·48	8·12	18·74	43·33	101·48	243·28	601·85	1532·03	3796·61	6195·13
+ *·95*	·34	·83	2·04	5·11	13·29	36·65	110·61	381·37	1590·43	7823·00
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0952	·1906	·2865	·3831	·4808	·5799	·6807	·7839	·8900
Mode	·0000	·1352	·2667	·3912	·5066	·6121	·7074	·7930	·8697	·9384
σ	·3162	·3138	·3064	·2938	·2758	·2519	·2214	·1833	·1360	·0767
$(1-\rho^2)/\sqrt{n-1}$	·3162	·3131	·3036	·2878	·2656	·2372	·2024	·1613	·1138	·0601
β_1	·0000	·0228	·0932	·2179	·4101	·6933	1·1112	1·7516	2·8193	4·9565
β_2	2·5000	2·5372	2·6527	2·8587	3·1798	3·6616	4·3914	5·5540	7·6188	12·2879

TABLE A. *Ordinates and Constants of Frequency Curves.*

$n = 12.$ ρ variate (correlation in population sampled).

r variate (correlation in sample).	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	·11	·04	·01	·00	·00	·00	—	—	—	—
− *·90*	1·60	·61	·22	·08	·02	·01	·00	—	—	—
− *·85*	7·30	2·93	1·12	·40	·13	·04	·01	—	—	—
− *·80*	20·67	8·71	3·47	1·28	·43	·12	·03	·00	—	—
− *·75*	45·08	19·94	8·29	3·18	1·10	·33	·08	·01	—	—
− *·70*	83·24	38·68	16·77	6·68	2·38	·73	·18	·03	·00	—
− *·65*	136·86	66·80	30·25	12·52	4·62	1·46	·37	·07	·01	—
− *·60*	206·44	105·87	50·10	21·57	8·24	2·69	·70	·13	·01	—
− *·55*	291·24	156·97	77·68	34·81	13·80	4·65	1·25	·23	·02	—
− *·50*	389·33	220·58	114·20	53·33	21·95	7·66	2·12	·41	·04	—
− *·45*	497·73	296·51	160·71	78·27	33·48	12·11	3·46	·69	·07	—
− *·40*	612·62	383·82	217·93	110·79	49·33	18·52	5·48	1·13	·12	—
− *·35*	729·56	480·84	286·20	152·04	70·53	27·52	8·45	1·80	·20	·00
− *·30*	843·79	585·17	365·35	203·01	98·26	39·90	12·72	2·80	·32	·01
− *·25*	950·51	693·78	454·68	264·55	133·77	56·63	18·79	4·30	·51	·01
− *·20*	1045·09	803·05	552·83	337·18	178·37	78·86	27·27	6·49	·80	·02
− *·15*	1123·41	908·99	657·79	421·03	233·38	107·94	39·00	9·69	1·24	·03
− *·10*	1181·98	1007·35	766·83	515·70	299·98	145·45	55·03	14·30	1·91	·05
− *·05*	1218·21	1093·82	876·57	620·13	379·19	193·12	76·70	20·91	2·93	·08
·00	1230·47	1164·30	983·02	732·48	471·64	252·88	105·70	30·32	4·46	·13
+ *·05*	1218·21	1215·06	1081·67	850·05	577·41	326·69	144·13	43·65	6·79	·21
+ *·10*	1181·98	1243·05	1167·70	969·15	695·83	416·49	194·57	62·45	10·30	·34
+ *·15*	1123·41	1246·03	1236·18	1085·08	825·16	523·93	260·05	88·83	15·62	·56
+ *·20*	1045·09	1222·86	1282·35	1192·20	962·35	650·07	344·11	125·68	23·70	·91
+ *·25*	950·51	1173·63	1301·99	1284·05	1102·71	794·98	450·63	176·90	35·99	1·49
+ *·30*	843·79	1099·72	1291·80	1353·61	1239·72	957·09	583·60	247·67	54·78	2·49
+ *·35*	729·56	1003·91	1249·80	1393·76	1364·85	1132·50	746·54	344·77	83·60	4·20
+ *·40*	612·62	890·31	1175·76	1397·87	1467·64	1314·07	941·64	476·75	127·99	7·20
+ *·45*	497·73	764·16	1071·56	1360·66	1536·09	1490·50	1168·15	653·92	196·63	12·58
+ *·50*	389·33	631·64	941·41	1279·15	1557·56	1645·55	1420·17	887·55	303·03	22·49
+ *·55*	291·24	499·45	791·98	1153·78	1520·32	1757·80	1683·36	1187·72	468·00	41·25
+ *·60*	206·44	374·33	632·17	989·48	1415·90	1801·62	1930·97	1558·25	722·68	77·95
+ *·65*	136·86	262·47	472·60	796·46	1242·33	1750·35	2120·06	1986·75	1110·84	152·41
+ *·70*	83·24	168·90	324·59	590·32	1007·76	1582·81	2190·55	2427·07	1685·48	309·54
+ *·75*	45·08	96·80	198·77	391·03	733·76	1294·33	2072·67	2774·00	2484·25	654·55
+ *·80*	20·67	46·98	103·20	220·15	456·19	911·23	1713·02	2842·00	3444·82	1435·84
+ *·85*	7·30	17·56	41·32	95·84	220·41	503·13	1129·13	2396·80	4191·11	3192·59
+ *·90*	1·60	4·09	10·31	26·08	66·93	176·39	481·91	1361·28	3744·80	6541·18
+ *·95*	·11	·30	·81	2·25	6·48	19·94	67·87	268·10	1310·98	7752·85
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0956	·1914	·2877	·3847	·4826	·5818	·6826	·7855	·8910
Mode	·0000	·1309	·2586	·3805	·4948	·6004	·6968	·7843	·8636	·9353
σ	·3015	·2991	·2919	·2797	·2622	·2390	·2096	·1729	·1276	·0714
$(1-\rho^2)/\sqrt{n-1}$	·3015	·2985	·2895	·2744	·2533	·2261	·1930	·1538	·1085	·0573
β_1	·0000	·0215	·0880	·2054	·3854	·6487	1·0329	1·6118	2·5509	4·3322
β_2	2·5385	2·5742	2·6848	2·8816	3·1870	3·6417	4·3231	5·3905	7·2330	11·1595

TABLE A—(*continued*).

$n = 13$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	·04	·01	·00	·00	·00	—	—	—	—	—
− *·90*	·73	·26	·09	·03	·01	·00	—	—	—	—
− *·85*	4·04	1·49	·52	·17	·05	·01	·00	—	—	—
− *·80*	13·03	5·06	1·85	·62	·19	·05	·01	—	—	—
− *·75*	31·34	12·84	4·91	1·72	·54	·14	·03	·00	—	—
− *·70*	62·49	27·00	10·82	3·95	1·28	·35	·08	·01	—	—
− *·65*	109·33	49·86	20·96	7·99	2·69	·76	·17	·03	—	—
− *·60*	173·60	83·57	36·87	14·67	5·13	1·51	·35	·05	·00	—
− *·55*	255·68	129·97	60·20	25·03	9·10	2·78	·66	·11	·01	—
− *·50*	354·43	190·29	92·61	40·28	15·26	4·83	1·19	·20	·02	—
− *·45*	467·24	265·03	135·62	61·76	24·42	8·04	2·05	·35	·03	—
− *·40*	590·21	353·79	190·50	90·93	37·56	12·88	3·41	·61	·05	—
− *·35*	718·39	455·19	258·08	129·26	55·84	19·97	5·50	1·01	·09	—
− *·30*	846·13	566·86	338·66	178·18	80·64	30·14	8·65	1·66	·15	—
− *·25*	967·43	685·47	431·85	238·96	113·45	44·38	13·30	2·66	·26	·00
− *·20*	1076·39	806·84	536·45	312·55	155·92	63·95	20·07	4·19	·42	·01
− *·15*	1167·55	926·10	650·33	399·46	209·73	90·38	29·75	6·51	·69	·01
− *·10*	1236·25	1037·95	770·44	499·56	276·51	125·48	43·44	9·98	1·11	·02
− *·05*	1278·96	1136·93	892·78	611·89	357·72	171·31	62·54	15·15	1·77	·04
·00	1293·45	1217·76	1012·46	734·51	454·39	230·21	88·89	22·76	2·82	·06
+ *·05*	1278·96	1275·64	1123·90	864·29	566·93	304·67	124·80	33·91	4·45	·10
+ *·10*	1236·25	1306·66	1221·04	996·83	694·79	397·09	173·18	50·15	7·03	·17
+ *·15*	1167·55	1308·10	1297·68	1126·41	836·11	509·76	237·58	73·66	11·07	·29
+ *·20*	1076·39	1278·72	1347·93	1246·04	987·33	644·20	322·16	107·48	17·43	·50
+ *·25*	967·43	1218·98	1366·66	1347·70	1142·82	800·73	431·61	155·83	27·47	·86
+ *·30*	846·13	1131·13	1350·13	1422·76	1294·58	977·68	570·83	224·46	43·35	1·49
+ *·35*	718·39	1019·23	1296·48	1462·65	1432·08	1170·41	744·26	321·04	68·58	2·63
+ *·40*	590·21	888·97	1206·29	1459·72	1542·51	1370·21	954·71	455·44	108·77	4·72
+ *·45*	467·24	747·35	1082·96	1408·44	1611·41	1563·16	1201·40	639·76	172·98	8·65
+ *·50*	354·43	602·21	932·90	1306·67	1624·11	1729·32	1477·09	887·39	275·75	16·22
+ *·55*	255·68	461·64	765·35	1157·00	1567·94	1842·84	1763·97	1210·31	440·01	31·24
+ *·60*	173·60	333·18	591·84	967·83	1435·55	1873·82	2029·03	1612·76	700·88	62·08
+ *·65*	109·33	223·11	425·11	753·80	1228·80	1793·32	2220·16	2078·63	1108·60	127·80
+ *·70*	62·49	135·64	277·57	534·99	962·72	1582·52	2267·17	2549·96	1724·34	273·57
+ *·75*	31·34	72·39	159·28	334·58	667·77	1246·49	2095·16	2898·15	2589·24	609·94
+ *·80*	13·03	32·04	75·91	174·24	387·67	829·19	1661·34	2907·54	3618·62	1408·68
+ *·85*	4·04	10·58	27·01	67·94	169·42	419·43	1020·26	2338·80	4348·30	3276·40
+ *·90*	·73	2·05	5·65	15·61	43·90	127·20	383·80	1203·06	3673·97	6869·84
+ *·95*	·04	·11	·32	·98	3·14	10·79	41·42	187·46	1074·87	7642·57
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0960	·1921	·2887	·3859	·4841	·5834	·6841	·7868	·8919
Mode	·0000	·1274	·2522	·3721	·4853	·5908	·6880	·7772	·8585	·9326
σ	·2887	·2863	·2793	·2674	·2504	·2279	·1994	·1640	·1206	·0671
$(1-\rho^2)/\sqrt{n-1}$	·2887	·2858	·2771	·2627	·2425	·2165	·1848	·1472	·1039	·0548
β_1	·0000	·0204	·0833	·1941	·3631	·6087	·9636	1·4902	2·3236	3·8337
β_2	2·5714	2·6057	2·7117	2·8999	3·1904	3·6201	4·2575	5·2404	6·8937	10·2454

TABLE A. *Ordinates and Constants of Frequency Curves.*

$n = 14$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	·01	·00	·00	·00	—	—	—	—	—	—
− *·90*	·34	·11	·03	·01	·00	—	—	—	—	—
− *·85*	2·23	·75	·24	·07	·02	·00	—	—	—	—
− *·80*	8·18	2·93	·98	·30	·08	·02	·00	—	—	—
− *·75*	21·69	8·22	2·90	·93	·26	·06	·01	—	—	—
− *·70*	46·70	18·77	6·95	2·33	·69	·17	·03	·00	—	—
− *·65*	86·94	37·04	14·45	5·07	1·55	·40	·08	·01	—	—
− *·60*	145·33	65·68	27·00	9·93	3·17	·84	·17	·02	—	—
− *·55*	223·45	107·13	46·44	17·91	5·98	1·65	·35	·05	·00	—
− *·50*	321·20	163·42	74·76	30·28	10·57	3·04	·66	·09	·01	—
− *·45*	436·63	235·83	113·94	48·51	17·73	5·31	1·21	·18	·01	—
− *·40*	566·06	324·64	165·76	74·28	28·46	8·92	2·11	·32	·02	—
− *·35*	704·20	428·97	231·67	109·39	44·02	14·43	3·57	·57	·04	—
− *·30*	844·64	546·64	312·50	155·68	65·88	22·66	5·86	·98	·07	—
− *·25*	980·21	674·21	408·32	214·86	95·78	34·62	9·38	1·64	·13	—
− *·20*	1103·62	806·97	518·19	288·41	135·67	51·63	14·70	2·69	·23	·00
− *·15*	1207·94	939·26	640·05	377·28	187·62	75·34	22·59	4·35	·38	·01
− *·10*	1287·18	1064·65	770·57	481·74	253·73	107·76	34·14	6·94	·64	·01
− *·05*	1336·68	1176·41	905·17	601·04	335·93	151·28	50·77	10·92	1·07	·02
·00	1353·52	1267·92	1038·07	733·22	435·79	208·62	74·41	17·01	1·77	·03
+ *·05*	1336·68	1333·18	1162·50	874·80	554·12	282·80	107·57	26·23	2·91	·05
+ *·10*	1287·18	1367·32	1271·05	1020·68	690·62	376·89	153·45	40·10	4·77	·09
+ *·15*	1207·94	1367·06	1356·11	1164·04	843·38	493·74	216·07	60·80	7·81	·15
+ *·20*	1103·62	1331·09	1410·48	1296·43	1008·40	635·51	300·25	91·50	12·76	·27
+ *·25*	980·21	1260·37	1428·09	1408·13	1179·07	802·90	411·53	136·65	20·87	·49
+ *·30*	844·64	1158·20	1404·74	1488·72	1345·78	994·23	555·84	202·51	34·16	·89
+ *·35*	704·20	1030·12	1338·84	1528·05	1495·87	1204·16	738·66	297·60	56·01	1·64
+ *·40*	566·06	883·63	1232·03	1517·46	1613·92	1422·35	963·63	433·15	92·02	3·08
+ *·45*	436·63	727·61	1089·56	1451·35	1682·84	1632·02	1230·08	623·12	151·50	5·92
+ *·50*	321·20	571·57	920·31	1328·79	1685·90	1809·23	1529·42	883·27	249·81	11·64
+ *·55*	223·45	424·77	736·29	1155·02	1609·82	1923·36	1840·21	1227·85	411·85	23·56
+ *·60*	145·33	295·23	551·59	942·42	1448·96	1940·22	2122·58	1661·76	676·71	49·22
+ *·65*	86·94	188·79	380·68	710·22	1209·98	1829·15	2314·64	2165·10	1101·47	106·69
+ *·70*	46·70	108·45	236·30	482·68	915·58	1575·19	2336·05	2667·21	1756·31	240·71
+ *·75*	21·69	53·89	127·07	284·99	605·00	1195·09	2108·50	3014·47	2686·77	565·87
+ *·80*	8·18	21·76	55·58	137·29	327·97	751·18	1604·06	2961·44	3784·47	1375·97
+ *·85*	2·23	6·34	17·57	47·94	129·65	348·10	917·81	2272·14	4491·55	3347·70
+ *·90*	·34	1·02	3·08	9·30	28·67	91·32	304·31	1058·56	3588·66	7183·43
+ *·95*	·01	·04	·13	·43	1·52	5·81	25·17	130·49	877·42	7501·01
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0963	·1927	·2896	·3870	·4853	·5847	·6854	·7879	·8926
Mode	·0000	·1246	·2470	·3652	·4775	·5828	·6808	·7711	·8541	·9303
σ	·2774	·2751	·2682	·2566	·2401	·2182	·1906	·1564	·1145	·0634
$(1-\rho^2)/\sqrt{n-1}$	·2774	·2746	·2663	·2524	·2330	·2080	·1775	·1414	·0998	·0527
β_1	·0000	·0194	·0790	·1838	·3430	·5729	·9020	1·3838	2·1298	3·4290
β_2	2·6000	2·6329	2·7346	2·9145	3·1912	3·5979	4·1955	5·1038	6·5961	9·4886

TABLE A—(*continued*).

$n = 15$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
– *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
– *·95*	·00	·00	·00	—	—	—	—	—	—	—
– *·90*	·15	·04	·01	·00	·00	—	—	—	—	—
– *·85*	1·22	·38	·11	·03	·01	·00	—	—	—	—
– *·80*	5·12	1·69	·52	·14	·04	·01	—	—	—	—
– *·75*	14·96	5·25	1·70	·50	·13	·03	·00	—	—	—
– *·70*	34·77	12·99	4·45	1·37	·37	·08	·01	—	—	—
– *·65*	68·88	27·42	9·93	3·21	·90	·21	·04	·00	—	—
– *·60*	121·21	51·42	19·70	6·69	1·96	·47	·08	·01	—	—
– *·55*	194·55	87·97	35·70	12·77	3·91	·97	·18	·02	—	—
– *·50*	289·98	139·82	60·13	22·68	7·29	1·90	·37	·05	·00	—
– *·45*	406·50	209·05	95·35	37·96	12·82	3·50	·71	·09	·01	—
– *·40*	540·85	296·76	143·69	60·46	21·49	6·15	1·30	·17	·01	—
– *·35*	687·70	402·73	207·17	92·23	34·56	10·39	2·30	·32	·02	—
– *·30*	839·97	525·16	287·27	135·50	53·62	16·97	3·95	·57	·04	—
– *·25*	989·42	660·62	384·61	192·47	80·56	26·90	6·59	1·00	·07	—
– *·20*	1127·28	804·07	498·68	265·12	117·61	41·52	10·72	1·72	·12	—
– *·15*	1245·03	949·02	627·56	354·99	167·21	62·56	17·09	2·90	·21	—
– *·10*	1335·15	1087·93	767·80	462·81	231·94	92·19	26·73	4·80	·37	·00
– *·05*	1391·74	1212·67	914·28	588·16	314·29	133·08	41·06	7·85	·64	·01
·00	1411·04	1315·18	1060·32	729·17	416·38	188·35	62·06	12·66	1·11	·01
+ *·05*	1391·74	1388·08	1197·90	882·11	539·56	261·56	92·37	20·21	1·89	·02
+ *·10*	1335·15	1425·42	1318·13	1041·17	683·89	356·36	135·46	31·94	3·23	·04
+ *·15*	1245·03	1423·30	1411·83	1198·41	847·52	476·43	195·77	50·00	5·49	·08
+ *·20*	1127·28	1380·40	1470·37	1343·80	1026·06	624·58	278·78	77·60	9·31	·15
+ *·25*	989·42	1298·27	1486·66	1465·74	1211·89	802·05	390·92	119·38	15·80	·28
+ *·30*	839·97	1181·45	1456·06	1551·88	1393·76	1007·27	539·22	182·03	26·81	·53
+ *·35*	687·70	1037·21	1377·39	1590·37	1556·64	1234·25	730·36	274·85	45·57	1·02
+ *·40*	540·85	875·02	1253·60	1571·56	1682·30	1470·94	968·99	410·40	77·56	2·01
+ *·45*	406·50	705·73	1092·09	1489·95	1750·86	1697·55	1254·73	604·65	132·19	4·04
+ *·50*	289·98	540·45	904·49	1346·21	1743·50	1885·77	1577·71	875·91	225·46	8·33
+ *·55*	194·55	389·38	705·68	1148·73	1646·64	1999·91	1912·58	1241·01	384·06	17·70
+ *·60*	121·21	260·61	512·15	914·23	1457·04	2001·47	2212·18	1705·89	650·97	38·88
+ *·65*	68·88	159·16	339·61	666·66	1187·01	1858·75	2404·17	2246·82	1090·34	88·74
+ *·70*	34·77	86·37	200·40	433·86	867·51	1562·05	2398·10	2779·52	1782·27	211·01
+ *·75*	14·96	39·97	100·99	241·85	546·09	1141·54	2114·05	3123·86	2777·70	523·05
+ *·80*	5·12	14·72	40·55	107·77	276·43	677·98	1543·04	3005·21	3943·34	1339·09
+ *·85*	1·22	3·79	11·39	33·71	98·85	287·83	822·59	2199·24	4622·48	3408·02
+ *·90*	·15	·51	1·67	5·52	18·65	65·32	240·39	927·98	3492·48	7483·96
+ *·95*	·00	·01	·05	·19	·73	3·12	15·23	90·51	713·62	7335·23
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0965	·1932	·2903	·3879	·4864	·5858	·6865	·7888	·8932
Mode	·0000	·1224	·2429	·3595	·4710	·5761	·6745	·7659	·8504	·9283
σ	·2673	·2650	·2583	·2470	·2309	·2096	·1828	·1496	·1093	·0602
$(1-\rho^2)/\sqrt{n-1}$	·2673	·2646	·2566	·2423	·2245	·2004	·1710	·1363	·0962	·0508
β_1	·0000	·0184	·0751	·1745	·3248	·5407	·8473	1·2904	1·9635	3·0956
β_2	2·6250	2·6566	2·7542	2·9265	3·1904	3·5759	4·1375	4·9799	6·3347	8·8548

TABLE A. *Ordinates and Constants of Frequency Curves.*

$n = 16$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	·00	—	—	—	—	—	—	—	—	—
− *·90*	·07	·02	·00	·00	—	—	—	—	—	—
− *·85*	·67	·19	·05	·01	·00	—	—	—	—	—
− *·80*	3·19	·97	·27	·07	·02	·00	—	—	—	—
− *·75*	10·28	3·34	1·00	·27	·06	·01	·00	—	—	—
− *·70*	25·80	8·97	2·84	·80	·19	·04	·01	—	—	—
− *·65*	54·39	20·23	6·80	2·02	·51	·11	·02	—	—	—
− *·60*	100·76	40·12	14·33	4·50	1·20	·26	·04	·00	—	—
− *·55*	168·85	72·01	27·35	9·08	2·55	·57	·09	·01	—	—
− *·50*	260·97	119·24	48·20	16·93	5·01	1·18	·20	·02	—	—
− *·45*	377·23	184·72	79·55	29·61	9·24	2·29	·41	·05	—	—
− *·40*	515·11	270·42	124·16	49·05	16·17	4·23	·80	·09	·00	—
− *·35*	669·43	376·88	184·68	77·51	27·05	7·46	1·48	·18	·01	—
− *·30*	832·67	502·91	263·24	117·56	43·50	12·67	2·65	·34	·02	—
− *·25*	995·53	645·25	361·12	171·85	67·54	20·84	4·61	·61	·03	—
− *·20*	1147·76	798·62	478·36	242·94	101·63	33·28	7·80	1·10	·06	—
− *·15*	1279·16	955·82	613·35	332·95	148·55	51·78	12·89	1·93	·12	—
− *·10*	1380·50	1108·16	762·59	443·20	211·35	78·63	20·86	3·32	·21	—
− *·05*	1444·45	1246·06	920·53	573·72	293·10	116·70	33·10	5·62	·38	—
·00	1466·31	1359·84	1079·59	722·83	396·57	169·51	51·59	9·40	·69	·00
+ *·05*	1444·45	1440·62	1230·45	886·63	523·71	241·10	79·06	15·52	1·23	·01
+ *·10*	1380·50	1481·25	1362·60	1058·69	675·07	335·89	119·19	25·36	2·18	·02
+ *·15*	1279·16	1477·14	1465·15	1229·86	848·97	458·26	176·81	40·99	3·84	·04
+ *·20*	1147·76	1426·97	1527·93	1388·46	1040·70	611·89	258·02	65·60	6·77	·08
+ *·25*	995·53	1333·04	1542·70	1520·86	1241·67	798·66	370·16	103·97	11·92	·16
+ *·30*	832·67	1201·32	1504·45	1612·58	1438·86	1017·24	521·43	163·10	20·98	·31
+ *·35*	669·43	1041·01	1412·54	1649·98	1614·73	1261·08	719·87	253·03	36·96	·63
+ *·40*	515·11	863·74	1271·49	1622·42	1748·02	1516·38	971·31	387·62	65·16	1·30
+ *·45*	377·23	682·33	1091·14	1524·73	1815·85	1760·11	1275·84	584·87	114·98	2·74
+ *·50*	260·97	509·39	886·11	1359·54	1797·36	1959·32	1622·38	865·86	202·85	5·94
+ *·55*	168·85	355·79	674·19	1138·85	1678·96	2072·94	1981·54	1250·36	357·03	13·25
+ *·60*	100·76	229·32	474·01	884·08	1460·52	2058·14	2298·30	1745·69	624·23	30·62
+ *·65*	54·39	133·74	302·02	623·79	1160·78	1882·87	2489·31	2324·29	1075·93	73·57
+ *·70*	25·80	68·58	169·43	388·74	819·36	1544·13	2454·04	2887·46	1802·95	184·41
+ *·75*	10·28	29·55	80·00	204·59	491·36	1086·96	2112·96	3227·08	2862·73	481·97
+ *·80*	3·19	9·92	29·49	84·33	232·25	609·99	1479·67	3040·08	4096·07	1299·15
+ *·85*	·67	2·25	7·36	23·62	75·13	237·24	734·93	2121·99	4742·40	3458·65
+ *·90*	·07	·25	·90	3·27	12·09	46·58	189·31	810·97	3388·31	7772·88
+ *·95*	·00	·00	·02	·08	·35	1·67	9·19	62·58	578·60	7150·91
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0968	·1937	·2909	·3888	·4873	·5868	·6875	·7896	·8937
Mode	·0000	·1206	·2394	·3548	·4655	·5705	·6692	·7614	·8471	·9265
σ	·2582	·2560	·2495	·2384	·2227	·2020	·1759	·1437	·1047	·0575
$(1-\rho^2)/\sqrt{n-1}$	·2582	·2556	·2479	·2350	·2169	·1936	·1652	·1310	·0930	·0491
β_1	·0000	·0176	·0716	·1660	·3083	·5117	·7983	1·2080	1·8195	2·8181
β_2	2·6471	2·6775	2·7712	2·9363	3·1883	3·5545	4·0836	4·8677	6·1046	8·3239

TABLE A—(*continued*).

$n = 17$. ρ variate (correlation in population sampled).

r variate (correlation in sample).	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	·00	·00	—	—	—	—	—	—	—	—
− *·90*	·03	·01	·00	—	—	—	—	—	—	—
− *·85*	·37	·10	·02	·01	·00	—	—	—	—	—
− *·80*	1·98	·56	·14	·03	·01	·00	—	—	—	—
− *·75*	7·05	2·12	·58	·14	·03	·01	—	—	—	—
− *·70*	19·10	6·17	1·81	·47	·10	·02	·00	—	—	—
− *·65*	42·84	14·88	4·64	1·27	·29	·05	·01	—	—	—
− *·60*	83·54	31·23	10·39	3·02	·74	·14	·02	—	—	—
− *·55*	146·13	58·78	20·89	6·43	1·66	·34	·05	·00	—	—
− *·50*	234·22	101·41	38·54	12·61	3·43	·74	·11	·01	—	—
− *·45*	349·12	162·78	66·18	23·04	6·64	1·50	·24	·02	—	—
− *·40*	489·26	245·73	106·99	39·68	12·14	2·90	·49	·05	—	—
− *·35*	649·87	351·73	164·17	64·96	21·11	5·33	·95	·10	·00	—
− *·30*	823·17	480·28	240·55	101·72	35·19	9·43	1·78	·20	·01	—
− *·25*	998·93	628·51	338·14	153·03	56·47	16·10	3·22	·37	·02	—
− *·20*	1165·43	791·03	457·61	222·01	87·57	26·61	5·66	·70	·03	—
− *·15*	1310·64	960·04	597·82	311·43	131·60	42·74	9·69	1·28	·06	—
− *·10*	1423·48	1125·69	755·35	423·25	192·06	66·87	16·23	2·28	·12	—
− *·05*	1495·05	1276·87	924·29	558·10	272·60	102·06	26·61	4·01	·23	—
·00	1519·58	1402·18	1096·21	714·59	376·67	152·13	42·77	6·95	·43	·00
+ *·05*	1495·05	1491·07	1260·42	888·75	506·94	221·67	67·49	11·89	·79	·01
+ *·10*	1423·48	1535·05	1404·71	1073·56	664·54	315·72	104·60	20·08	1·46	·01
+ *·15*	1310·64	1528·81	1516·33	1258·69	848·09	439·58	159·26	33·51	2·68	·02
+ *·20*	1165·43	1471·07	1583·40	1430·69	1052·66	597·82	238·16	55·31	4·91	·04
+ *·25*	998·93	1365·00	1596·49	1573·74	1268·71	793·12	349·55	90·30	8·97	·09
+ *·30*	823·17	1218·19	1550·21	1671·08	1481·38	1024·51	502·86	145·74	16·37	·18
+ *·35*	649·87	1041·98	1444·64	1707·15	1670·43	1284·98	707·60	232·31	29·89	·39
+ *·40*	489·26	850·26	1286·11	1670·36	1811·36	1558·97	970·98	365·12	54·60	·84
+ *·45*	349·12	657·90	1087·22	1556·06	1878·12	1820·03	1293·78	564·21	99·74	1·86
+ *·50*	234·22	478·81	865·73	1369·26	1847·84	2030·22	1663·80	853·62	182·02	4·22
+ *·55*	146·13	324·22	642·35	1125·98	1707·27	2142·80	2047·42	1256·38	331·00	9·90
+ *·60*	83·54	201·23	437·52	852·59	1460·03	2110·68	2381·31	1781·59	596·99	24·05
+ *·65*	42·84	112·08	267·85	582·09	1132·06	1902·13	2570·49	2397·95	1058·86	60·84
+ *·70*	19·10	54·30	142·84	347·36	771·78	1522·30	2504·51	2991·52	1818·97	160·72
+ *·75*	7·05	21·79	63·21	172·59	440·91	1032·18	2106·17	3324·73	2942·45	442·92
+ *·80*	1·98	6·67	21·38	65·80	194·61	547·33	1415·08	3067·10	4243·31	1257·03
+ *·85*	·37	1·34	4·74	16·51	56·94	195·02	654·85	2042·00	4852·41	3500·67
+ *·90*	·03	·12	·49	1·93	7·82	33·12	148·67	706·81	3278·45	8051·44
+ *·95*	·00	·00	·01	·03	·17	·89	5·53	43·15	467·87	6952·68
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0970	·1941	·2915	·3895	·4881	·5876	·6883	·7903	·8941
Mode	·0000	·1190	·2364	·3507	·4608	·5656	·6646	·7575	·8442	·9250
σ	·2500	·2479	·2415	·2307	·2153	·1951	·1696	·1384	·1006	·0551
$(1-\rho^2)/\sqrt{n-1}$	·2500	·2475	·2400	·2275	·2100	·1875	·1600	·1275	·0900	·0475
β_1	·0000	·0168	·0683	·1582	·2934	·4855	·7543	1·1348	1·6940	2·5832
β_2	2·6667	2·6960	2·7861	2·9446	3·1855	3·5340	4·0337	4·7661	5·9012	7·8748

TABLE A. *Ordinates and Constants of Frequency Curves.*

$n = 18$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	·00	—	—	—	—	—	—	—	—	—
− *·90*	·01	·00	·00	—	—	—	—	—	—	—
− *·85*	·20	·05	·01	·00	—	—	—	—	—	—
− *·80*	1·23	·32	·08	·02	·00	—	—	—	—	—
− *·75*	4·82	1·34	·34	·08	·01	·00	—	—	—	—
− *·70*	14·10	4·24	1·15	·27	·05	·01	—	—	—	—
− *·65*	33·66	10·93	3·16	·80	·17	·03	—	—	—	—
− *·60*	69·10	24·24	7·52	2·02	·45	·08	·01	—	—	—
− *·55*	126·18	47·87	15·93	4·55	1·08	·20	·03	—	—	—
− *·50*	209·71	86·05	30·73	9·36	2·35	·46	·06	—	—	—
− *·45*	322·33	143·10	54·93	17·88	4·76	·98	·14	·01	—	—
− *·40*	463·60	222·77	91·98	32·03	9·09	1·98	·30	·03	—	—
− *·35*	629·39	327·48	145·60	54·32	16·44	3·80	·61	·06	—	—
− *·30*	811·85	457·58	219·30	87·80	28·40	7·01	1·19	·11	—	—
− *·25*	999·97	610·75	315·87	135·94	47·10	12·40	2·24	·23	·01	—
− *·20*	1180·56	781·66	436·73	202·39	75·28	21·22	4·09	·44	·02	—
− *·15*	1339·70	961·98	581·30	290·60	116·32	35·20	7·27	·84	·03	—
− *·10*	1464·32	1140·77	746·40	403·25	174·11	56·74	12·60	1·57	·07	—
− *·05*	1543·76	1305·33	925·87	541·62	252·92	89·04	21·34	2·86	·14	—
·00	1571·04	1442·41	1110·44	704·76	356·91	136·21	35·38	5·13	·27	—
+ *·05*	1543·76	1539·62	1288·06	888·76	489·54	203·29	57·47	9·09	·51	·00
+ *·10*	1464·32	1587·04	1444·69	1086·05	652·63	296·06	91·57	15·86	·98	·01
+ *·15*	1339·70	1578·54	1565·59	1285·14	845·22	420·66	143·10	27·33	1·87	·01
+ *·20*	1180·56	1512·94	1637·00	1470·71	1062·25	582·69	219·31	46·52	3·55	·02
+ *·25*	999·97	1394·42	1648·23	1624·60	1293·27	785·76	329·31	78·24	6·73	·05
+ *·30*	811·85	1232·37	1593·57	1727·60	1521·54	1029·39	483·81	129·92	12·75	·11
+ *·35*	629·39	1040·48	1473·96	1762·12	1723·97	1306·25	693·90	212·79	24·12	·24
+ *·40*	463·60	835·02	1297·82	1715·64	1872·57	1598·97	968·37	343·11	45·64	·54
+ *·45*	322·33	632·84	1080·74	1584·29	1937·95	1877·54	1308·88	543·00	86·32	1·26
+ *·50*	209·71	449·00	843·82	1375·79	1895·25	2098·72	1702·25	839·57	162·94	3·00
+ *·55*	126·18	294·75	610·56	1110·63	1731·96	2209·80	2110·51	1259·46	306·15	7·37
+ *·60*	69·10	176·17	402·90	820·29	1456·10	2159·46	2461·52	1813·97	569·59	18·84
+ *·65*	33·66	93·71	236·98	541·90	1101·45	1917·07	2648·10	2468·15	1039·63	50·19
+ *·70*	14·10	42·89	120·15	309·66	725·26	1497·24	2550·03	3092·09	1830·85	139·75
+ *·75*	4·82	16·03	49·82	145·26	394·72	977·87	2094·49	3417·34	3017·33	406·10
+ *·80*	1·23	4·48	15·47	51·23	162·68	489·96	1350·14	3087·13	4385·61	1213·45
+ *·85*	·20	·79	3·05	11·51	43·06	159·94	582·14	1960·44	4953·44	3534·99
+ *·90*	·01	·05	·26	1·14	5·05	23·50	116·49	614·60	3164·79	8320·69
+ *·95*	·00	·00	·00	·01	·08	·47	3·32	29·69	377·45	6744·32
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0971	·1944	·2920	·3901	·4888	·5884	·6890	·7909	·8945
Mode	·0000	·1176	·2338	·3472	·4567	·5613	·6605	·7540	·8417	·9236
σ	·2425	·2405	·2342	·2237	·2086	·1889	·1641	·1337	·0970	·0530
$(1-\rho^2)/\sqrt{n-1}$	·2425	·2401	·2328	·2207	·2037	·1819	·1552	·1237	·0873	·0461
β_1	·0000	·0161	·0653	·1511	·2797	·4617	·7147	1·0695	1·5839	2·3830
β_2	2·6842	2·7124	2·7992	2·9515	3·1823	3·5144	3·9873	4·6737	5·7207	7·4908

TABLE A—(*continued*).

$n = 19$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	·00	—	—	—	—	—	—	—	—	—
− *·90*	·01	·00	—	—	—	—	—	—	—	—
− *·85*	·11	·02	·00	·00	—	—	—	—	—	—
− *·80*	·76	·18	·04	·01	·00	—	—	—	—	—
− *·75*	3·29	·85	·20	·04	·01	—	—	—	—	—
− *·70*	10·39	2·90	·73	·16	·03	·00	—	—	—	—
− *·65*	26·39	8·00	2·15	·50	·10	·01	—	—	—	—
− *·60*	57·03	18·78	5·43	1·35	·27	·04	·00	—	—	—
− *·55*	108·73	38·90	12·11	3·21	·70	·12	·01	—	—	—
− *·50*	187·37	72·86	24·46	6·94	1·60	·28	·03	·00	—	—
− *·45*	296·98	125·54	45·49	13·84	3·41	·64	·08	·01	—	—
− *·40*	438·38	201·54	78·91	25·79	6·79	1·35	·18	·01	—	—
− *·35*	608·28	304·27	128·86	45·32	12·77	2·71	·39	·03	—	—
− *·30*	799·03	435·05	199·51	75·63	22·88	5·19	·79	·07	—	—
− *·25*	998·93	592·25	294·45	120·51	39·21	9·54	1·56	·14	·00	—
− *·20*	1193·40	770·79	415·93	184·13	64·59	16·89	2·96	·28	·01	—
− *·15*	1366·56	961·93	564·06	270·60	102·59	28·93	5·45	·56	·02	—
− *·10*	1503·20	1153·66	736·02	383·39	157·51	48·04	9·76	1·07	·04	—
− *·05*	1590·74	1331·66	925·52	524·53	234·18	77·52	17·08	2·03	·08	—
·00	1620·88	1480·70	1122·52	693·62	337·49	121·70	29·20	3·78	·16	—
+ *·05*	1590·74	1586·45	1313·57	886·92	471·76	186·07	48·84	6·93	·33	—
+ *·10*	1503·20	1637·37	1482·73	1096·42	639·60	277·05	80·00	12·50	·66	·00
+ *·15*	1366·56	1626·51	1613·08	1309·42	840·60	401·73	128·32	22·24	1·30	·01
+ *·20*	1193·40	1552·77	1688·90	1508·71	1069·70	566·77	201·53	39·05	2·57	·01
+ *·25*	998·93	1421·51	1698·12	1673·64	1315·58	776·85	309·60	67·65	5·04	·03
+ *·30*	799·03	1244·13	1634·75	1782·33	1559·56	1032·16	464·51	115·58	9·90	·06
+ *·35*	608·28	1036·82	1500·76	1815·10	1775·54	1325·12	679·05	194·50	19·43	·15
+ *·40*	438·38	818·35	1306·92	1758·51	1931·83	1636·61	963·78	321·76	38·08	·35
+ *·45*	296·98	607·48	1072·08	1609·70	1995·55	1932·88	1321·43	521·51	74·55	·85
+ *·50*	187·37	420·16	820·76	1379·49	1939·87	2165·06	1738·01	824·05	145·56	2·12
+ *·55*	108·73	267·40	579·15	1093·22	1753·38	2274·19	2171·08	1259·96	282·59	5·48
+ *·60*	57·03	153·91	370·23	787·57	1449·19	2204·82	2539·21	1843·14	542·34	14·73
+ *·65*	26·39	78·18	209·24	503·43	1069·45	1928·16	2722·44	2535·20	1018·65	41·32
+ *·70*	10·39	33·81	100·85	275·47	680·14	1469·57	2591·04	3189·48	1839·04	121·27
+ *·75*	3·29	11·76	39·19	122·01	352·63	924·51	2078·60	3505·34	3087·80	371·57
+ *·80*	·76	3·00	11·17	39·80	135·72	437·70	1285·55	3100·93	4523·43	1169·00
+ *·85*	·11	·47	1·96	8·01	32·49	130·90	516·43	1878·29	5046·26	3562·39
+ *·90*	·01	·02	·14	·67	3·25	16·63	91·09	533·33	3048·86	8581·50
+ *·95*	·00	·00	·00	·01	·04	·25	1·99	20·38	303·89	6528·96
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0973	·1947	·2924	·3906	·4894	·5890	·6896	·7915	·8948
Mode	·0000	·1165	·2316	·3442	·4531	·5576	·6569	·7509	·8394	·9224
σ	·2357	·2337	·2275	·2172	·2025	·1832	·1590	·1294	·0937	·0511
$(1-\rho^2)/\sqrt{n-1}$	·2357	·2333	·2263	·2145	·1980	·1768	·1508	·1202	·0849	·0448
β_1	·0000	·0154	·0626	·1446	·2672	·4400	·6789	1·0110	1·4866	2·2105
β_2	2·7000	2·7272	2·8109	2·9573	3·1787	3·4958	3·9447	4·5897	5·5597	7·1586

TABLE A. *Ordinates and Constants of Frequency Curves.*

$n = 20$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	—	—	—	—	—	—	—	—	—	—
− *·90*	·00	·00	—	—	—	—	—	—	—	—
− *·85*	·06	·01	·00	—	—	—	—	—	—	—
− *·80*	·47	·10	·02	·00	—	—	—	—	—	—
− *·75*	2·24	·53	·11	·02	·00	—	—	—	—	—
− *·70*	7·64	1·99	·46	·10	·02	·00	—	—	—	—
− *·65*	20·65	5·85	1·46	·31	·05	·01	—	—	—	—
− *·60*	46·98	14·53	3·91	·90	·17	·02	·00	—	—	—
− *·55*	93·51	31·56	9·20	2·26	·45	·07	·01	—	—	—
− *·50*	167·11	61·57	19·43	5·14	1·09	·17	·02	—	—	—
− *·45*	273·13	109·93	37·61	10·70	2·44	·42	·05	·00	—	—
− *·40*	413·76	181·99	67·57	20·74	5·06	·92	·11	·01	—	—
− *·35*	586·81	282·18	113·83	37·75	9·91	1·93	·25	·02	—	—
− *·30*	784·96	412·86	181·17	65·03	18·39	3·84	·53	·04	—	—
− *·25*	996·07	573·27	273·98	106·64	32·57	7·32	1·08	·08	—	—
− *·20*	1204·17	758·68	395·39	167·20	55·31	13·42	2·13	·18	·00	—
− *·15*	1391·40	960·11	546·33	251·51	90·32	23·73	4·07	·37	·01	—
− *·10*	1540·28	1164·55	724·46	363·84	142·24	40·60	7·55	·73	·02	—
− *·05*	1636·14	1356·03	923·47	507·05	216·43	67·37	13·64	1·44	·05	—
·00	1669·24	1517·23	1132·65	681·41	318·54	108·54	24·06	2·78	·10	—
+ *·05*	1636·14	1631·71	1337·14	883·46	453·78	169·98	41·43	5·27	·21	—
+ *·10*	1540·28	1686·21	1518·98	1104·85	625·68	258·79	69·76	9·84	·44	—
+ *·15*	1391·40	1672·86	1658·98	1331·72	834·48	382·94	114·86	18·07	·90	·00
+ *·20*	1204·17	1590·74	1739·26	1544·87	1075·23	550·27	184·86	32·72	1·85	·01
+ *·25*	996·07	1446·47	1746·32	1720·99	1335·82	766·63	290·54	58·39	3·77	·02
+ *·30*	784·96	1253·70	1673·93	1835·44	1595·60	1033·05	445·18	102·63	7·68	·04
+ *·35*	586·81	1031·28	1525·25	1866·25	1825·32	1341·82	663·32	177·47	15·62	·09
+ *·40*	413·76	800·54	1313·68	1799·15	1989·34	1672·08	957·45	301·19	31·71	·22
+ *·45*	273·13	582·06	1061·54	1632·51	2051·11	1986·22	1331·67	499·96	64·26	·57
+ *·50*	167·11	392·47	796·88	1380·68	1981·92	2229·43	1771·29	807·35	129·80	1·50
+ *·55*	93·51	242·15	548·35	1074·11	1771·83	2336·21	2229·32	1258·17	260·36	4·07
+ *·60*	46·98	134·22	339·60	754·79	1439·68	2247·04	2614·58	1869·39	515·46	11·50
+ *·65*	20·65	65·11	184·41	466·85	1036·50	1935·78	2793·79	2599·35	996·30	33·96
+ *·70*	7·64	26·60	84·50	244·62	636·66	1439·79	2627·94	3283·99	1843·93	105·05
+ *·75*	2·24	8·62	30·77	102·29	314·46	872·48	2059·10	3589·11	3154·22	339·37
+ *·80*	·47	2·00	8·05	30·87	113·01	390·31	1221·83	3109·18	4657·18	1124·15
+ *·85*	·06	·28	1·25	5·56	24·47	106·93	457·32	1796·33	5131·57	3583·56
+ *·90*	·00	·00	·08	·39	2·09	11·76	71·10	461·97	2931·90	8834·61
+ *·95*	—	—	·00	·00	·02	·13	1·19	13·97	244·23	6309·15
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0974	·1950	·2928	·3911	·4900	·5896	·6902	·7919	·8951
Mode	·0000	·1154	·2297	·3415	·4500	·5543	·6538	·7482	·8374	·9213
σ	·2294	·2274	·2214	·2113	·1969	·1780	·1543	·1254	·0907	·0493
$(1-\rho^2)/\sqrt{n-1}$	·2294	·2271	·2202	·2088	·1927	·1721	·1468	·1170	·0826	·0436
β_1	·0000	·0148	·0600	·1386	·2557	·4202	·6464	·9584	1·4001	2·0603
β_2	2·7143	2·7406	2·8213	2·9623	3·1749	3·4783	3·9055	4·5131	5·4154	6·8681

TABLE A—(*continued*).

$n = 21$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
– *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
– *·95*	·00	—	—	—	—	—	—	—	—	—
– *·90*	·00	·00	—	—	—	—	—	—	—	—
– *·85*	·03	·01	·00	—	—	—	—	—	—	—
– *·80*	·29	·06	·01	·00	—	—	—	—	—	—
– *·75*	1·52	·34	·07	·01	·00	—	—	—	—	—
– *·70*	5·61	1·36	·29	·05	·01	—	—	—	—	—
– *·65*	16·13	4·27	·99	·19	·03	·00	—	—	—	—
– *·60*	38·65	11·22	2·82	·60	·10	·01	—	—	—	—
– *·55*	80·30	25·56	6·97	1·59	·29	·04	·00	—	—	—
– *·50*	148·80	51·95	15·41	3·79	·74	·11	·01	—	—	—
– *·45*	250·78	96·10	31·04	8·26	1·74	·27	·03	—	—	—
– *·40*	389·90	164·07	57·77	16·64	3·77	·63	·07	·00	—	—
– *·35*	565·17	261·27	100·39	31·39	7·67	1·37	·16	·01	—	—
– *·30*	769·89	391·17	164·25	55·82	14·76	2·84	·35	·02	—	—
– *·25*	991·59	553·98	254·51	94·20	27·02	5·61	·75	·05	—	—
– *·20*	1213·06	745·68	375·26	151·59	47·28	10·64	1·53	·11	·00	—
– *·15*	1414·38	956·73	528·29	233·39	79·38	19·43	3·04	·24	·01	—
– *·10*	1575·70	1173·63	711·91	344·73	128·23	34·26	5·83	·50	·01	—
– *·05*	1680·10	1378·59	919·93	489·35	199·70	58·45	10·88	1·02	·03	—
·00	1716·23	1552·13	1141·00	668·33	300·17	96·65	19·79	2·04	·06	—
+ *·05*	1680·10	1675·52	1358·90	878·59	435·79	155·05	35·09	4·01	·14	—
+ *·10*	1575·70	1733·68	1553·58	1111·53	611·07	241·33	60·74	7·73	·29	—
+ *·15*	1414·38	1717·73	1703·39	1352·20	827·05	364·44	102·64	14·66	·63	—
+ *·20*	1213·06	1626·97	1788·20	1579·31	1079·03	533·39	169·29	27·37	1·33	·00
+ *·25*	991·59	1469·48	1792·97	1766·81	1354·17	755·33	272·20	50·31	2·82	·01
+ *·30*	769·89	1261·29	1711·25	1887·05	1629·83	1032·26	425·95	90·99	5·95	·02
+ *·35*	565·17	1024·10	1547·62	1915·72	1873·44	1356·52	646·90	161·66	12·53	·06
+ *·40*	389·90	781·85	1318·32	1837·75	2045·23	1705·56	949·63	281·48	26·36	·14
+ *·45*	250·78	556·80	1049·40	1652·96	2104·80	2037·72	1339·82	478·52	55·31	·38
+ *·50*	148·80	366·00	772·43	1379·62	2021·59	2291·99	1802·29	789·71	115·56	1·06
+ *·55*	80·30	218·92	518·34	1053·63	1787·57	2396·03	2285·43	1254·35	239·50	3·02
+ *·60*	38·65	116·85	311·00	722·19	1427·92	2286·37	2687·85	1892·96	489·12	8·96
+ *·65*	16·13	54·13	162·26	432·22	1002·93	1940·29	2862·39	2660·83	972·87	27·86
+ *·70*	5·61	20·90	70·68	216·87	595·00	1408·33	2661·07	3375·86	1845·86	90·85
+ *·75*	1·52	6·31	24·12	85·61	279·97	822·04	2036·49	3668·97	3216·90	309·47
+ *·80*	·29	1·34	5·79	23·90	93·95	347·48	1159·40	3112·44	4787·20	1079·30
+ *·85*	·03	·16	·80	3·86	18·41	87·22	404·32	1715·20	5209·97	3599·09
+ *·90*	·00	·00	·04	·23	1·34	8·29	55·40	399·51	2814·91	9080·68
+ *·95*	—	—	·00	·00	·01	·07	·71	9·56	195·96	6087·03
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0976	·1952	·2932	·3916	·4905	·5902	·6907	·7924	·8954
Mode	·0000	·1145	·2279	·3391	·4472	·5515	·6509	·7457	·8354	·9203
σ	·2236	·2216	·2157	·2058	·1917	·1732	·1500	·1218	·0880	·0478
$(1-\rho^2)/\sqrt{n-1}$	·2236	·2214	·2147	·2035	·1878	·1677	·1431	·1140	·0805	·0425
β_1	·0000	·0142	·0577	·1331	·2451	·4020	·6166	·9107	1·3227	1·9288
β_2	2·7273	2·7527	2·8306	2·9666	3·1711	3·4617	3·8683	4·4432	5·2858	6·6169

TABLE A. *Ordinates and Constants of Frequency Curves.*

$n = 22$. ρ variate (correlation in population sampled).

r variate (correlation in sample).	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	·00	—	—	—	—	—	—	—	—	—
− *·90*	·01	—	—	—	—	—	—	—	—	—
− *·85*	·02	·00	·00	—	—	—	—	—	—	—
− *·80*	·18	·03	·01	·00	—	—	—	—	—	—
− *·75*	1·03	·21	·04	·01	—	—	—	—	—	—
− *·70*	4·11	·92	·18	·03	·00	—	—	—	—	—
− *·65*	12·59	3·11	·67	·12	·02	·00	—	—	—	—
− *·60*	31·74	8·65	2·02	·40	·06	·01	—	—	—	—
− *·55*	68·85	20·67	5·28	1·12	·19	·02	·00	—	—	—
− *·50*	132·30	43·77	12·21	2·80	·50	·07	·01	—	—	—
− *·45*	229·92	83·90	25·58	6·36	1·24	·17	·02	—	—	—
− *·40*	366·87	147·70	49·31	13·34	2·80	·43	·04	·00	—	—
− *·35*	543·53	241·56	88·41	26·06	5·93	·97	·10	·01	—	—
− *·30*	754·00	370·08	148·69	47·84	11·83	2·09	·23	·01	—	—
− *·25*	985·69	534·57	236·09	83·10	22·38	4·29	·52	·03	—	—
− *·20*	1220·22	731·56	355·63	137·23	40·36	8·43	1·10	·07	—	—
− *·15*	1435·65	951·98	510·10	216·26	69·67	15·89	2·26	·16	·00	—
− *·10*	1609·59	1181·06	698·57	326·14	115·44	28·86	4·49	·34	·01	—
− *·05*	1722·72	1399·48	915·06	471·58	183·99	50·64	8·67	·72	·02	—
·00	1761·97	1585·51	1147·75	654·54	282·44	85·93	16·25	1·50	·04	—
+ *·05*	1722·72	1718·00	1379·01	872·46	417·89	141·20	29·67	3·04	·09	—
+ *·10*	1609·59	1779·88	1586·65	1116·63	595·93	224·73	52·80	6·06	·20	—
+ *·15*	1435·65	1761·23	1746·45	1370·99	818·50	346·33	91·59	11·87	·43	—
+ *·20*	1220·22	1661·60	1835·83	1612·18	1081·27	516·27	154·80	22·86	·96	—
+ *·25*	985·69	1490·67	1838·17	1811·21	1370·78	743·10	254·65	43·29	2·10	·00
+ *·30*	754·00	1267·07	1746·86	1937·29	1662·37	1029·97	406·96	80·55	4·60	·01
+ *·35*	543·53	1015·49	1568·03	1963·65	1920·03	1369·38	629·96	147·04	10·04	·03
+ *·40*	366·87	762·48	1321·06	1874·43	2099·64	1737·17	940·51	262·68	21·88	·09
+ *·45*	229·92	531·86	1035·89	1671·23	2156·76	2087·52	1346·06	457·35	47·54	·26
+ *·50*	132·30	340·82	747·64	1376·56	2059·06	2352·90	1831·17	771·34	102·73	·75
+ *·55*	68·85	197·63	489·26	1032·04	1800·84	2453·82	2339·55	1248·74	219·99	2·23
+ *·60*	31·74	101·58	284·38	690·00	1414·20	2323·01	2759·17	1914·05	463·46	6·97
+ *·65*	12·59	44·94	142·57	399·58	969·04	1942·00	2928·43	2719·83	948·62	22·83
+ *·70*	4·11	16·39	59·04	191·99	555·25	1375·57	2690·72	3465·29	1845·14	78·45
+ *·75*	1·03	4·60	18·88	71·56	248·90	773·40	2011·22	3745·20	3276·10	281·79
+ *·80*	·18	·89	4·16	18·48	78·00	308·91	1098·57	3111·21	4913·78	1034·74
+ *·85*	·02	·10	·51	2·67	13·82	71·03	356·95	1635·37	5281·97	3609·52
+ *·90*	·01	·00	·02	·13	·86	5·84	43·11	345·00	2698·71	9320·25
+ *·95*	·00	—	·00	·00	·00	·04	·42	6·53	157·01	5864·34
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0977	·1955	·2935	·3920	·4910	·5906	·6912	·7928	·8956
Mode	·0000	·1137	·2264	·3369	·4447	·5486	·6484	·7435	·8339	·9194
σ	·2182	·2162	·2105	·2007	·1869	·1688	·1461	·1185	·0855	·0464
$(1-\rho^2)/\sqrt{n-1}$	·2182	·2160	·2095	·1986	·1833	·1637	·1396	·1113	·0786	·0415
β_1	·0000	·0137	·0555	·1279	·2354	·3853	·5893	·8674	1·2532	1·8125
β_2	2·7391	2·7630	2·8390	2·9703	3·1672	3·4461	3·8328	4·3790	5·1687	6·3926

TABLE A—(*continued*).

$n = 23$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	·00	—	—	—	—	—	—	—	—	—
− *·90*	·00	—	—	—	—	—	—	—	—	—
− *·85*	·01	·00	—	—	—	—	—	—	—	—
− *·80*	·11	·02	—	—	—	—	—	—	—	—
− *·75*	·70	·13	·00	—	—	—	—	—	—	—
− *·70*	3·01	·63	·11	·00	·00	—	—	—	—	—
− *·65*	9·81	2·26	·45	·08	·01	—	—	—	—	—
− *·60*	26·04	6·66	1·45	·26	·04	·00	—	—	—	—
− *·55*	58·96	16·69	3·99	·78	·12	·01	—	—	—	—
− *·50*	117·47	36·83	9·65	2·06	·34	·04	·00	—	—	—
− *·45*	210·52	73·14	21·06	4·90	·88	·11	·01	—	—	—
− *·40*	344·75	132·79	42·04	10·68	2·08	·29	·02	—	—	—
− *·35*	522·03	223·04	77·76	21·61	4·58	·69	·06	·00	—	—
− *·30*	737·47	349·67	134·43	40·95	9·47	1·54	·15	·01	—	—
− *·25*	978·54	515·16	218·71	73·21	18·51	3·28	·36	·02	—	—
− *·20*	1225·82	716·90	336·58	124·07	34·41	6·67	·79	·04	—	—
− *·15*	1455·33	946·00	491·89	200·13	61·07	12·98	1·68	·10	—	—
− *·10*	1642·05	1186·97	684·57	308·15	103·79	24·29	3·46	·23	·00	—
− *·05*	1764·10	1418·82	909·02	453·86	169·29	43·81	6·89	·51	·01	—
·00	1806·56	1617·48	1153·02	640·19	265·41	76·30	13·33	1·10	·02	—
+ *·05*	1764·10	1759·25	1397·58	865·25	400·21	128·44	25·06	2·30	·06	—
+ *·10*	1642·05	1824·91	1618·30	1120·27	580·40	208·99	45·84	4·75	·13	—
+ *·15*	1455·33	1803·46	1788·25	1388·22	808·98	328·68	81·62	9·60	·30	—
+ *·20*	1225·82	1694·75	1882·27	1643·56	1082·09	499·05	141·37	19·07	·69	—
+ *·25*	978·54	1510·18	1882·05	1854·28	1385·76	730·12	237·92	37·20	1·56	·00
+ *·30*	737·47	1271·20	1780·86	1986·26	1693·34	1026·34	388·31	71·21	3·55	·01
+ *·35*	522·03	1005·63	1586·63	2010·13	1965·21	1380·56	612·67	133·58	8·04	·02
+ *·40*	344·75	742·61	1322·06	1909·35	2152·67	1767·06	930·25	244·82	18·14	·06
+ *·45*	210·52	507·37	1021·21	1687·50	2207·11	2135·75	1350·57	436·53	40·80	·17
+ *·50*	117·47	316·95	722·70	1371·71	2094·49	2412·26	1858·09	752·42	91·21	·52
+ *·55*	58·96	178·18	461·21	1009·57	1811·83	2509·73	2391·84	1241·53	201·81	1·65
+ *·60*	26·04	88·20	259·71	658·38	1398·78	2357·16	2828·70	1932·86	438·57	5·42
+ *·65*	9·81	37·26	125·10	368·92	935·07	1941·17	2992·09	2776·53	923·77	18·68
+ *·70*	3·01	12·84	49·25	169·74	517·49	1341·82	2717·16	3552·48	1842·02	67·66
+ *·75*	·70	3·36	14·76	59·73	220·99	726·69	1983·68	3818·06	3332·08	256·26
+ *·80*	·11	·59	2·99	14·27	64·66	274·26	1039·58	3105·96	5037·18	990·75
+ *·85*	·01	·06	·33	1·85	10·37	57·78	314·71	1557·24	5348·05	3615·31
+ *·90*	·00	·00	·01	·08	·55	4·11	33·50	297·55	2583·97	9553·82
+ *·95*	·00	·00	·00	·00	·00	·02	·25	4·46	125·64	5642·53
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0978	·1957	·2938	·3923	·4914	·5911	·6916	·7931	·8958
Mode	·0000	·1130	·2250	·3351	·4424	·5462	·6461	·7415	·8324	·9185
σ	·2132	·2113	·2056	·1960	·1825	·1647	·1425	·1155	·0832	·0450
$(1-\rho^2)/\sqrt{n-1}$	·2132	·2111	·2047	·1940	·1791	·1599	·1364	·1087	·0768	·0405
β_1	·0000	·0132	·0535	·1232	·2264	·3698	·5645	·8279	1·1905	1·7092
β_2	2·7500	2·7738	2·8467	2·9735	3·1633	3·4313	3·8024	4·3197	5·0623	6·1951

TABLE A. *Ordinates and Constants of Frequency Curves.*

$n = 24$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
– *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
– *·95*	—	—	—	—	—	—	—	—	—	—
– *·90*	·00	—	—	—	—	—	—	—	—	—
– *·85*	·01	—	—	—	—	—	—	—	—	—
– *·80*	·07	·00	·00	—	—	—	—	—	—	—
– *·75*	·48	·08	·01	—	—	—	—	—	—	—
– *·70*	2·20	·43	·07	·00	—	—	—	—	—	—
– *·65*	7·63	1·65	·30	·05	·00	—	—	—	—	—
– *·60*	21·33	5·12	1·04	·17	·02	·00	—	—	—	—
– *·55*	50·42	13·46	3·01	·55	·08	·01	—	—	—	—
– *·50*	104·18	30·95	7·63	1·52	·23	·03	·00	—	—	—
– *·45*	192·53	63·69	17·31	3·76	·63	·08	·01	—	—	—
– *·40*	323·58	119·24	35·80	8·54	1·54	·20	·02	—	—	—
– *·35*	500·79	205·69	68·31	17·90	3·53	·49	·04	—	—	—
– *·30*	720·45	329·99	121·39	35·02	7·57	1·13	·10	·00	—	—
– *·25*	970·29	495·86	202·37	64·42	15·29	2·51	·25	·01	—	—
– *·20*	1229·99	701·70	318·18	112·03	29·30	5·27	·57	·03	—	—
– *·15*	1473·52	938·94	473·77	184·97	53·46	10·58	1·25	·07	—	—
– *·10*	1673·17	1191·49	670·05	290·81	93·20	20·41	2·66	·16	·00	—
– *·05*	1804·33	1436·72	901·95	436·29	155·59	37·86	5·48	·36	·01	—
·00	1850·07	1648·13	1156·93	625·41	249·11	67·67	10·92	·80	·01	—
+ *·05*	1804·33	1799·35	1414·71	857·07	382·82	116·68	21·14	1·74	·04	—
+ *·10*	1673·17	1868·86	1648·62	1122·60	564·61	194·13	39·75	3·71	·09	—
+ *·15*	1473·52	1844·51	1828·88	1403·99	798·61	311·57	72·65	7·76	·21	—
+ *·20*	1229·99	1726·50	1927·58	1673·57	1081·63	481·83	128·95	15·89	·49	—
+ *·25*	970·29	1528·14	1924·68	1896·13	1399·26	716·52	222·03	31·93	1·16	—
+ *·30*	720·45	1273·84	1813·38	2034·06	1722·83	1021·51	370·08	62·89	2·74	·00
+ *·35*	500·79	994·68	1603·54	2055·28	2009·07	1390·19	595·15	121·20	6·42	·01
+ *·40*	323·58	722·40	1321·50	1942·61	2204·43	1795·33	919·02	227·90	15·02	·04
+ *·45*	192·53	483·43	1005·54	1701·90	2255·95	2182·51	1353·49	416·18	34·98	·12
+ *·50*	104·18	294·40	697·76	1365·25	2128·00	2470·20	1883·18	733·09	80·88	·37
+ *·55*	50·42	160·45	434·25	986·41	1820·73	2563·87	2442·41	1232·91	184·91	1·22
+ *·60*	21·33	76·48	236·89	627·46	1381·90	2388·99	2896·56	1949·56	414·53	4·21
+ *·65*	7·63	30·86	109·64	340·21	901·23	1938·06	3053·53	2831·08	898·52	15·27
+ *·70*	2·20	10·05	41·03	149·89	481·72	1307·36	2740·64	3637·57	1836·75	58·29
+ *·75*	·48	2·45	11·52	49·80	195·98	682·00	1954·22	3887·76	3385·03	232·77
+ *·80*	·07	·39	2·14	11·00	53·55	243·21	982·60	3097·07	5157·63	947·52
+ *·85*	·01	·03	·21	1·28	7·76	46·94	277·15	1481·10	5408·62	3616·87
+ *·90*	·00	·00	·01	·04	·35	2·89	26·01	256·32	2471·22	9781·81
+ *·95*	·00	·00	·00	·00	·00	·01	·15	3·04	100·42	5422·78
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0979	·1959	·2941	·3927	·4918	·5915	·6920	·7934	·8960
Mode	·0000	·1124	·2238	·3334	·4404	·5441	·6440	·7397	·8310	·9178
σ	·2085	·2067	·2011	·1916	·1783	·1609	·1391	·1127	·0811	·0438
$(1-\rho^2)/\sqrt{n-1}$	·2085	·2064	·2002	·1897	·1752	·1564	·1334	·1063	·0751	·0396
β_1	·0000	·0127	·0516	·1187	·2180	·3557	·5419	·7918	1·1335	1·6167
β_2	2·7600	2·7826	2·8537	2·9764	3·1596	3·4174	3·7774	4·2653	4·9654	6·0161

TABLE A—(*continued*).

$n = 25$. ρ variate (correlation in population sampled).

r variate (correlation in sample).	*0*	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*	*·9*
− *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
− *·95*	—	—	—	—	—	—	—	—	—	—
− *·90*	—	—	—	—	—	—	—	—	—	—
− *·85*	·00	—	—	—	—	—	—	—	—	—
− *·80*	·04	·00	—	—	—	—	—	—	—	—
− *·75*	·32	·05	·00	—	—	—	—	—	—	—
− *·70*	1·61	·29	·05	·00	—	—	—	—	—	—
− *·65*	5·93	1·20	·21	·03	·00	—	—	—	—	—
− *·60*	17·46	3·93	·75	·12	·01	—	—	—	—	—
− *·55*	43·08	10·85	2·27	·38	·05	·00	—	—	—	—
− *·50*	92·30	25·99	6·01	1·11	·16	·01	—	—	—	—
− *·45*	175·88	55·40	14·22	2·89	·44	·05	·00	—	—	—
− *·40*	303·38	106·96	30·45	6·82	1·14	·13	·01	—	—	—
− *·35*	479·90	189·49	59·94	14·80	2·72	·34	·03	—	—	—
− *·30*	703·06	311·08	109·50	29·91	6·05	·83	·07	·00	—	—
− *·25*	961·07	476·77	187·04	56·62	12·62	1·80	·17	·01	—	—
− *·20*	1232·83	686·08	300·46	101·06	24·92	4·15	·41	·02	—	—
− *·15*	1490·32	930·93	455·82	170·78	46·75	8·62	·93	·04	—	—
− *·10*	1703·04	1194·73	655·13	274·14	83·60	17·14	2·04	·11	—	—
− *·05*	1843·49	1453·28	893·96	418·94	142·84	32·68	4·35	·25	·00	—
·00	1892·58	1677·56	1159·60	610·31	233·56	59·95	8·94	·59	·01	—
+ *·05*	1843·49	1838·37	1430·51	848·05	365·78	105·89	17·81	1·32	·02	—
+ *·10*	1703·04	1911·80	1677·69	1123·71	548·65	180·12	34·44	2·90	·06	—
+ *·15*	1490·32	1884·46	1868·40	1418·41	787·53	295·02	64·59	6·26	·14	—
+ *·20*	1232·83	1756·95	1971·86	1702·29	1080·00	464·70	117·49	13·22	·35	—
+ *·25*	961·07	1544·63	1966·16	1936·84	1411·36	702·41	206·98	27·37	·86	—
+ *·30*	703·06	1275·10	1844·50	2080·76	1750·96	1015·61	352·32	55·47	2·11	·00
+ *·35*	479·90	982·79	1618·89	2099·17	2051·69	1398·37	577·51	109·85	5·13	·01
+ *·40*	303·38	701·99	1319·51	1974·33	2255·00	1822·10	906·95	211·92	12·43	·02
+ *·45*	175·88	460·12	989·05	1714·58	2303·40	2227·89	1354·96	396·34	29·95	·08
+ *·50*	92·30	273·17	672·95	1357·37	2159·72	2526·88	1906·56	713·49	71·65	·26
+ *·55*	43·08	144·33	408·43	962·75	1827·71	2616·37	2491·37	1223·04	169·24	·90
+ *·60*	17·46	66·25	215·85	597·36	1363·75	2418·65	2962·87	1964·30	391·39	3·26
+ *·65*	5·93	25·53	95·99	313·40	867·68	1932·87	3112·90	2883·60	873·02	12·46
+ *·70*	1·61	7·85	34·15	132·22	447·95	1272·41	2761·35	3720·72	1829·54	50·16
+ *·75*	·32	1·78	8·99	41·47	173·61	639·37	1923·13	3954·50	3435·16	211·20
+ *·80*	·04	·26	1·53	8·48	44·30	215·45	927·75	3084·92	5275·33	905·21
+ *·85*	·00	·02	·13	·88	5·81	38·09	243·81	1407·18	5464·05	3614·59
+ *·90*	—	·00	·00	·00	·22	2·03	20·16	220·57	2360·87	10004·61
+ *·95*	—	—	—	—	·00	·01	·09	2·07	80·18	5206·06
+ *1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	·00
Mean	·0000	·0980	·1960	·2943	·3930	·4921	·5918	·6923	·7937	·8962
Mode	·0000	·1118	·2227	·3318	·4385	·5421	·6420	·7380	·8297	·9170
σ	·2041	·2023	·1968	·1875	·1744	·1573	·1359	·1100	·0791	·0427
$(1-\rho^2)/\sqrt{n-1}$	·2041	·2021	·1960	·1858	·1715	·1531	·1306	·1041	·0735	·0388
β_1	·0000	·0123	·0499	·1146	·2102	·3423	·5203	·7586	1·0816	1·5334
β_2	2·7692	2·7916	2·8601	2·9788	3·1559	3·4042	3·7453	4·2149	4·8769	5·8584

TABLE A. *Ordinates and Constants of Frequency Curves.*

$n = 50$. ρ variate (correlation in population sampled).

r variate (correlation in sample).

	0	*·1*	*·2*	*·3*	*·4*	*·5*	*·6*	*·7*	*·8*		*·9*
−*1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00	*·67*	·2
− *·95*	—	—	—	—	—	—	—	—	—	*·68*	·3
− *·90*	—	—	—	—	—	—	—	—	—	*·69*	·6
− *·85*	—	—	—	—	—	—	—	—	—	*·70*	1·0
− *·80*	—	—	—	—	—	—	—	—	—	*·71*	1·7
− *·75*	—	—	—	—	—	—	—	—	—	*·72*	2·9
− *·70*	·00	—	—	—	—	—	—	—	—	*·73*	5·1
− *·65*	·01	—	—	—	—	—	—	—	—	*·74*	8·8
− *·60*	·10	·00	—	—	—	—	—	—	—	*·75*	15·3
− *·55*	·69	·04	·00	—	—	—	—	—	—	*·76*	26·7
− *·50*	3·68	·27	·01	—	—	—	—	—	—	*·77*	46·4
− *·45*	15·10	1·40	·08	·00	—	—	—	—	—	*·78*	80·4
− *·40*	49·85	5·82	·44	·02	—	—	—	—	—	*·79*	138·9
− *·35*	136·13	20·06	1·88	·11	·00	—	—	—	—	*·80*	238·4
− *·30*	314·21	58·57	6·85	·48	·02	—	—	—	—	*·81*	406·0
− *·25*	623·17	147·07	21·50	1·85	·09	—	—	—	—	*·82*	683·7
− *·20*	1075·24	321·70	59·02	6·32	·36	·00	—	—	—	*·83*	1134·8
− *·15*	1629·13	618·58	142·88	19·11	1·35	·04	—	—	—	*·84*	1848·2
− *·10*	2182·12	1052·79	307·18	51·61	4·55	·18	·00	—	—	*·85*	2936·7
− *·05*	2595·77	1593·19	589·27	125·03	13·87	·68	·01	—	—	*·86*	4519·5
·00	2749·60	2149·47	1011·38	272·76	38·38	2·39	·05	—	—	*·87*	6673·8
+ *·05*	2595·77	2587·70	1554·59	535·97	96·53	7·70	·20	·00	—	*·88*	9340·1
+ *·10*	2182·12	2777·44	2138·36	948·41	220·60	22·82	·78	·01	—	*·89*	12187·5
+ *·15*	1629·13	2650·80	2625·45	1507·84	457·26	62·11	2·82	·02	—	*·90*	14502·0
+ *·20*	1075·24	2239·38	2864·46	2144·80	856·50	154·86	9·46	·11	—	*·91*	15261·3
+ *·25*	623·17	1663·32	2758·76	2712·27	1441·46	351·85	29·47	·48	—	*·92*	13599·9
+ *·30*	314·21	1076·19	2323·98	3022·24	2161·66	723·72	84·89	1·99	·00	*·93*	9630·6
+ *·35*	136·13	599·07	1691·43	2932·02	2856·13	1333·93	224·22	7·75	·02	*·94*	4918·7
+ *·40*	49·85	282·26	1046·45	2437·43	3274·68	2172·97	536·92	28·37	·09	*·95*	1550·4
+ *·45*	15·10	110·16	538·66	1700·04	3192·62	3070·40	1147·17	96·36	·51	*·96*	230·3
+ *·50*	3·68	34·61	224·22	967·26	2575·61	3668·50	2138·79	298·71	2·85	*·97*	9·6
+ *·55*	·69	8·42	72·63	432·21	1656·86	3578·51	3372·39	824·60	15·26	*·98*	·3
+ *·60*	·10	1·51	17·37	143·93	807·35	2713·26	4300·25	1954·75	76·77	*·99*	·0
+ *·65*	·01	·18	2·84	33·16	276·92	1489·70	4152·55	3764·57	350·56		
+ *·70*	·00	·01	·29	4·73	59·97	532·70	2747·94	5398·21	1367·63		
+ *·75*	—	·00	·01	·35	6·91	104·96	1061·91	4990·96	4092·05		
+ *·80*	—	—	·00	·01	·32	8·58	181·98	2306·06	7650·44		
+ *·85*	—	—	—	·00	·00	·17	8·16	322·77	5817·30		
+ *·90*	—	—	—	—	—	·00	·03	4·26	621·81		
+ *·95*	—	—	—	—	—	—	·00	·00	·24		
+*1·00*	·00	·00	·00	·00	·00	·00	·00	·00	·00		

r variate (correlation in sample).

For the constants of the curves for $n=50$: see p. 372 above.

TABLE A—(*continued*).

$n = 100$. ρ variate (correlation in population sampled).

	0	·1	·2	·3	·4	·5	·6	·7	·8	r variate (correlation in sample).	·9	·9 (normal curve)*
−1·00	·00	·00	·00	·00	·00	·00	·00	·00	·00	·75	·1	·0
− ·95	—	—	—	—	—	—	—	—	—	·76	·2	—
− ·90	—	—	—	—	—	—	—	—	—	·77	·5	—
− ·85	—	—	—	—	—	—	—	—	—	·78	1·4	—
− ·80	—	—	—	—	—	—	—	—	—	·79	4·0	·0
− ·75	—	—	—	—	—	—	—	—	—	·80	11·3	·2
− ·70	—	—	—	—	—	—	—	—	—	·81	31·3	·3
− ·65	—	—	—	—	—	—	—	—	—	·82	84·7	3·2
− ·60	—	—	—	—	—	—	—	—	—	·83	221·8	25·3
− ·55	—	—	—	—	—	—	—	—	—	·84	556·2	150·2
− ·50	·00	—	—	—	—	—	—	—	—	·85	1320·0	678·6
− ·45	·08	·00	—	—	—	—	—	—	—	·86	2919·0	2330·1
− ·40	·91	·01	—	—	—	—	—	—	—	·87	5895·9	6082·6
− ·35	7·43	·15	·00	—	—	—	—	—	—	·88	10597·3	11944·8
− ·30	42·60	1·41	·02	—	—	—	—	—	—	·89	16373·8	18211·3
− ·25	177·84	9·50	·18	·00	—	—	—	—	—	·90	20754·4	20887·0
− ·20	555·18	48·00	1·55	·02	—	—	—	—	—	·91	20233·5	18211·3
− ·15	1321·36	185·37	9·53	·16	·00	—	—	—	—	·92	13848·4	11944·8
− ·10	2431·67	554·86	45·84	1·24	·01	—	—	—	—	·93	5823·1	6082·6
− ·05	3493·29	1299·62	173·78	7·57	·09	—	—	—	—	·94	1227·8	2330·1
·00	3939·27	2395·29	522·21	36·98	·70	·00	—	—	—	·95	93·7	678·6
+ ·05	3493·29	3480·22	1246·23	145·32	4·56	·03	—	—	—	·96	1·5	150·2
+ ·10	2431·67	3979·11	2358·16	458·62	24·21	·25	—	—	—	·97	—	25·3
+ ·15	1321·36	3560·42	3519·22	1156·81	104·66	1·87	·00	—	—	·98	—	3·2
+ ·20	555·18	2469·56	4103·61	2311·65	365·76	11·67	·04	—	—	·99	—	·3
+ ·25	177·84	1309·30	3687·36	3611·32	1021·01	59·96	·41	—	—	1·00	—	·2
+ ·30	42·60	520·41	2504·77	4329·36	2237·43	249·60	3·35	·00	—			
+ ·35	7·43	151·11	1253·64	3884·32	3759·08	824·50	22·96	·03	—			
+ ·40	·91	30·98	446·88	2522·86	4690·55	2099·42	127·86	·35	—			
+ ·45	·08	4·29	108·49	1135·08	4166·27	3962·12	558·78	3·87	—			
+ ·50	·00	·38	16·90	333·60	2488·44	5254·02	1829·25	35·59	·00			
+ ·55	—	·02	1·56	59·16	925·05	4549·13	4200·10	254·84	·02			
+ ·60	—	·00	·08	5·68	192·25	2320·57	6157·92	1316·29	2·01			
+ ·65	—	—	·00	·25	19·17	601·45	5023·96	4363·45	38·45			
+ ·70	—	—	—	·00	·73	63·47	1850·41	7728·87	519·97			
+ ·75	—	—	—	—	·01	1·92	220·19	5408·23	3951·47			
+ ·80	—	—	—	—	·00	·01	4·76	876·76	10951·32			
+ ·85	—	—	—	—	—	·00	·01	11·56	4488·65			
+ ·90	—	—	—	—	—	—	·00	·00	29·37			
+ ·95	—	—	—	—	—	—	—	—	·00			
+1·00	·00	·00	·00	·00	·00	·00	·00	·00	·00			

* These ordinates indicate how poor is the approximation of a normal curve to the frequency of r, where n is 100, but ρ is large.

For the constants of the curves for $n = 100$: see p. 372 above.

TABLE A. *Ordinates and Constants of Frequency Curves.*

$n = 400$.

ρ variate (correlation in population sampled).

r variate (correlation in sample)	*0*	·*1*	·*2*	·*3*	·*4*	·*5*	·*6*
− ·*40*	·00	·00	·00	·00	·00	·00	·00
− ·*35*	—	—	—	—	—	—	—
− ·*30*	—	—	—	—	—	—	—
− ·*25*	·00	—	—	—	—	—	—
− ·*20*	2·46	·00	—	—	—	—	—
− ·*15*	87·84	·03	—	—	—	—	—
− ·*10*	1087·30	6·72	·00	—	—	—	—
− ·*05*	4845·66	89·41	·03	—	—	—	—
·*00*	7953·88	1071·03	2·31	·00	—	—	—
+ ·*05*	4845·66	4809·08	77·24	·01	—	—	—
+ ·*10*	1087·30	8034·24	990·50	1·37	—	—	—
+ ·*15*	87·84	4878·55	4767·51	55·10	·00	—	—
+ ·*20*	2·46	1037·29	8285·39	846·48	·52	—	—
+ ·*25*	·00	72·74	4910·62	4701·88	30·13	·00	—
+ ·*30*	—	1·55	917·19	8740·76	642·95	·01	—
+ ·*35*	—	·01	48·55	4906·74	4567·01	10·75	·00
+ ·*40*	—	·00	·63	724·97	9469·37	399·28	·01
+ ·*45*	—	—	·00	23·50	4810·20	4278·49	1·75
+ ·*50*	—	—	—	·13	473·26	10606·00	167·53
+ ·*55*	—	—	—	·00	6·55	4491·43	3668·41
+ ·*60*	—	—	—	—	·08	212·53	12429·40
+ ·*65*	—	—	—	—	·00	·16	3688·70
+ ·*70*	—	—	—	—	—	·00	40·04
+ ·*75*	—	—	—	—	—	—	·00
+ ·*80*	—	—	—	—	—	—	—
+ ·*85*	—	—	—	—	—	—	—
+ ·*90*	—	—	—	—	—	—	—
+ ·*95*	—	—	—	—	—	—	—
+*1·00*	·00	·00	·00	·00	·00	·00	·00

r variate (correlation in sample)	·*7*	·*7* normal*	·*8*	·*8* normal*	·*9*	·*9* normal*
·*54*	·0	·0	·0	·0	·0	·0
·*55*	·1	—	—	—	—	—
·*56*	·2	—	—	—	—	—
·*57*	·8	·0	—	—	—	—
·*58*	2·8	·2	—	—	—	—
·*59*	9·3	1·4	—	—	—	—
·*60*	28·7	7·3	—	—	—	—
·*61*	82·9	31·3	—	—	—	—
·*62*	221·2	115·3	—	—	—	—
·*63*	542·5	364·4	—	—	—	—
·*64*	1212·0	987·7	—	—	—	—
·*65*	2478·7	2296·4	—	—	—	—
·*66*	4553·8	4579·8	—	—	—	—
·*67*	7487·9	7834·8	—	—	—	—
·*68*	10922·9	11497·0	·0	—	—	—
·*69*	13999·7	14471·6	·1	—	—	—
·*70*	15598·3	15625·2	·2	·0	—	—
·*71*	14931·6	14471·6	2·2	·1	—	—
·*72*	12106·9	11497·0	11·2	1·2	—	—
·*73*	8212·0	7834·8	52·2	11·7	—	—
·*74*	4572·5	4579·8	213·0	86·8	—	—
·*75*	2052·1	2296·4	751·2	471·8	—	—
·*76*	726·6	987·7	2248·2	1885·6	—	—
·*77*	198·1	364·4	5595·6	5538·9	—	—
·*78*	40·5	115·3	11311·7	11958·6	—	—
·*79*	6·0	31·3	18071·0	18977·4	—	—
·*80*	·0	7·3	22139·3	22135·7	—	—
·*81*	—	1·4	19920·7	18977·4	—	—
·*82*	—	·2	12665·6	11958·6	—	—
·*83*	—	·0	5383·5	5538·9	·0	—
·*84*	—	—	1435·4	1885·6	·1	—
·*85*	—	—	222·2	471·8	2·6	·0
·*86*	—	—	18·2	86·8	49·7	·6
·*87*	—	—	·7	11·7	658·1	290·2
·*88*	—	—	·0	1·2	5308·0	4598·7
·*89*	—	—	—	·1	22606·9	24134·5
·*90*	—	—	—	·0	41873·1	41941·4
·*91*	—	—	—	—	25806·1	24134·5
·*92*	—	—	—	—	3626·0	4598·7
·*93*	—	—	—	—	66·8	290·2
·*94*	—	—	—	—	·1	·6
·*95*	—	—	—	—	·0	·0
·*96*	—	—	—	—	—	—
·*97*	—	—	—	—	—	—
·*98*	—	—	—	—	—	—
·*99*	—	—	—	—	—	—
1·00	·0	·0	·0	·0	·0	·0

TABLE B*. *To assist the calculation of the Ordinates of the Correlation Frequency Curves from Expansion Formulae.*

$\log \frac{n-2}{\sqrt{n-1}}$.

$\rho = 0$.

$\log (1-\rho^2)^{\frac{3}{2}} = 0$.

n	$\log \frac{n-2}{\sqrt{n-1}}$	n	$\log \frac{n-2}{\sqrt{n-1}}$	n	$\log \frac{n-2}{\sqrt{n-1}}$	r	$\log \chi_1$	$\log \chi_2$	ϕ_1	ϕ_2	ϕ_3	ϕ_4
3	$\bar{1}$·8494850	42	·7956681	81	·9460821	$-$·95	·5054977	$\bar{2}$·8825969	$\phi_1 = $ ·25	$\phi_2 = $ ·03125	$\phi_3 = -$ ·0390625	$\phi_4 = -$ ·0102539
4	·0624694	43	·8011592	82	·9488475	$-$·90	·3606232	$\bar{1}$·3172203				
5	·1760913	44	·8065151	83	·9515781	$-$·85	·2783685	$\bar{1}$·5639844				
6	·2525750	45	·8117421	84	·9542748	$-$·80	·2218487	$\bar{1}$·7335437				
7	·3098944	46	·8168464	85	·9569384	$-$·75	·1795110	$\bar{1}$·8605570				
8	·3556022	47	·8218336	86	·9595698	$-$·70	·1462149	$\bar{1}$·9604452				
9	·3935530	48	·8267089	87	·9621697	$-$·65	·1192240	·0414179				
10	·4259687	49	·8314772	88	·9647388	$-$·60	·0969100	·1083599				
11	·4542425	50	·8361432	89	·9672779	$-$·55	·0782279	·1644063				
12	·4793037	51	·8407111	90	·9697877	$-$·50	·0624694	·2116818				
13	·5018021	52	·8451849	91	·9722688	$-$·45	·0491347	·2516860				
14	·5222096	53	·8495685	92	·9747218	$-$·40	·0378604	·2855089				
15	·5408793	54	·8538654	93	·9771475	$-$·35	·0283764	·3139606				
16	·5580824	55	·8580790	94	·9795464	$-$·30	·0204793	·3376520				
17	·5740313	56	·8622124	95	·9819190	$-$·25	·0140144	·3570469				
18	·5888955	57	·8662687	96	·9842661	$-$·20	·0088644	·3724968				
19	·6028127	58	·8702506	97	·9865880	$-$·15	·0049416	·3842651				
20	·6158957	59	·8741609	98	·9888854	$-$·10	·0021824	·3925427				
21	·6282386	60	·8780020	99	·9911587	$-$·05	·0005435	·3974593				
22	·6399203	61	·8817764	100	·9934085	·00	·0	·3990899				
23	·6510080	62	·8854863	400	1·2993966	$+$·05	·0005435	·3974593				
24	·6615588	63	·8891340			$+$·10	·0021824	·3925427				
25	·6716222	64	·8927214			$+$·15	·0049416	·3842651				
26	·6812412	65	·8962506			$+$·20	·0088644	·3724968				
27	·6904533	66	·8997233			$+$·25	·0140144	·3570469				
28	·6992915	67	·9031414			$+$·30	·0204793	·3376520				
29	·7077847	68	·9065065			$+$·35	·0283764	·3139606				
30	·7159590	69	·9098203			$+$·40	·0378604	·2855089				
31	·7238374	70	·9130844			$+$·45	·0491347	·2516860				
32	·7314404	71	·9163001			$+$·50	·0624694	·2116818				
33	·7387867	72	·9194689			$+$·55	·0782279	·1644063				
34	·7458930	73	·9225921			$+$·60	·0969100	·1083599				
35	·7527745	74	·9256711			$+$·65	·1192240	·0414179				
36	·7594449	75	·9287070			$+$·70	·1462149	$\bar{1}$·9604452				
37	·7659168	76	·9317011			$+$·75	·1795110	$\bar{1}$·8605570				
38	·7722016	77	·9346545			$+$·80	·2218487	$\bar{1}$·7335437				
39	·7783099	78	·9375682			$+$·85	·2783685	$\bar{1}$·5639844				
40	·7842513	79	·9404434			$+$·90	·3606232	$\bar{1}$·3172203				
41	·7900346	80	·9432811			$+$·95	·5054977	$\bar{2}$·8825969				

* If the ordinate at r be y, then (see p. 348)

$$y = Y\left(1 + \frac{\phi_1}{n-1} + \frac{\phi_2}{(n-1)^2} + \frac{\phi_3}{(n-1)^3} + \frac{\phi_4}{(n-1)^4}\right),$$

where

$$\log Y = \log \frac{n-2}{\sqrt{n-1}} + \log (1-\rho^2)^{\frac{3}{2}} - (n-1)\log \chi_1 - \log \chi_2.$$

All these quantities are given for $r = -$·95 to $+$·95 and $\rho = 0$·0 to 0·9 in this Table B.

TABLE B. *To assist the calculation of the Ordinates of the Correlation Frequency Curves from Expansion Formulae.*

$\rho = \cdot 1.$ $\log (1-\rho^2)^{\frac{3}{2}} = \bar{1}\cdot 9934528.$

$\rho = \cdot 2.$ $\log (1-\rho^2)^{\frac{3}{2}} = \bar{1}\cdot 9734068.$

$\rho = \cdot 1.$							$\rho = \cdot 2.$			
r	$\log \chi_1$	$\log \chi_2$	ϕ_1	ϕ_2	ϕ_3	ϕ_4	r	$\log \chi_1$	$\log \chi_2$	ϕ_1
– ·95	·5470942	$\bar{1}$·8563426	·238125	·0229783	– ·0364766	– ·0013075	– ·95	·5899090	$\bar{1}$·8182302	·22625
– ·90	·4002321	$\bar{1}$·2919599	·23875	·0233820	– ·0366462	– ·0017344	– ·90	·4413696	$\bar{1}$·2546862	·2275
– ·85	·3159806	$\bar{1}$·5397223	·239375	·0237893	– ·0368123	– ·0021666	– ·85	·3554188	$\bar{1}$·5032983	·22875
– ·80	·2574549	$\bar{1}$·7102846	·24	·0242	– ·0369750	– ·0026040	– ·80	·2951711	$\bar{1}$·6747215	·23
– ·75	·2131018	$\bar{1}$·8383056	·240625	·0246143	– ·0371342	– ·0030466	– ·75	·2490732	$\bar{1}$·8036150	·23125
– ·70	·1777811	$\bar{1}$·9392061	·24125	·0250320	– ·0372898	– ·0034943	– ·70	·2119841	$\bar{1}$·9053996	·2325
– ·65	·1487560	·0211959	·241875	·0254533	– ·0374417	– ·0039470	– ·65	·1811668	$\bar{1}$·9882856	·23375
– ·60	·1243983	·0891598	·2425	·0258781	– ·0375900	– ·0044047	– ·60	·1549924	·0571577	·235
– ·55	·1036628	·1462328	·243125	·0263064	– ·0377346	– ·0048672	– ·55	·1324153	·1151516	·23625
– ·50	·0858411	·1945400	·24375	·0267383	– ·0378754	– ·0053345	– ·50	·1127264	·1643923	·2375
– ·45	·0704333	·2355806	·244375	·0271736	– ·0380123	– ·0058066	– ·45	·0954255	·2063796	·23875
– ·40	·0570761	·2704450	·245	·0276125	– ·0381453	– ·0062833	– ·40	·0801485	·2422038	·24
– ·35	·0454992	·2999432	·245625	·0280549	– ·0382744	– ·0067646	– ·35	·0666246	·2726756	·24125
– ·30	·0354989	·3246862	·24625	·0285008	– ·0383994	– ·0072503	– ·30	·0546496	·2984059	·2425
– ·25	·0269206	·3451377	·246875	·0289502	– ·0385204	– ·0077405	– ·25	·0440680	·3198590	·24375
– ·20	·0196470	·3616495	·2475	·0294031	– ·0386373	– ·0082349	– ·20	·0347621	·3373870	·245
– ·15	·0135901	·3744849	·248125	·0298596	– ·0387500	– ·0087336	– ·15	·0266432	·3512533	·24625
– ·10	·0086862	·3838348	·24875	·0303195	– ·0388585	– ·0092363	– ·10	·0196470	·3616495	·2475
– ·05	·0048920	·3898291	·249375	·0307830	– ·0389627	– ·0097431	– ·05	·0137293	·3687055	·24875
·00	·0021824	·3925427	·25	·03125	– ·0390625	– ·0102539	·00	·0088644	·3724968	·25
+ ·05	·0005490	·3920005	·250625	·0317205	– ·0391579	– ·0107685	+ ·05	·0050431	·3730485	·25125
+ ·10	·0	·3881779	·25125	·0321945	– ·0392490	– ·0112869	+ ·10	·0022729	·3703365	·2525
+ ·15	·0005603	·3809998	·251875	·0326721	– ·0393354	– ·0118089	+ ·15	·0005777	·3642861	·25375
+ ·20	·0022729	·3703365	·2525	·0331531	– ·0394174	– ·0123344	+ ·20	·0	·3547680	·255
+ ·25	·0052014	·3559973	·253125	·0336377	– ·0394947	– ·0128634	+ ·25	·0006024	·3415919	·25625
+ ·30	·0094334	·3377189	·25375	·0341258	– ·0395674	– ·0133957	+ ·30	·0024715	·3244949	·2575
+ ·35	·0150862	·3151498	·254375	·0346174	– ·0396353	– ·0139313	+ ·35	·0057238	·3031260	·25875
+ ·40	·0223140	·2878260	·255	·0351125	– ·0396984	– ·0144700	+ ·40	·0105126	·2770218	·26
+ ·45	·0313204	·2551371	·255625	·0356111	– ·0397568	– ·0150118	+ ·45	·0170404	·2455721	·26125
+ ·50	·0423754	·2162728	·25625	·0361133	– ·0398102	– ·0155564	+ ·50	·0255763	·2079674	·2625
+ ·55	·0558421	·1701431	·256875	·0366189	– ·0398587	– ·0161038	+ ·55	·0364823	·1631181	·26375
+ ·60	·0722203	·1152488	·2575	·0371281	– ·0399021	– ·0166540	+ ·60	·0502571	·1095254	·265
+ ·65	·0922180	·0494649	·258125	·0376408	– ·0399406	– ·0172067	+ ·65	·0676076	·0450651	·26625
+ ·70	·1168803	$\bar{1}$·9696565	·25875	·0381570	– ·0399739	– ·0177618	+ ·70	·0895777	$\bar{1}$·9666028	·2675
+ ·75	·1478351	$\bar{1}$·8709390	·259375	·0386768	– ·0400021	– ·0183193	+ ·75	·1177943	$\bar{1}$·8692544	·26875
+ ·80	·1878190	$\bar{1}$·7451026	·26	·0392	– ·0400250	– ·0188790	+ ·80	·1549924	$\bar{1}$·7448109	·27
+ ·85	·2419720	$\bar{1}$·5767267	·260625	·0397268	– ·0400427	– ·0194408	+ ·85	·2063110	$\bar{1}$·5778522	·27125
+ ·90	·3218470	$\bar{1}$·3311524	·26125	·0402570	– ·0400550	– ·0200045	+ ·90	·2833014	$\bar{1}$·3337203	·2725
+ ·95	·4643287	$\bar{2}$·8977254	·261875	·0407908	– ·0400620	– ·0205702	+ ·95	·4228471	$\bar{2}$·9017612	·27375

TABLE B—(*continued*).

$\rho = \cdot 2$. $\log (1-\rho^2)^{\frac{3}{2}} = \bar{1}\cdot 9734068$.				$\rho = \cdot 3$. $\log (1-\rho^2)^{\frac{3}{2}} = \bar{1}\cdot 9385621$.					
ϕ_2	ϕ_3	ϕ_4	r	$\log \chi_1$	$\log \chi_2$	ϕ_1	ϕ_2	ϕ_3	ϕ_4
·0159758	− ·0326811	+ ·0057193	*− ·95*	·6348801	$\bar{2}$·7667074	·214375	·0102424	− ·0280527	+ ·0104960
·0166531	− ·0331264	+ ·0050816	*− ·90*	·4849062	$\bar{1}$·2038806	·21625	·0110633	− ·0288235	+ ·0098985
·0173445	− ·0335620	+ ·0044192	*− ·85*	·3974915	$\bar{1}$·4532246	·218125	·0119158	− ·0295824	+ ·0092418
·01805	− ·0339875	+ ·0037321	*− ·80*	·3357497	$\bar{1}$·6253949	·22	·0128	− ·0303250	+ ·0085260
·0187695	− ·0344025	+ ·0030208	*− ·75*	·2881264	$\bar{1}$·7550511	·221875	·0137158	− ·0310528	+ ·0077516
·0195031	− ·0348065	+ ·0022856	*− ·70*	·2494796	$\bar{1}$·8576146	·22375	·0146633	− ·0317633	+ ·0069189
·0202508	− ·0351990	+ ·0015269	*− ·65*	·2170712	$\bar{1}$·9412961	·225625	·0156424	− ·0324549	+ ·0060287
·0210125	− ·0355797	+ ·0007451	*− ·60*	·1892713	·0109810	·2275	·0166531	− ·0331264	+ ·0050816
·0217883	− ·0359481	− ·0000593	*− ·55*	·1650331	·0698054	·229375	·0176955	− ·0337760	+ ·0040787
·0225781	− ·0363037	− ·0008860	*− ·50*	·1436465	·1198950	·23125	·0187695	− ·0344025	+ ·0030208
·0233820	− ·0366462	− ·0017344	*− ·45*	·1246098	·1627501	·233125	·0198752	− ·0350042	+ ·0019092
·0242	− ·0369750	− ·0026040	*− ·40*	·1075577	·1994619	·235	·0210125	− ·0355797	+ ·0007451
·0250320	− ·0372898	− ·0034943	*− ·35*	·0922180	·2308416	·236875	·0221814	− ·0361275	− ·0004699
·0258781	− ·0375900	− ·0044047	*− ·30*	·0783851	·2575009	·23875	·0233820	− ·0366462	− ·0017344
·0267383	− ·0378754	− ·0053345	*− ·25*	·0659021	·2799047	·240625	·0246143	− ·0371342	− ·0030466
·0276125	− ·0381453	− ·0062833	*− ·20*	·0546496	·2984059	·2425	·0258781	− ·0375900	− ·0044047
·0285008	− ·0383994	− ·0072503	*− ·15*	·0445372	·3132690	·244375	·0271736	− ·0380123	− ·0058066
·0294031	− ·0386373	− ·0082349	*− ·10*	·0354989	·3246862	·24625	·0285008	− ·0383994	− ·0072503
·0303195	− ·0388585	− ·0092363	*− ·05*	·0274889	·3327884	·248125	·0298596	− ·0387500	− ·0087336
·03125	− ·0390625	− ·0102539	*·00*	·0204793	·3376520	·25	·03125	− ·0390625	− ·0102539
·0321945	− ·0392490	− ·0112869	*+ ·05*	·0144591	·3393033	·251875	·0326721	− ·0393354	− ·0118089
·0331531	− ·0394170	− ·0123344	*+ ·10*	·0094334	·3377189	·25375	·0341258	− ·0395674	− ·0133957
·0341258	− ·0395674	− ·0133957	*+ ·15*	·0054243	·3328255	·255625	·0356111	− ·0397568	− ·0150118
·0351125	− ·0396984	− ·0144700	*+ ·20*	·0024715	·3244949	·2575	·0371281	− ·0399021	− ·0166540
·0361133	− ·0398102	− ·0155564	*+ ·25*	·0006354	·3125381	·259375	·0386768	− ·0400021	− ·0183193
·0371281	− ·0399021	− ·0166540	*+ ·30*	·0	·2966934	·26125	·0402570	− ·0400550	− ·0200045
·0381570	− ·0399739	− ·0177618	*+ ·35*	·0006788	·2766112	·263125	·0418689	− ·0400595	− ·0217064
·0392	− ·0400250	− ·0188790	*+ ·40*	·0028223	·2518296	·265	·0435125	− ·0400141	− ·0234213
·0402570	− ·0400550	− ·0200045	*+ ·45*	·0066301	·2217400	·266875	·0451877	− ·0399172	− ·0251457
·0413281	− ·0400635	− ·0211375	*+ ·50*	·0123676	·1855345	·26875	·0468945	− ·0397675	− ·0268757
·0424133	− ·0400500	− ·0222767	*+ ·55*	·0203937	·1421251	·270625	·0486330	− ·0395633	− ·0286077
·0435125	− ·0400141	− ·0234213	*+ ·60*	·0312032	·0900151	·2725	·0504031	− ·0393033	− ·0303374
·0446258	− ·0399553	− ·0245700	*+ ·65*	·0454992	·0270821	·274375	·0522049	− ·0389860	− ·0320608
·0457531	− ·0398732	− ·0257219	*+ ·70*	·0643213	$\bar{1}$·9501937	·27625	·0540383	− ·0386098	− ·0337735
·0468945	− ·0397675	− ·0268757	*+ ·75*	·0892920	$\bar{1}$·8544683	·278125	·0559033	− ·0381733	− ·0354711
·04805	− ·0396375	− ·0280304	*+ ·80*	·1231416	$\bar{1}$·7316990	·28	·0578	− ·0376750	− ·0371490
·0492195	− ·0394829	− ·0291847	*+ ·85*	·1710041	$\bar{1}$·5664684	·281875	·0597283	− ·0371134	− ·0388026
·0504031	− ·0393033	− ·0303374	*+ ·90*	·2444254	$\bar{1}$·3241210	·28375	·0616883	− ·0364871	− ·0404269
·0516008	− ·0390982	− ·0314873	*+ ·95*	·3802830	1·8940059	·285625	·0636799	− ·0357945	− ·0420171

TABLE B. *To assist the calculation of the Ordinates of the Correlation Frequency Curves from Expansion Formulae.*

$\rho = \cdot 4.$ $\log (1-\rho^2)^{\frac{3}{2}} = \bar{1}\cdot 8864189.$							$\rho = \cdot 5.$ $\log (1-\rho^2)^{\frac{3}{2}} = \bar{1}\cdot 8125919.$			
r	$\log \chi_1$	$\log \chi_2$	ϕ_1	ϕ_2	ϕ_3	ϕ_4	r	$\log \chi_1$	$\log \chi_2$	ϕ_1
− *·95*	·6832371	$\bar{2}$·6990762	·2025	·0057781	− ·0229682	+ ·0129113	− *·95*	·7367591	$\bar{2}$·6107927	·190625
− *·90*	·5320225	$\bar{1}$·1368698	·205	·0066125	− ·0240578	+ ·0125970	− *·90*	·5844606	$\bar{1}$·0491282	·19375
− *·85*	·4433337	$\bar{1}$·3868509	·2075	·0075031	− ·0251404	+ ·0121802	− *·85*	·4946527	$\bar{1}$·2996689	·196875
− *·80*	·3802830	$\bar{1}$·5596756	·21	·00845	− ·0262125	+ ·0116596	− *·80*	·4304462	$\bar{1}$·4730716	·2
− *·75*	·3313147	$\bar{1}$·6900043	·2125	·0094531	− ·0272705	+ ·0110342	− *·75*	·3802830	$\bar{1}$·6039976	·203125
− *·70*	·2912852	$\bar{1}$·7932591	·215	·0105125	− ·0283109	+ ·0103034	− *·70*	·3390180	$\bar{1}$·7078702	·20625
− *·65*	·2574549	$\bar{1}$·8776516	·2175	·0116281	− ·0293303	+ ·0094672	− *·65*	·3039093	$\bar{1}$·7929019	·209375
− *·60*	·2281921	$\bar{1}$·9480680	·22	·0128	− ·0303250	+ ·0085260	− *·60*	·2733227	$\bar{1}$·8639801	·2125
− *·55*	·2024481	·0076453	·2225	·0140281	− ·0312916	+ ·0074804	− *·55*	·2462074	$\bar{1}$·9242431	·215625
− *·50*	·1795110	·0585101	·225	·0153125	− ·0322266	+ ·0063318	− *·50*	·2218487	$\bar{1}$·9758187	·21875
− *·45*	·1588770	·1021639	·2275	·0166531	− ·0331264	+ ·0050816	− *·45*	·1997401	·0202098	·221875
− *·40*	·1401787	·1396988	·23	·01805	− ·0339875	+ ·0037321	− *·40*	·1795110	·0585101	·225
− *·35*	·1231416	·1719271	·2325	·0195031	− ·0348065	+ ·0022856	− *·35*	·1608837	·0915336	·228125
− *·30*	·1075577	·1994619	·235	·0210125	− ·0355797	+ ·0007451	− *·30*	·1436465	·1198950	·23125
− *·25*	·0932674	·2227694	·2375	·0225781	− ·0363037	− ·0008860	− *·25*	·1276363	·1440625	·234375
− *·20*	·0801485	·2422038	·24	·0242	− ·0369750	− ·0026040	− *·20*	·1127264	·1643923	·2375
− *·15*	·0681078	·2580318	·2425	·0258781	− ·0375900	− ·0044047	− *·15*	·0988194	·1811527	·240625
− *·10*	·0570761	·2704450	·245	·0276125	− ·0381453	− ·0062833	− *·10*	·0858411	·1945400	·24375
− *·05*	·0470041	·2795781	·2475	·0294031	− ·0386373	− ·0082349	− *·05*	·0737368	·2046893	·246875
·00	·0378604	·2855089	·25	·03125	− ·0390625	− ·0102539	*·00*	·0624694	·2116818	·25
+ *·05*	·0296300	·2882652	·2525	·0331531	− ·0394174	− ·0123344	+ *·05*	·0520175	·2155489	·253125
+ *·10*	·0223140	·2878260	·255	·0351125	− ·0396984	− ·0144700	+ *·10*	·0423754	·2162728	·25625
+ *·15*	·0159298	·2841201	·2575	·0371281	− ·0399021	− ·0166540	+ *·15*	·0335527	·2137861	·259375
+ *·20*	·0105126	·2770218	·26	·0392	− ·0400250	− ·0188790	+ *·20*	·0255763	·2079674	·2625
+ *·25*	·0061172	·2663445	·2625	·0413281	− ·0400635	− ·0211375	+ *·25*	·0184918	·1986347	·265625
+ *·30*	·0028223	·2518296	·265	·0435125	− ·0400141	− ·0234213	+ *·30*	·0123676	·1855345	·26875
+ *·35*	·0007352	·2331303	·2675	·0457531	− ·0398732	− ·0257219	+ *·35*	·0072998	·1683255	·271875
+ *·40*	·0	·2097881	·27	·04805	− ·0396375	− ·0280304	+ *·40*	·0034197	·1465558	·275
+ *·45*	·0008089	·1811980	·2725	·0504031	− ·0393033	− ·0303374	+ *·45*	·0009057	·1196270	·278125
+ *·50*	·0034197	·1465558	·275	·0528125	− ·0388672	− ·0326331	+ *·50*	·0	·0867431	·28125
+ *·55*	·0081829	·1047779	·2775	·0552781	− ·0383256	− ·0349072	+ *·55*	·0010353	·0468291	·284375
+ *·60*	·0155840	·0543720	·28	·0578	− ·0376750	− ·0371490	+ *·60*	·0044774	$\bar{1}$·9984028	·2875
+ *·65*	·0263161	$\bar{1}$·9932210	·2825	·0603781	− ·0369119	− ·0393475	+ *·65*	·0109971	$\bar{1}$·9393579	·290625
+ *·70*	·0414078	$\bar{1}$·9181979	·285	·0630125	− ·0360328	− ·0414911	+ *·70*	·0215976	$\bar{1}$·8665804	·29375
+ *·75*	·0624694	$\bar{1}$·8244269	·2875	·0657031	− ·0350342	− ·0435679	+ *·75*	·0378604	$\bar{1}$·7752089	·296875
+ *·80*	·0922180	$\bar{1}$·7037082	·29	·06845	− ·0339125	− ·0455654	+ *·80*	·0624694	$\bar{1}$·6570600	·3
+ *·85*	·1357728	$\bar{1}$·5406314	·2925	·0712531	− ·0326643	− ·0474709	+ *·85*	·1005057	$\bar{1}$·4967424	·303125
+ *·90*	·2046635	$\bar{1}$·3005493	·295	·0741125	− ·0312859	− ·0492710	+ *·90*	·1634553	$\bar{1}$·2596309	30625
+ *·95*	·3357497	$\bar{2}$·8728199	·2975	·0770281	− ·0297740	− ·0509521	+ *·95*	·2881264	$\bar{2}$·8351091	·309375

TABLE B—(*continued*).

$\rho = \cdot5.$
$\log(1-\rho^2)^{\frac{3}{2}} = \bar{1}\cdot8125919.$

$\rho = \cdot6.$
$\log(1-\rho^2)^{\frac{3}{2}} = \bar{1}\cdot7092700.$

ϕ_2	ϕ_3	ϕ_4	r	$\log \chi_1$	$\log \chi_2$	ϕ_1	ϕ_2	ϕ_3	ϕ_4
·0025830	−·0178043	+·0130733	−·*95*	·7983074	$\bar{2}\cdot4939170$	·17875	·0006570	−·0129380	+·0113090
·0033008	−·0191498	+·0132346	−·*90*	·6450539	$\bar{2}\cdot9327299$	·1825	·0011281	−·0148103	+·0120455
·0041064	−·0205090	+·0132540	−·*85*	·5542555	$\bar{1}\cdot1837659$	·18625	·0017258	−·0159569	+·0126231
·005	−·0218750	+·0131250	−·*80*	·4890205	$\bar{1}\cdot3576828$	·19	·00245	−·0175375	+·0130246
·0059814	−·0232410	+·0128423	−·*75*	·4377890	$\bar{1}\cdot4891430$	·19375	·0033008	−·0191498	+·0132346
·0070508	−·0246002	+·0124015	−·*70*	·3954124	$\bar{1}\cdot5935710$	·1975	·0042781	−·0207818	+·0132403
·0082080	−·0259457	+·0117995	−·*65*	·3591488	$\bar{1}\cdot6791805$	·20125	·0053820	−·0224218	+·0130306
·0094531	−·0272705	+·0110342	−·*60*	·3273589	$\bar{1}\cdot7508604$	·205	·0066125	−·0240578	+·0125970
·0107861	−·0285679	+·0101042	−·*55*	·2989895	$\bar{1}\cdot8117504$	·20875	·0079695	−·0256780	+·0119330
·0122070	−·0298309	+·0090097	−·*50*	·2733227	$\bar{1}\cdot8639801$	·2125	·0094531	−·0272705	+·0110342
·0137158	−·0310528	+·0077516	−·*45*	·2498484	$\bar{1}\cdot9090541$	·21625	·0110633	−·0288235	+·0098985
·0153125	−·0322266	+·0063318	−·*40*	·2281921	$\bar{1}\cdot9480680$	·22	·0128	−·0303250	+·0085260
·0169971	−·0333454	+·0047535	−·*35*	·2080718	$\bar{1}\cdot9818379$	·22375	·0146633	−·0317633	+·0069189
·0187695	−·0344025	+·0030208	−·*30*	·1892713	·0109810	·2275	·0166531	−·0331264	+·0050816
·0206299	−·0353909	+·0011389	−·*25*	·1716222	·0359679	·23125	·0187695	−·0344025	+·0030208
·0225781	−·0363037	−·0008860	−·*20*	·1549924	·0571577	·235	·0210125	−·0355797	+·0007451
·0246143	−·0371342	−·0030466	−·*15*	·1392781	·0748218	·23875	·0233820	−·0366462	−·0017344
·0267383	−·0378754	−·0053345	−·*10*	·1243983	·0891598	·2425	·0258781	−·0375900	−·0044047
·0289502	−·0385204	−·0077405	−·*05*	·1102908	·1003106	·24625	0285008	−·0383994	−·0072503
·03125	−·0390625	−·0102539	·*00*	·0969100	·1083599	·25	·03125	−·0390625	−·0102539
·0336377	−·0394947	−·0128634	+·*05*	·0842253	·1133434	·25375	·0341258	−·0395674	−·0133957
·0361133	−·0398102	−·0155564	+·*10*	·0722203	·1152488	·2575	·0371281	−·0399021	−·0166540
·0386768	−·0400021	−·0183193	+·*15*	·0608930	·1140143	·26125	·0402570	−·0400550	−·0200045
·0413281	−·0400635	−·0211375	+·*20*	·0502571	·1095254	·265	·0435125	−·0400141	−·0234213
·0440674	−·0399876	−·0239952	+·*25*	·0403433	·1016073	·26875	·0468945	−·0397675	−·0268757
·0468945	−·0397675	−·0268757	+·*30*	·0312032	·0900151	·2725	·0504031	−·0393033	−·0303374
·0498096	−·0393963	−·0297613	+·*35*	·0229135	·0744170	·27625	·0540383	−·0386098	−·0337735
·0528125	−·0388672	−·0326331	+·*40*	·0155840	·0543720	·28	·0578	−·0376750	−·0371490
·0559033	−·0381733	−·0354711	+·*45*	·0093675	·0292945	·28375	·0616883	−·0364871	−·0404269
·0590820	−·0373077	−·0382544	+·*50*	·0044774	$\bar{1}\cdot9984028$	·2875	·0657031	−·0350342	−·0435679
·0623486	−·0362637	−·0409610	+·*55*	·0012127	$\bar{1}\cdot9606388$	·29125	·0698445	−·0333044	−·0465305
·0657031	−·0350342	−·0435679	+·*60*	·0	$\bar{1}\cdot9145399$	·295	·0741125	−·0312859	−·0492710
·0691455	−·0336124	−·0460509	+·*65*	·0014639	$\bar{1}\cdot8580230$	·29875	·0785070	−·0289669	−·0517436
·0726758	−·0319916	−·0483849	+·*70*	·0065529	$\bar{1}\cdot7880012$	·3025	·0830281	−·0263353	−·0539003
·0762939	−·0301647	−·0505437	+·*75*	·0167837	$\bar{1}\cdot6996456$	·30625	·0876758	−·0233795	−·0556908
·08	−·0281250	−·0525000	+·*80*	·0347621	$\bar{1}\cdot5848120$	·31	·09245	−·0200875	−·0570629
·0837939	−·0258656	−·0542255	+·*85*	·0654746	$\bar{1}\cdot4281563$	·31375	·0973508	−·0164474	−·0579619
·0876758	−·0233795	−·0556908	+·*90*	·1202910	$\bar{1}\cdot1951114$	·3175	·1023781	−·0120568	−·0583312
·0916455	−·0206600	−·0568656	+·*95*	·2358762	$\bar{2}\cdot7751326$	·32125	·1075320	−·0080757	−·0581117

TABLE B. *To assist the calculation of the Ordinates of the Correlation Frequency Curves from Expansion Formulae.*

$\rho = \cdot 7$. $\log(1-\rho^2)^{\frac{3}{2}} = \bar{1}\cdot 5613553$.							$\rho = \cdot 8$. $\log(1-\rho^2)^{\frac{3}{2}} = \bar{1}\cdot 3344538$.			
r	$\log \chi_1$	$\log \chi_2$	ϕ_1	ϕ_2	ϕ_3	ϕ_4	r	$\log \chi_1$	$\log \chi_2$	ϕ_1
−·95	·8731268	$\bar{2}$·3332450	·166875	·0000002	−·0087458	+·0081651	−·95	·9728591	$\bar{2}$·0942943	·155
−·90	·7190257	$\bar{2}$·7724818	·17125	·0000945	−·0101880	+·0094402	−·90	·8180004	$\bar{2}$·5339099	·16
−·85	·6273441	$\bar{1}$·0239593	·175625	·0003611	−·0117540	+·0105881	−·85	·7255265	$\bar{2}$·7857835	·165
−·80	·5611883	$\bar{1}$·1983367	·18	·0008	−·0134250	+·0115710	−·80	·6585413	$\bar{2}$·9605755	·17
−·75	·5089957	$\bar{1}$·3302774	·184375	·0014111	−·0151821	+·0123553	−·75	·6054797	$\bar{1}$·0929508	·175
−·70	·4656161	$\bar{1}$·4352073	·18875	·0021945	−·0170065	+·0129113	−·70	·5611883	$\bar{1}$·1983367	·18
−·65	·4283019	$\bar{1}$·5213417	·193125	·0031502	−·0188794	+·0132135	−·65	·5229163	$\bar{1}$·2849499	·185
−·60	·3954124	$\bar{1}$·5935710	·1975	·0042781	−·0207818	+·0132403	−·60	·4890205	$\bar{1}$·3576828	·19
−·55	·3658926	$\bar{1}$·6550366	·201875	·0055783	−·0226951	+·0129741	−·55	·4584391	$\bar{1}$·4196788	·195
−·50	·3390180	$\bar{1}$·7078702	·20625	·0070508	−·0246002	+·0124015	−·50	·4304462	$\bar{1}$·4730716	·2
−·45	·3142753	$\bar{1}$·7535784	·210625	·0086955	−·0264785	+·0115131	−·45	·4045223	$\bar{1}$·5193703	·205
−·40	·2912852	$\bar{1}$·7932591	·215	·0105125	−·0283109	+·0103034	−·40	·3802830	$\bar{1}$·5596756	·21
−·35	·2697607	$\bar{1}$·8277312	·219375	·0125018	−·0300788	+·0087711	−·35	·3574352	$\bar{1}$·5948094	·215
−·30	·2494796	$\bar{1}$·8576146	·22375	·0146633	−·0317633	+·0069189	−·30	·3357497	$\bar{1}$·6253949	·22
−·25	·2302671	$\bar{1}$·8833832	·228125	·0169971	−·0333454	+·0047535	−·25	·3150444	$\bar{1}$·6519100	·225
−·20	·2119841	$\bar{1}$·9053996	·2325	·0195031	−·0348065	+·0022856	−·20	·2951711	$\bar{1}$·6747215	·23
−·15	·1945188	$\bar{1}$·9239392	·236875	·0221814	−·0361275	−·0004699	−·15	·2760084	$\bar{1}$·6941098	·235
−·10	·1777811	$\bar{1}$·9392061	·24125	·0250320	−·0372898	−·0034943	−·10	·2574549	$\bar{1}$·7102846	·24
−·05	·1616988	$\bar{1}$·9513444	·245625	·0280549	−·0382744	−·0067646	−·05	·2394256	$\bar{1}$·7233964	·245
·00	·1462149	$\bar{1}$·9604452	·25	·03125	−·0390625	−·0102539	·00	·2218487	$\bar{1}$·7335437	·25
+·05	·1312858	$\bar{1}$·9665509	·254375	·0346174	−·0396353	−·0139313	+·05	·2046635	$\bar{1}$·7407774	·255
+·10	·1168803	$\bar{1}$·9696565	·25875	·0381570	−·0399739	−·0177618	+·10	·1878190	$\bar{1}$·7451026	·26
+·15	·1029796	$\bar{1}$·9697088	·263125	·0418689	−·0400595	−·0217064	+·15	·1712730	$\bar{1}$·7464775	·265
+·20	·0895777	$\bar{1}$·9666028	·2675	·0457531	−·0398732	−·0257219	+·20	·1549924	$\bar{1}$·7448109	·27
+·25	·0766832	$\bar{1}$·9601751	·271875	·0498096	−·0393963	−·0297613	+·25	·1389531	$\bar{1}$·7399556	·275
+·30	·0643213	$\bar{1}$·9501937	·27625	·0540383	−·0386098	−·0337735	+·30	·1231416	$\bar{1}$·7316990	·28
+·35	·0525383	$\bar{1}$·9363424	·280625	·0584393	−·0374949	−·0377031	+·35	·1075577	$\bar{1}$·7197481	·285
+·40	·0414078	$\bar{1}$·9181979	·285	·0630125	−·0360328	−·0414911	+·40	·0922180	$\bar{1}$·7037082	·29
+·45	·0310401	$\bar{1}$·8951960	·289375	·0677580	−·0342047	−·0450741	+·45	·0771634	$\bar{1}$·6830497	·295
+·50	·0215976	$\bar{1}$·8665804	·29375	·0726758	−·0319916	−·0483849	+·50	·0624694	$\bar{1}$·6570600	·30
+·55	·0133179	$\bar{1}$·8313240	·298125	·0777658	−·0293748	−·0513521	+·55	·0482647	$\bar{1}$·6247660	·305
+·60	·0065529	$\bar{1}$·7880012	·3025	·0830281	−·0263353	−·0539003	+·60	·0347621	$\bar{1}$·5848120	·31
+·65	·0018354	$\bar{1}$·7345749	·306875	·0884627	−·0228545	−·0559500	+·65	·0223140	$\bar{1}$·5352510	·315
+·70	·0	$\bar{1}$·6680154	·31125	·0940695	−·0189134	−·0574179	+·70	·0115163	$\bar{1}$·4731726	·32
+·75	·0024195	$\bar{1}$·5835655	·315625	·0998486	−·0144932	−·0582164	+·75	·0034197	$\bar{1}$·3939808	·325
+·80	·0115163	$\bar{1}$·4731726	·32	·1058	−·0095750	−·0582540	+·80	·0	$\bar{1}$·2898462	·33
+·85	·0320384	$\bar{1}$·3216122	·324375	·1119236	−·0041400	−·0574351	+·85	·0053672	$\bar{1}$·1458632	·335
+·90	·0750398	$\bar{1}$·0944747	·32875	·1182195	+·0018306	−·0556601	+·90	·0296300	$\bar{2}$·9280951	·34
+·95	·1767574	$\bar{2}$·6814297	·333125	·1246877	+·0083557	−·0528253	+·95	·1075577	$\bar{2}$·5269450	·345

TABLE B—(*continued*).

$\rho = \cdot 8.$
$\log (1-\rho^2)^{\frac{3}{2}} = \bar{1}\cdot 3344538.$

$\rho = \cdot 9.$
$\log (1-\rho^2)^{\frac{3}{2}} = \bar{2}\cdot 9181304.$

ϕ_2	ϕ_3	ϕ_4	r	$\log \chi_1$	$\log \chi_2$	ϕ_1	ϕ_2	ϕ_3	ϕ_4
·0006125	−·0056047	+·0044070	*−·95*	1·1344648	$\bar{3}$·6665553	·143125	·0024939	−·0038913	+·0010198
·0002	−·0067750	+·0060060	*−·90*	·9789250	$\bar{2}$·1065114	·14875	·0014445	−·0045010	+·0025068
·0000125	−·0081703	+·0075898	*−·85*	·8857364	$\bar{2}$·3587425	·154375	·0006799	−·0054756	+·0042099
·00005	−·0097625	+·0090871	*−·80*	·8180004	$\bar{2}$·5339099	·16	·0002	−·0067750	+·0060060
·0003125	−·0115234	+·0104333	*−·75*	·7641490	$\bar{2}$·6666800	·165625	·0000049	−·0083591	+·0077831
·0008	−·0134250	+·0115710	*−·70*	·7190257	$\bar{2}$·7724818	·17125	·0000945	−·0101880	+·0094402
·0015125	−·0154391	+·0124493	*−·65*	·6798765	$\bar{2}$·8595337	·176875	·0004689	−·0122216	+·0108873
·00245	−·0175375	+·0130246	*−·60*	·6450539	$\bar{2}$·9327299	·1825	·0011281	−·0148103	+·0120455
·0036125	−·0196922	+·0132598	*−·55*	·6134923	$\bar{2}$·9952161	·188125	·0020721	−·0167425	+·0128469
·005	−·0218750	+·0131250	*−·50*	·5844606	$\bar{1}$·0491282	·19375	·0033008	−·0191498	+·0132346
·0066125	−·0240578	+·0125970	*−·45*	·5574342	$\bar{1}$·0959782	·199375	·0048143	−·0216015	+·0131629
·00845	−·0262125	+·0116596	*−·40*	·5320225	$\bar{1}$·1368698	·205	·0066125	−·0240578	+·0125970
·0105125	−·0283109	+·0103034	*−·35*	·5079254	$\bar{1}$·1726281	·210625	·0086955	−·0264785	+·0115131
·0128	−·0303250	+·0085260	*−·30*	·4849062	$\bar{1}$·2038806	·21625	·0110633	−·0288235	+·0098985
·0153125	−·0322266	+·0063318	*−·25*	·4627737	$\bar{1}$·2311092	·221875	·0137158	−·0310528	+·0077516
·01805	−·0339875	+·0037321	*−·20*	·4413696	$\bar{1}$·2546862	·2275	·0166531	−·0331264	+·0050816
·0210125	−·0355797	+·0007451	*−·15*	·4205607	$\bar{1}$·2748976	·233125	·0198752	−·0350042	+·0019092
·0242	−·0369750	−·0026040	*−·10*	·4002321	$\bar{1}$·2919599	·23875	·0233820	−·0366462	−·0017344
·0276125	−·0381453	−·0062833	*−·05*	·3802830	$\bar{1}$·3060315	·244375	·0271736	−·0380123	−·0058066
·03125	−·0390625	−·0102539	*·00*	·3606232	$\bar{1}$·3172203	·25	·03125	−·0390625	−·0102539
·0351125	−·0396984	−·0144700	*+·05*	·3411701	$\bar{1}$·3255880	·255625	·0356111	−·0397568	−·0150118
·0392	−·0400250	−·0188790	*+·10*	·3218470	$\bar{1}$·3311524	·26125	·0402570	−·0400550	−·0200045
·0435125	−·0400141	−·0234213	*+·15*	·3025809	$\bar{1}$·3338874	·266875	·0451877	−·0399172	−·0251457
·04805	−·0396375	−·0280304	*+·20*	·2833014	$\bar{1}$·3337203	·2725	·0504031	−·0393033	−·0303374
·0528125	−·0388672	−·0326331	*+·25*	·2639393	$\bar{1}$·3305264	·278125	·0559033	−·0381733	−·0354711
·0578	−·0376750	−·0371490	*+·30*	·2444254	$\bar{1}$·3241210	·28375	·0616883	−·0364871	−·0404269
·0630125	−·0360328	−·0414911	*+·35*	·2246902	$\bar{1}$·3142457	·289375	·0677580	−·0342047	−·0450741
·06845	−·0339125	−·0455654	*+·40*	·2046635	$\bar{1}$·3005493	·295	·0741125	−·0312859	−·0492710
·0741125	−·0312859	−·0492710	*+·45*	·1842748	$\bar{1}$·2825579	·300625	·0807518	−·0276909	−·0528645
·08	−·0281250	−·0525000	*+·50*	·1634553	$\bar{1}$·2596309	·30625	·0876758	−·0233795	−·0556908
·0861125	−·0244016	−·0551378	*+·55*	·1421425	$\bar{1}$·2308910	·311875	·0948846	−·0183117	−·0575750
·09245	−·0200875	−·0570629	*+·60*	·1202910	$\bar{1}$·1951114	·3175	·1023781	−·0120568	−·0583312
·0990125	−·0151547	−·0581467	*+·65*	·0978953	$\bar{1}$·1505243	·323125	·1101564	−·0057467	−·0577622
·1058	−·0095750	−·0582540	*+·70*	·0750398	$\bar{1}$·0944747	·32875	·1182195	+·0018306	−·0556601
·1128125	−·0033203	−·0572424	*+·75*	·0520175	$\bar{1}$·0227457	·334375	·1265674	+·0103245	−·0518057
·12005	+·0036375	−·0549629	*+·80*	·0296300	$\bar{2}$·9280951	·34	·1352	+·0197750	−·0459690
·1275125	+·0113266	−·0512594	*+·85*	·0100596	$\bar{2}$·7965809	·345625	·1441174	+·0302222	−·0379088
·1352	+·0197750	−·0459690	*+·90*	·0	$\bar{2}$·5959739	·35125	·1533195	+·0417061	−·0273728
·1431125	+·0290109	−·0389219	*+·95*	·0274889	$\bar{2}$·2200433	·356875	·1628064	+·0542668	−·0140979

TABLE C. *Position of Origin and Abscissal Unit in terms of Standard Deviation. The first Number gives the Position of the Origin, the second the Abscissal Unit.*

n	$\rho = 0$	$\rho = \cdot 1$	$\rho = \cdot 2$	$\rho = \cdot 3$	$\rho = \cdot 4$
3	0 ·0707107	·1116225 ·0709720	·2266490 ·0717802	·3490329 ·0732142	·4840501 ·0754314
4	0 ·0866025	·1478819 ·0870223	·3011028 ·0883242	·4659475 ·0906481	·6510044 ·0942760
5	0 ·1000000	·1778638 ·1005663	·3643600 ·1023258	·5634956 ·1054792	·7915154 ·1104330
6	0 ·1118034	·2038711 ·1125038	·4165411 ·1146829	·6485234 ·1185985	·9145610 ·1247759
7	0 ·1224745	·2271065 ·1232981	·4645401 ·1258629	·7246924 ·1304802	1·0250662 ·1377861
8	0 ·1322876	·2482790 ·1332252	·5083017 ·1361468	·7942029 ·1414136	1·1260506 ·1497650
9	0 ·1414214	·2678441 ·1424651	·5487533 ·1457192	·8584888 ·1515911	1·2195137 ·1609168
10	0 ·1500000	·2861133 ·1511433	·5865316 ·1547090	·9185423 ·1611481	1·3068548 ·1713870
11	0 ·1581139	·3033102 ·1593510	·6220953 ·1632105	·9750815 ·1701841	1·3890961 ·1812835
12	0 ·1658312	·3196014 ·1671572	·6557863 ·1712949	1·0286451 ·1787749	1·4670090 ·1906890
13	0 ·1732051	·3351148 ·1746157	·6878684 ·1790180	1·0796486 ·1869796	1·5411920 ·1996684
14	0 ·1802776	·3499533 ·1817701	·7185489 ·1864240	1·1284203 ·1948454	1·6121199 ·2082735
15	0 ·1870829	·3641907 ·1886516	·7479946 ·1935489	1·1752248 ·2024107	1·6801759 ·2165466
16	0 ·1936492	·3779001 ·1952922	·7763419 ·2004222	1·2202786 ·2097070	1·7456751 ·2245228
17	0 ·2000000	·3911338 ·2017147	·8037041 ·2070688	1·2637617 ·2167611	1·8088795 ·2322314
18	0 ·2061553	·4039379 ·2079391	·8301762 ·2135096	1·3052186 ·2234913	1·8700099 ·2396972
19	0 ·2121320	·4163514 ·2139828	·8558391 ·2197626	1·3465992 ·2302287	1·9292543 ·2469416
20	0 ·2179449	·4284077 ·2198606	·8807623 ·2258433	1·3861929 ·2366781	1·9867744 ·2539828
21	0 ·2236068	·4401360 ·2255854	·9050057 ·2317651	1·4247026 ·2429578	2·0427101 ·2608369
22	0 ·2291288	·4515616 ·2311687	·9286218 ·2375398	1·4622122 ·2490805	2·0971840 ·2675179
23	0 ·2345208	·4627064 ·2366202	·9516565 ·2431779	1·4987939 ·2550572	2·1503024 ·2740380
24	0 ·2397916	·4735903 ·2419491	·9741508 ·2486887	1·5345139 ·2608980	2·2021616 ·2804084
25	0 ·2449490	·4842308 ·2471633	·9961404 ·2540801	1·5694295 ·2666117	2·2528457 ·2866387
50	0 ·3500000	·6995857 ·3533454	1·4409868 ·3637983	2·2751863 ·3827476	3·2760792 ·4130572
100	0 ·4974937	·9997548 ·5023813	2·0606127 ·5176589	3·2571060 ·5453562	4·6973149 ·5896685
400	0 ·9987492	2·0150354 1·0087684	4·1553109 1·0400784	6·5735909 1·0968491	9·4914497 1·1876817

TABLE C—(*continued*).

n	$\rho=\cdot5$	$\rho=\cdot6$	$\rho=\cdot7$	$\rho=\cdot8$	$\rho=\cdot9$
3	·6397035 ·0787233	·8298069 ·0836496	1·0821815 ·0914099	1·4670408 ·1051565	2·2572614 ·1375647
4	·8697069 ·0997391	1·1461037 ·1080827	1·5308211 ·1216223	2·1592865 ·1467433	3·6084918 ·2112748
5	1·0657049 ·1179621	1·4204694 ·1296152	1·9303003 ·1488972	2·8019830 ·1857778	4·9700548 ·2860585
6	1·2384801 ·1342232	1·6646016 ·1489755	2·2906234 ·1737018	3·3941363 ·2219447	6·2758785 ·3579595
7	1·3941989 ·1490074	1·8857159 ·1666372	2·6192262 ·1964454	3·9396570 ·2553750	7·4992321 ·4253943
8	1·5367713 ·1626318	2·0886726 ·1829343	2·9218412 ·2174712	4·4441908 ·2863637	8·6363301 ·4881142
9	1·6688524 ·1753174	2·2769195 ·1981116	3·2029089 ·2370570	4·9133944 ·3152309	9·6932562 ·5464383
10	1·7923354 ·1872248	2·4529927 ·2123532	3·4658872 ·2554247	5·3522889 ·3422692	10·6791012 ·6008602
11	1·9086231 ·1984751	2·6188112 ·2258005	3·7134918 ·2727512	5·7651162 ·3677294	11·6030258 ·6518808
12	2·0191978 ·2092046	2·7758616 ·2385646	3·9478808 ·2891787	6·1553948 ·3918213	12·4732325 ·6999496
13	2·1236943 ·2193627	2·9253194 ·2507341	4·1707880 ·3048225	6·5260261 ·4147185	13·2967143 ·7454499
14	2·2238984 ·2291249	3·0681290 ·2623809	4·3836241 ·3197770	6·8794080 ·4365653	14·0793562 ·7887049
15	2·3200595 ·2385095	3·2050605 ·2735636	4·5875494 ·3341195	7·2175324 ·4574817	14·8260439 ·8299826
16	2·4125823 ·2475526	3·3367521 ·2843315	4·7835297 ·3479156	7·5420659 ·4775684	15·5412908 ·8695312
17	2·5018385 ·2562879	3·4637416 ·2947261	4·9723756 ·3612200	7·8544126 ·4969104	16·2273352 ·9074724
18	2·5881420 ·2647443	3·5864801 ·3047823	5·1547751 ·3740794	8·1557645 ·5155798	16·8882836 ·9440329
19	2·6717601 ·2729462	3·7053440 ·3145295	5·3313166 ·3865338	8·4471433 ·5336387	17·5262030 ·9793257
20	2·7529232 ·2809148	3·8206677 ·3239936	5·5025071 ·3986177	8·7294265 ·5511403	18·1432348 1·0134685
21	2·8318307 ·2886686	3·9341863 ·3333199	5·6687868 ·4103611	9·0033775 ·5681311	18·7411717 1·0465598
22	2·9086578 ·2962240	4·0418750 ·3421658	5·8305409 ·4217904	9·2696624 ·5846516	19·3216440 1·0786891
23	2·9835558 ·3035949	4·1481734 ·3509062	5·9881082 ·4329287	9·5288659 ·6007373	19·8860017 1·1099306
24	3·0566623 ·3107942	4·2518695 ·3594371	6·1417876 ·4437967	9·7815042 ·6164198	20·4355030 1·1403534
25	3·1280980 ·3178324	4·3532587 ·3677825	6·2918434 ·4544124	10·0281639 ·6317351	20·9711894 1·1700149
50	4·5677713 ·4603212	6·3911451 ·5361355	9·2990780 ·6677547	14·9462808 ·9376711	31·5884211 1·7584334
100	6·5628599 ·6587893	9·2058648 ·7696570	13·4374982 ·9623213	21·6797456 1·3574820	46·0257637 2·5594824
400	13·2812941 1·3293798	18·6652227 1·5566854	27·3067557 1·9517326	44·1732879 2·7620803	94·0676645 5·2272309

MISCELLANEA.

I. Tables for estimating the Probability that the Mean of a unique Sample of Observations lies between $-\infty$ and any given Distance of the Mean of the Population from which the Sample is drawn.

By "STUDENT."

In the last number of *Biometrika* Mr Young completes the table given in Vol. X. p. 522 of the standard deviation frequency curves for small samples by working out the cases where the numbers in the sample are as low as two and three.

In the course of his note he writes "The smallest sample considered is that of $n = 4$ but samples of two and three are of occasional occurrence, especially in physical work, and now and again a value of the probable error of an experimental result is deduced from a set of two or of three observations."

Further on he states "it is evident that the probable error determined from a set of three observations is very untrustworthy and that when there are only two observations it is very much worse."

Now in my original paper (*Biometrika*, Vol. VI. p. 1) I stopped at $n = 4$ because I had not realised that anyone would be foolish enough to work with probable errors deduced from a smaller number of observations, but now I too will complete my tables which will I think emphasise the moderation of the second quotation from Mr Young's note.

Generally speaking there are two objects in determining the standard deviation of a set of observations, namely (1) to compare it with the standard deviation of similar sets of observations, and (2) to estimate the accuracy with which the mean of the observations represents the mean of the population from which the sample is drawn.

The former purpose is served by the table which Mr Young was engaged in completing, the latter, which is by far the most common use of the S.D., by the table which I gave in my original paper and which I now propose to complete downwards by including $n = 2$ and $n = 3$ and to extend upwards as far as $n = 30$.

In the tables the probability is given (to four places of decimals) that the mean of a unique sample shall lie between $-\infty$ and a distance z from the mean of the population, z being measured in terms of the S.D. (s) of the sample.

[By unique I mean to say that all the information which we have (or at all events intend to use) about the distribution of the population is given by the sample in question.]

To compare with the last column of the table ($n = 30$) I have given the corresponding probability calculated from the nearest normal curve, namely the one with S.D. $\frac{s}{\sqrt{n-3}}$ (not $\frac{s}{\sqrt{n-1}}$ as is usually given) and this shows I think that for ordinary purposes Sheppard's tables may be used with $n > 30$.

With regard to samples of two it will be seen that odds of 9 : 1 are reached at a little more than three times the S.D., of 99 : 1 at a little more than thirty times, of 999 : 1 at a little more than 300 times, while 9999 : 1 is reached at in or about 3000 times the S.D.!

Perhaps I may be permitted to restate my opinion as to the best way of judging the accuracy of physical or chemical determinations.

After considerable experience I have not encountered any determination which is not influenced by the date on which it is made, from this it follows that a number of determinations of the same thing made on the same day are likely to lie more closely together than if the repetitions had been made on different days.

It also follows that if the probable error is calculated from a number of observations made close together in point of time much of the secular error will be left out and for general use the probable error will be too small.

Where then the materials are sufficiently stable it is well to run a number of determinations on the same material through any series of routine determinations which have to be made, spreading them over the whole period.

Thus if an analyst is determining the percentage of nitrogen in different samples of seed corn and wishes to know the probable error of the determination, i.e. how accurately his figures give the percentage of nitrogen in a bulk of corn.

Let us suppose that he makes ten determinations a day for sixty days and that it is of some real importance to him to get a clear idea of his error; he will do well to get sixty *different* samples from the same bulk of corn and analyse one of these on each of the sixty days; unless I am much mistaken he will have a more modest idea of his infallibility than he had before he compared the sixty results together. He will also, in so far as his repeated sample is representative, get a close approximation to the probable error of a single determination.

In some cases it is not possible to obtain a sufficient bulk of material and then it may be better to determine each result in duplicate, the repetitions being separated as widely as possible in point of time. Then the square root of the mean of the squares of the differences between corresponding pairs gives twice the standard deviation of the average of a pair, and if enough pairs can be taken and the determinations made on different samples this is a better method than the other, as the error of the sampling is better sampled.

In the preparation of the tables a slight mistake was discovered in the second row of the odd numbers in the original table by Mr W. L. Bowie to whom I am indebted for the calculation of the new figures.

$z\ (=x/\varepsilon)$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$	$n=11$	$n=12$	$n=13$	$n=14$	$n=15$	$n=16$	$n=17$	$n=18$	$n=19$
·1	·5317	·5498	·5633	·5745	·5841	·5928	·6006	·6079	·6146	·6210	·6269	·6326	·6380	·6432	·6481	·6529	·6575	·6619
·2	·5628	·5981	·6241	·6453	·6634	·6795	·6936	·7066	·7185	·7295	·7397	·7493	·7583	·7668	·7748	·7824	·7896	·7965
·3	·5928	·6437	·6804	·7096	·7340	·7549	·7733	·7896	·8042	·8174	·8294	·8404	·8505	·8598	·8683	·8762	·8836	·8904
·4	·6211	·6857	·7309	·7657	·7940	·8175	·8375	·8547	·8697	·8827	·8942	·9045	·9135	·9217	·9289	·9354	·9413	·9466
·5	·6476	·7236	·7749	·8131	·8428	·8667	·8863	·9025	·9161	·9275	·9373	·9455	·9527	·9588	·9640	·9686	·9725	·9760
·6	·6720	·7573	·8125	·8518	·8813	·9040	·9218	·9360	·9473	·9565	·9640	·9701	·9752	·9793	·9827	·9856	·9879	·9899
·7	·6944	·7867	·8440	·8830	·9109	·9314	·9468	·9585	·9675	·9744	·9798	·9840	·9873	·9900	·9920	·9936	·9949	·9960
·8	·7148	·8123	·8701	·9076	·9332	·9512	·9640	·9733	·9801	·9851	·9888	·9916	·9937	·9952	·9964	·9972	·9979	·9984
·9	·7333	·8345	·8915	·9269	·9498	·9652	·9756	·9828	·9878	·9913	·9938	·9956	·9968	·9977	·9983	·9988	·9991	·9994
1·0	·7500	·8536	·9092	·9419	·9622	·9751	·9834	·9889	·9925	·9949	·9966	·9977	·9984	·9989	·9993	·9995	·9997	·9998
1·1	·7651	·8700	·9236	·9537	·9714	·9821	·9887	·9928	·9954	·9970	·9981	·9988	·9992	·9995	·9997	·9998	·9999	·9999
1·2	·7789	·8841	·9354	·9628	·9782	·9870	·9922	·9953	·9971	·9982	·9989	·9993	·9996	·9997	·9998	·9999		
1·3	·7913	·8963	·9451	·9700	·9832	·9905	·9946	·9969	·9982	·9990	·9994	·9996	·9998	·9999	·9999			
1·4	·8026	·9069	·9531	·9756	·9870	·9930	·9962	·9979	·9989	·9994	·9997	·9998	·9999					
1·5	·8128	·9160	·9598	·9800	·9899	·9948	·9973	·9986	·9993	·9996	·9998	·9999						
1·6	·8222	·9240	·9652	·9836	·9920	·9961	·9981	·9990	·9995	·9997	·9999							
1·7	·8308	·9310	·9699	·9864	·9937	·9970	·9986	·9993	·9997	·9998								
1·8	·8386	·9371	·9737	·9886	·9950	·9977	·9990	·9995	·9998	·9999								
1·9	·8458	·9425	·9770	·9904	·9959	·9983	·9992	·9997	·9999									
2·0	·8524	·9472	·9797	·9919	·9967	·9986	·9994	·9998										
2·1	·8585	·9514	·9821	·9931	·9973	·9989	·9996	·9998										
2·2	·8642	·9552	·9841	·9941	·9978	·9992	·9997	·9999										
2·3	·8695	·9585	·9858	·9950	·9982	·9993	·9998											
2·4	·8743	·9615	·9873	·9957	·9985	·9995	·9998											
2·5	·8789	·9642	·9886	·9963	·9987	·9996	·9998											
2·6	·8831	·9667	·9898	·9967	·9989	·9996	·9999											
2·7	·8871	·9689	·9908	·9972	·9991	·9997												
2·8	·8908	·9709	·9916	·9975	·9992	·9998												
2·9	·8943	·9727	·9924	·9978	·9993	·9998												
3·0	·8976	·9743	·9931	·9981	·9994	·9998												
3·1	·9007	·9759	·9937	·9983	·9995	·9999												
3·2	·9036	·9772	·9942	·9984	·9996													
3·3	·9063	·9785	·9947	·9986	·9996													
3·4	·9089	·9797	·9951	·9988	·9997													
3·5	·9114	·9808	·9955	·9989	·9997													

4·0	·9220	·9851	·9969	·9994	·9998
4·5	·9303	·9881	·9977	·9996	·9998
5·0	·9372	·9903	·9984	·9997	·9999
5·5	·9428	·9919	·9988	·9998	
6·0	·9474	·9932	·9990	·9999	
6·5	·9514	·9942	·9992		
7·0	·9548	·9950	·9994		
7·5	·9578	·9956	·9995		
8·0	·9604	·9961	·9996		
8·5	·9627	·9966	·9996		
9·0	·9648	·9969	·9997		
9·5	·9666	·9973	·9997		
10·0	·9683	·9975	·9998		
15·0	·9788	·9989	·9999		
20·0	·9841	·9994			
25·0	·9873	·9996			
30·0	·9894	·9997			
35·0	·9909	·9998			
40·0	·9920	·9998			
45·0	·9929	·9999			
50·0	·9935				
60·0	·9947				
70·0	·9955				
80·0	·9960				
90·0	·9965				
100·0	·9968				
120·0	·9973				
140·0	·9977				
150·0	·9979				
160·0	·9980				
180·0	·9982				
200·0	·9984				
250·0	·9987				
300·0	·9989				
350·0	·9991				
400·0	·9992				
450·0	·9993				
500·0	·9994				
600·0	·9995				
700·0	·9995				
1000·0	·9997				
1500·0	·9998				
2000·0	·9998				
3000·0	·9999				

$z\left(=\frac{x}{s}\right)$	$n=20$	$n=21$	$n=22$	$n=23$	$n=24$	$n=25$	$n=26$	$n=27$	$n=28$	$n=29$	$n=30$	For comparison $\left(\frac{\sqrt{27}}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{27x^2}{2}}\,dx\right)$
·1	·6662	·6703	·6744	·6783	·6821	·6858	·6894	·6929	·6964	·6997	·7030	·6983
·2	·8030	·8093	·8152	·8209	·8264	·8316	·8367	·8415	·8462	·8506	·8550	·8507
·3	·8967	·9026	·9082	·9133	·9182	·9227	·9270	·9309	·9347	·9383	·9415	·9405
·4	·9513	·9556	·9595	·9630	·9662	·9691	·9717	·9742	·9763	·9784	·9801	·9802
·5	·9789	·9815	·9838	·9858	·9875	·9890	·9903	·9915	·9925	·9934	·9942	·9953
·6	·9915	·9929	·9940	·9950	·9958	·9964	·9970	·9975	·9979	·9982	·9985	·9991
·7	·9967	·9974	·9979	·9984	·9987	·9990	·9992	·9994	·9995	·9996	·9997	·9999
·8	·9988	·9991	·9993	·9995	·9996	·9997	·9998	·9999	·9999	·9999	·9999	
·9	·9995	·9996	·9997	·9998	·9999	·9999	·9999					
1·0	·9999	·9999	·9999	·9999								

II. On the Representation of Statistical Data.

By L. ISSERLIS, D.Sc.

§ 1. In a paper entitled "On the Mathematical Representation of Statistical Data*" Professor Edgeworth gives several examples of the fitting of what he calls the generalized curve of error and some of his recent modifications of it to frequency data. The examples to which he applies his methods are taken, with a single exception, from a paper by the present writer, in which double hypergeometrical series were fitted to frequency distributions in two variables†. Professor Edgeworth describes these data as "*well suited to test and illustrate methods of representing frequency distributions.*" Judged by the usual criterion for the testing of goodness of fit these examples indicate that Professor Edgeworth's curves are decidedly unsatisfactory, but he maintains that the fault lies in the criterion and not in the curves. Professor Edgeworth proposes therefore a new criterion for the testing of the goodness of fit of theory to observation, viz. the magnitude of the sum $S(e^2/U)$, where U is the ratio of the frequency in any category of the data to the total frequency (called by Professor Edgeworth the relative frequency) and e is the difference between this and the theoretical relative frequency. He adds erroneously that this is Professor Pearson's χ^2 divided by the total frequency N. To quote from Professor Edgeworth's paper (p. 461) "His (i.e. Professor Pearson's) χ^2 being, in our notation, equal to $S(N^2e^2)/S(NU)$, where N is the total number of observations, is N times as great as our criterion $S(e^2/U)$." As a matter of fact Professor Pearson's χ^2 is equal to $S\{(m'-m)^2/m'\}$ where m' is the *theoretical* and m the observed frequency, and in Professor Edgeworth's notation this would be $NS(e^2/u)$ where u is the *theoretical* relative frequency. The fact that the denominator U employed by Professor Edgeworth is the *observed* relative frequency must make the test nugatory in practice, for $S(e^2/U)$ will be infinite whenever the data contain one category with zero frequency. Very little difference results in Professor Edgeworth's examples when the U's are replaced by u's in the criterion proposed by him, but the later part of his paper claims to give methods of curve fitting which‡ "not only do or may give better values for the coefficients than the use of moments, but also *must*§ give better results than moments or any other process." Professor Edgeworth attempts in the section entitled "some new constructions" to achieve this highly desirable end by, as he says, minimising the Pearsonian criterion χ^2. Actually he minimises $S(e^2/U)$, a very much easier thing to do since the denominators are constant, but leading to equations which are altogether irrelevant.

Professor Edgeworth's proposed criterion is an absolute one, independent of the number of categories. The value $S(e^2/U)$ is usually small and apparently we are to judge of the goodness of fit by the number of zeroes between the decimal point and the first significant figure.

In what follows we fit Pearsonian frequency curves of the appropriate type to the examples used by Professor Edgeworth. In addition to calculating χ^2 and the corresponding value of P for each, we give also the values of $S(e^2/U)$ and $S(e^2/u)$. All these are compared with the corresponding values for the curves fitted by Professor Edgeworth||. It will appear that Professor Edgeworth's curves are a poorer fit than the Pearsonian types not only when judged by the legitimate criterion but even according to the criterion proposed by Professor Edgeworth, and this in spite of the fact that he does not as a rule attempt to fit his curve to the whole of each distribution but confines himself to the central compartments.

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[*] *Journal of the Royal Statistical Society*, Vol. LXXIX. Part IV. (July, 1916).

[†] *Phil. Mag.* September, 1914.

[‡] *Loc. cit.* p. 476.

[§] Professor Edgeworth's italics.

[||] So far as that was possible. Several of Professor Edgeworth's calculations in the later sections of the paper are not finished, and some of the arithmetic would have to be rectified before completing the calculations.

§ 2. EXAMPLE I. *Distribution of* 25,000 *Deals at Whist according to the Number of Trumps held by the First Hand.*

TABLE I.

Number of trumps	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	Total
Frequency ...	215	1724	5262	7440	6371	2950	852	166	20	25000
Relative frequency	·00860	·06896	·21048	·29760*	·25484	·11800	·03408	·00664	·00080	1·00000

We quote from the *Phil. Mag.* paper previously referred to the following values: Mean at 3·24944 trumps, moments about the mean $\mu_2 = 1\cdot66130$, $\mu_3 = \cdot39094$, also $\beta_1 = \cdot03333$, $\beta_2 = 2\cdot93469$. These values lead to the Type I curve

$$y = 7596\cdot005 \left(1 + \frac{x}{6\cdot52180}\right)^{16\cdot21927} \left(1 - \frac{x}{12\cdot76682}\right)^{31\cdot75022}$$

with origin at the mode (= 3·12682 trumps) represented by the continuous line in Fig. 1.

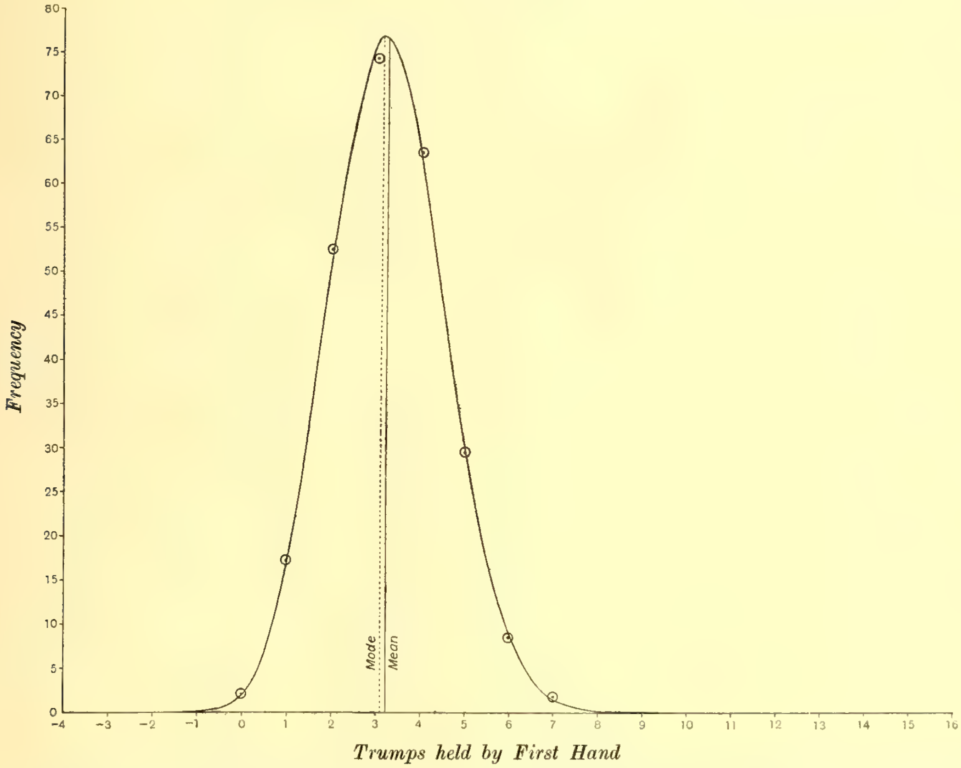

FIG. 1. ⊙ ⊙ ⊙ Distribution of 25,000 deals at Whist according to the number of Trumps held by the First Hand according to theory.

Continuous Line: Type I Frequency Curve

$$y = 7596\cdot005 \left(1 + \frac{x}{6\cdot52180}\right)^{16\cdot21927} \left(1 - \frac{x}{12\cdot76682}\right)^{31\cdot75022}.$$

---

* ·29776 in Prof. Edgeworth's paper is clearly a misprint.

The following table gives the theoretical frequencies of this curve compared with the observed frequencies and the theoretical frequencies for the two curves fitted by Professor Edgeworth. These last being obtained by multiplying the values he gives of the theoretical relative frequencies by 25,000.

TABLE II.

Number of trumps	*-1*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	Totals
Observed frequency	—	215	1724	5262	7440	6371	2950	852	166	20	—	—	25000
Type I	4·91	202·85	1698·32	5201·23	7643·49	6218·30	2993·16	871·65	152·89	15·34	·85	·02	25003
	Sum = 207·76									Sum = 16·21			
Prof. Edgeworth's 1st curve, p. 461	—	—	—	5120	7338	6128	3028	—	—	—	—	—	—
Prof. Edgeworth's 2nd curve, p. 467	—	258	1778	5093	7420	6183	3098	965	208	—	—	—	25003

For the Type I curve $\chi^2 = 13{\cdot}60$ while $n' = 9$ so that $P = {\cdot}0938$.

Professor Edgeworth's first curve gives $\chi^2 = 17{\cdot}48$, $n' = 4$ and $P = {\cdot}000574$*. His second curve gives $\chi^2 = 41{\cdot}55$ for $n' = 8$ so that P is less than ·000001. If in the second curve we take the four central compartments only (i.e. 2 to 5 trumps) we have $\chi^2 = 19{\cdot}48$, $n' = 4$ so that $P = {\cdot}000222$.

Judged by the usual criterion then, both of Professor Edgeworth's curves are a very much poorer fit than Pearson's Type I. If we calculate the value of the criterion proposed by Professor Edgeworth we find that

$S(e^2/U)$ is ·000814 for Professor Edgeworth's first curve (four compartments),
·00181 for eight compartments of his second curve,
and ·00077 for the four central compartments of the second curve.

These values of $S(e^2/U)$ may be compared with ·000544 for nine compartments or ·000420 for four compartments for Pearson's Type I.

Professor Edgeworth says (p. 467) of this example that "the correspondence between fact and theory is very satisfactory."

EXAMPLE II. *The theoretical Frequency Distribution of black Balls in* 210 *Draws of four Balls out of a Bag containing four white and six black Balls.*

The frequencies are

TABLE III.

0	*1*	*2*	*3*	*4*	Total
1	24	90	80	15	210

The mean is at 2·4 black balls, the moments about the mean are $\mu_2 = {\cdot}64$, $\mu_3 = -{\cdot}032$, $\mu_4 = 1\frac{1}{7}$ so that $\beta_1 = {\cdot}00390625$, $\beta_2 = 2{\cdot}747768$. The corresponding Pearsonian frequency curve is of Type II,

$$y = 101{\cdot}06905\left[1 - \frac{x^2}{(3{\cdot}7341772)^2}\right]^{9{\cdot}3938124},$$

represented by the continuous line in Fig. 2. The origin is at mode = mean = 2·4.

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* Professor Edgeworth calls this $\chi^2 = 40$, $n' = 6$ but only states the calculated (relative) frequencies for four compartments.

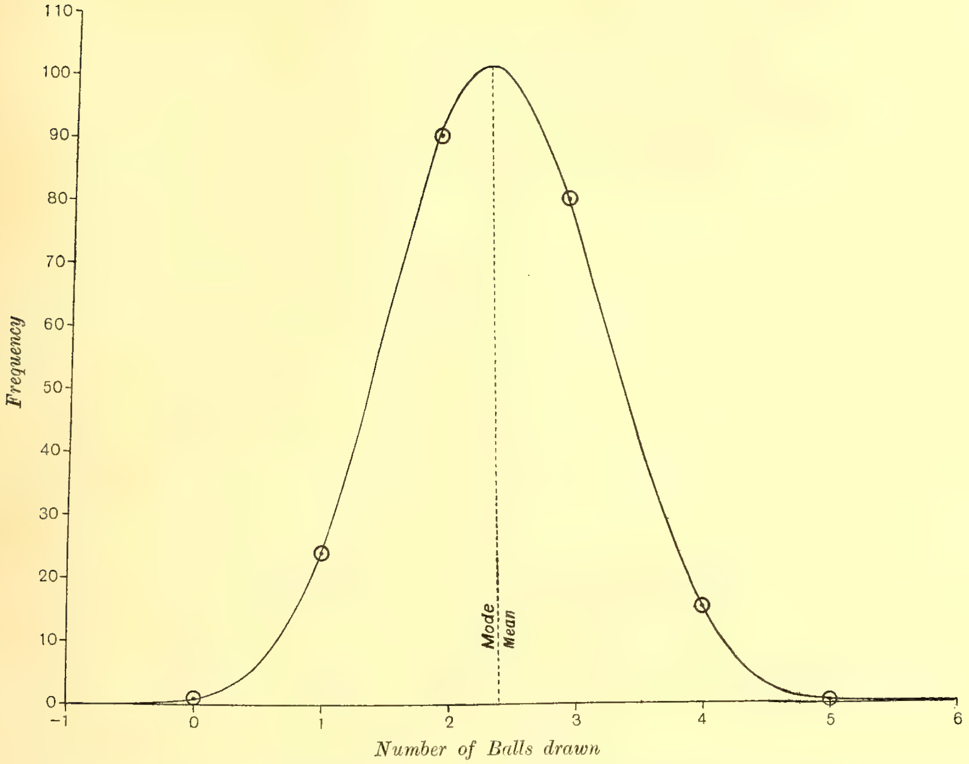

Fig. 2. ⊙ ⊙ ⊙ Theoretical Distribution of Black Balls in 210 draws of 4 balls out of a bag containing 4 white and 6 black balls, i.e. terms of the hypergeometrical series $F(-4, -6, 1, 1)$.

Continuous Line: Type II Frequency Curve

$$y = 101{\cdot}06905 \left[1 - \left(\frac{x}{3{\cdot}734177}\right)^2\right]^{9{\cdot}393812}.$$

The table below gives the theoretical frequencies deduced from this equation and the values given by Professor Edgeworth on p. 471 of his paper*.

TABLE IV.

Number of Black Balls drawn	*0*	*1*	*2*	*3*	*4*	Totals
Data	1	24	90	80	15	210
	Sum = 25					
Type II	·677	24·322	90·669	79·051	15·036	209·755
	Sum = 24·999					
Professor Edgeworth's curve, p. 471	28·39		86·71	76·56	28·44	220·10

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* Professor Edgeworth's are relative frequencies. These have been multiplied by 210 to obtain the entries in the table.

The value of χ^2 for the Type II curve is only ·016 while $n' = 4$. This value is so small as to be outside Elderton's table, so we put

$$P = \sqrt{\frac{2}{\pi}} \int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2} d\chi + \sqrt{\frac{2}{\pi}} e^{-\frac{1}{2}\chi^2} \chi,$$

and from Sheppard's table find the first term to be ·89894 while the second is ·10052 so that $P = \cdot 99946$. The fit is practically perfect. This is not surprising since we have fitted a curve which is known to be the so-called "parallel" to the hypergeometrical series to a hypergeometrical distribution. The value of χ^2 for Professor Edgeworth's curve is 7·035 which with $n' = 4$ gives $P = \cdot 071$. The sum $S(e^2/U)$ is ·000077 for the Type II curve and ·061[*] for Professor Edgeworth's curve. He says (p. 470) that "the calculated frequencies correspond fairly well with the observed frequencies." The divergence between the total for the data (210) and for the calculated frequencies (220·1) is extraordinarily great and suggests that some errors may have crept into Professor Edgeworth's arithmetic. The values of $S(e^2/u)$ are ·000077 for the Type II curve and ·0335 for Professor Edgeworth's curve.

EXAMPLE III. *Distribution according to ages, of the Marriages of 235,252 Spinsters.*

This example is described by Professor Edgeworth as one of "considerable abnormality," and he states (p. 462) "I should be considerably surprised if the Pearsonian types would stand the Pearsonian criterion in the case of Dr Isserlis' second example which consists of the distribution of 235,252 ages at marriage."

The answer is that at any rate they stand the test much better than the curve fitted to this example by Professor Edgeworth. That the fit is a poor one is probably due not so much to the size of the total frequency (as Professor Edgeworth suggests) but to heterogeneity of the material[†], due to misstatement of age.

The observed frequencies are as follows:

TABLE V.

Age	*15—*	*20—*	*25—*	*30—*	*35—*	*40—*	*45—*	*50—*	*55—*	*60—*	*65—*	*70—*	Total
Frequency	17546	118542	70411	20241	5873	1706	636	171	64	28	23	11	235252

Again quoting from the *Phil. Mag.* (*loc. cit.* p. 385): with arbitrary origin at 22·5 years and unit 5 years, the mean is at ·5227798, the moments are $\mu_2 = \cdot 8294195$, $\mu_3 = 1\cdot 2482896$, $\mu_4 = 5\cdot 9016221$ whence $\beta_1 = 2\cdot 730913$, $\beta_2 = 8\cdot 578766$[‡]. These values of β_1, β_2 indicate a curve of Type VI, but the criterion

$$\kappa = \beta_1 (\beta_2 + 3)^2 / 4 (4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6) = 1\cdot 151975$$

is so nearly unity, that (having regard to the large values of β_1, β_2) we may expect the transition curve of Type V to give nearly as good a fit.

The curve of Type VI is

$$y = (4\cdot 29787)\, 10^{14} (x - 2\cdot 305660)^{6\cdot 599764} x^{-18\cdot 410607}$$

with origin at 4·655444 years, mode at 22·625624 years, mean at 25·113899 years and x measured in 5 year units.

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[*] Not ·07 (Prof. Edgeworth's value on p. 470).

[†] On this point cf. W. Palin Elderton, *Frequency Curves and Correlation*, pp. 142–3.

[‡] The *Phil. Mag.* value, $\beta_1 = 8\cdot 77876$ is wrong (though μ_2, μ_3, μ_4 are correct). Professor Edgeworth has used the wrong value, but the error is not enough seriously to influence his figures.

The curve of Type V is $y = (1{\cdot}43044)\, 10^{13}\, x^{-10{\cdot}7294715}\, e^{-\frac{22{\cdot}1029363}{x}}$.

Here also x is in 5 year units, the origin is at 12·453948 years and the mode at 22·754053 years. The two curves are shown in Fig. 3.

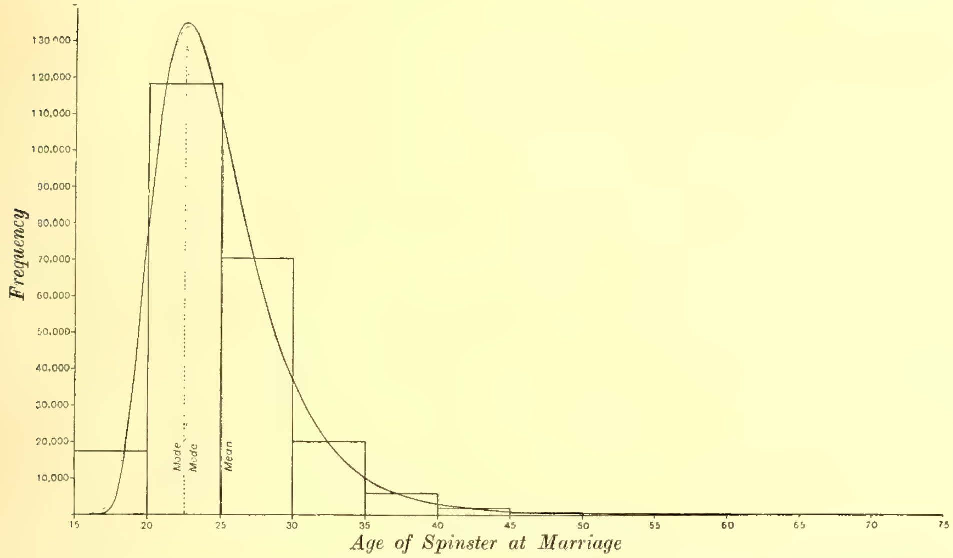

Fig. 3. Distribution of Ages at Marriage of 235,252 Spinsters. Actual Frequencies represented by the histogram.

Continuous Curve: Type VI Frequency Curve

$$y = (4{\cdot}29787)\, 10^{14}\, \frac{(x - 2{\cdot}305660)^{6{\cdot}599764}}{x^{18{\cdot}410607}}.$$

Dotted Curve: Type V Frequency Curve

$$y = (1{\cdot}43044)\, 10^{13}\, x^{-10{\cdot}729472}\, e^{-\frac{22{\cdot}1029363}{x}}.$$

The observed and calculated frequencies are given in the table below.

TABLE VI.

Age at Marriage of Spinsters.

Age	Observed frequency		Type VI		Type V		Prof. Edgeworth's* curve, p. 469
15—	17546		15969†		16448‡		14633
20—	118542		120458		120061		123696
25—	70411		69289		69508		68811
30—	20241		21009		20907		19596
35—	5873		5896		5770		5740
40—	1706	Sum = 2639	1732	Sum = 2632	1692	Sum = 2556	2752
45—	636		582		544		
50—	171		191		191		
55—	64		71		73		
60—	28		28		30		
65—	23		12		13		
70—	11		16§		13§		
Totals	235252		235253		235250		235228

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* I.e. Professor Edgeworth's relative theoretical frequencies multiplied by 235252.

† 16·184 years to 20 years. ‡ 12·454 years to 20 years. § All beyond 70 years.

It is worth noting that with the exception of one age group (30—35) the theoretical frequencies corresponding to Professor Edgeworth's curve are consistently worse than both the Type V and Type VI frequencies.

The values of χ^2, $S(e^2/U)$, and $S(e^2/u)$ for the three curves are given by the following table:

TABLE VII.

	χ^2	$S(e^2/U)$	$S(e^2/u)$
Type V	129	·00053	·00055
Type VI	232	·00090	·00099
Prof. Edgeworth's curve	868	·00366	·00369

Neither Type V nor Type VI fits the data well, but all the criteria agree in making Professor Edgeworth's curve a much poorer fit than either of the others. It should be observed that in order to make comparison with Professor Edgeworth's curve possible we have followed him in grouping all ages above 40 together. This gives a fictitious improvement to the Type V curve, for Type VI which depends on the first four moments will fit the "tail" of the experience better than Type V whose constants depend on the first three moments only. The value of μ_4 is of course heavily influenced by the marriages at the higher ages.

The source of heterogeneity in the spinster curve is misstatement (false or in ignorance) of age. I am indebted to Professor Pearson for the remark that Table V shows a lumping up of frequency before the decades. If we compare Type VI against the observed, we find a redundancy of women just below 30, i.e. at 25, and this is repeated at 45. Again the group 15—20 is increased by the minors who misstated their age as over 15. Professor Pearson informs me that these points have come up for consideration in the treatment of very ample statistics not only English but Italian, and in both cases the males give better results than the females.

We may attempt to circumvent the women who object to entering a new decade by grouping in ten year periods round the decades. The following table shows that the removal of this source of heterogeneity at once renders Type VI a very good fit, the transition curve Type V a fair fit, but leaves Professor Edgeworth's curve a very poor fit.

TABLE VIII.

Age	Observed frequency	Type VI	Type V	Prof. Edgeworth's curve
15—	136088	136427	136509	138329
25—	90652	90298	90415	88407
35—	7579	7628	7462	8492
45—	807	773	735	
55—	92 Sum	99 Sum	103 Sum	
65—	34 = 126	28 = 127	26 = 129	
Totals	235252	235253	235250	235228

We now have $\chi^2 = 4{\cdot}56$ when $n' = 5$ for the Type VI curve so that $P = {\cdot}34$ indicating quite a good fit. For Type V the value of χ^2 is 11·01 with $n' = 5$ giving $P = {\cdot}027$, while for Professor Edgeworth's curve χ^2 is 96·31 with $n' = 3$ so that P is still negligible.

§ 3. Professor Edgeworth has on several occasions advocated the value of his generalized frequency curve, and endeavoured to illustrate its numerical advantages. On the present occasion he has introduced what appears to be an erroneous formula for testing its "goodness of fit." In his paper he gives no theoretical discussion of the test. It would be well if Professor Edgeworth tried on some future occasion to give a demonstration of its validity. Any such attempt would have to account for the departure from Bernoulli's law that accuracy varies as the square root of the number of observations. According to Professor Edgeworth's test results obtained from 100 observations and from 1,000,000 would be equally accurate provided the relative frequencies were the same*! However taking him on his own ground we find his curves give a very poor fit compared with Pearson's well-known types, and offer no easy systematical methods of treating frequency. He escapes what is the real hard work of curve fitting by adopting the higher moments calculated by the biometricians and then appears to say how very much shorter his methods are! We are prepared on any occasion to race him in fitting a frequency curve *ab initio* and getting a better fit as a result.

Added April 12th, 1917. Professor Edgeworth in reply to a letter in which I suggested that there might be an error in his numerical values quoted in my Table IV, has replied that owing to a clerical error the frequency in the last column is wrong. On using his amended relative frequency, that entry becomes 18·35 instead of 28·44. With this alteration χ^2 becomes 1·297 instead of 7·035, so that the corresponding value of P is ·733. This of course denotes a good fit, but one which is still very inferior to the Pearsonian Type II curve for which $P = \cdot 999$.

III. Relation of the Mode, Median and Mean in Frequency Curves.

By ARTHUR T. DOODSON, M.Sc.

§ 1. It is well known that in frequency curves of a moderate degree of asymmetry the distance from the mode to the median is approximately two-thirds of the distance from the mode to the mean. So far as the author knows there has not been published any investigation of this property except in the case of one type of curve where

$$y = y_0 x^p e^{-x}.$$

Professor Karl Pearson, in his memoir on "Skew Variation in Homogeneous Material" (*Phil. Trans.* 1895), remarks on this property of the curve and he gives an expression for the ratio of the two distances in the form

$$c = \cdot 6691 + \cdot 0094/p \quad \ldots\ldots\ldots\ldots\ldots\ldots (1).$$

This was obtained from a few particular cases by the method of least squares.

§ 2. Let $y = y_0 \phi(x)$ represent a frequency curve which has a maximum at $x = 0$ and which is zero at $x = -a_1$ and at $x = a_2$. As a rule there is only one maximum in frequency curves of this type. On these assumptions Laplace has given a method of approximating to the definite integral, the limits of integration lying between $x = -a_1$ and $x = a_2$. Following Laplace, we assume

$$y = y_0 \phi(x) = y_0 e^{-t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2),$$

where y_0 is the maximum ordinate, or the mode. Since x vanishes with t we may assume

$$x = B_1 t + B_2 t^2 + B_3 t^3 + \ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots (3).$$

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[*] Cf. on the other hand Pearson's χ^2 test. Let there be two series of observations with populations N_1, N_2 and same relative frequency leading to $S(e^2/u) = \chi_0^2$. The two values of χ^2 would be $\chi_1^2 = N_1 \chi_0^2$, $\chi_2^2 = N_2 \chi_0^2$ so that the probabilities would be proportional to $e^{\frac{-\frac{1}{2}\chi_0^2}{(1/\sqrt{N_1})^2}}$ and $e^{\frac{-\frac{1}{2}\chi_0^2}{(1/\sqrt{N_2})^2}}$, comparing with standard deviations of form $1/\sqrt{N_1}$ and $1/\sqrt{N_2}$.

Then, taken between appropriate limits,

$$\int y\,dx = \int y_0 e^{-t^2} \cdot \frac{dx}{dt} \cdot dt = \int y_0 e^{-t^2} (B_1 + 2B_2 t + 3B_3 t^2 + \ldots)\, dt \quad \ldots\ldots(4).$$

This method is most useful where the range of t is either $-\infty$ to $+\infty$ or 0 to $+\infty$. The first case gives the "whole" area N; the second gives N_m, the area from the mode to the upper limit of x. Therefore

$$N = \int_{-\infty}^{+\infty} y_0 e^{-t^2} (B_1 + 3B_3 t^2 + \ldots)\, dt = y_0 \sqrt{\pi}\, (B_1 + \tfrac{3}{2} B_3 + \ldots) \quad \ldots\ldots(5),$$

$$N_m = \tfrac{1}{2} N + \int_0^{\infty} y_0 e^{-t^2} (2B_2 t + 4B_4 t^3 + \ldots)\, dt = \tfrac{1}{2} N + y_0 (B_2 + 2!\, B_4 + 3!\, B_6 + \ldots) \ldots\ldots(6).$$

Hence the area from the mode to the median is

$$y_0 (B_2 + 2!\, B_4 + 3!\, B_6 + \ldots) \quad \ldots\ldots(7).$$

The evaluation of $B_1, B_2, \ldots$ we shall perform by means of the method of indeterminate coefficients, using (3) and

$$\frac{dx}{dt} \cdot \frac{d\phi}{dx} + 2t\phi = 0 \quad \ldots\ldots(8)$$

which is obtained by logarithmic differentiation of (2) with respect to t. De Morgan* gives the values of the first five coefficients in terms of the differential coefficients at the mode of $\log \phi(x)$. It may be shown that when $\phi(x)$ contains high exponents or a large number of factors the coefficients $B_1, B_2, B_3, \ldots$ are of decreasing magnitude and the series (5) and (7) are convergent enough for use. This is generally the case with frequency curves. When the skewness is small the coefficients $B_2, B_4, \ldots$ must be small; in this case, then, if d be the abscissa of the median we have the area from the mode to the median approximately equal to $y_0 d$, and usually we may neglect $B_4, B_6, \ldots$ in comparison with B_2. Hence approximately

$$d = B_2 \quad \ldots\ldots(9).$$

If we let D be the distance of the mean from the mode we have

$$D = \frac{1}{N} \int_{-\alpha_1}^{\alpha_2} xy\,dx = \frac{1}{N} \int_{-\infty}^{+\infty} y_0 e^{-t^2} \tfrac{1}{2} (2C_2 t + 3C_3 t^2 + \ldots)\, dt \quad \ldots\ldots(10),$$

where C_r is the coefficient of t^r in the expansion of x^2 in powers of t. Then from (5) we have

$$D = \frac{y_0}{N} \sqrt{\pi}\, (\tfrac{3}{4} C_3 + \ldots) = \frac{\tfrac{3}{2} B_1 B_2 + \ldots}{B_1 + \ldots} \quad \ldots\ldots(11).$$

Therefore to a first approximation $D = \tfrac{3}{2} B_2 = \tfrac{3}{2} d$ ……(12).

This is the relation that has been noticed in practical statistical work and it holds under the conditions stated.

§ 3. The relation just found is quite general provided that the frequency curves are moderately asymmetrical. It would, of course, be possible to give general formulae for the more accurate representation of the relation between the distances of the mean and median from the mode in the very general type of curve already used. But in practice nearly all the types of frequency curves are included in Professor Pearson's system and it is for these curves that we shall now obtain a closer approximation to the true value of d/D. In the memoir already quoted Professor Pearson takes the differential equation of frequency curves in the form

$$\frac{1}{y} \frac{dy}{dx} = \frac{x}{a_0 + a_1 x + a_2 x^2} \quad \ldots\ldots(13),$$

the origin of x being at the mode.

If we write $y = e^{-t^2}$ we obtain, as in (8),

$$\frac{dx^2}{dt} + 4t a_2 x + 4t (a_0 + a_1 x) = 0 \quad \ldots\ldots(14),$$

and writing

$$x^2 = C_2 t^2 + C_3 t^3 + \ldots \quad \ldots\ldots(15),$$

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* *Differential and Integral Calculus*, p. 602.

we have the following equations for the determination of B_1, B_2, ...

$$\left.\begin{array}{rl} C_2 + 2a_0 = 0 & C_2 = B_1^2 \\ 3C_3 + 4a_1B_1 = 0 & C_3 = 2B_1B_2 \\ 4C_4 + 4a_2C_2 + 4a_1B_2 = 0 & C_4 = 2B_1B_3 + B_2^2 \\ 5C_5 + 4a_2C_3 + 4a_1B_3 = 0 & C_5 = 2B_1B_4 + 2B_2B_3 \\ \cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots \end{array}\right\} \quad \cdots\cdots\cdots\cdots(16).$$

The solution of these equations gives the following values of B_1, B_2, ...

$$\left.\begin{array}{ll} B_1^2 = -2a_0 & B_2 = -\dfrac{2}{3}a_1 \\ B_3 = \dfrac{1}{B_1}\left(\dfrac{1}{9}a_0^2 + a_0a_2\right) & B_4 = -\dfrac{2}{135}\dfrac{a_1^3}{a_0} + \dfrac{2}{5}a_1a_2 \\ B_5 = -\dfrac{1}{B_1}\left(\dfrac{1}{540}\dfrac{a_1^4}{a_0} + \dfrac{a_1^2a_2}{30} + \dfrac{5}{12}a_0a_2^2\right) & B_6 = -\dfrac{4}{8505}\dfrac{a_1^5}{a_0^2} + \dfrac{8}{945}\dfrac{a_1^3a_2}{a_0} - \dfrac{44}{315}a_1a_2^2 \\ B_7 = -\dfrac{1}{B_1}\left(\dfrac{139}{340200}\dfrac{a_1^6}{a_0^2} - \dfrac{53}{12600}\dfrac{a_1^4a_2}{a_0} + \dfrac{a_1^2a_2^2}{4200} - \dfrac{a_0a_2^3}{8}\right) & B_8 = \dfrac{2}{25515}\dfrac{a_1^7}{a_0^3} - \dfrac{2}{2835}\dfrac{a_1^5a_2}{a_0^2} + \dfrac{2}{2835}\dfrac{a_1^3a_2^2}{a_0} + \dfrac{2}{63}a_1a_2^3 \\ \cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots \end{array}\right\} \quad \cdots\cdots\cdots\cdots(17).$$

The area from the mode to the median is then

$$y_0\left\{-a_1\left(\frac{2}{3} - \frac{4}{5}a_2 + \frac{88}{105}a_2^2 - \frac{16}{21}a_2^3 + \ldots\right) - \frac{a_1^3}{a_0}\left(\frac{4}{135} - \frac{16}{315}a_2 - \frac{16}{945}a_2^2 + \ldots\right)\right.$$
$$\left. - \frac{a_1^5}{a_0^2}\left(\frac{8}{2835} + \frac{16}{945}a_2 + \ldots\right) + \frac{a_1^7}{a_0^3}\left(\frac{16}{8505} + \ldots\right) + \ldots\right\} \quad \cdots\cdots\cdots(18).$$

§ 4. Now this area can be found in another way in terms of d, the distance from the mode to the median, by assuming that $\phi(x)$ can be expanded in powers of x by Maclaurin's Theorem. Professor Pearson* has shown that a high order parabola does not adequately fit these curves throughout the range, but if we assume that the skewness is small and therefore d small, we may use Maclaurin's expansion legitimately within the range $x = 0$ to $x = d$. Since $\dfrac{d\phi}{dx}$ is zero at the mode we have

$$y = y_0(1 + A_2x^2 + A_3x^3 + \ldots) \quad \cdots\cdots\cdots\cdots(19),$$

and substituting in $\int_0^d y\,dx$ we have the required area equal to

$$y_0\left(d + \frac{A_2}{3}d^3 + \frac{A_3}{4}d^4 + \ldots\right) \quad \cdots\cdots\cdots\cdots(20).$$

The values of the coefficients A_2, A_3, ... may be obtained by using the differential equation (13) and the expansion (19); whence we obtain

$$A_s = (s+2)A_{s+2}a_0 + (s+1)A_{s+1}a_1 + sA_s a_2,$$

and therefore $A_2 = \dfrac{1}{2a_0}$, $A_3 = -\dfrac{a_1}{3a_0^2}$, $A_4 = \dfrac{a_1^2}{4a_0^3} + \dfrac{1-2a_2}{8a_0^2}$, $A_5 = -\dfrac{a_1}{6a_0^3} + \ldots$.

Thus the area required is

$$y_0\left\{d + \frac{d^3}{6a_0} - \frac{d^4a_1}{12a_0^2} + \frac{d^5(1-2a_2)}{40a_0^2} + \frac{d^5a_1^2}{20a_0^3} - \frac{d^6a_1}{36a_0^3} + \ldots\right\} \quad \cdots\cdots\cdots\cdots(21).$$

This expression is in descending powers of a_0 and so is the one obtained in (18). Hence we may assume

$$d = d_0 + \frac{d_1}{a_0} + \frac{d_2}{a_0^2} + \frac{d_3}{a_0^3} + \ldots \quad \cdots\cdots\cdots\cdots(22),$$

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* *Biometrika*, Vol. II. p. 19.

and substituting for d in (21) we may then equate to (18). After considerable reduction, we have, to the approximation taken throughout the course of the work,

$$\left.\begin{aligned} d_0 &= -a_1\left(\frac{2}{3}-\frac{4}{5}a_2+\frac{88}{105}a_2^2-\frac{16}{21}a_2^3+\dots\right) & d_2 &= a_1^5\left(\frac{64}{5103}-\frac{1184}{14175}a_2+\dots\right) \\ d_1 &= a_1^3\left(\frac{8}{405}-\frac{8}{63}a_2+\frac{656}{1575}a_2^2+\dots\right) & d_3 &= a_1^7\left(\frac{21688}{3444525}+\dots\right) \end{aligned}\right\} \dots(23).$$

§ 5. We have assumed that the skewness is small; this is so if a_1 be small, and as the coefficient of a_0^{-3} is approximately $a_1^7/170$ it will be seen that we have a fairly rapidly convergent series for the value of d expressed in terms of the coefficients of the differential equation. We now proceed to transform the series so as to obtain the value of d/D in terms of the skewness.

It is easily shown from the differential equation that

$$\left.\begin{aligned} a_1 &= -D(1+2a_2) \\ a_0 &= -\mu_2(1+3a_2)+D^2(1+a_2) \end{aligned}\right\} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(24),$$

where μ_2 is the second moment coefficient about the mean.

Hence
$$\frac{a_1^2}{a_0} = -\frac{D^2}{\mu_2}\cdot\frac{(1+2a_2)^2}{1+3a_2}\cdot\left(1-\frac{D^2}{\mu_2}\cdot\frac{1+a_2}{1+3a_2}\right)^{-1} \dots\dots\dots\dots\dots\dots(25).$$

Now the differential equation is obtained on the assumption that $\frac{1}{y}\frac{dy}{dx}=\frac{x}{f(x)}$ and that $f(x)$ is represented for all practical purposes by $a_0+a_1x+a_2x^2$; this implies that a_3, a_4, ... of the complete expansion are negligibly small and usually a_2 is small also. If we restrict a_2 and $\frac{D^2}{\mu_2}$ (the square of the skewness) to the cases where they are small we may expand by the binomial theorem all the quantities involved in (22), and after some reduction we obtain

$$\frac{d}{D} = \left(\frac{2}{3}+\frac{8}{15}a_2-\frac{16}{21}a_2^2+\frac{32}{35}a_2^3-\dots\right)+\left(\frac{8}{405}-\frac{64}{945}a_2+\frac{64}{675}a_2^2+\dots\right)(sk.)^2$$
$$+\left(\frac{184}{25515}-\frac{9424}{127575}a_2\right)(sk.)^4+\frac{1328}{3444525}(sk.)^6+\dots\dots\dots\dots\dots\dots(26),$$

where $(sk.)$ = skewness.

The values of the coefficients of the successive powers of the skewness are given in the following table:

$(sk.)^0$	$(sk.)^2$	$(sk.)^4$	$(sk.)^6$	
·66666667	·01975309	·00721144	·00038554	—
·53333333	− ·06772486	− ·07387027	—	a_2
− ·76190476	·09481481	—	—	a_2^2
·91428571	—	—	—	a_2^3

§ 6. The types of curves for which the value of d/D is given by (26) are given below* with the values of a_2:

Type I. $y = y_0\left(1+\frac{x}{a_1}\right)^{m_1}\left(1-\frac{x}{a_2}\right)^{m_2}$, $a_2 = \frac{1}{m_1+m_2}$.

Type III. $y = y_0e^{-\gamma x}\left(1+\frac{x}{a}\right)^{\gamma a}$, $a_2 = 0$.

Type IV. $y = \frac{y_0}{\left(1+\frac{x^2}{a^2}\right)^m}e^{-\gamma\tan^{-1}\frac{x}{a}}$, $a_2 = -\frac{1}{2m}$.

Type V. $y = y_0x^{-p}e^{-\gamma/x}$, $a_2 = -\frac{1}{p}$.

Type VI. $y = y_0(x-a)^{m_1}x^{-m_2}$, $a_2 = \frac{1}{m_1-m_2}$.

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* These are not in all cases referred to the mode as the origin of x.

It is very doubtful whether the series can be simplified any further and the question of convergency is a very difficult one, for no simple law can be given for any of the coefficients involved. The series has been tested with curves of Types I and III and the results are tabled below:

Type	a_2	$(sk.)^2$	d/D from formula	d/D (correct)	Error
I	0·1	·1733333	·715706	·715702	+ ·000004
	0·1	·4160000	·719087	·719048	+ ·000039
	0·05	·2582456	·696072	·696072	·000000
	0·05	·4657500	·700078	·700181	– ·000103
III	0	0·1	·66871448	·66871460	– ·00000012
	0	0·2	·670909	·670909	·000000
	0	0·5	·678394	·678347	+ ·000047
	0	1·0	·69402	·69315	+ ·00087

These show that the formula gives very good results even for high values of the skewness. It will be noted that in the case of Type III curves we have quite good results even when the skewness is unity. If the equation of Type III be taken in the form

$$y = y_0 e^{-x}\left(1 + \frac{x}{p}\right)^p$$

then the square of the skewness is $\frac{1}{p+1}$ and when the skewness is unity p is zero. But the value of d as given by (22) and (23) is

$$d_0 - \frac{d_1}{p} + \frac{d_2}{p^2} - \frac{d_3}{p^3} + \dots,$$

and this formula is of no great value if p be small. The processes of § 5 give

$$d = d_0' + \frac{d_1'}{p+1} + \frac{d_2'}{(p+1)^2} + \frac{d_3'}{(p+1)^3} + \dots$$

by what is practically Montfort's Theorem, and the transformed series is simpler and of more value than the untransformed series.

The formula given in § 3 for the area from the mode to the median has also been tested and gives results similar to those for the ratio of d to D. This may be useful if the area on either side of the mode be required.

In conclusion I tender my thanks to Professor Pearson for his valuable criticism and help.

IV. The Probable Error of a Mendelian Class Frequency.

EDITORIAL.

Dr Raymond Pearl in a recent paper entitled "The Probable Error of a Mendelian Class Frequency*," provides a striking illustration of how the capable biologist needs a long continued training in the logic of mathematics before he ventures into the field of probability. Dr Pearl writes: "With the increased volume of Mendelian experimentation there is an ever-growing need for adequate and clearly understood tests for the statistical significance of differences between observed results and expectation." Now there is absolutely no obscurity about the test for a *single* class frequency in a Mendelian result. I cannot conceive that there ever could be or ever has been such obscurity in the mind of a trained mathematician. If on the basis of Mendelian theory the relative frequency of that class be $\bar{p}$ and the relative frequency of all other classes be $1 - \bar{p} = \bar{q}$, then in observation samples of m, the frequencies of this particular class distribute themselves according to the binomial law $N(\bar{p} + \bar{q})^m$ in N samples. To determine the improbability of any observed result we have only to calculate the terms of this series. The mean of this frequency distribution is well known to be $m\bar{p}$, and its standard deviation $\sqrt{m\bar{p}\bar{q}}$. When neither $\bar{p}$ nor $\bar{q}$ is very small, then for practical purposes (when m of course is of a fair size, not a few units) it is quite adequate to calculate the probabilities of occurrence in such a single class frequency by the tables of the probability integral. The Mendelian worker who wishes to be more exact has merely to evaluate the terms of the binomial, or if m be too large, to approximate to those terms by the aid of my Type III curve, which falls for the cases excepted above much closer to the binomial than the normal curve does*.

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* *American Naturalist*, Vol. LI. p. 144, March, 1917.

Two observations may here be made: First, if according to Mendelian theory there must be zero frequency in any particular class, then the improbability of any occurrence in that class is infinite. Secondly, the χ^2 test can be applied to any number whatever of Mendelian classes, to s classes individually and all the remainder, or in a particular case to *one* class and all the remainder. When applied to one class and all the remainder, it reduces to exactly the process given above, i.e. to the application of the probability table on the basis of the standard deviation $\sqrt{m\bar{p}\bar{q}}$; or, referring to the genesis of the χ^2 test, which started by replacing the binomial by a normal curve, to using if we please the binomial. There is thus no contradiction between the two methods. This absence of contradiction was indicated by me in *Biometrika*, Vol. IX. p. 312, and was perfectly familiar to Weldon, who applied the binomial test to cases where it is proper to apply it. Now let us examine Dr Pearl's remarks in the light of these observations. He writes of Weldon's and Johannsen's use of the standard deviation of the binomial for entering the probability tables†: "In the first place it assumes the Gaussian distribution of errors, an assumption not often strictly warranted, as Pearson has clearly shown, and in many cases grossly in error." Now this statement requires considerable modification, (i) it does not assume the Gaussian law of errors, but the fact that the Gaussian or better Laplacian integral for practical purposes gives adequately the frequencies of the binomial when extreme values of m, $\bar{p}$ and $\bar{q}$ are not included. Dr Pearl makes no attempt to show that Weldon (or Johannsen for the matter of that) was dealing with such extreme cases, but repeats the suggestion of his previous paper‡ that Weldon somehow blundered. (ii) Further Dr Pearl cites for my view on the subject my paper "On the Influence of Past Experience on Future Expectation§." In that paper, which deals as we shall show below with an entirely different matter, the standard deviation is not $\sqrt{m\bar{p}\bar{q}}$, but the standard deviation characteristic of a *second* sample, so that criticism on this ground is out of place. Incidentally, however, I do recite in that paper the warning as to the use of the Laplacian integral when m, $\bar{p}$, or $\bar{q}$ are extreme, and it was open to Dr Pearl to investigate whether Weldon had used the χ^2 test in such cases.

Let us look again at Dr Pearl. He writes: "The χ^2 test gives a measure of the goodness of fit of the *whole* distribution, and *only* that. Now besides being interested in that point the Mendelian worker quite as often wants to know, in addition, something about the probability that *particular classes* observed are significantly different from the expected. To that sort of knowledge the χ^2 test helps him not at all. It is an 'all or none' sort of method||."

Now Dr Pearl's italics absolutely give him away, for they demonstrate that he does not in the least understand how to handle the χ^2 test or what it means! Any test applied to s out of n classes must of course include the remaining $n - s$ classes as a single group, for their total frequency is *fixed*, when the frequencies of the s classes are given. The χ^2 test can be applied to a single class or to any number of classes up to $n - 1$, and it can only be a total misunderstanding of the test which can lead Dr Pearl to say that it is an "all or none" sort of method. Every test is an "all or none" test, if Dr Pearl means that it involves the *total number observed*.

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* The area of Type III curve up to any deviation from the mean is given by the incomplete Γ-function. Tables of that function have been long in hand and are approaching completion and publication. Delay has arisen only from urgent war work.

† *Loc. cit.*, p. 144.

‡ *Journal of Experimental Zoology*, Vol. XIII. p. 203 *et seq.*

§ *Phil. Mag.*, March, 1907, pp. 365–378.

|| *Loc. cit.*, p. 145.

Again let me cite Dr Pearl: "The χ^2 test leads to this absurdity: if I perform a Mendelian experiment in which I get ten thousand million offspring agreeing *perfectly* with expectation save for *one* lone individual (perhaps a mutation, perhaps a mistake in the record, or what not) which is of a sort not expected, then Pearson and the χ^2 test agree that the probability is infinitely great that the ten thousand million *do not* follow Mendelian law!*"

Now this paragraph is I fear "characteristically" partizan, for in my criticism of Dr Pearl's paper I said nothing whatever about ten thousand millions obeying or not obeying the Mendelian law. Dr Pearl in his paper on the inheritance of fertility in hens got very frequently (from my standpoint) exceptions to his Mendelian theory. He drew in fact blue balls from a bag which his theory asserted contained only green or red balls. As a result he rejected the "goodness of fit" theory, which provided as it ought to do an infinite improbability. I asserted that he must either "remould his theory or explain away his observations†"; it was not a question of my test being fallacious. That he appears to be doing now when he states that exceptions may be "mutations" or "mistakes in records" or "what nots," or as he did in his original paper when he remarked of such exceptions that they are of "a type which is continually arising in Mendelian work." In other words he then emphasised and now again emphasises the "elasticity" of his observations. That is certainly the horn of the dilemma upon which I was prepared to drive him. But he is not content with such elasticity of Mendelism, or with badness of records or with "what nots," he seeks to show that a true test of Mendelian theory will be as plastic as the observations are elastic, in which case he will have successfully polled both horns.

In order to achieve this end he starts with a paper of mine published in the *Philosophical Magazine* for 1907, which he describes as "a very important paper, which is apparently almost entirely unknown to biologists." While feeling grateful for the compliment, I regret that it should be brought to the notice of biologists in regard to a matter which it cannot possibly cover. Let me briefly state the purport of that paper. A bag, a population, or a theoretical frequency is presented, the distribution law of which is *absolutely unknown.* The number in this fundamental distribution is supposed indefinitely great or else individuals must be returned before each draw. A first sample of n is drawn of which p present one characteristic and q do not. A second sample of m is now drawn, what is the chance that r individuals of this m will present the characteristic?

I showed that the mean of second samples would be

$$m\frac{p}{n}+\frac{m}{n+2}\,\frac{q-p}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)},$$

and the standard deviation σ would be given by

$$\sigma^2 = m\left(\frac{p}{n}+\frac{q-p}{n(n+2)}\right)\left(\frac{q}{n}-\frac{q-p}{n(n+2)}\right)\left(1+\frac{m-1}{n+3}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)},$$

the whole frequency distribution being provided by a hypergeometrical series. It is this hypergeometrical series which Dr Pearl proposes to employ as a test of "goodness of fit" in Mendelian class frequencies.

At first sight it seems almost impossible to determine how it can be applied, for the Mendelian Law which *is supposed known* is the theoretical frequency, the bag or original population, which my memoir supposes to be *unknown* except as suggested by the first sample. The only adequate method of applying the theory—and then it would be using a sledge hammer to crack nuts—would be to suppose the *first* sample indefinitely large. In this case p/n represents the class frequency of Mendelian theory $\bar{p}$, and the ratio of the observed sample m to the total possible Mendelian population n is vanishingly small, and accordingly there results

$$\text{mean} = m\bar{p}, \qquad \sigma^2 = m\bar{p}\bar{q},$$

and the hypergeometrical series collapses into $N(\bar{p}+\bar{q})^m$. Thus we fall back on the obvious binomial test with its infinite improbability for the cases of $\bar{p}=0$, which Dr Pearl finds not

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* *Loc. cit.*, p. 145.

† *Biometrika*, Vol. IX. p. 312.

"plastic" enough for his observations. But this, the logical application of the method, is not what Dr Pearl adopts although he has commenced his paper by emphasising the "ever-growing need for adequate and clearly understood tests for the statistical significance of differences between *observed results* and *expectation**" and cited in this respect the χ^2 and the $\sqrt{mpq}$ tests both of which are criteria of goodness of fit of observed results as a sample of the results to be expected from an indefinitely large and rigidly proportioned theoretical population.

What Dr Pearl actually does had better be stated in his own words (italics his): "The proposal which I wish to make for the expression of a Mendelian result is that *the expectation be expressed as the quartile limits of each class of frequency in a second sample of the same size as the observed sample*†."

Now it is very difficult to understand this sentence, for we are not told what the "expectation" is. If it is the Mendelian expectation, then why is the Mendelian expectation the second sample and not the first sample? And further why should its size be the *same* as that of the observation and not indefinitely large?

If we seek enlightenment in the working of the illustrations provided we can find it on pp. 150–151. Here to begin with we have the observations of F. L. Platt on the mating of Blue Andalusian fowls set out. There are 58 offspring of the mating Blue × Blue of which almost 7 % instead of being the white, blue or black of Mendelian theory are "dark red." These "mutations," "errors of record" or "what nots" are thrown in with the blacks and classed as "pigmented not blue" (elastic observations!). Dr Pearl now assumes a *first* sample of 58 to be given in the actual Mendelian proportions of 1 : 2 : 1, i.e., 14·5 : 29 : 14·5, and he then uses (i) and (ii) to obtain the mean = 14·9833 and σ = 4·6364 for testing the actual data.

Now what is the real significance of this procedure? Clearly he has made the Mendelian theory absolutely *plastic*. He has taken it not as the rigid law of the population sampled, but as the variable distribution of a very small first sample of 58 from a population of which the actual law is *absolutely unknown* or rather merely suggested by a very small sample. The smaller the observed series, the weaker will be the theory. The artifice is clear when it is clearly expressed, but no biologist with the average biologist's knowledge of mathematics would realise what Dr Pearl has done and he would thus run the danger of accepting Dr Pearl's test as a valid one. Suppose the *sampled* population to contain 1000 blue balls, 800 green balls, 500 red balls and 10 yellow balls. A sample of 50 would as a rule produce no yellow balls, say it produced 22 blue, 17 green and 11 red. This would be used to represent the "Mendelian" ratio 22 : 17 : 11 but later samples of 50 might occasionally produce yellow balls. These it is true would not appear in the "Mendelian theory" of 22 : 17 : 11, but would give no indefinite improbability, because the Mendelian theory has been shifted from being the rigid law of an indefinitely large population, to being the chance distribution of a sample, *of just the same size as the observations themselves*. Its accuracy or weight fluctuates with the observations themselves, and the "goodness of fit" test suggested by this process becomes no test whatever of the accordance of theory and observation, for the theory is made to have just the same weight as the observations, and the fewer the observed data, the greater *ceteris paribus* will be the likelihood of accordance between the two samples.

Shortly Dr Pearl's method is entirely fallacious, as any trained mathematician would have informed Dr Pearl had he sought advice before publication. It is most regrettable that such extensions of biometric theory should be lightly published, without any due sense of responsibility, not solely in biological but in psychological journals. It can only bring biometry into contempt as a science if, professing a mathematical foundation, it yet shows in its manifestations most inadequate mathematical reasoning.

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* *Loc. cit.*, p. 144. The italics are mine.

† *Loc. cit.*, p. 148.

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The great delay in the issue of this part of *Biometrika* has been due to a variety of causes which it is desirable to indicate here. In the first place the Editor and the whole of his Staff have been engaged on war work, which since July 1916 has become so incessant and engrossing that all research investigations have had to be placed on one side. In the next place three members of the Staff, who were originally engaged on the very heavy calculations of the memoir in this part dealing with the correlation coefficient in small samples have left to undertake special work in the present crisis. And lastly the war depletion of the staff of the Cambridge University Press has inevitably delayed the setting up of a part involving such a great mass of numerical matter.

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